VSD-MOEA: A Novel Dominance-Based Multi-Objective Evolutionary Algorithm with Explicit Variable Space Diversity Management

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***Abstract*—Most state-of-the-art Multi-objective Evolutionary Algorithms (MOEAs) promote the preservation of the diversity in the objective space, whereas the information about the diversity in the decision variables is almost neglected. However, in the case of single-objective optimization, it has been shown that explicitly managing the diversity in the decision variables usually leads to higher quality solutions. In this paper, the Variable-Space-Diversity based MOEA (VSD-MOEA) is presented. VSD-MOEA is a dominance-based MOEA whose main novelty is that it explicitly considers the diversity in the variable space. Note that the diversity in the objective space is also taken into account. The simultaneous use of information of both spaces allows to properly adapt the balance between exploration and intensification. Particularly, at the initial stages, the decisions taken by the approach are more biased by the information of the diversity in the decision variables, whereas in the last stages decisions are based on the information of the objective space. The new method is compared with state-of-art MOEAs using several benchmarks. The novel proposal attains quite better results showing a more stable and robust behaviour. Additionally, a scalability study in the decision variable reports important benefits of the novel proposal.**

I. Introduction

MULTI-OBJECTIVE Optimization Problems (MOPs) involve the simultaneous optimization of several objective functions that are usually in conflict [1]. A continuous box-constrained minimization MOP, which is the kind of problem addressed in this paper, can be defined as follows:

$$\begin{aligned} &minimize \quad \vec{F} = [f_1(\vec{\mathbf{x}}), f_2(\vec{\mathbf{x}}), ..., f_M(\vec{\mathbf{x}})] \\ &subject \quad to \quad x_i^{(L)} \le x_i \le x_i^{(U)}, \quad i = 1, 2, ..., n. \end{aligned} \tag{1}$$

where n corresponds to the dimension of the variable space, $\vec{\mathbf{x}}$ is a vector of n decision variables $\vec{\mathbf{x}} = (x_1, ..., x_n) \in R^n$, which are constrained by $x_i^{(L)}$ and $x_i^{(U)}$, i.e. the lower bound and upper bound, and M is the number of objective functions to optimize. The feasible space bounded by $x_i^{(L)}$ and $x_i^{(U)}$ is denoted by Ω, each solution is mapped to the objective space with the function $F : \Omega \rightarrow R^M$, which consist of M real-valued objective functions and R^M is called the *objective space*.

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Given two solutions $\vec{\mathbf{x}}$, $\vec{\mathbf{y}} \in \Omega$, $\vec{\mathbf{x}}$ dominates $\vec{\mathbf{y}}$, mathematically denoted by $\vec{\mathbf{x}} \prec \vec{\mathbf{y}}$, iff $\forall m \in 1, 2, ..., M : f_m(\vec{\mathbf{x}}) \le f_m(\vec{\mathbf{y}})$ and $\exists m \in 1, 2, ..., M : f_m(\vec{\mathbf{x}}) < f_m(\vec{\mathbf{y}})$. The best solutions of a MOP are those whose objective vectors are not dominated by any other feasible vector. These solutions are known as the Pareto optimal solutions. The Pareto set is the set of all Pareto optimal solutions, and the Pareto front are the images of the Pareto set. The goal of multi-objective optimization approaches is to obtain a proper approximation of the Pareto front, i.e., a set of well distributed solutions that are close to the Pareto front.

One of the most popular metaheuristics used to deal with MOPs is the Evolutionary Algorithm (EA). In single-objective EAs, it has been shown that taking into account the diversity of the variable space to properly balance between exploration and exploitation is highly important to attain high quality solutions [2]. Diversity can be taken into account in the design of several components such as in the variation stage [3], [4], replacement phase [5] and/or population model [6]. The explicit consideration of diversity leads to improvements in terms of premature convergence avoidance, meaning that taking into account the diversity in the design of EAs is specially important when dealing with long-term executions. Recently, some diversity management algorithms that combine the information of diversity, stopping criterion and elapsed generations have been devised. They have allowed to provide a gradual loss of diversity that depends on the time or evaluations granted to the execution [5]. Particularly the aim of such methodology is to promote exploration in the initial generations and gradually alter the behaviour towards intensification. These schemes have provided really promising results. For instance, new best-known solutions for some well-known variants of the frequency assignment problem [7], and for a two-dimensional packing problem [5] have been attained using such methodology. Additionally, this principle guided the design of the winning strategy of the Second Wind Farm Layout Optimization Competition[1], which was held in the Genetic and Evolutionary Computation Conference. Thus, the benefits of such methodology have been shown in several different single-objective optimization problems.

One of the goals in the design of Multi-objective Evolutionary Algorithms (MOEAs) is to obtain a well-spread set of solutions in the objective space. The maintenance of some degree of diversity in the objective space implies that complete

[1]https://www.irit.fr/wind-competition/

convergence does not appear in the variable space [8]. In some way, the variable space inherits some degree of diversity due to the way in which the objective space is taken into account. However, this is just an indirect way of preserving the diversity in the variable space, so in some cases the level of diversity might not be large enough to ensure a high degree of exploration. For instance, it has been shown that with some of the WFG benchmarks, in most of the state-of-the-art MOEAs the distance parameters quickly converge, meaning that the approach focuses just on optimizing the position parameters for a long period of the optimization process [8]. Thus, while some degree of diversity is maintained, a similar situation to premature convergence is presented meaning that genetic operators might not be able to generate better trade-offs.

Attending to the differences between state-of-the-art single-objective EAs and MOEAs, this paper proposes a novel MOEA, the Variable-Space-Diversity based MOEA (VSD-MOEA), that is based on controlling the amount of diversity in the variable space in an explicit way. Similarly to the successful methodology applied in single-objective optimization, the stopping criterion and the amount of evaluations performed are used to vary the amount of desired diversity. The main difference with respect to the single-objective case is that the objective space is simultaneously considered. Particularly, the approach grants more importance to the diversity of the variable space in the initial stages, whereas as the generations evolve, it gradually grants more importance to the diversity of the objective space. In fact, at the end of the execution, the diversity of the variable space is neglected, so in the last phases the proposal is similar to current state-of-the-art approaches. To our knowledge, this is the first MOEA whose design follows this principle. Since there exist currently a quite large amount of different MOEAs [9], three popular schemes have been selected to validate our proposal. This validation has been performed with several well-known benchmarks and proper quality metrics. The important benefits of properly taking into account the diversity of the variable space is clearly shown in this paper. Particularly, the advantages are clearer in the most complex problems. Note that this is consistent with the single-objective case, where the most important benefits have been obtained in complex multi-modal cases [7].

The rest of this paper is organized as follows. Section II provides a review of related papers. Several critical components that are highly related with diversity, and the VSD-MOEA proposal are detailed in section III. Section IV is devoted to the experimental validation of the novel proposal. Finally, conclusions and some lines of future work are given in Section V. Note also that some supplementary materials are given. They include details of the experimental results with additional metrics as well as some explanatory videos.

