VSD-MOEA: A Novel Dominance-Based Multi-Objective Evolutionary Algorithm with Explicit Variable Space Diversity Management

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***Abstract*—Most state-of-the-art Multi-objective Evolutionary Algorithms (MOEAs) promote the preservation of the diversity in the objective space, whereas the information about the diversity in the decision variables is almost neglected. However, in the case of single-objective optimization, it has been shown that explicitly managing the diversity in the decision variables usually leads to higher quality solutions. In this paper, the Variable-Space-Diversity based MOEA (VSD-MOEA) is presented. VSD-MOEA is a dominance-based MOEA whose main novelty is that it explicitly considers the diversity in the variable space. Note that the diversity in the objective space is also taken into account. The simultaneous use of information of both spaces allows to properly adapt the balance between exploration and intensification. Particularly, at the initial stages, the decisions taken by the approach are more biased by the information of the diversity in the decision variables, whereas in the last stages decisions are based on the information of the objective space. The new method is compared with state-of-art MOEAs using several benchmarks. The novel proposal attains quite better results showing a more stable and robust behaviour. Additionally, a scalability study in the decision variable reports important benefits of the novel proposal.**

I. Introduction

MULTI-OBJECTIVE Optimization Problems (MOPs) involve the simultaneous optimization of several objective functions that are usually in conflict [1]. A continuous box-constrained minimization MOP, which is the kind of problem addressed in this paper, can be defined as follows:

$$\begin{aligned} &minimize \quad \vec{F} = [f_1(\vec{\mathbf{x}}), f_2(\vec{\mathbf{x}}), ..., f_M(\vec{\mathbf{x}})] \\ &subject \quad to \quad x_i^{(L)} \le x_i \le x_i^{(U)}, \quad i = 1, 2, ..., n. \end{aligned} \tag{1}$$

where n corresponds to the dimension of the variable space, $\vec{\mathbf{x}}$ is a vector of n decision variables $\vec{\mathbf{x}} = (x_1, ..., x_n) \in R^n$, which are constrained by $x_i^{(L)}$ and $x_i^{(U)}$, i.e. the lower bound and upper bound, and M is the number of objective functions to optimize. The feasible space bounded by $x_i^{(L)}$ and $x_i^{(U)}$ is denoted by Ω, each solution is mapped to the objective space with the function $F : \Omega \to R^M$, which consist of M real-valued objective functions and R^M is called the *objective space*.

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Given two solutions $\vec{\mathbf{x}}, \vec{\mathbf{y}} \in \Omega$, $\vec{\mathbf{x}}$ dominates $\vec{\mathbf{y}}$, mathematically denoted by $\vec{\mathbf{x}} \prec \vec{\mathbf{y}}$, iff $\forall m \in 1, 2, ..., M : f_m(\vec{\mathbf{x}}) \le f_m(\vec{\mathbf{y}})$ and $\exists m \in 1, 2, ..., M : f_m(\vec{\mathbf{x}}) < f_m(\vec{\mathbf{y}})$. The best solutions of a MOP are those whose objective vectors are not dominated by any other feasible vector. These solutions are known as the Pareto optimal solutions. The Pareto set is the set of all Pareto optimal solutions, and the Pareto front are the images of the Pareto set. The goal of multi-objective optimization approaches is to obtain a proper approximation of the Pareto front, i.e., a set of well distributed solutions that are close to the Pareto front.

One of the most popular metaheuristics used to deal with MOPs is the Evolutionary Algorithm (EA). In single-objective EAs, it has been shown that taking into account the diversity of the variable space to properly balance between exploration and exploitation is highly important to attain high quality solutions [2]. Diversity can be taken into account in the design of several components such as in the variation stage [3], [4], replacement phase [5] and/or population model [6]. The explicit consideration of diversity leads to improvements in terms of premature convergence avoidance, meaning that taking into account the diversity in the design of EAs is specially important when dealing with long-term executions. Recently, some diversity management algorithms that combine the information of diversity, stopping criterion and elapsed generations have been devised. They have allowed to provide a gradual loss of diversity that depends on the time or evaluations granted to the execution [5]. Particularly the aim of such methodology is to promote exploration in the initial generations and gradually alter the behaviour towards intensification. These schemes have provided really promising results. For instance, new best-known solutions for some well-known variants of the frequency assignment problem [7], and for a two-dimensional packing problem [5] have been attained using such methodology. Additionally, this principle guided the design of the winning strategy of the Second Wind Farm Layout Optimization Competition[1], which was held in the Genetic and Evolutionary Computation Conference. Thus, the benefits of such methodology have been shown in several different single-objective optimization problems.

One of the goals in the design of Multi-objective Evolutionary Algorithms (MOEAs) is to obtain a well-spread set of solutions in the objective space. The maintenance of some degree of diversity in the objective space implies that complete

[1] https://www.irit.fr/wind-competition/

convergence does not appear in the variable space [8]. In some way, the variable space inherits some degree of diversity due to the way in which the objective space is taken into account. However, this is just an indirect way of preserving the diversity in the variable space, so in some cases the level of diversity might not be large enough to ensure a high degree of exploration. For instance, it has been shown that with some of the WFG benchmarks, in most of the state-of-the-art MOEAs the distance parameters quickly converge, meaning that the approach focuses just on optimizing the position parameters for a long period of the optimization process [8]. Thus, while some degree of diversity is maintained, a similar situation to premature convergence is presented meaning that genetic operators might not be able to generate better trade-offs.

Attending to the differences between state-of-the-art single-objective EAs and MOEAs, this paper proposes a novel MOEA, the Variable-Space-Diversity based MOEA (VSD-MOEA), that is based on controlling the amount of diversity in the variable space in an explicit way. Similarly to the successful methodology applied in single-objective optimization, the stopping criterion and the amount of evaluations performed are used to vary the amount of desired diversity. The main difference with respect to the single-objective case is that the objective space is simultaneously considered. Particularly, the approach grants more importance to the diversity of the variable space in the initial stages, whereas as the generations evolve, it gradually grants more importance to the diversity of the objective space. In fact, at the end of the execution, the diversity of the variable space is neglected, so in the last phases the proposal is similar to current state-of-the-art approaches. To our knowledge, this is the first MOEA whose design follows this principle. Since there exist currently a quite large amount of different MOEAs [9], three popular schemes have been selected to validate our proposal. This validation has been performed with several well-known benchmarks and proper quality metrics. The important benefits of properly taking into account the diversity of the variable space is clearly shown in this paper. Particularly, the advantages are clearer in the most complex problems. Note that this is consistent with the single-objective case, where the most important benefits have been obtained in complex multi-modal cases [7].

The rest of this paper is organized as follows. Section II provides a review of related papers. Several critical components that are highly related with diversity, and the VSD-MOEA proposal are detailed in section III. Section IV is devoted to the experimental validation of the novel proposal. Finally, conclusions and some lines of future work are given in Section V. Note also that some supplementary materials are given. They include details of the experimental results with additional metrics as well as some explanatory videos.

II. Literature Review

This section is devoted to review some of the most important papers that are closely related with the trend dealt by our proposal. First, some of the most popular ways of managing diversity in EAs are presented. Then, a brief explanation related to the state-of-the-art in MOEAs is showed.

A. Diversity Management in Evolutionary Algorithms

The proper balance between exploration and exploitation is one of the keys to success in the design of EAs. In the single-objective domain it is known that properly managing the diversity in the variable space is a way to control such balance, and as a consequence, a large amount of diversity management techniques have been devised [10]. Particularly, these methods are classified depending on the component(s) of the EA that is modified to alter the amount of maintained diversity. A popular taxonomy identifies the following groups [11]: *selection-based*, *population-based*, *crossover/mutation-based*, *fitness-based*, and *replacement-based*. Additionally, the methods are referred to as *uniprocess-driven* when a single component is altered, whereas the term *multiprocess-driven* is used to refer to those methods that act on more than one component.

Among the previous proposals, the replacement-based methods have attained very high-quality results in last years [7], so this alternative was selected with the aim of designing a novel MOEA incorporating an explicit way to control the diversity in the variable space. The basic principle of these methods is to manage the level of exploration in successive generations by controlling the diversity of the survivors of the population [7]. Since the most common problem is the premature convergence, the modifications are usually performed with the aim of slowing down the convergence. One of the most popular proposals belonging to this group is the *crowding* method which is based on the principle that offspring should replace similar individuals from the previous generation [12]. Several replacement strategies that do not rely on crowding have also been devised. In some methods, diversity is considered as an objective. For instance, in the hybrid genetic search with adaptive diversity control (HGSADC) [13], individuals are sorted by their contribution to diversity and by their original cost. Then, the rankings of the individuals are used in the fitness assignment phase. A more recent proposal [7] incorporates a penalty approach to alter gradually the amount of diversity maintained in the population. Particularly, initial phases preserve a larger amount of diversity than the final phases of the optimization. This last method has inspired the design of the novel proposal put forth in this paper for multi-objective optimization.

