Supplementary Document for "VSD-MOEA: A Dominance-Based Multi-Objective Evolutionary Algorithm with Explicit Variable Space Diversity Management"

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I. Multimedia Material

This section is devoted to provide a detailed description of the included video [1]. Specifically, to have a better understanding of the behaviour of each algorithm with the WFG5 problem a simulation is recorded. The WFG5 problem is selected given its properties being perhaps the most difficult one –under standard parameterization– of the WFG problems. The main peculiarity of this problem dwell in its desceptiveness, which involves several local optimal regions that mislead the search process of the algorithms. Principally, the Pareto geometry of this problem is convex, and such Pareto optimal solutions of the distance parameters have the following values:

$$x_{i=k+1:n} = 2i \times 0.35 \quad (1)$$

Therefore, in this simulation is taken into account the standard parameterization indicated in the main document, as well the specified configuration. Each algorithm was run with two objectives and two decision variables, whose number of position and distance parameters was set to one, and the number of generations was set to 1000.

The video is vertically divided in two sides, the left-side and right-side representing each one the objective space and the decision variable space respectively. In the decision variable space (right-side) each local optimal region is remarked with a horizontal blue line, and the global optimal region is remarked with a horizontal red line. Particularly, the position and distance parameters are denoted by x_1 and x_2 respectively. The video shows that after ten generations the state-of-the-art-MOEAs have converged prematurely to the local optimal regions, contrarely to the VSD-MOEA which is still exploring. Approximatelly on the 30% of total generations, VSD-MOEA has found three individuals in the global optimal region, and few generations before (in the 40%) has located several individuals around the optimal region. At the 50% of total generations, VSD-MOEA has converged to the optimal region (red horizontal line) avoiding the remaining sub-optimals (blue horizontal lines). Finally, the remaining 50% of generations VSD-MOEA keeps improving quality in the objective space.

II. Comparison agains State-of-the-art MOEAs in long-term

In this section the statistical and test results of the IGD+ are shown [1]. Particularly, the stopping criterion was set to $250,000$ generations. Table I shows the attained IGD+ for the benchmark functions with two objectives. Especifically, the minimum, maximum, mean and standard deviation of the IGD+ for each tested method and function is presented. The last row shows the results considering all the functions together. In each function, the data of the method that attained the largest mean is shown in bold face. Additionally, all the methods that are not statistically inferior than such method are shown in bold face. Thus, the methods shown in bold face in a give problem are referred to as the winning methods. Therefore, the amount of functions where each method attained the best results with two objectives are VSD-MOEA and R2-EMOA with 8 and 13 respectively. Evenmore, the mean IGD+ of all the function attained by the VSD-MOEA is quite superior than the remaining algorithms (including the R2-EMOA). In fact, the total mean of R2-EMOA (0.060), NSGA-II (0.051) and MOEA/D (0.062) are similar. In contrast VSD-MOEA achieved a better value (0.021). Futhermore, in the cases where VSD-MOEA loses, the difference with respect to the best methods is not large. In fact, the IGD+ attained by VSD-MOEA and by the best method was never larger than 0.05. However, all the other methods presented a deterioration larger than 0.05 in several cases. The counting of functions with deterioration larger than 0.05 is 7, 5 and 8 for MOEA/D, NSGA-II and R2-EMOA respectively. Therefore, if VSD-MOEA loses in some cases, its deterioration is always small resulting in a robust behavior.

In order to have a better understading, several pair-wise statistical tests were done among each tested method in each function. As it is explained in the main document, the table III shows statistical test for the two-objective cases. The calculated data confirms that although VSD-MOEA loses in some cases, the overall numbers of wins and losses favor VSD-MOEA.

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[1] Alternatively, the video can be consulted in this link https://www.youtube.com/watch?v=ElxK_95n4lE&feature=youtu.be

TABLE III
STATISTICAL TESTS AND DETERIORATION LEVEL OF THE IGD+ FOR TWO OBJECTIVES

	↑	↓	↔	**Deterioration**
MOEA/D	23	34	12	0.979
NSGA-II	11	49	9	0.725
R2-EMOA	34	22	13	0.922
VSD-MOEA	51	14	4	0.036

TABLE IV
STATISTICAL TESTS AND DETERIORATION LEVEL OF THE IGD+ FOR THREE OBJECTIVES

	↑	↓	↔	**Deterioration**
MOEA/D	15	37	5	0.787
NSGA-II	6	46	5	1.214
R2-EMOA	35	16	6	0.669
VSD-MOEA	49	6	2	0.039

Tables II and IV shows the same information for the problems conformed of three objectives. In this case, the superiority of VSD-MOEA is even clearer. Takng into account the mean of all functions, VSD-MOEA attained much larger mean of the IGD+ in comparison to the other methods. Particularly, VSD-MOEA attained the value 0.059, whereas the second ranked algorithm (R2-EMOA) attained a value 0.093. The difference between IGD+ attained by VSD-MOEA was never larger than 0.05. However, all the other methods presented a deterioration larger than 0.05 in at least one problem. Particularly, it happened in 5, 8 and 7 problems for MOEA/D, NSGA-II and R2-EMOA respectively. Evemore, VSD-MOEA is notably superior than the other methods in terms of both: total deterioration and statistical-tests (wins). VSD-MOEA won in 49 pair-wise comparisons, whereas the second best ranked algorithm (R2-EMOA) won in 35 pair-wise comparisons.