II. LITERATURE REVIEW

This section is devoted to review some of the most important papers that are closely related with the trend dealt by our proposal. First, some of the most popular ways of managing diversity in EAs are presented. Then, a brief explanation related to the state-of-the-art in MOEAs is showed.

A. Diversity Management in Evolutionary Algorithms

The proper balance between exploration and exploitation is one of the keys to success in the design of EAs. In the single-objective domain it is known that properly managing the diversity in the variable space is a way to control such balance, and as a consequence, a large amount of diversity management techniques have been devised [10]. Particularly, these methods are classified depending on the component(s) of the EA which is modified to alter the amount of maintained diversity. A popular taxonomy identifies the following groups [11]: *selection-based*, *population-based*, *crossover/mutation-based*, *fitness-based*, and *replacement-based*. Additionally, the methods are referred to as *uniprocess-driven* when a single component is altered, whereas the term *multiprocess-driven* is used to refer to those methods that act on more than one component.

Among the previous proposals, the replacement-based methods have attained very high-quality results in last years [7], so this alternative was selected with the aim of designing a novel MOEA incorporating an explicit way to control the diversity in the variable space. The basic principle of these methods is to manage the level of exploration in successive generations by controlling the diversity of the survivors of the population [7]. Since the most common problem is the premature convergence, the modifications are usually performed with the aim of slowing down the convergence. One of the most popular proposals belonging to this group is the *crowding* method which is based on the principle that offspring should replace similar individuals from the previous generation [12]. Several replacement strategies that do not rely on crowding have also been devised. In some methods, diversity is considered as an objective. For instance, in the hybrid genetic search with adaptive diversity control (HGSADC) [13], individuals are sorted by their contribution to diversity and by their original cost. Then, the rankings of the individuals are used in the fitness assignment phase. A more recent proposal [7] incorporates a penalty approach to alter gradually the amount of diversity maintained in the population. Particularly, initial phases preserve a larger amount of diversity than the final phases of the optimization. This last method has inspired the design of the novel proposal put forth in this paper for multi-objective optimization.

Its important to remark that in the case of multi-objective optimization, few works related to the maintenance of diversity in the variable space have been developed. The following section reviews some of the most important MOEAs and introduces some of the works that consider the maintenance of diversity in the variable space.

B. Multi-objective Evolutionary Algorithms

In recent decades, several MOEAs have been proposed. While the purpose of most of them is to provide a well-spread set of solutions close to the Pareto front, several ways of facing this purpose have been devised. Therefore, several taxonomies have been proposed with the aim of better classifying the different schemes [14]. Particularly, a MOEA can be designed based on Pareto dominance, indicators and/or decomposition [15]. Since none of the groups has a remarkable superiority over the others, in this work all of them are taken into account to validate our proposal. This section introduces the three types of schemes and some of the most popular approaches belonging to each category. Thus, at least one MOEA of each category is selected to carry out the validation of VSD-MOEA.

The dominance-based category includes those schemes where the Pareto dominance relation is used to guide the design of some of its components such as the fitness assignment, parent selection and replacement phase. The dominance relation does not inherently promotes the preservation of diversity in the objective space, therefore additional techniques such as niching, crowding and/or clustering are usually integrated with the aim of obtaining a proper spread and convergence to the Pareto front. The most popular dominance-based MOEA is the Non-Dominated Sorting Genetic Algorithm II (NSGA-II) [16].

In order to assess the performance of MOEAs, several quality indicators have been devised. In the indicator-based MOEAs, the use of the Pareto dominance relation is substituted by some quality indicators to guide the decisions performed by the MOEA. An advantage of indicator-based algorithms is that the indicators usually take into account both the quality and diversity in objective space, so incorporating additional mechanisms to promote diversity in the objective space is not required. Among the different indicators, hypervolume is a widely accepted Pareto-compliance quality indicator. The Indicator-Based Evolutionary Algorithm (IBEA) [17] was the first method belonging to this category. A more recent one is the R2-Indicator-Based Evolutionary Multi-objetive Algorithm (R2-MOEA) [18], which has reported a quite promising performance in multi-objective problems. Its most important feature is the use of the R2 indicator, which compute the mean difference in utilites through a set of weight vectors.

Finally, decomposition-based MOEAs [19] are based on transforming the MOP into a set of single-objective optimization problems that are tackled simultaneously. This transformation can be performed in several ways, e.g. with a linear weighted sum or with a weighted Tchebycheff function. Given a set of weights to establish different single-objective functions, the MOEA searches for a single high-quality solution for each of them. The weight vectors should be selected with the aim of obtaining a well-spread set of solutions [1]. The Multi-objective Evolutionary Algorithm Based on Decomposition (MOEA/D) [20] is the most popular decomposition-based MOEA. Its main principles include problem decomposition, weighted aggregation of objectives and mating restrictions through the use of neighborhoods. Different ways of aggregating the objectives have been tested with MOEA/D. Among them, the use of the Tchebycheff approach is quite popular.

It is important to stand out that none of the most popular algorithms in the multi-objective field introduce special mechanisms to promote diversity in variable space. However, some efforts have been dedicated to this principle. A popular approach to promote the diversity in the decision space is the application of fitness sharing [21] in a similar way than in single-objective optimization. Although, fitness sharing might be used to promote diversity both in objective and decision variable space, most popular variants consider only distances in the objective space. Another MOEA designed to promote diversity in both the decision and the objective space is the Genetic Diversity Evolutionary Algorithm (GDEA) [22]. In this case, each individual is assigned with a diversity-based objective which is calculated as the Euclidean distance in the genotype space to the remaining individuals in the population. Then, a ranking that considers both the original objectives and the diversity objective is used to sort individuals. Another somewhat popular approach is to calculate distances between candidate solutions by taking into account both the objective and variable space [23], [24] with the aim of promoting diversity in both spaces. A different proposal combines the use of two selection operators [25]. The first one promotes diversity and quality in the objective space whereas the second one promotes diversity in the decision space. In the same line, modifying the hypervolume to integrate the decision space diversity in a single metric was proposed in [26]. In this approach, the proposed metric is used to guide the selection in the MOEA. Finally, some indirect mechanisms that might affect the diversity have also been introduced in some schemes. Probably, the most popular one is the use of mating restrictions [27], [19].

Attending to the analyses of the previous approaches, it is clear that they might bring benefits to decision makers because the final solutions obtained by these methods present a larger decision space diversity than the ones obtained by traditional approaches [23], [28]. Thus, while clear improvements are obtained when taking into account metrics related to the Pareto Set, the benefits in terms of the obtained Pareto front are not so clear. We claim that one of the reasons of this behaviour might be that the diversity in the variable space is considered in the whole optimization process. However, in a similar way that in the single objective domain, reducing the importance granted to the diversity in the decision space as the generations progress is really important [5]. Currently, no MOEA considers this idea, so this principle has guided the design of our novel MOEA.