Its important to remark that in the case of multi-objective optimization, few works related to the maintenance of diversity in the variable space have been developed. The following section reviews some of the most important MOEAs and introduces some of the works that consider the maintenance of diversity in the variable space.

B. Multi-objective Evolutionary Algorithms

In recent decades, several MOEAs have been proposed. While the purpose of most of them is to provide a well-spread set of solutions close to the Pareto front, several ways of facing this purpose have been devised. Therefore, several taxonomies have been proposed with the aim of better classifying the different schemes [14]. Particularly, a MOEA can be designed based on Pareto dominance, indicators and/or decomposition [15]. Since none of the groups has a remarkable superiority over the others, in this work all of them are taken into account to validate our proposal. This section introduces the three types of schemes and some of the most popular approaches belonging to each category. Thus, at least one MOEA of each category is selected to carry out the validation of VSD-MOEA.

The dominance-based category includes those schemes where the Pareto dominance relation is used to guide the design of some of its components such as the fitness assignment, parent selection and replacement phase. The dominance relation does not inherently promotes the preservation of diversity in the objective space, therefore additional techniques such as niching, crowding and/or clustering are usually integrated with the aim of obtaining a proper spread and convergence to the Pareto front. The most popular dominance-based MOEA is the Non-Dominated Sorting Genetic Algorithm II (NSGA-II) [16].

In order to assess the performance of MOEAs, several quality indicators have been devised. In the indicator-based MOEAs, the use of the Pareto dominance relation is substituted by some quality indicators to guide the decisions performed by the MOEA. An advantage of indicator-based algorithms is that the indicators usually take into account both the quality and diversity in objective space, so incorporating additional mechanisms to promote diversity in the objective space is not required. Among the different indicators, hypervolume is a widely accepted Pareto-compliance quality indicator. The Indicator-Based Evolutionary Algorithm (IBEA) [17] was the first method belonging to this category. A more recent one is the R2-Indicator-Based Evolutionary Multi-objetive Algorithm (R2-EMOA) [18], which has reported a quite promising performance in multi-objective problems. Its most important feature is the use of the R2 indicator, which compute the mean difference in utilites through a set of weight vectors.

Finally, decomposition-based MOEAs [19] are based on transforming the MOP into a set of single-objective optimization problems that are tackled simultaneously. This transformation can be performed in several ways, e.g. with a linear weighted sum or with a weighted Tchebycheff function. Given a set of weights to establish different single-objective functions, the MOEA searches for a single high-quality solution for each of them. The weight vectors should be selected with the aim of obtaining a well-spread set of solutions [1]. The Multi-objective Evolutionary Algorithm Based on Decomposition (MOEA/D) [20] is the most popular decomposition-based MOEA. Its main principles include problem decomposition, weighted aggregation of objectives and mating restrictions through the use of neighborhoods. Different ways of aggregating the objectives have been tested with MOEA/D. Among them, the use of the Tchebycheff approach is quite popular.

It is important to stand out that none of the most popular algorithms in the multi-objective field introduce special mechanisms to promote diversity in variable space. However, some efforts have been dedicated to this principle. A popular approach to promote the diversity in the decision space is the application of fitness sharing [21] in a similar way than in single-objective optimization. Although, fitness sharing might be used to promote diversity both in objective and decision variable space, most popular variants consider only distances in the objective space. Another MOEA designed to promote diversity in both the decision and the objective space is the Genetic Diversity Evolutionary Algorithm (GDEA) [22]. In this case, each individual is assigned with a diversity-based objective which is calculated as the Euclidean distance in the genotype space to the remaining individuals in the population. Then, a ranking that considers both the original objectives and the diversity objective is used to sort individuals. Another somewhat popular approach is to calculate distances between candidate solutions by taking into account both the objective and variable space [23], [24] with the aim of promoting diversity in both spaces. A different proposal combines the use of two selection operators [25]. The first one promotes diversity and quality in the objective space whereas the second one promotes diversity in the decision space. In the same line, modifying the hypervolume to integrate the decision space diversity in a single metric was proposed in [26]. In this approach, the proposed metric is used to guide the selection in the MOEA. Finally, some indirect mechanisms that might affect the diversity have also been introduced in some schemes. Probably, the most popular one is the use of mating restrictions [27], [19].

Attending to the analyses of the previous approaches, it is clear that they might bring benefits to decision makers because the final solutions obtained by these methods present a larger decision space diversity than the ones obtained by traditional approaches [23], [28]. Thus, while clear improvements are obtained when taking into account metrics related to the Pareto Set, the benefits in terms of the obtained Pareto front are not so clear. We claim that one of the reasons of this behaviour might be that the diversity in the variable space is considered in the whole optimization process. However, in a similar way that in the single objective domain, reducing the importance granted to the diversity in the decision space as the generations progress is really important [5]. Currently, no MOEA considers this idea, so this principle has guided the design of our novel MOEA.

III. PROPOSAL

This section is devoted to explain our proposal, whose main contributions rely in the replacement phase procedure and the density estimator, the latter is considered in the objective space. VSD-MOEA is based on the principle of considering the diversity and the stopping criterion explicitly. Thus, the optimization search capabilities are altered gradually provoking a balance from exploration to explotation. This principle might be incorporated with any of the three categories of MOEAs. In this paper, our decision was to incorporate it in a dominance-based approach. Particularly, our proposal considers the application of the *fast-non-dominated-sort* procedure, so it might be considered as an extension of NSGA-II [16]. Also, it takes into account a more advanced way to control the diversity in the objective space. Overall, VSD-MOEA is a dominance-based MOEA in which a novel replacement phase is provided, i.e. no novel components are incorporated in the parent selection and variation stages.

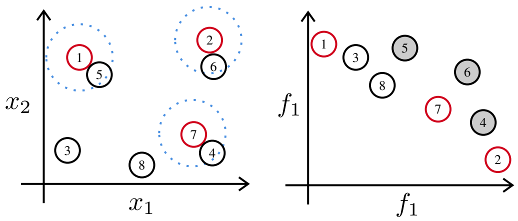

Fig. 1. Replacement Phase - The left side represents the variables space and the right side the respectively objective space of the individuals.

A. Replacement Phase of VSD-MOEA

The methodology used in the replacement phase considers several principles that were provided in the development of replacement phases in the single-objective domain [5]. In these approaches, at the begin of each generation the parents and offspring are joined. Then, N individuals must be selected to survive. In order to take into account the diversity in the decision space, the Distance to Closest Neighbor (DCN) is used. In each step, this normalized distance (Eqn. 2) is calculated with respect to those individuals that have already been selected to survive.

$$Distance(A,B) = \left(\sum_{i=1}^{n} \left(\frac{A_i - B_i}{x_i^{(U)} - x_i^{(L)}} \right)^2 \right)^{1/2} \tag{2}$$

Thus, individuals with large DCN values are those that contribute in a significant way to preserve exploration in different regions of the search space. In order to avoid an excessive decrease of the exploration degree, individuals with a DCN value lower than a threshold value are penalized, however they can be selected if non-penalized individual exist. In order to better visualize this principle, it can be considered that in each selection of a survivor, a hypersphere centered in such survivor is created. Then, all the individuals that are inside a hypersphere are penalized. One of the key novelties of the methodology is that the radius of the hyperspheres are modified dynamically by taking into account the stopping criterion and elapsed generations. Particularly, the radius is decreased in a linear way, therefore it alters the behaviour from exploration towards intensification. Note that, this method requires a parameter which is the initial radius of the hyperspheres which is denoted as D_I. Particularly, setting this parameter with a large value might provokes the penalization of the whole set of individuals, thus enough non-useful diversity might be maintained in the initial generations. However, too small values might not penalize to all the individuals, therefore this approach might behave as a traditional non-diversity based approach. Thus, this parameter depends of each MOP.