REFERENCES

[1] H. Ishibuchi, H. Masuda, Y. Tanigaki, and Y. Nojima, *Modified Distance Calculation in Generational Distance and Inverted Generational Distance*. Cham: Springer International Publishing, 2015, pp. 110–125. [Online]. Available: https://doi.org/10.1007/978-3-319-15892-1_8

TABLE I
SUMMARY OF THE IGD+ RESULTS ATTAINED FOR PROBLEMS WITH TWO OBJECTIVES

	MOEA/D				**NSGA-II**				**R2-EMOA**				**VSD-MOEA**			
	Min	**Max**	**Mean**	**Std**	**Min**	**Max**	**Mean**	**Std**	**Min**	**Max**	**Mean**	**Std**	**Min**	**Max**	**Mean**	**Std**
WFG1	0.006	0.015	0.008	0.002	0.006	0.014	0.008	0.002	0.006	0.061	0.013	0.014	**0.006**	**0.019**	**0.008**	**0.003**
WFG2	0.006	0.055	0.052	0.011	0.003	0.053	0.040	0.022	0.053	0.055	0.054	0.000	**0.003**	**0.003**	**0.003**	**0.000**
WFG3	0.008	0.008	0.008	0.000	0.011	0.013	0.012	0.000	0.008	0.009	0.008	0.000	**0.007**	**0.007**	**0.007**	**0.000**
WFG4	0.007	0.007	0.007	0.000	0.007	0.010	0.008	0.001	**0.005**	**0.005**	**0.005**	**0.000**	0.006	0.006	0.006	0.000
WFG5	0.060	0.069	0.065	0.002	0.060	0.068	0.066	0.002	0.064	0.066	0.065	0.000	**0.038**	**0.057**	**0.047**	**0.006**
WFG6	**0.034**	**0.073**	**0.050**	**0.010**	**0.034**	**0.064**	**0.051**	**0.007**	**0.034**	**0.076**	**0.053**	**0.010**	0.068	0.088	0.081	0.004
WFG7	0.007	0.007	0.007	0.000	0.008	0.010	0.009	0.000	**0.005**	**0.006**	**0.005**	**0.000**	0.006	0.006	0.006	0.000
WFG8	0.103	0.120	0.112	0.005	0.116	0.139	0.125	0.005	0.103	0.120	0.110	0.004	**0.026**	**0.099**	**0.043**	**0.025**
WFG9	0.011	0.125	0.067	0.053	0.014	0.127	0.101	0.046	0.009	0.125	0.067	0.053	**0.009**	**0.014**	**0.011**	**0.001**
DTLZ1	**0.001**	**0.001**	**0.001**	**0.000**	0.002	0.002	0.002	0.000	0.001	0.001	0.001	0.000	0.001	0.001	0.001	0.000
DTLZ2	0.002	0.002	0.002	0.000	0.002	0.003	0.003	0.000	**0.002**	**0.002**	**0.002**	**0.000**	0.002	0.002	0.002	0.000
DTLZ3	0.002	0.002	0.002	0.000	0.002	0.003	0.002	0.000	**0.002**	**0.002**	**0.002**	**0.000**	0.002	0.002	0.002	0.000
DTLZ4	0.002	0.363	0.105	0.163	0.002	0.363	0.064	0.136	0.002	0.363	0.167	0.180	**0.002**	**0.002**	**0.002**	**0.000**
DTLZ5	0.002	0.002	0.002	0.000	0.002	0.003	0.003	0.000	**0.002**	**0.002**	**0.002**	**0.000**	0.002	0.002	0.002	0.000
DTLZ6	0.022	0.149	0.076	0.027	0.126	0.315	0.205	0.036	0.019	0.128	0.078	0.027	**0.002**	**0.002**	**0.002**	**0.000**
DTLZ7	0.003	0.003	0.003	0.000	0.002	0.003	0.003	0.000	**0.002**	**0.002**	**0.002**	**0.000**	0.003	0.003	0.003	0.000
UF1	0.004	0.004	0.004	0.000	0.005	0.006	0.006	0.000	0.003	0.005	0.004	0.001	**0.003**	**0.003**	**0.003**	**0.000**
UF2	**0.003**	**0.005**	**0.004**	**0.000**	0.008	0.010	0.010	0.000	0.004	0.006	0.005	0.001	0.004	0.007	0.005	0.001
UF3	0.141	0.237	0.180	0.022	0.052	0.127	0.084	0.020	0.119	0.210	0.183	0.021	**0.038**	**0.095**	**0.057**	**0.013**
UF4	0.024	0.031	0.026	0.001	0.027	0.039	0.033	0.003	**0.019**	**0.023**	**0.021**	**0.001**	0.020	0.024	0.022	0.001
UF5	0.079	0.593	0.265	0.120	**0.091**	**0.254**	**0.142**	**0.033**	0.079	0.521	0.215	0.131	**0.088**	**0.154**	**0.132**	**0.014**
UF6	0.066	0.529	0.380	0.108	0.037	0.542	0.193	0.114	0.064	0.432	0.266	0.103	**0.021**	**0.065**	**0.038**	**0.011**
UF7	**0.003**	**0.005**	**0.004**	**0.000**	0.007	0.008	0.007	0.000	0.003	0.242	0.046	0.082	**0.003**	**0.009**	**0.004**	**0.001**
Mean	0.026	0.105	0.062	0.023	0.027	0.095	0.051	0.019	0.026	0.107	0.060	0.027	0.016	0.029	0.021	0.003