III. PROPOSAL

This section is devoted to explain our proposal, whose main contributions rely in the replacement phase procedure and the density estimator approach of the objective space. VSD-MOEA is based on the principle of considering the diversity and the stopping criterion explicitly. Thus, the optimization search capabilities are altered gradually provoking a balance from exploration to explotation. This principle might be incorporated with any of the three categories of MOEAs. In this paper, our decision was to incorporate it in a dominance-based approach. Particularly, our proposal considers the application of the fast-non-dominated-sort procedure so it might be considered as an extension of NSGA-II [16]. Also, it takes into account a more advanced way to control the diversity in the objective space. Overall, VSD-MOEA is a dominance-based MOEA in which a novel replacement phase is provided, i.e. no novel components are incorporated in the parent selection and variation stages.

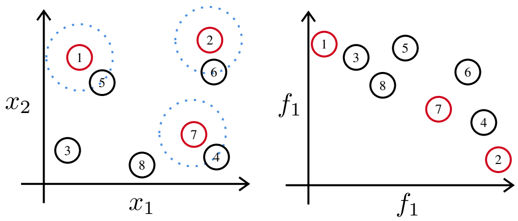

Fig. 1. Replacement Phase - The left side represents the variables space and the right side the respectively objective space of the individuals.

Algorithm 1 Replacement Phase of VSD-MOEA

1: Input: P_t (Population of current generation), Q_t (Offspring of current Generation)
2: Output: P_{t+1}
3: $R_t = P_t \cup Q_t$
4: $P_{t+1} = \emptyset$
5: $Penalized = \emptyset$
6: $D = D_I - D_I * \frac{G_{Elapsed}}{0.9 * G_{End}}$
7: move(R_t, P_{t+1}, Best in each objective according Equation (3))
8: **while** $|P_{t+1}| \leq$ N **do**
9: Compute **Diversity_Variable_Space** (R_t, P_{t+1})
10: move(R_t, Penalized, Variable space diversity $<$ D)
11: **if** R_t is empty **then**
12: Compute **Diversity_Variable_Space** ($Penalized$, P_{t+1})
13: move(Penalized, R_t, Largest variable space diversity)
14: $conditionally-non-dominated-sort(R_t \cup P_{t+1})$
15: Compute **Diversity_Objective_Space**(R_t, P_{t+1}))
16: **return** P_{t+1}

A. Replacement Phase of VSD-MOEA

The methodology used in the replacement phase considers several principles that were provided in the development of replacement phases for the single-objective domain [5]. In these approaches, at the begin of each generation the parents and offspring are joined. Then, N individuals must be selected to survive. In order to take into account the diversity in the decision space, the Distance to Closest Neighbor (DCN) is used. In each step, this normalized distance (Eqn. 2) is calculated with respect to those individuals that have already been selected to survive.

$$Distance(A, B) = \left(\sum_{i=1}^{n} \left(\frac{A_i - B_i}{x_i^{(U)} - x_i^{(L)}} \right)^2 \right)^{1/2} \quad (2)$$

Thus, individuals with large DCN values are those that contribute in a significant way to preserve exploration in different regions of the search space. In order to avoid an excessive decrease of the exploration degree, individuals with a DCN value lower than a threshold value are penalized, meaning that they are just selected if non-penalized individuals exist. In order to better visualize this principle, it can be considered that in each selection of a survivor, a hypersphere centered in such survivor is created. Then, all the individuals that are inside a hypersphere are penalized. One of the key novelties of the methodology is that the sizes of the hyperspheres are modified dynamically by taking into account the stopping criterion and elapsed generations. Particularly, the sizes are decreased in a linear way, to alter the behaviour from exploration towards intensification. Note that, this method requires a parameter which is the initial radius of the hyperspheres and is denoted as D_I. A too large value might provoke the penalization of the whole set of individuals, meaning that a non-useful diversity might be maintained in the initial generations. However, too small values might not penalize at all the individuals, meaning that the approach might behave as a traditional non-diversity based approach.

In order to better describe the principle of the replacement phase, we denote the individuals in the following way. The *reference individuals* are those that have been selected to survive as parents of the next generation. The remaining ones are classified in *penalized individuals* and *candidate individuals*. A representation of this scheme can be visualized in Fig. 1, where in the left part is showed the decision space, and in the right side its corresponding objective space. The reference individuals are marked with a red border. It can be visualized that each one of them is surrounded by a circle with radius D. Following this principle, a larger dimension of this circle would be a hypersphere. Then, any candidate individual that is inside of this circle is penalized. In this figure the penalized individuals are shown in gray in the right part. Once that the individuals are penalized, any of the typical approaches that are used to select survivors might be taken into account without considering the penalized individuals.

The specific pseudocode of the replacement phase that is used in VSD-MOEA is shown in Algorithm 1. Basically, it is the approach previously described which is integrated through a procedure to meassure the contribution of each individual to the objective space. In each step, the candidate individual with lowest rank and, in case of a tie, with the largest contribution to the diversity in the objective space is selected. In each generation, the first step (line 3) is to join the parents (P_t) and the offspring (Q_t) in R_t. Then, the set of penalized individuals and the parents of the next generation (P_{t+1}) are emptied (lines 4 and 5). Additionally, the radius of the hyperspheres (D) is uptated (line 6). Note that in the formula applied D_I is the initial size of the hypersphere, $G_{Elapsed}$ is the amount of generations that have been evolved, and G_{End} is the stopping criterion, i.e. the number of generations that are to be evolved in the execution of VSD-MOEA. It is clear that after the 90% of the generations, the D value is lower than 0, meaning that no penalties are performed. This means that in the first 90% of the generations, more exploration than in traditional MOEAs is induced, whereas in the final stages, a traditional MOEA is applied. The reason to induce such behaviour is that, we are not interested in obtaining a diverse set of solutions in the decision space –if it is not requiered by the problem– at the end of the run. Maintaining diverse solutions in the initial stages is just a way to promote exploration which results in better approximations to the Pareto front at the end of the execution. Finally, the next population is filled with the boundary solutions (line 7), i.e. for each k-objective the best candidate solution is selected to survive as is indicated by Equation (3) where $\lambda_j = 1$ if $k = j$ and $\lambda_j = 1 \times 10^5$ if $j \neq k$.

$$\begin{aligned} Best_{k \in \{1,..,m\}} =& min_{\vec{x} \in P} \{ max_{j \in \{1,...,m\}} \{ \lambda_j |f_j(\vec{x})| \} \\ & + 1 \times 10^{-4} \sum_{j=1}^{m} |f_j(\vec{x})| \} \end{aligned} \quad (3)$$

Then, until N individuals are selected (line 8), the following steps are carried out. First, the DCN value of each individual that has not been selected is calculated (line 9). Then, those individuals with a DCN value lower than D are penalized (line 10). If all the candidate individuals are penalized (line 11), it means that the amount of exploration is lower than expected. Thus, the individual with largest DCN values is recovered, i.e. moved to the non-penalized individuals set (lines 12 and 13). Finally, the objective space is taken into account. Specifically, the candidate individuals and the reference set are joined. Then, the *fast-non-dominated-sort* procedure is executed with such a set, stopping as soon as a front with a candidate individual is found (line 14). Then, for each candidate individual that belongs to the lowest front, the individual with higher contribution to the diversity in the objective space is selected (line 15). The specific way in which the diversity in the objective space is measured is described in the next section.