In order to better describe the principle of the replacement phase, we denote the individuals in the following way. The *reference individuals* are those that have been selected to survive as parents of the next generation. The remaining ones are classified in *penalized individuals* and *candidate individuals*. A representation of this scheme can be visualized in Fig. 1, where the decision and objective space are showed (left and right sides). The reference individuals are marked with a red border. It can be visualized that each one of them is surrounded by a blue dash circle with radius D_t. Following this principle, a larger dimension of this circle would be a hypersphere. Then, any candidate individual that is inside of this circle is penalized. In this figure the penalized individuals are shown with gray background in the right side. Once that the individuals are penalized, any of the typical approaches that are used to select survivors might be taken into account without considering the penalized individuals.

Algorithm 1 Replacement Phase of VSD-MOEA

```
1:  Input: P_t (Population of current generation), Q_t (Offspring of current Generation)
2:  Output: P_{t+1}
3:  R_t = P_t ∪ Q_t
4:  P_{t+1} = ∅
5:  Penalized = ∅
6:  D_t = D_I − D_I * G_Elapsed / (0.9*G_End)
7:  move(R_t, P_{t+1}, Best in each objective according the AASF)
8:  while |P_{t+1}| ≤ N do
9:      Compute Diversity_Variable_Space (R_t, P_{t+1})
10:     move(R_t, Penalized, Variable space diversity < D_t)
11:     if R_t is empty then
12:         Compute Diversity_Variable_Space (Penalized, P_{t+1})
13:         move(Penalized, R_t, Largest variable space diversity)
14:     conditionally − non − dominated − sort(R_t ∪ P_{t+1})
15:     Compute Diversity_Objective_Space(R_t, P_{t+1}))
16: return P_{t+1}
```

The specific pseudocode of the replacement phase that is used in VSD-MOEA is shown in Algorithm 1. Basically, it is the approach previously described, it is integrated with a sophisticated density estimator, which qualifies the contribution of each individual in the objective space. In each step, the candidate individual with lowest rank is selected. In case of tie is selected the one with the largest contribution to the diversity in the objective space. In each generation, the first step (line 3) consists in joinning the parents (P_t) and the offspring (Q_t) individuals denoted by R_t. Thereafter, the set of penalized individuals and the parents of the next generation (P_{t+1}) are emptied (lines 4 and 5). Additionally, the minimum threshold value or radius (D_t) is uptated (line 6). Note that the symbol D_I denotes the initial radius or minimum threshold value, the amount of generations that have been evolved is indicated with $G_{Elapsed}$, and G_{End} is the stopping criterion, i.e. the number of generations that are to be evolved in the execution of VSD-MOEA. It is clear that after the 90% of the generations, the D_t value is lower than 0, meaning that no penalties are performed.n This means that in the first 90% of the generations, more exploration than in traditional MOEAs is induced, whereas in the final stages, a traditional MOEA is applied. The reason to induce such behaviour is that, we are not interested in obtaining a diverse set of solutions in the decision space –if it is not requiered by the problem– at the end of the run. Maintaining diverse solutions in the initial stages is just a way to promote exploration which results in better approximations to the Pareto front at the end of the execution. Finally, the next population is filled with the boundary solutions (line 7), i.e. for each k-objective the best candidate solution is selected to survive is indicated by the *Augmented Achievement Scalarizing Function* (AASF) [29].

Then, until N individuals are selected (line 8), the following steps are carried out. First, the DCN value of each individual that has not been selected is calculated (line 9). Then, those individuals with a DCN value lower than D_t are penalized (line 10). If all the candidate individuals are penalized (line 11), it means that the amount of exploration is lower than expected. Thus, the individual with largest DCN values is recovered, i.e. moved to the non-penalized individuals set (lines 12 and 13). Finally, the objective space is taken into account. Specifically, the candidate individuals and the reference set are joined. Then, the *fast-non-dominated-sort* procedure is executed with such a set, stopping as soon as a front with a candidate individual is found (line 14). Then, for each candidate individual that belongs to the lowest front, the individual with higher contribution to the diversity in the objective space is selected (line 15). The specific way in which the diversity in the objective space is measured is described in the next section.

Note also that, as part of the diversity calculation of the variable space, a metric should be selected. Since our experimental validation is performed with a continuous domain, the normalized Euclidean distance is used. However, in discrete domains other distance metrics such as the Manhattan, or the Hamming distance might be considered, and the definition of such distance might affect the performance of the approach [7].

B. A Sophisticated Density Estimator for the Objective Space

Since the dominance definition is not related to the preservation of diversity in the objective space, dominance-based MOEAs incorporate special procedures to maintain diverse solutions, such as clustering and/or crowding. In this paper, we define a novel distance metric, and an iteratvely heristic selection approach, which selects an individual of the best front and with the largest defined distance. Specifically, the novel distance is called "Improvement Distance" (ID) and it follows the same principles that guided the design of both indicators the IGD+ and $I_{\epsilon+}$ [30], [17], [31]. The main idea is to prefer those individuals whose quality in all objectives is similarly preserved. Particularly, a non-dominated individual can be very distant to the Pareto Front due that such individual could be the best in one objective but meaningly deteriorated in the rest of objectives, so as result it has high diversity in objective space. In fact, high improvements in one objective value are related to larger selection probabilities and not the opposite, so this behaviour should be avoided.

The key idea takes into account the dominance relation between the candidate and reference individuals. Consequently, the reference and the candidate individuals are compared. If the reference individual is dominated by the candidate individual, then the euclidean distance with no modification is implemented. However if they are non-dominated with each other, then is calculated the minimum distance from the reference individual to the dominated region by the candidate individual. Additionally, if the candidate individual is dominanted by the reference individual, then is computed the I_ϵ indicator, that gives the minimum distance by which the candidate individual needs to or can be traslated in each dimension in objective space such that the reference individual is dominated. Therefore, this distance can be viewed as an amount of inferiority of the solution in comparison with the reference individual. The improvement distance is defined in Equation (3) which incorporates the I_ϵ indicador (Equation 4) where R and C are the reference and candidate solutions respectively.

$$ID(R,C) = \left(\sum_{i=1}^{M} (max(0, R_i - C_i))^2 \right)^{1/2} - I_\epsilon(R,C) \quad (3)$$

$$I_\epsilon(R,C) = \begin{cases} min_\epsilon\{f_i(C) - \epsilon \leq f_i(R)\}\} & R \preceq C \\ 0 & otherwise \end{cases} \quad (4)$$

Specifically, this distance is considered as a Weakly Pareto Compliant Indicator. In addition, this metric relaxes some difficulties encountered when the number of objectives is increased, given that the solutions in many objectives are usually non-dominated with each other by using the Pareto dominance relation. This means a very low selection pressure toward the Pareto front in Pareto-dominance-based MOEAS [32]. Principally, the improvement distance is effective with over-prioritization of dominance-resist solutions i.e., solutions with exceptional performance in one objective and extremely poor performance in many others [33].

Given that the design of this MOEA is considered for long-term executions, the algorithm 1 is implemented effciently. Thus, the distances are pre-computed and are updated in each iteration, the same for the dominance count information that is used in the *conditionally-non-dominated-sort*. In fact, the worst case complexity of this algorithm is $O((3N)^2 \times n)$, since that the dimension of the decision variable space is usually bigger than the number of objectives.

IV. Experimental Validation

In this section the experimental validation is carried out, showing that controlling the diversity in the variable space is a way to improve further some of the results obtained by state-of-art-MOEAs. At the beginning, several technical specifications taken into account in our comparison outline are explained. Therafter, to have a broad perception of the VSD-MOEA some scenaries are analyzed. Between them an analyzes driven to probe the scalability in the decision variable space is taken into account. This analyzes is narrowed through some specific and well known problems. In the same line, with the intention to have a better understanding of the critial parameter that induces the initial amount of diversity is taken into account. This is carried out through several settings and with all the test-problems. Finally, since that the mechanism imposed in our proposal, which avoid the premature convergence, highly depends on the elapsed time, we reported the benefits of our proposal considering both short, and long term executions. The latter experiments shows that the VSD-MOEA has a decent performance in short-term executions too.