TABLE II
SUMMARY OF THE IGD+ RESULTS ATTAINED FOR PROBLEMS WITH THREE OBJECTIVES

	MOEA/D				**NSGA-II**				**R2-EMOA**				**VSD-MOEA**			
	Min	**Max**	**Mean**	**Std**	**Min**	**Max**	**Mean**	**Std**	**Min**	**Max**	**Mean**	**Std**	**Min**	**Max**	**Mean**	**Std**
WFG1	0.080	0.100	0.090	0.005	0.142	0.179	0.160	0.010	0.058	0.098	0.079	0.010	**0.049**	**0.070**	**0.058**	**0.006**
WFG2	0.057	0.068	0.063	0.002	0.073	0.133	0.097	0.014	0.102	0.104	0.103	0.000	**0.031**	**0.048**	**0.037**	**0.004**
WFG3	0.023	0.023	0.023	0.000	0.031	0.061	0.039	0.005	**0.022**	**0.023**	**0.022**	**0.000**	0.033	0.033	0.033	0.000
WFG4	0.127	0.127	0.127	0.000	0.121	0.144	0.132	0.005	0.095	0.098	0.097	0.001	**0.090**	**0.094**	**0.093**	**0.001**
WFG5	0.177	0.184	0.181	0.002	0.160	0.186	0.170	0.005	0.147	0.158	0.153	0.003	**0.140**	**0.150**	**0.146**	**0.003**
WFG6	0.155	0.205	0.175	0.012	0.159	0.196	0.177	0.009	**0.122**	**0.151**	**0.140**	**0.007**	0.156	0.173	0.166	0.005
WFG7	0.127	0.127	0.127	0.000	0.113	0.138	0.123	0.007	0.094	0.102	0.097	0.001	**0.092**	**0.094**	**0.094**	**0.001**
WFG8	0.189	0.194	0.192	0.001	0.244	0.274	0.256	0.008	0.161	0.166	0.163	0.001	**0.099**	**0.154**	**0.109**	**0.015**
WFG9	0.130	0.240	0.154	0.036	0.138	0.246	0.224	0.025	0.099	0.211	0.119	0.037	**0.099**	**0.210**	**0.118**	**0.036**
DTLZ1	0.014	0.014	0.014	0.000	0.017	0.020	0.018	0.001	**0.013**	**0.014**	**0.014**	**0.000**	0.014	0.014	0.014	0.000
DTLZ2	0.027	0.027	0.027	0.000	0.030	0.036	0.032	0.001	**0.023**	**0.024**	**0.023**	**0.000**	0.024	0.025	0.024	0.000
DTLZ3	0.027	0.027	0.027	0.000	0.027	0.032	0.030	0.001	**0.023**	**0.023**	**0.023**	**0.000**	0.024	0.025	0.024	0.000
DTLZ4	0.027	0.595	0.092	0.181	0.028	0.036	0.032	0.001	0.023	0.595	0.190	0.225	**0.024**	**0.025**	**0.024**	**0.000**
DTLZ5	0.003	0.003	0.003	0.000	0.003	0.003	0.003	0.000	0.002	0.002	0.002	0.000	**0.002**	**0.002**	**0.002**	**0.000**
DTLZ6	0.022	0.163	0.087	0.032	0.126	0.224	0.187	0.027	0.003	0.136	0.069	0.033	**0.002**	**0.002**	**0.002**	**0.000**
DTLZ7	0.045	0.045	0.045	0.000	0.038	0.052	0.044	0.003	0.060	0.087	0.079	0.008	**0.027**	**0.029**	**0.028**	**0.000**
UF8	0.048	0.365	0.069	0.051	0.093	0.220	0.178	0.031	0.027	0.159	0.033	0.022	**0.025**	**0.034**	**0.029**	**0.002**
UF9	0.041	0.151	0.086	0.049	0.106	0.314	0.139	0.049	0.025	0.137	0.094	0.053	**0.022**	**0.028**	**0.024**	**0.001**
UF10	0.163	0.565	0.294	0.125	0.198	0.658	0.261	0.080	0.159	0.553	0.257	0.131	**0.070**	**0.187**	**0.103**	**0.026**
Mean	0.078	0.170	0.099	0.026	0.097	0.166	0.121	0.015	0.066	0.150	0.093	0.028	0.054	0.074	0.059	0.005