Note also that, as part of the diversity calculation of the variable space, a metric should be selected. Since our experimental validation is performed with a continuous domain, the normalized Euclidean distance is used. However, in discrete domains other distance metrics such as the Manhattan, or the Hamming distance might be considered, and the definition of such distance might affect the performance of the approach [7].

B. Improvement distance in objective space

Since the dominance definition is not related to the preservation of diversity in the objective space, dominance-based MOEAs incorporate special procedures to maintain diverse solutions. Several different approaches have been defined, such as clustering and/or crowding with several different variants each of them. In this paper, we define a novel distance metric, and then, as previously described a greedy approach, which selects an individual of the best front and with the largest distance. Specifically, the novel distance is called "Improvement Distance" (ID) and it follows the same principles that guided the design of the IGD+ and $I_{\epsilon+}$ indicators [29], [17], [30]. The main idea is to prefer those individuals whose quality in all objectives is similarly preserved. Particularly, a non-dominated individual can be very distant to the Pareto Front due that such individual could be the best in one objective but meaningly deteriorated in the rest of objectives, so as result it has high diversity in objective space. In fact, high improvements in one objective value are related to larger selection probabilities and not the opposite, so this behaviour should be avoided.

The key idea takes into account the dominance relation between the candidate and reference individuals. Consequently, the reference and the candidate individuals are compared. If the reference individual is dominated by the candidate individual, then the euclidean distance with no modification is implemented. However if they are non-dominated with each other, then is calculated the minimum distance from the reference individual to the dominated region by the candidate individual. Additionally, if the candidate individual is dominanted by the reference individual, then is computed the I_ϵ indicator, that gives the minimum distance by which the candidate individual needs to or can be traslated in each dimension in objective space such that the reference individual is dominated. Therefore, this distance can be viewed as an amount of inferiority of the solution in comparison with the reference individual. The improvement distance is defined in Equation (4) which incorporates the I_ϵ indicador (Equation 5) where R and C are the reference and candidate solutions respectively.

$$ID(R,C) = \left(\sum_{i=1}^{M} (max(0, R_i - C_i))^2 \right)^{1/2} - I_\epsilon(R,C) \quad (4)$$

$$I_\epsilon(R,C) = \begin{cases} min_\epsilon\{f_i(C) - \epsilon \leq f_i(R)\} & R \preceq C \\ 0 & otherwise \end{cases} \quad (5)$$

Specifically, this distance is considered as a Weakly Pareto Compliant Indicator. In addition, this metric relaxes some difficulties encountered when the number of objectives is increased, given that the solutions in many objectives are usually non-dominated with each other by using the Pareto dominance relation. This means a very low selection pressure toward the Pareto front in Pareto-dominance-based MOEAS [31]. Principally, the improvement distance is effective with over-prioritization of dominance-resist solutions i.e., solutions with exceptional performance in one objective and extremely poor performance in many others [32].

IV. Experimental Validation

In this section the experimental validation is carried out, showing that controlling the diversity in the variable space is a way to improve further some of the results obtained by state-of-art-MOEAs.In the same way, the scalability of each algorithm is analyzed with respect an increasing number of decision variables. The WFG [33], DTLZ [34] and UF [35] test problems have been used for our purpose.

Through the literature several crossover operators have been proposed in MOEAs [36], a popular operator is the Simulated Binary Crossover (SBX).4[37]In addition based in diversity-related properties the No-Reflection Dynamic SBX is implemented [38], which is considered as a variant of the SBX operator.

Additionally, our experimental validation includes the VSD-MOEA, as well as four well-known state-of-the-art algorithms. Given that all of them are stochastic algorithms, each execution was repeated 35 times with different seeds. The common configuration in all of them was the following: the stopping criterion was set to $250,000$ generations, the population size was fixed to 100, the WFG test problems were configured with two and three objectives, setting 24 parameters, where 20 of them are distance parameters and 4 are position parameters. Specifically, in the DTLZ test instances, the number of decision variables is set to $n = M + r - 1$, where $r = \{5, 10, 20\}$ for DTLZ1, DTLZ2 to DTLZ6 and DTLZ7 respectively, as is suggested by the authors [34]. The UF benchmark is composed of ten test instances, where the first seven are of two objectives and the rest with three objectives, the number of decision variables is assigned to $n = 10$.

In general (except for GDE3), the crossover and mutation operators are DNR-SBX and polynomial respectively, with

TABLE I
PARAMETRIZATION

Algorithm	Configuration
GDE3	CR = 0.9 and F = 0.5
MOMBI-II	$\epsilon = 1e-3$, $\alpha = 0.5$, record size = 5 generations
MOEA/D	size of neighborhood,= 20, max updates by sub-problem (nr) = 2 and $\delta = 0.9$
VSD-MOEA	$D_I = \sqrt{n} * 0.25$

TABLE II
REFERENCES POINTS FOR THE HV INDICATOR

Instances	Reference Point
WFG1-WFG9	$[2.1, ..., 2m + 0.1]$
DTLZ 1, 2, 4	$[1.1, ..., 1.1]$
DTLZ 3, 5, 6	$[3, ..., 3]$
DTLZ7	$[1.1, ..., 1.1, 2m]$
UF 1-10	$[2, ..., 2]$

crossover probability of 0.9 and mutation of $1/n$ [39], also the crossover and mutation distribution indexes were assigned to 20 and 50 respectively. The extra-parametrization of each algorithm is showed in the table I.

Particularly, the algorithms MOEA/D and MOMBI-II require a set of vectors uniformly scattered on the unit-simplex, therefore the number of vectors generated increases nonlinearly with the number of objectives. Consequently, is applied the method proposed in [40], [41] where the uniform design (UD) [42] and good lattice point (glp) are combined. Thus the number weight of vectors is not affected by the number of objectives.

In addition, our experimental analysis has been carry out with the hypervolume indicator, since the WFG benchmark variate the Pareto front shape with different distance parameters [43].

In this work the reference points are chosen to be a vector with values slightly larger than the nadir point, also for complex problems the reference point is considered as nadir point plus the unity. Therefore the reference points implemented in the hypervolume indicator are showed in the table II as used in [41], [44].

In order to statistically compare the hypervolume results, a similar guideline than the proposed in [45] was used. First a Shapiro-Wilk test was performed to check whatever or not the values of the results followed a Gaussian distribution. If, so, the Levene test was used to check for the homogeneity of the variances. If samples had equal variance, an ANOVA test was done; if not, a Welch test was performed. For non-Gaussian distributions, the nonparametric Kruskal-Wallis test was used to test whether samples are drawn from the same distribution. An algorithm X is said to win algorithm Y when the differences between them are statistically significant, if the mean and median obtained by X are higher than the mean and median achieved by Y.