To validate the proposed MOEA in this work are considered some of the most popular benchmark in the multi-objective field. Particularly, the WFG [34], DTLZ [35], and UF [36] test problems have been used for our purpose. Additionally, our experimental validation includes the VSD-MOEA, as well as three well-known state-of-the-art algorithms. Given that all of them are stochastic algorithms, each execution was repeated 35 times with different seeds. The common configuration in

TABLE I
PARAMETERIZATION OF EACH MOEA CONSIDERED

Algorithm	Configuration
MOEA/D	Max. updates by sub-problem (η_r) = 2, tour selection = 10, neighbour size = 10, period utility updating = 30 generations probability local selection (δ) = 0.9,
VSD-MOEA	$D_I = \sqrt{n} * 0.25$
R2-EMOA	$\rho = 1$, offspring by iteration = 1

all of them was the following: the stopping criterion was set to $250,000$ generations, the population size was fixed to 100, the WFG test problems were configured with two and three objectives, which are setted to 24 parameters, where 20 of them are distance parameters, and 4 are position parameters. Additinally, in the DTLZ test instances, the number of decision variables is setted to $n = M + r - 1$, where $r = \{5, 10, 20\}$ for DTLZ1, DTLZ2 to DTLZ6 and DTLZ7 respectively, as is suggested by the authors [35]. The UF benchmark is composed of ten test instances, which is categorized in two groups that are based in the number of objetive functions to solve, thus the first seven consists of two objectives, and the reaming of three, where the number of decision variables taken into account in each one is $n = 30$. In general, the crossover and mutation operators are the Simulated Binary Crossover (SBX), and polynomial [37], [38], which probabilities are setted to 0.9 and $1/n$ respectively. Also, the crossover and mutation distribution indexes were assigned to 20 and 50 respectively. The extra-parameterization of each algorithm is showed in the table I.

Despite the fact that the MOEA/D, and R2-EMOA can be employed with the same utility function –in this case the Tchebycheff function– each one of them is designed through a notorious dissimilar paradigm. Thus, the weight vectors taken into consideration for each one of them were different.The main reason of this, is that the R2-EMOA can be configured with a different population size than the number of weight vectors without been significantly affected on its performance. Particularly, the R2-EMOA employes 501 and 496 weight vectors for two and three objectives respectively. Contrarily, in the MOEA/D each weight vector is identified as a subproblem, therefore the population should correpond to the number of weight vectors. Also, the weight vectors used in the MOEA/D should be uniformly scattered on the unit-simplex, however it can be a complication since that the number of vectors requiered for this task increases non-linearly according the number of objectives. Therefore, in this version is appled the method proposed in [39], [40] where the uniform design (UD) [41] and good lattice point (GLP) are combined. In this way, the number of weight vectors that is required by this MOEA is not affected by the number of objectives.

Mainly, the experimental analyzes is carried out considering the hypervolume indicator (HV). The HV metric measures the size of the objective space dominated by the approximated solutions given a reference point, so the solutions dominated by the reference point are not considered. Particularly, the reference point is chosen to be a vector which values are sightly larger (ten percent) than the nadir point as is suggested in [42]. Similarly that in [43], and to have a fair comparison the normalized HV is taken into account. Specifically, the HV reported is obtained as the division between the HV reached by a set of solutions and the HV of the optimal Pareto Front. In this way, the more approximate to unity this metric is, the more a MOEA reaches to the Pareto Front.

In order to statistically compare the HV results, a similar guideline than the proposed in [44] was used. First a Shapiro-Wilk test was performed to check whatever or not the values of the results followed a Gaussian distribution. If, so, the Levene test was used to check for the homogeneity of the variances. If samples had equal variance, an ANOVA test was done; if not, a Welch test was performed. For non-Gaussian distributions, the nonparametric Kruskal-Wallis test was used to test whether samples are drawn from the same distribution. An algorithm X is said to win algorithm Y when the differences between them are statistically significant, if the mean and median obtained by X are higher than the mean and median achieved by Y.

In the tables II, III are showed the normalized hypervolume with two an three objectives respectively. From this empirical results, it is clear that the VSD-MOEA obteained the highest general mean HV with both two and three objectives. Even more, the standard deviation is the lowest in almost all the problems, thus this MOEA shows to be stable, meaning that reach similar results with differents seeds. The best general mean (last row) considering two objectives is achieved by the VSD-MOEA with 0.955. Also, the second general mean is obtained by the NSGA-II with 0.886. Its important to remark that the general mean can be unsteady, since that if one problem is far a way from the Pareto front then this measure could be highly degraded. However, all the state-of-the-art-MOEAs achieved a general mean of 0.88 considering two objectives, whilst the VSD-MOEA obtained 0.95. Considering three objectives the performance of the NSGA-II is highly affected, this might occurs since that density estimator employed in the NSGA-II highly depends on the dominance-relation, thus in some problems (e.g. multi-frontal problems UF10) the solutions do not converge adequatly to the Pareto front. In the same line, the VSD-MOEA achieved the best HV values in almost all the problems, in fact such values that are higher than 0.9 are close enough to the Pareto Front. The second best MOEA based in the general mean is the R2-EMOA with 0.855, despite that it adopts the same utility function that the MOEA/D, the latter is lightly lower with 0.835, this might occurs given that the MOEA/D has several paremeters to be tunned, and the selected configuration could be insuficient to long-term executions.

In the tables IV and V are showed the statistical tests with two an three objectives respectively. In the column tagged with "Diff" is computed the difference between the mean of each algorithm and the best mean achieved. Taking into account two objectives the VSD-MOEA achieved 52 wins, in comparison the R2-EMOA had 34 wins. In spite that the NSGA-II achieved a better general mean it won less times that the remaining MOEAs. Meaning that the NSGA-II obtains values close enough than the best MOEA, this can be viewed in the last row where is computed the total sum of all the problems. Generally speaking, the best results are obtained by the VSD-

MOEA, since that the total sum of the "Diff" column is 0.061, thus when this algorithm does not achieved the best results, it obtained near solutions to the best results. Futhermore, the algorithms R2-EMOA and MOEA/D achieved a similar total "Diff" values. Particularly, the worst "Diff" value achieved by the VSD-MOEA is in the problem WFG6. This problem is unimodal and non-separable, and this occurs since that the initial factor distance is very high, in fact through other experimental analyzes this instance was correctly solved with $D_I = 0.1$ which mean achieved in this problem was of 0.917 and 0.868 for two and three objectievs respectively. In addition, inspecting three objectives (table V) it can be seen similar results, notably the VSD-MOEA improves its performance respectively to the remaining MOEAs. Particularly, the most complicated problems are better solved by the VSD-MOEA as are UF3, UF4, UF5, and UF6 in two objectives, and UF9, UF10 in three objectives. The UF5 is considered as one of the most difficult problems since that the optimal Pareto front is conformed by 21 points, also it has several suboptimal regions where the solutions can be stuck. Neverthless, the MOEAs that considers weight vectors have more chances to get stuck, since that it is a highly disconected front. It is showed, since that the NSGA-II has a lower "Diff" value (0.048) than the MOEA/D and R2-EMOA (0.205 and 0.122). Diversely, the UF10 is a multi-frontal problem, this means that there exists different optimal non-dominated fronts that correspond to different locally optimal values [45], this characteristic increases the difficultness of the problem as the number of objectives increases. Indeed, the latter problem has notably converged better in VSD-MOEA than the reamining MOEAs.

A. Decision Variable Scalability Experiments

The scalability of each MOEA is also evaluated respect with the number of decision variables [46]. The figures **??**, **??** show the hypervolume performance of 30, 100, 250 and 500 variables respectively. Particularly, the scalability study was realized in DTLZ4, UF5 with two objectives and DTLZ4, UF10 with three objectives. In some instances the GDE3 degrades relatively fast according increases the number of decision variables as is showed in UF5, UF10 and DTLZ4 with three objectives. Also, the GDE3 show a non-stable performance with the parameter configurations, as is explained by J. Lampinen et al.[8], where they indicate that a high value for CR might lead to premature convergence with respect to one objective compared to another.

Specifically, the instance UF10 with GDE3 has an increment of HV for 100 variables, this irregularity is related with diversity issues [8] On the other hand the VSD-MOEA is enough stable, also it provides the best HV values. It is interesting that the MOEA/D and MOMBI-II required an extra-parametrization, hence the stability could be compromised.

The 50% attainment surfaces WFG2, WFG8 and DTLZ7 instances are showed in the figure **??**. The analyses shows that our proposal provide solutions approximated to the Pareto front. Although the GDE3 approximate the WFG2 and DTLZ7, this this algorithm is far in some regions of the Pareto front with the WFG8 instance.