In the tables III, IV are showed the hypervolume statistics with two an three objectives respectively. The column *Diff* indicate how far is the best HV mean from each algorithm, this field is computed as the difference between the mean of the each algorithm and the best mean. Consequently, the best algorithm in each instance has assigned a zero.

Considering two objectives the GDE3 provides the best results, however the VSD-MOEA is very close to the GDE3 as is showed in the *Diff* columns. It is important to highlight that the DTLZ test suites have the next weaknesses [35]. The global optimum lies in the center or bounds, all are separable and the global optimum has the same parameter values for different dimensions. Particularly the DTLZ5 and DTLZ6 are easy for the GDE3, since the global optimal are located in the low bound, thus if the differential operators locate a solution outside of bounds, the repair procedure could move the point among the optimal.

Additionally, the GDE3 get worse according increases the number of objectives, even more the *Diff* results are in average high, except for the DTLZ5 and DTLZ6 where the optimal set are located along the bounds. Despite the fact that the VSD-MOEA does not have a repair procedure because it implements genetic operators, the results are fairly stable. Also improves the hypervolume with three objectives, hence can be considered a robust algorithm, in fact considering more objectives it provides better results than the dominance-based algorithms[2]. A remarkable characteristic that can be appreciated in two and three objectives is that our proposal provide the best results in the most difficult problems as are dependence, multi-modal and deceptiveness.

On average the VSD-EMOA has lower *Diff* value than the rest of algorithms, therefore considering all best mean of each instance, the VSD-EMOA is not too far from the best means. Even more, the min and max averages are better than the min and max of the state-of-art algorithms.

The effective test showed in the tables V, VI are conformed by two an three objectives respectively, these metrics qualify the superiority of each algorithm with the rest through pair comparisons. Thus, an algorithm **A** is compared with algorithm **B**, if **A** wins, the difference with **B** is accumulated in wins $\uparrow$ of the algorithm **A**, the same process is for the algorithm **B** but it is accumulated in the lost field $\downarrow$. The column *Score* is conformed by the difference between the win and lost values, therefore a high positive score indicate the superiority of the algorithm.

In addition, our proposal has low negative scores in the DTLZ6 and WFG6 with two objectives, it might occurs because these instances are not a problem to long term executions, therefore they are totally defined by the crowding procedure. However, the negative scores in VSD-MOEA are not highly significant. Additionally, considering three objectives, the VSD-MOEA provides the best and positive scores. In general our proposal has the best scores in difficult instances as are the UF and some WFG problems.

A. Decision Variable Scalability Experiments

The scalability of each MOEA is also evaluated respect with the number of decision variables [43]. The figures 2, 3 show the hypervolume performance of 30, 100, 250 and 500 variables respectively. Particularly, the scalability study was realized in DTLZ4, UF5 with two objectives and DTLZ4,

[2]Experimentally with ten objectives the algorithm is best than dominance-based MOEAs.

UF10 with three objectives. In some instances the GDE3 degrades relatively fast according increases the number of decision variables as is showed in UF5, UF10 and DTLZ4 with three objectives. Also, the GDE3 show a non-stable performance with the parameter configurations, as is explained by J. Lampinen et al.[8], where they indicate that a high value for CR might lead to premature convergence with respect to one objective compared to another.

Specifically, the instance UF10 with GDE3 has an increment of HV for 100 variables, this irregularity is related with diversity issues [8] On the other hand the VSD-MOEA is enough stable, also it provides the best HV values. It is interesting that the MOEA/D and MOMBI-II required an extra-parametrization, hence the stability could be compromised.

The 50% attainment surfaces WFG2, WFG8 and DTLZ7 instances are showed in the figure 4. The analyses shows that our proposal provide solutions approximated to the Pareto front. Although the GDE3 approximate the WFG2 and DTLZ7, this this algorithm is far in some regions of the Pareto front with the WFG8 instance.

V. Conclusion

The evolutionary algorithms have been a most popular approaches to deal with complex optimization problems. Particularly, the MOEAs works with different principles where the objective space is involved. As it is seen in single-objective the diversity provides quality solutions specifically in complex problems.

In this work we have provided an algorithm with a particular replacement phase. This phase considers the diversity in both spaces, specifically in the variable space the diversity is based in a decremental dynamic concept. Thus, in the first stages the diversity in variables space is induced, and in a gradual way the last stages the replacement phase works as is usual.

Additionally a improvement distance is suggested, which is based in the IGD+ indicator and is considered as weakly Pareto compliant.

The experimental validation is carried out with long-term executions and the three popular benchmarks. This validation shows that the VSD-MOEA is able to properly solve overall test problems, also in the most complex test instances shows the best hypervolume values.

In addition, some scalability experiments with the decision variables are carry out through long-term executions, results indicate the superiority and stability provided with the hypervolume indicator.

We show the relevance of diversity in the variable space an established a way to preserve the diversity in explicitly in variable space.

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Fig. 2. Scalability study of decision variables with two objectives (HV)

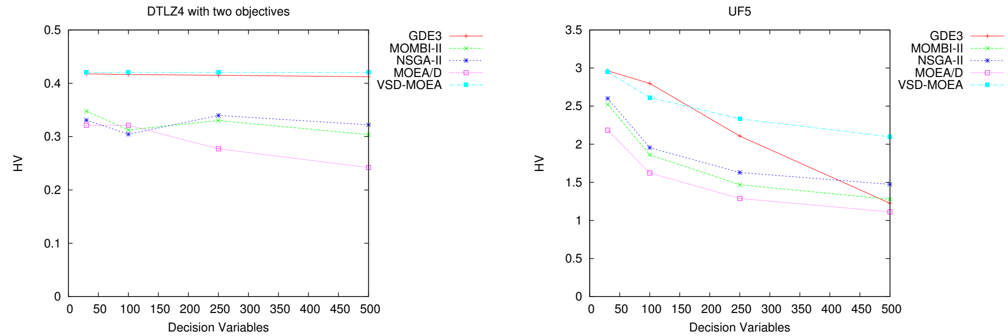

Fig. 3. Scalability study of decision variables with three objectives (HV)

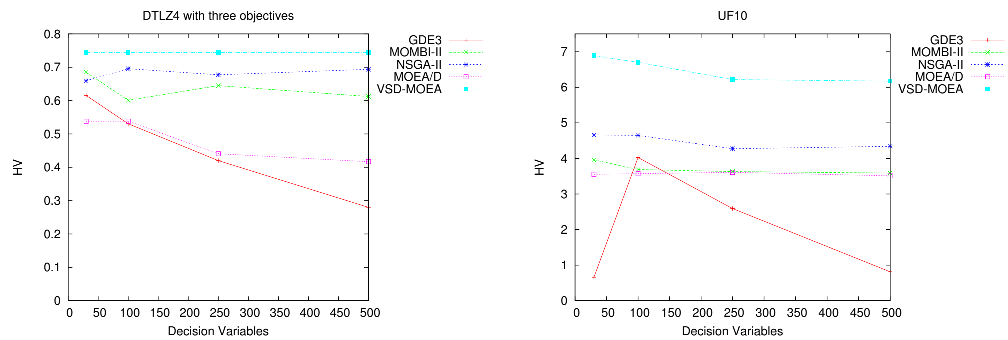

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Fig. 4. 50% Attainment Surfaces Achieved