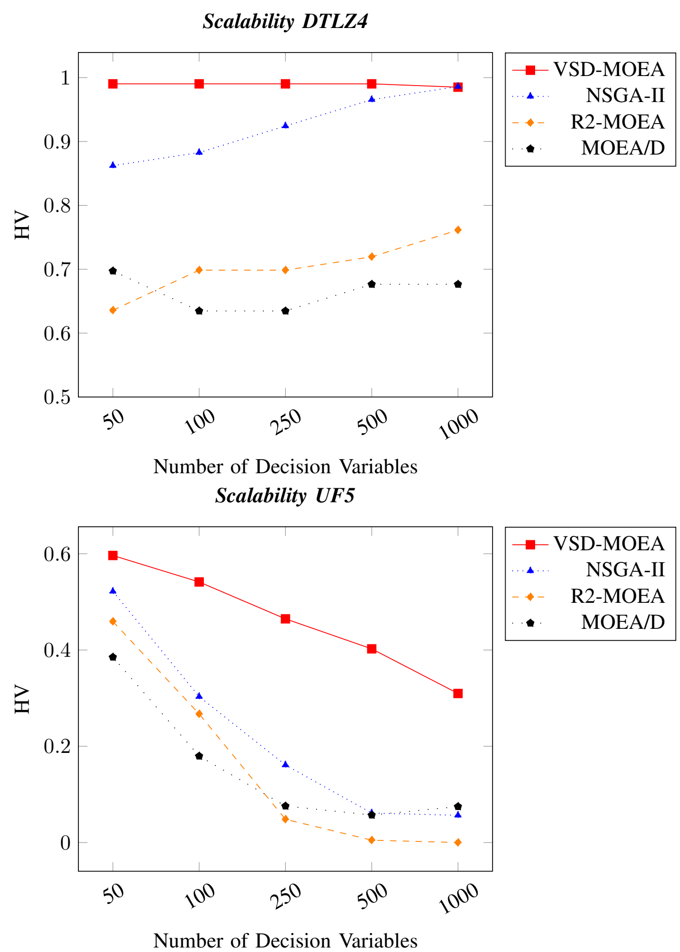

Fig. 2. Scalability study of decision variables with two objectives

B. Analyzes of the Intial Factor Distance

C. Convergence of the Diversity of WFG1

D. Improving Dependece on the Execution

V. CONCLUSION

The evolutionary algorithms have been a most popular approaches to deal with complex optimization problems. Particularly, the MOEAs works with different principles where the objective space is involved. As it is seen in single-objective the diversity provides quality solutions specifically in complex problems.

In this work we have provided an algorithm with a particular replacement phase. This phase considers the diversity in both spaces, specifically in the variable space the diversity is based in a decremental dynamic concept. Thus, in the first stages the diversity in variables space is induced, and in a gradual way the last stages the replacement phase works as is usual.

Additionally a improvement distance is suggested, which is based in the IGD+ indicator and is considered as weakly Pareto compliant.

The experimental validation is carried out with long-term executions and the three popular benchmarks. This validation shows that the VSD-MOEA is able to properly solve overall

TABLE II
STATISTICS HV WITH TWO OBJECTIVES

	MOEA/D				NSGA-II				R2-MOEA				VSD-MOEA			
	Min	**Max**	**Mean**	**Std**	**Min**	**Max**	**Mean**	**Std**	**Min**	**Max**	**Mean**	**Std**	**Min**	**Max**	**Mean**	**Std**
WFG1	0.984	0.993	0.992	0.002	0.987	0.993	0.992	0.002	0.946	0.994	0.988	0.012	0.975	0.994	**0.993**	0.003
WFG2	0.965	0.996	0.967	0.007	0.966	0.998	0.974	0.014	0.965	0.966	0.966	0.000	0.998	0.998	**0.998**	0.000
WFG3	0.992	0.992	0.992	0.000	0.987	0.988	0.987	0.000	0.991	0.992	0.991	0.000	0.992	0.992	**0.992**	0.000
WFG4	0.988	0.988	0.988	0.000	0.983	0.987	0.985	0.001	0.991	0.991	**0.991**	0.000	0.990	0.990	0.990	0.000
WFG5	0.876	0.893	0.882	0.005	0.884	0.899	0.890	0.002	0.886	0.895	0.891	0.003	0.901	0.937	**0.923**	0.008
WFG6	0.879	0.940	**0.914**	0.016	0.894	0.942	0.913	0.012	0.875	0.942	0.912	0.015	0.852	0.886	0.868	0.008
WFG7	0.988	0.988	0.988	0.000	0.983	0.987	0.984	0.001	0.991	0.991	**0.991**	0.000	0.990	0.990	0.990	0.000
WFG8	0.800	0.822	0.811	0.006	0.771	0.801	0.789	0.006	0.803	0.824	0.815	0.005	0.945	0.959	**0.953**	0.003
WFG9	0.795	0.972	0.883	0.082	0.793	0.966	0.832	0.070	0.797	0.976	0.884	0.079	0.960	0.976	**0.969**	0.004
DTLZ1	0.993	0.993	**0.993**	0.000	0.990	0.992	0.991	0.000	0.992	0.992	0.992	0.000	0.992	0.992	0.992	0.000
DTLZ2	0.989	0.989	0.989	0.000	0.986	0.988	0.987	0.000	0.991	0.992	**0.992**	0.000	0.990	0.990	0.990	0.000
DTLZ3	0.989	0.989	0.989	0.000	0.987	0.989	0.989	0.001	0.991	0.992	**0.992**	0.000	0.990	0.990	0.990	0.000
DTLZ4	0.259	0.989	0.781	0.330	0.259	0.988	0.863	0.274	0.259	0.992	0.657	0.365	0.990	0.990	**0.990**	0.000
DTLZ5	0.989	0.989	0.989	0.000	0.986	0.988	0.987	0.000	0.991	0.992	**0.992**	0.000	0.990	0.990	0.990	0.000
DTLZ6	0.448	0.910	0.700	0.105	0.138	0.511	0.322	0.075	0.510	0.922	0.691	0.107	0.990	0.990	**0.990**	0.000
DTLZ7	0.996	0.996	0.996	0.000	0.996	0.997	0.996	0.000	0.997	0.997	**0.997**	0.000	0.996	0.996	0.996	0.000
UF1	0.991	0.993	0.992	0.000	0.986	0.989	0.988	0.000	0.978	0.994	0.990	0.005	0.994	0.995	**0.994**	0.000
UF2	0.987	0.993	**0.991**	0.002	0.980	0.983	0.981	0.001	0.984	0.991	0.988	0.002	0.983	0.991	0.988	0.002
UF3	0.481	0.674	0.597	0.043	0.678	0.871	0.784	0.048	0.531	0.704	0.589	0.041	0.822	0.904	**0.881**	0.015
UF4	0.881	0.917	0.908	0.006	0.875	0.910	0.889	0.008	0.923	0.935	**0.929**	0.003	0.920	0.931	0.925	0.002
UF5	0.035	0.792	0.484	0.165	0.256	0.766	0.641	0.104	0.123	0.792	0.566	0.192	0.628	0.787	**0.688**	0.041
UF6	0.255	0.711	0.447	0.114	0.235	0.801	0.635	0.120	0.349	0.767	0.568	0.113	0.813	0.919	**0.888**	0.022
UF7	0.987	0.991	**0.990**	0.001	0.980	0.983	0.981	0.001	0.557	0.991	0.910	0.150	0.987	0.992	**0.990**	0.001
Mean	**0.806**	**0.935**	**0.881**	**0.038**	**0.808**	**0.927**	**0.886**	**0.032**	**0.801**	**0.940**	**0.882**	**0.048**	**0.943**	**0.964**	**0.955**	**0.005**

TABLE III
STATISTICS HV WITH THREE OBJECTIVES

	MOEA/D				NSGA-II				R2-MOEA				VSD-MOEA			
	Min	**Max**	**Mean**	**Std**	**Min**	**Max**	**Mean**	**Std**	**Min**	**Max**	**Mean**	**Std**	**Min**	**Max**	**Mean**	**Std**
WFG1	0.958	0.969	0.966	0.002	0.925	0.945	0.935	0.005	0.968	0.979	0.975	0.002	0.979	0.984	**0.982**	0.001
WFG2	0.973	0.978	0.976	0.001	0.959	0.974	0.968	0.004	0.962	0.963	0.963	0.000	0.987	0.991	**0.989**	0.001
WFG3	0.992	0.992	**0.992**	0.000	0.976	0.988	0.985	0.002	0.991	0.992	0.992	0.000	0.989	0.989	0.989	0.000
WFG4	0.864	0.865	0.865	0.000	0.854	0.883	0.868	0.007	0.903	0.905	0.904	0.000	0.919	0.921	**0.919**	0.001
WFG5	0.795	0.804	0.797	0.002	0.806	0.836	0.821	0.008	0.843	0.853	0.848	0.002	0.835	0.859	**0.853**	0.006
WFG6	0.777	0.832	0.809	0.013	0.788	0.836	0.815	0.011	0.847	0.875	**0.857**	0.007	0.825	0.856	0.835	0.009
WFG7	0.864	0.865	0.865	0.000	0.858	0.889	0.875	0.008	0.901	0.905	0.904	0.001	0.918	0.920	**0.919**	0.000
WFG8	0.778	0.785	0.782	0.002	0.697	0.730	0.716	0.008	0.816	0.821	0.819	0.001	0.877	0.910	**0.903**	0.008
WFG9	0.726	0.851	0.819	0.039	0.720	0.833	0.746	0.027	0.773	0.895	0.872	0.038	0.813	0.881	**0.874**	0.011
DTLZ1	0.950	0.950	0.950	0.000	0.935	0.950	0.943	0.004	0.939	0.943	0.941	0.001	0.963	0.966	**0.964**	0.001
DTLZ2	0.899	0.899	0.899	0.000	0.871	0.901	0.886	0.007	0.913	0.916	0.915	0.001	0.929	0.930	**0.930**	0.000
DTLZ3	0.899	0.899	0.899	0.000	0.876	0.901	0.890	0.006	0.914	0.916	0.915	0.000	0.929	0.930	**0.930**	0.000
DTLZ4	0.151	0.899	0.813	0.238	0.871	0.904	0.888	0.007	0.151	0.916	0.675	0.298	0.928	0.930	**0.930**	0.001
DTLZ5	0.978	0.978	0.978	0.000	0.982	0.984	0.983	0.001	0.985	0.986	**0.986**	0.000	0.986	0.986	**0.986**	0.000
DTLZ6	0.310	0.889	0.591	0.142	0.183	0.382	0.243	0.056	0.400	0.946	0.672	0.143	0.986	0.986	**0.986**	0.000
DTLZ7	0.914	0.914	0.914	0.000	0.907	0.935	0.924	0.006	0.837	0.893	0.860	0.014	0.962	0.966	**0.964**	0.001
UF8	0.151	0.830	0.773	0.107	0.324	0.646	0.463	0.069	0.578	0.917	0.898	0.057	0.905	0.925	**0.918**	0.006
UF9	0.753	0.916	0.846	0.067	0.368	0.782	0.728	0.096	0.778	0.954	0.844	0.079	0.937	0.975	**0.963**	0.010
UF10	0.145	0.555	0.341	0.162	0.060	0.391	0.242	0.067	0.143	0.578	0.413	0.166	0.469	0.762	**0.627**	0.086
Mean	**0.730**	**0.877**	**0.835**	**0.041**	**0.735**	**0.826**	**0.785**	**0.021**	**0.771**	**0.903**	**0.855**	**0.043**	**0.902**	**0.930**	**0.919**	**0.007**

test problems, also in the most complex test instances shows the best hypervolume values.

In addition, some scalability experiments with the decision variables are carry out through long-term executions, results indicate the superiority and stability provided with the hypervolume indicator.

We show the relevance of diversity in the variable space an established a way to preserve the diversity in explicitly in variable space.

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TABLE IV
STATISTICAL TESTS OF HV WITH TWO OBJECTIVES

	MOEA/D				**NSGA-II**				**R2-MOEA**				**VSD-MOEA**			
	$\uparrow$	$\downarrow$	$\leftrightarrow$	**Diff**	$\uparrow$	$\downarrow$	$\leftrightarrow$	**Diff**	$\uparrow$	$\downarrow$	$\leftrightarrow$	**Diff**	$\uparrow$	$\downarrow$	$\leftrightarrow$	**Diff**
WFG1	1	1	1	0.000	0	2	1	0.001	0	1	2	0.005	3	0	0	0.000
WFG2	1	2	0	0.032	2	1	0	0.024	0	3	0	0.033	3	0	0	0.000
WFG3	2	1	0	0.001	0	3	0	0.005	1	2	0	0.001	3	0	0	0.000
WFG4	1	2	0	0.003	0	3	0	0.006	3	0	0	0.000	2	1	0	0.001
WFG5	0	3	0	0.041	1	1	1	0.033	1	1	1	0.032	3	0	0	0.000
WFG6	1	0	2	0.000	1	0	2	0.001	1	0	2	0.002	0	3	0	0.046
WFG7	1	2	0	0.003	0	3	0	0.007	3	0	0	0.000	2	1	0	0.001
WFG8	1	2	0	0.141	0	3	0	0.163	2	1	0	0.138	3	0	0	0.000
WFG9	1	1	1	0.086	0	3	0	0.137	1	1	1	0.085	3	0	0	0.000
DTLZ1	3	0	0	0.000	0	3	0	0.002	2	1	0	0.001	1	2	0	0.001
DTLZ2	1	2	0	0.002	0	3	0	0.004	3	0	0	0.000	2	1	0	0.001
DTLZ3	1	2	0	0.002	0	3	0	0.003	3	0	0	0.000	2	1	0	0.001
DTLZ4	0	2	1	0.209	1	1	1	0.128	0	0	3	0.334	2	0	1	0.000
DTLZ5	1	2	0	0.002	0	3	0	0.004	3	0	0	0.000	2	1	0	0.001
DTLZ6	1	1	1	0.291	0	3	0	0.668	1	1	1	0.299	3	0	0	0.000
DTLZ7	0	3	0	0.001	2	1	0	0.001	3	0	0	0.000	1	2	0	0.001
UF1	1	1	1	0.002	0	3	0	0.007	1	1	1	0.004	3	0	0	0.000
UF2	3	0	0	0.000	0	3	0	0.010	1	1	1	0.003	1	1	1	0.003
UF3	0	2	1	0.284	2	1	0	0.097	0	2	1	0.292	3	0	0	0.000
UF4	1	2	0	0.020	0	3	0	0.040	3	0	0	0.000	2	1	0	0.003
UF5	0	3	0	0.205	1	1	1	0.048	1	1	1	0.122	3	0	0	0.000
UF6	0	3	0	0.442	2	1	0	0.253	1	2	0	0.320	3	0	0	0.000
UF7	2	0	1	0.000	1	2	0	0.009	0	3	0	0.079	2	0	1	0.000
Total	**23**	**37**	**9**	**1.768**	**13**	**50**	**6**	**1.649**	**34**	**21**	**14**	**1.749**	**52**	**14**	**3**	**0.061**

TABLE V
STATISTICAL TESTS OF HV WITH THREE OBJECTIVES

	MOEA/D				**NSGA-II**				**R2-MOEA**				**VSD-MOEA**			
	$\uparrow$	$\downarrow$	$\leftrightarrow$	**Diff**	$\uparrow$	$\downarrow$	$\leftrightarrow$	**Diff**	$\uparrow$	$\downarrow$	$\leftrightarrow$	**Diff**	$\uparrow$	$\downarrow$	$\leftrightarrow$	**Diff**
WFG1	1	2	0	0.016	0	3	0	0.047	2	1	0	0.007	3	0	0	0.000
WFG2	2	1	0	0.014	1	2	0	0.022	0	3	0	0.027	3	0	0	0.000
WFG3	3	0	0	0.000	0	3	0	0.008	2	1	0	0.001	1	2	0	0.004
WFG4	0	3	0	0.055	1	2	0	0.052	2	1	0	0.015	3	0	0	0.000
WFG5	0	3	0	0.055	1	2	0	0.032	2	1	0	0.005	3	0	0	0.000
WFG6	0	2	1	0.048	0	2	1	0.043	3	0	0	0.000	2	1	0	0.022
WFG7	0	3	0	0.055	1	2	0	0.044	2	1	0	0.016	3	0	0	0.000
WFG8	1	2	0	0.121	0	3	0	0.187	2	1	0	0.084	3	0	0	0.000
WFG9	1	2	0	0.055	0	3	0	0.128	2	1	0	0.002	3	0	0	0.000
DTLZ1	2	1	0	0.014	1	2	0	0.022	0	3	0	0.024	3	0	0	0.000
DTLZ2	1	2	0	0.031	0	3	0	0.044	2	1	0	0.015	3	0	0	0.000
DTLZ3	1	2	0	0.031	0	3	0	0.039	2	1	0	0.015	3	0	0	0.000
DTLZ4	0	2	1	0.117	1	1	1	0.041	0	1	2	0.254	3	0	0	0.000
DTLZ5	0	3	0	0.007	1	2	0	0.003	2	0	1	0.000	2	0	1	0.000
DTLZ6	1	2	0	0.395	0	3	0	0.743	2	1	0	0.314	3	0	0	0.000
DTLZ7	1	2	0	0.050	2	1	0	0.040	0	3	0	0.104	3	0	0	0.000
UF8	1	2	0	0.145	0	3	0	0.455	2	1	0	0.020	3	0	0	0.000
UF9	1	1	1	0.117	0	3	0	0.235	1	1	1	0.119	3	0	0	0.000
UF10	0	2	1	0.287	0	2	1	0.386	2	1	0	0.214	3	0	0	0.000
Total	**16**	**37**	**4**	**1.615**	**9**	**45**	**3**	**2.571**	**30**	**23**	**4**	**1.237**	**53**	**3**	**1**	**0.026**