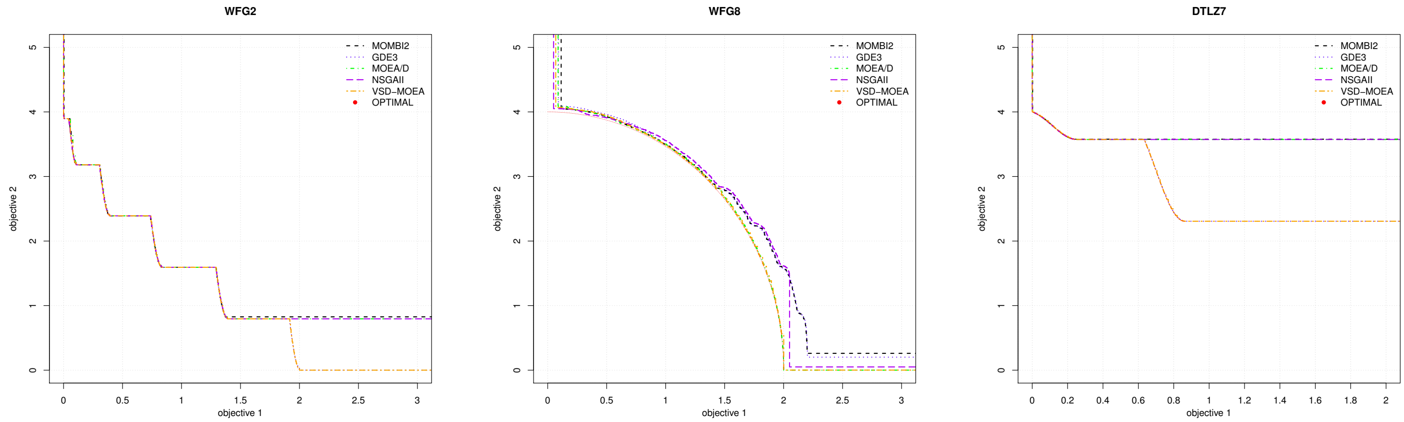

TABLE III
STATISTICS HV WITH TWO OBJECTIVES

	GDE3				MOMBI-II				NSGAII				MOEA/D				VSD-MOEA			
	Min	Max	Mean	Diff	Min	Max	Mean	Diff	Min	Max	Mean	Diff	Min	Max	Mean	Diff	Min	Max	Mean	Diff
DTLZ1	1.084	1.084	**1.084**	0.000	1.078	1.078	1.078	0.006	1.083	1.083	1.083	0.000	1.078	1.084	1.081	0.003	1.084	1.084	**1.084**	0.000
DTLZ2	0.421	0.421	**0.421**	0.000	0.418	0.418	0.418	0.003	0.419	0.420	0.419	0.001	0.420	0.420	0.420	0.001	0.420	0.420	0.420	0.000
DTLZ3	8.211	8.211	**8.211**	0.000	8.170	8.170	8.170	0.041	8.209	8.210	8.209	0.001	8.169	8.210	8.200	0.011	8.210	8.210	8.210	0.000
DTLZ4	0.421	0.421	**0.421**	0.000	0.110	0.418	0.383	0.038	0.110	0.420	0.313	0.107	0.110	0.418	0.365	0.055	0.420	0.420	0.420	0.000
DTLZ5	8.211	8.211	**8.211**	0.000	8.170	8.170	8.170	0.041	8.209	8.210	8.209	0.001	8.210	8.210	8.210	0.001	8.210	8.210	**8.210**	0.000
DTLZ6	8.211	8.211	**8.211**	0.000	7.989	8.170	8.073	0.138	8.062	8.209	8.128	0.083	8.027	8.210	8.095	0.116	7.989	8.210	8.123	0.088
DTLZ7	0.894	0.894	**0.894**	0.000	0.417	0.417	0.417	0.478	0.420	0.420	0.420	0.474	0.420	0.420	0.420	0.474	0.893	0.893	0.893	0.001
UF1	3.657	3.659	3.658	0.002	3.327	3.517	3.490	0.171	3.650	3.652	3.651	0.010	3.428	3.660	3.588	0.072	3.655	3.662	**3.661**	0.000
UF2	3.651	3.654	3.653	0.005	3.406	3.628	3.594	0.064	3.643	3.647	3.645	0.013	3.428	3.649	3.533	0.124	3.655	3.660	**3.658**	0.000
UF3	3.371	3.660	**3.642**	0.000	3.328	3.595	3.499	0.143	3.524	3.639	3.602	0.041	2.850	3.642	3.451	0.191	3.549	3.620	3.593	0.049
UF4	3.219	3.237	3.224	0.036	3.194	3.205	3.197	0.062	3.198	3.207	3.200	0.060	3.210	3.243	3.228	0.032	3.235	3.280	**3.260**	0.000
UF5	2.532	3.000	**2.964**	0.000	2.047	2.746	2.522	0.442	1.861	2.897	2.602	0.362	1.800	2.550	2.185	0.778	2.591	3.267	2.951	0.013
UF6	2.000	3.325	**3.098**	0.000	2.013	2.893	2.638	0.460	2.007	2.896	2.518	0.580	0.726	2.884	2.070	1.028	2.893	3.306	3.058	0.040
UF7	3.490	3.492	**3.491**	0.000	2.474	3.470	2.592	0.899	2.168	3.485	3.190	0.301	2.015	3.493	2.743	0.748	3.473	3.493	3.489	0.002
WFG1	4.263	5.256	4.848	0.408	4.623	5.666	**5.256**	0.000	4.716	5.250	5.156	0.100	4.480	5.243	5.037	0.219	4.717	5.250	5.205	0.051
WFG2	5.072	5.072	**5.072**	0.000	4.925	4.942	4.927	0.145	4.948	5.068	4.953	0.119	4.943	4.943	4.943	0.128	5.069	5.069	5.069	0.003
WFG3	4.513	4.530	4.522	0.043	4.561	4.563	4.562	0.003	4.533	4.543	4.539	0.026	4.562	4.563	4.563	0.002	4.565	4.565	**4.565**	0.000
WFG4	2.280	2.286	2.283	0.009	2.284	2.285	2.285	0.007	2.271	2.283	2.277	0.014	2.287	2.287	2.287	0.005	2.291	2.291	**2.291**	0.000
WFG5	1.984	1.990	**1.986**	0.000	1.976	1.996	1.980	0.005	1.970	1.977	1.975	0.011	1.972	2.010	1.977	0.009	1.976	1.984	1.980	0.005
WFG6	2.092	2.246	**2.183**	0.000	2.055	2.207	2.141	0.043	2.059	2.207	2.138	0.046	2.017	2.188	2.129	0.054	2.082	2.198	2.131	0.053
WFG7	2.272	2.280	2.276	0.015	2.284	2.285	2.284	0.007	2.269	2.282	2.275	0.016	2.287	2.287	2.287	0.005	2.291	2.291	**2.291**	0.000
WFG8	1.860	1.876	1.868	0.363	1.868	2.237	1.951	0.280	1.803	2.140	1.906	0.325	1.938	2.252	2.225	0.006	2.050	2.248	**2.231**	0.000
WFG9	1.711	1.714	1.713	0.542	2.197	2.264	2.244	0.010	2.168	2.258	2.229	0.026	1.706	2.264	2.225	0.030	2.242	2.271	**2.255**	0.000
Average	3.279	3.423	3.388	**0.062**	3.170	3.406	3.299	**0.151**	3.187	3.409	3.332	**0.118**	3.047	3.397	3.272	**0.178**	3.372	3.474	3.437	**0.013**

The bold values correspond to the best HV mean of each instance, in case of repeated they are also in bold.