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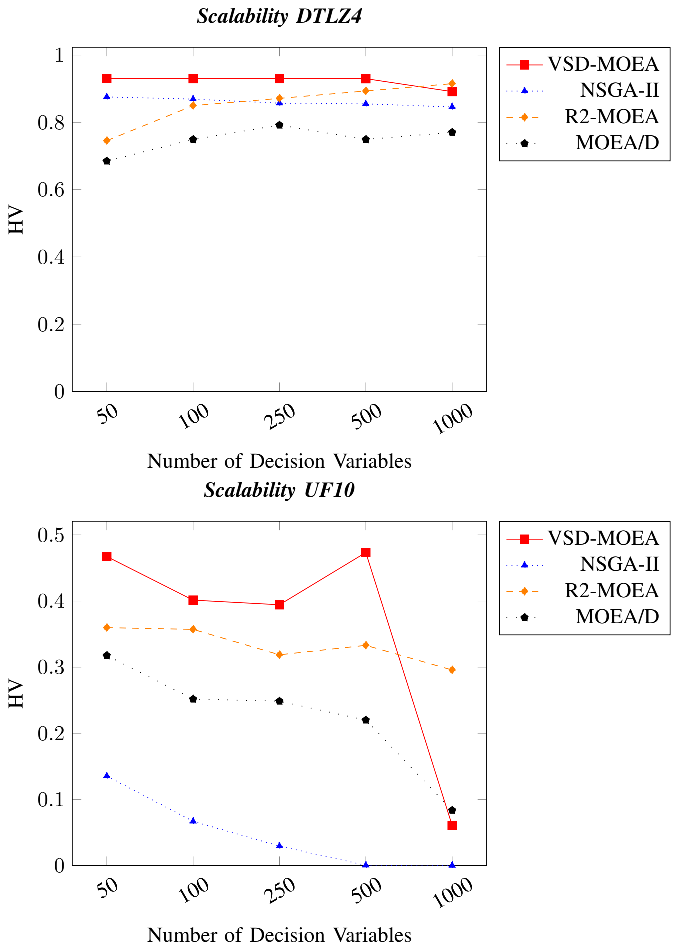

Fig. 3. Scalability study of decision variables with three objectives

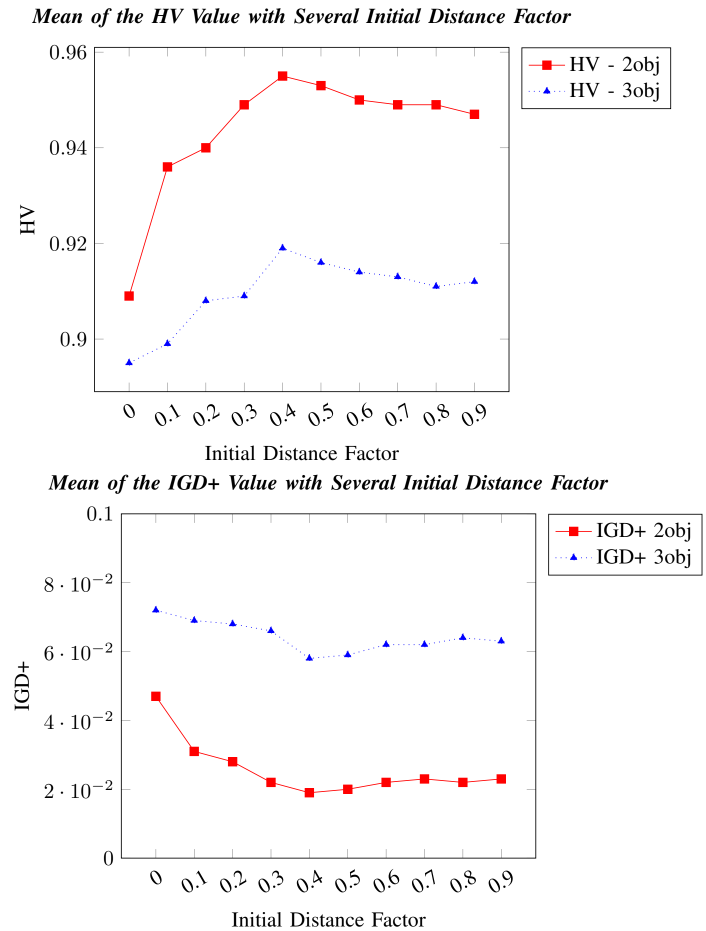

Fig. 4. Mean of Inidicator Considering All Instances with Several Initial Distance Factors

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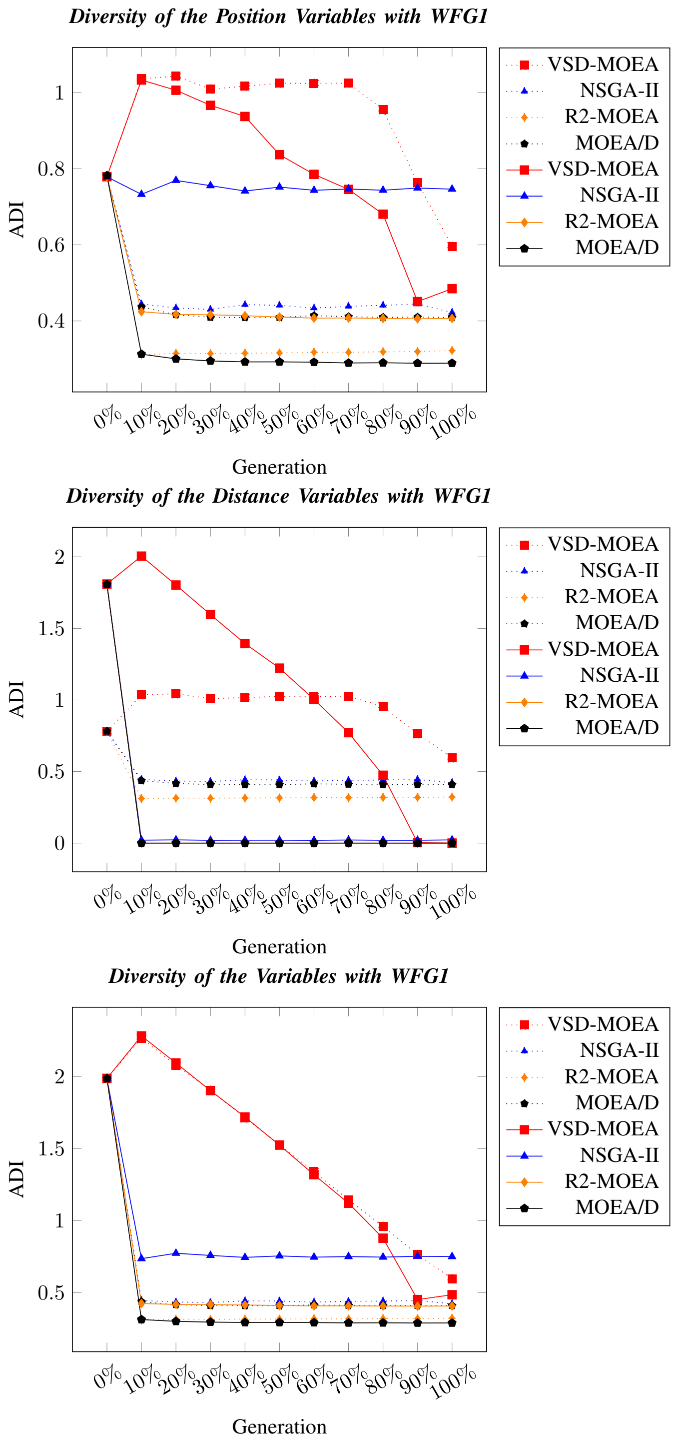