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Dr. Segura is a *Member* of the IEEE and a member of the ACM. He is currently an Associate Researcher of the Computer Science area at the Centre for Research in Mathematics.

TABLE IV
STATISTICS HV WITH THREE OBJECTIVES

	GDE3				MOMBI-II				NSGAII				MOEA/D				VSD-MOEA			
	Min	Max	Mean	Diff	Min	Max	Mean	Diff	Min	Max	Mean	Diff	Min	Max	Mean	Diff	Min	Max	Mean	Diff
DTLZ1	1.301	1.303	1.302	0.002	1.290	1.290	1.290	0.015	1.300	1.302	1.301	0.003	1.280	1.294	1.292	0.013	1.304	1.305	**1.304**	0.000
DTLZ2	0.707	0.724	0.713	0.031	0.724	0.724	0.724	0.020	0.691	0.724	0.709	0.035	0.709	0.709	0.709	0.035	0.742	0.746	**0.744**	0.000
DTLZ3	26.368	26.389	26.380	0.033	26.154	26.155	26.154	0.259	26.362	26.389	26.378	0.035	26.171	26.274	26.267	0.146	26.413	26.414	**26.413**	0.000
DTLZ4	0.707	0.720	0.714	0.031	0.455	0.724	0.701	0.044	0.121	0.726	0.699	0.046	0.121	0.705	0.603	0.142	0.744	0.745	**0.745**	0.000
DTLZ5	23.987	23.987	**23.987**	0.000	23.813	23.819	23.814	0.173	23.978	23.982	23.981	0.006	23.878	23.878	23.878	0.109	23.986	23.986	23.986	0.001
DTLZ6	23.987	23.987	**23.987**	0.000	23.358	23.813	23.576	0.411	23.701	23.979	23.817	0.170	23.268	23.713	23.533	0.454	23.482	23.986	23.737	0.250
DTLZ7	1.783	1.841	1.815	0.054	0.901	0.904	0.902	0.967	0.895	0.907	0.903	0.967	0.900	0.907	0.905	0.964	1.864	1.880	**1.869**	0.000
UF10	0.010	3.886	0.658	6.232	3.148	5.585	3.961	2.929	3.762	6.260	4.660	2.230	2.914	4.079	3.554	3.336	6.000	7.237	**6.890**	0.000
UF8	0.052	4.855	1.973	5.416	4.000	7.358	6.907	0.482	7.156	7.267	7.231	0.158	4.000	7.321	6.414	0.975	7.316	7.413	**7.389**	0.000
UF9	0.238	4.217	1.488	6.261	7.131	7.653	7.260	0.489	6.895	7.597	7.350	0.399	7.107	7.649	7.233	0.515	7.724	7.758	**7.748**	0.000
WFG1	16.338	18.105	17.099	28.884	40.294	48.674	**45.983**	0.000	43.052	44.553	43.642	2.341	40.351	44.994	43.646	2.337	45.437	45.940	45.763	0.220
WFG2	45.956	47.224	46.709	1.819	40.196	48.213	45.175	3.352	40.091	47.465	45.542	2.985	40.043	47.803	43.552	4.975	48.345	48.671	**48.528**	0.000
WFG3	30.389	30.999	30.730	0.448	31.165	31.191	**31.178**	0.000	30.587	31.144	30.905	0.273	31.145	31.157	31.152	0.025	31.029	31.068	31.048	0.130
WFG4	18.012	20.150	19.178	4.956	23.632	23.661	23.636	0.498	21.724	22.754	22.296	1.838	21.931	22.465	22.127	2.007	23.976	24.320	**24.134**	0.000
WFG5	20.143	21.167	20.679	1.212	21.390	21.405	21.395	0.496	19.985	21.045	20.591	1.299	19.676	20.357	19.760	2.130	21.730	22.082	**21.891**	0.000
WFG6	17.308	20.484	18.927	4.117	22.014	23.025	22.654	0.389	19.835	22.018	21.021	2.022	20.342	21.584	21.045	1.999	22.500	23.420	**23.044**	0.000
WFG7	18.964	20.969	19.861	4.268	23.632	23.650	23.636	0.492	21.552	22.972	22.442	1.687	22.260	22.261	22.260	1.868	23.911	24.336	**24.129**	0.000
WFG8	14.140	15.489	14.730	8.423	20.798	23.725	23.133	0.020	16.287	18.402	17.761	5.392	21.685	22.104	21.855	1.298	18.991	24.078	**23.153**	0.000
WFG9	17.519	18.364	17.997	5.072	19.111	23.270	22.850	0.219	17.276	21.262	18.062	5.007	17.684	21.796	21.243	1.826	19.260	23.788	**23.069**	0.000
Average	14.627	16.045	15.207	**4.066**	17.537	19.202	18.680	**0.592**	17.118	18.460	17.857	**1.415**	17.130	18.476	17.949	**1.324**	18.671	19.430	19.241	**0.032**

The bold values correspond to the best HV mean of each instance, in case of repeated they are also in bold.

TABLE V
EFFECTIVE TESTS HV WITH TWO OBJECTIVES

	GDE3			MOMBI-II			NSGA-II			MOEA/D			VSD-MOEA		
	↑	↓	Score	↑	↓	Score	↑	↓	Score	↑	↓	Score	↑	↓	Score
DTLZ1	0.006	0.000	**0.006**	0.000	0.017	-0.017	0.005	0.001	0.005	0.000	0.000	0.000	0.006	0.000	**0.006**
DTLZ2	0.005	0.000	**0.005**	0.000	0.008	-0.008	0.001	0.003	-0.002	0.003	0.001	0.002	0.004	0.000	0.003
DTLZ3	0.053	0.000	**0.053**	0.000	0.150	-0.150	0.049	0.002	0.047	0.030	0.032	-0.002	0.052	0.000	0.052
DTLZ4	0.201	0.000	**0.201**	0.087	0.075	0.012	0.000	0.336	-0.336	0.052	0.128	-0.076	0.200	0.000	0.199
DTLZ5	0.043	0.000	**0.043**	0.000	0.161	-0.161	0.039	0.003	0.036	0.041	0.001	0.040	0.042	0.000	0.042
DTLZ6	0.424	0.000	**0.424**	0.000	0.266	-0.266	0.088	0.083	0.005	0.022	0.177	-0.155	0.078	0.088	-0.009
DTLZ7	1.428	0.000	**1.428**	0.000	0.961	-0.961	0.003	0.947	-0.944	0.004	0.947	-0.943	1.422	0.001	1.421
UF1	0.246	0.002	0.244	0.000	0.598	-0.598	0.224	0.017	0.206	0.098	0.205	-0.107	0.255	0.000	**0.255**
UF2	0.187	0.005	0.182	0.061	0.174	-0.113	0.163	0.021	0.142	0.000	0.416	-0.416	0.206	0.000	**0.206**
UF3	0.424	0.000	**0.424**	0.000	0.340	-0.340	0.261	0.041	0.221	0.000	0.485	-0.485	0.237	0.057	0.180
UF4	0.051	0.040	0.011	0.000	0.122	-0.122	0.003	0.112	-0.109	0.063	0.032	0.031	0.189	0.000	**0.189**
UF5	1.582	0.000	**1.582**	0.337	0.950	-0.613	0.496	0.711	-0.215	0.000	2.297	-2.297	1.543	0.000	1.543
UF6	2.109	0.000	**2.109**	0.567	0.881	-0.313	0.448	1.120	-0.672	0.000	3.031	-3.031	1.947	0.040	1.907
UF7	1.950	0.000	**1.950**	0.000	2.393	-2.393	1.044	0.600	0.443	0.000	1.940	-1.940	1.942	0.002	1.939
WFG1	0.000	1.263	-1.263	0.627	0.000	**0.627**	0.426	0.049	0.377	0.190	0.504	-0.315	0.574	0.000	0.574
WFG2	0.395	0.000	**0.395**	0.000	0.329	-0.329	0.035	0.235	-0.200	0.016	0.264	-0.247	0.384	0.003	0.381
WFG3	0.000	0.142	-0.142	0.064	0.003	0.061	0.017	0.074	-0.057	0.065	0.002	0.063	0.075	0.000	**0.075**
WFG4	0.006	0.015	-0.009	0.010	0.008	0.001	0.000	0.037	-0.037	0.015	0.005	0.010	0.034	0.000	**0.034**
WFG5	0.030	0.000	**0.030**	0.009	0.005	0.004	0.000	0.024	-0.024	0.002	0.016	-0.014	0.009	0.005	0.004
WFG6	0.195	0.000	**0.195**	0.000	0.043	-0.043	0.000	0.046	-0.046	0.000	0.054	-0.054	0.000	0.053	-0.053
WFG7	0.000	0.034	-0.034	0.017	0.009	0.007	0.000	0.035	-0.035	0.024	0.005	0.019	0.043	0.000	**0.043**
WFG8	0.000	0.802	-0.802	0.128	0.554	-0.426	0.000	0.690	-0.690	0.951	0.000	0.951	0.968	0.000	**0.968**
WFG9	0.000	2.103	-2.103	0.547	0.010	0.537	0.521	0.041	0.480	0.512	0.034	0.478	0.608	0.000	**0.608**
Total	9.333	4.405	**4.929**	2.454	8.057	**-5.603**	3.824	5.230	**-1.406**	2.088	10.575	**-8.487**	10.818	0.250	**10.568**

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Joel Chacón Biography text here.

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TABLE VI
EFFECTIVE TESTS HV WITH THREE OBJECTIVES

	GDE3			MOMBI-II			NSGA-II			MOEA/D			VSD-MOEA		
	↑	↓	Score	↑	↓	Score	↑	↓	Score	↑	↓	Score	↑	↓	Score
DTLZ1	0.024	0.002	0.021	0.000	0.040	-0.040	0.021	0.004	0.017	0.002	0.033	-0.031	0.033	0.000	**0.033**
DTLZ2	0.008	0.041	-0.033	0.040	0.020	0.019	0.000	0.055	-0.055	0.000	0.053	-0.053	0.121	0.000	**0.121**
DTLZ3	0.339	0.033	0.306	0.000	0.821	-0.821	0.335	0.035	0.300	0.113	0.371	-0.258	0.474	0.000	**0.474**
DTLZ4	0.124	0.031	0.093	0.100	0.057	0.043	0.096	0.047	0.049	0.000	0.447	-0.447	0.262	0.000	**0.262**
DTLZ5	0.289	0.000	**0.289**	0.000	0.577	-0.577	0.270	0.011	0.258	0.064	0.320	-0.256	0.286	0.001	0.285
DTLZ6	1.285	0.000	**1.285**	0.000	0.813	-0.813	0.607	0.170	0.437	0.000	0.942	-0.942	0.365	0.331	0.034
DTLZ7	2.737	0.054	2.683	0.000	1.884	-1.884	0.000	1.882	-1.882	0.005	1.875	-1.870	2.952	0.000	**2.952**
UF10	0.000	16.434	-16.434	3.711	3.627	0.084	5.807	2.230	3.578	2.896	4.850	-1.954	14.727	0.000	**14.727**
UF8	0.000	20.048	-20.048	5.427	0.806	4.621	5.582	0.158	5.424	4.441	1.469	2.972	7.032	0.000	**7.032**
UF9	0.000	23.640	-23.640	5.772	0.489	5.283	5.862	0.399	5.464	5.745	0.515	5.230	7.663	0.000	**7.663**
WFG1	0.000	110.639	-110.639	33.562	0.000	**33.562**	26.543	4.463	22.081	26.547	4.454	22.093	32.903	0.000	32.903
WFG2	1.167	1.819	-0.652	1.623	3.720	-2.097	0.367	4.152	-3.785	0.000	6.598	-6.598	13.132	0.000	**13.132**
WFG3	0.000	1.363	-1.363	0.875	0.000	**0.875**	0.175	0.663	-0.488	0.774	0.025	0.749	0.461	0.234	0.227
WFG4	0.000	15.481	-15.481	7.307	0.498	6.809	3.287	3.177	0.110	2.949	3.685	-0.736	9.297	0.000	**9.297**
WFG5	0.918	1.928	-1.010	3.155	0.496	2.659	0.831	2.103	-1.272	0.000	5.514	-5.514	5.137	0.000	**5.137**
WFG6	0.000	12.058	-12.058	6.970	0.389	6.581	2.095	3.655	-1.560	2.118	3.609	-1.491	8.528	0.000	**8.528**
WFG7	0.000	13.025	-13.025	6.346	0.492	5.854	2.762	2.882	-0.119	2.400	3.426	-1.026	8.315	0.000	**8.315**
WFG8	0.000	26.981	-26.981	15.052	0.020	15.032	3.031	14.858	-11.828	11.219	2.576	8.644	15.133	0.000	**15.133**
WFG9	0.000	13.172	-13.172	11.249	0.219	11.030	0.000	12.976	-12.976	6.427	3.434	2.993	12.124	0.000	**12.124**
Total	6.892	256.749	**-249.857**	101.189	14.969	**86.220**	57.672	53.920	**3.751**	65.700	44.196	**21.504**	138.947	0.566	**138.381**