Fig. 5. Mean of Distances of MOEAs

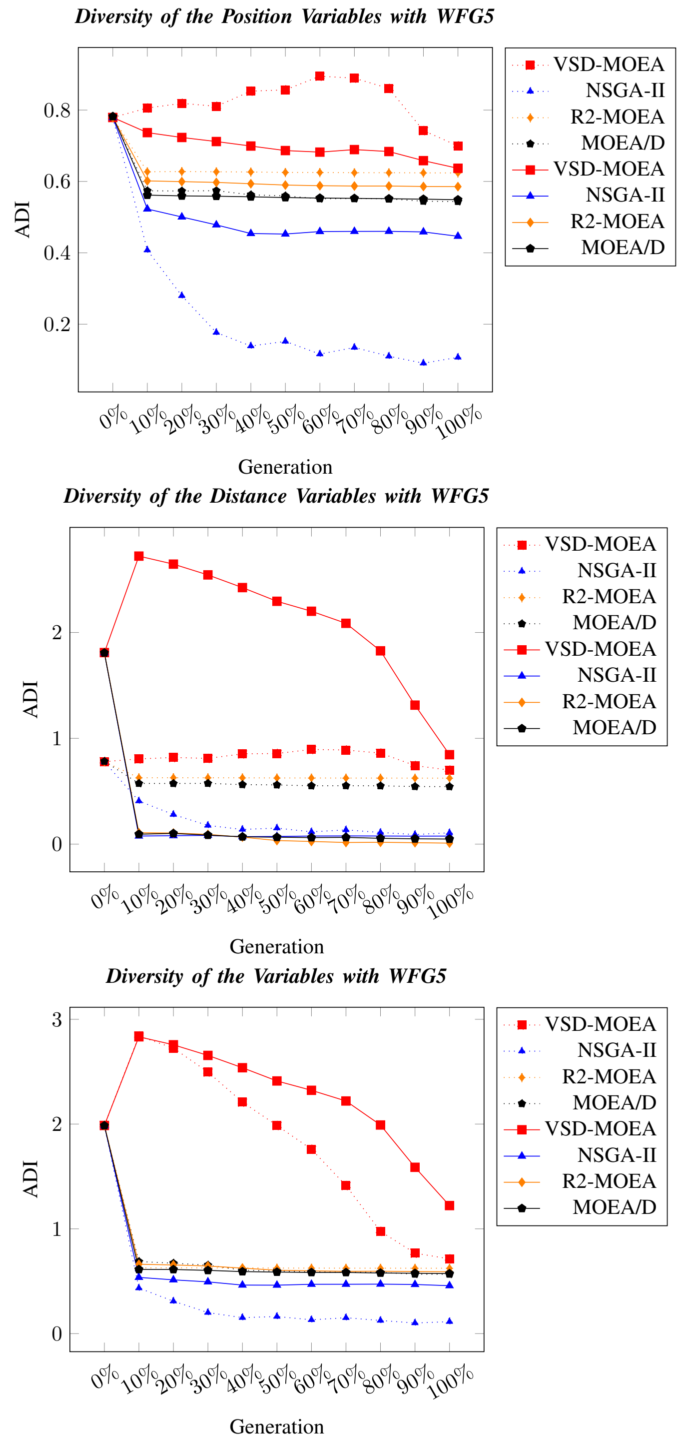

Fig. 6. Mean of Distances of MOEAs

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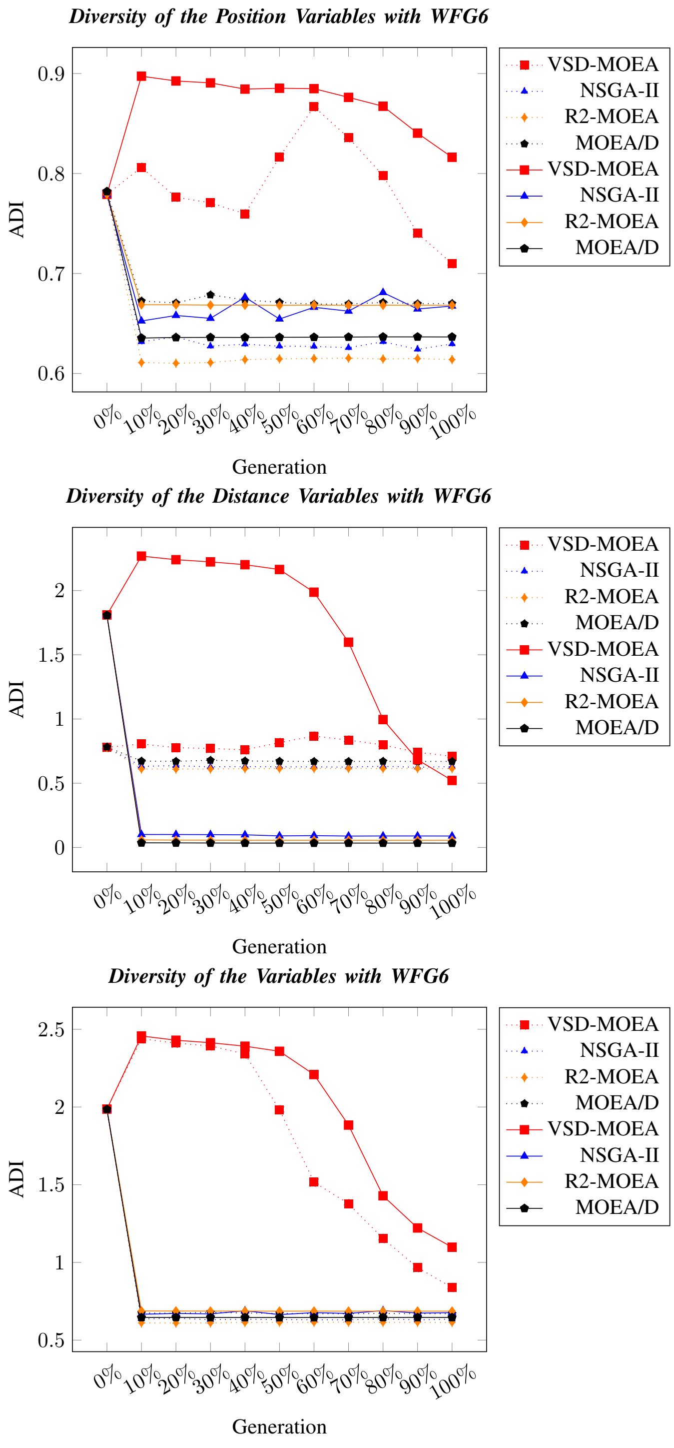

Fig. 7. Mean of Distances of MOEAs

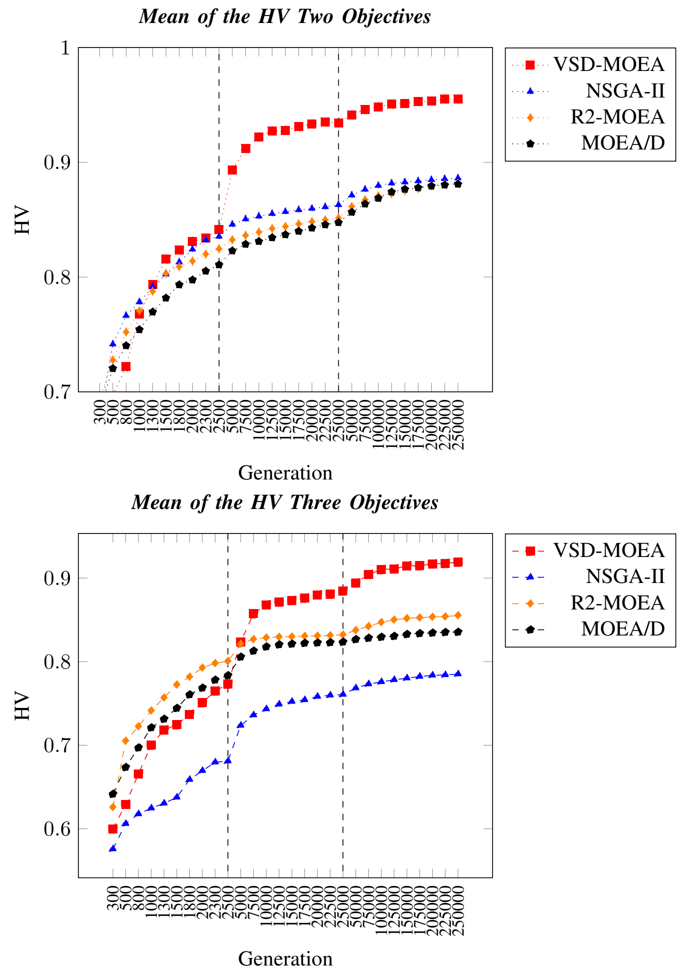

Fig. 8. Mean of Distances of MOEAs

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Joel Chacón Biography text here.

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