Supplementary Document for "VSD-MOEA: A Dominance-Based Multi-Objective Evolutionary Algorithm with Explicit Variable Space Diversity Management"

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This document contains supplementary material to provide a clearer understanding of the specifics of VSD-MOEA. First, a multimedia material that provides a visualization of the internal behavior of VSD-MOEA in comparison to other state-of-the-art schemes is provided. Additionally, some of the results described in the main document are analyzed in terms of the Modified Inverted Generational Distance (IGD+) proposed by Ishibuchi et al. (2015), with the conclusions being quite similar to those obtained in the main document with the hypervolume.

1 Multimedia Material

This section provides a description of the video included as supplementary material[1]. The main aim of this video is to show the large differences between the way in which VSD-MOEA and state-of-the-art MOEAs explore the search space. The video shows a single execution of VSD-MOEA and R2-EMOA to solve WFG5. Note that NSGA-II and MOEA/D were also executed and their behaviors were similar to that of R2-EMOA. Thus, in order to avoid saturating the video, they were not included. In order to allow a proper visualization, WFG5 was configured with two decision variables, meaning that one distance parameter and one position parameter are used. The main peculiarity of WFG5 is its deceptiveness (Huband et al., 2005). In this configuration, a solution x is part of the Pareto set when $x_2 = 1.4$. However, most of the descent directions point towards two local optimal fronts, which might appear in both $x_2 = 0$ and $x_2 = 4$. The stopping criterion was set to $100,000$ function evaluations and the remaining parameters were set as in the main document. It is worth noting that VSD-MOEA explicitly promotes decision variable space diversity until the halfway point of the execution.

The video is divided into two sides. The left-side represents the objective space and the right-side represents the decision variable space. In the decision variable space,

[1] Alternatively, the video can be accessed at https://youtu.be/dbk5DaFJ8y0

Table 1: Summary of the IGD+ results attained for problems with two objectives

	VSD-MOEA				**MOEA/D**				**NSGA-II**				**R2-EMOA**			
	Min	**Max**	**Mean**	**Std**	**Min**	**Max**	**Mean**	**Std**	**Min**	**Max**	**Mean**	**Std**	**Min**	**Max**	**Mean**	**Std**
WFG1	**0.006**	**0.019**	**0.008**	**0.003**	0.006	0.015	0.008	0.002	0.006	0.014	0.008	0.002	0.006	0.061	0.013	0.014
WFG2	**0.003**	**0.003**	**0.003**	**0.000**	0.006	0.055	0.052	0.011	0.003	0.053	0.040	0.022	0.053	0.055	0.054	0.000
WFG3	**0.007**	**0.007**	**0.007**	**0.000**	0.008	0.008	0.008	0.000	0.011	0.013	0.012	0.000	0.008	0.009	0.008	0.000
WFG4	0.006	0.006	0.006	0.000	0.007	0.007	0.007	0.000	0.007	0.010	0.008	0.001	**0.005**	**0.005**	**0.005**	**0.000**
WFG5	**0.038**	**0.057**	**0.047**	**0.006**	0.060	0.069	0.065	0.002	0.060	0.068	0.066	0.002	0.064	0.066	0.065	0.000
WFG6	0.068	0.088	0.081	0.004	**0.034**	**0.073**	**0.050**	**0.010**	**0.034**	**0.064**	**0.051**	**0.007**	**0.034**	**0.076**	**0.053**	**0.010**
WFG7	0.006	0.006	0.006	0.000	0.007	0.007	0.007	0.000	0.008	0.010	0.009	0.000	**0.005**	**0.006**	**0.005**	**0.000**
WFG8	**0.026**	**0.099**	**0.043**	**0.025**	0.103	0.120	0.112	0.005	0.116	0.139	0.125	0.005	0.103	0.120	0.110	0.004
WFG9	**0.009**	**0.014**	**0.011**	**0.001**	0.011	0.125	0.067	0.053	0.014	0.127	0.101	0.046	0.009	0.125	0.067	0.053
DTLZ1	0.001	0.001	0.001	0.000	**0.001**	**0.001**	**0.001**	**0.000**	0.002	0.002	0.002	0.000	0.001	0.001	0.001	0.000
DTLZ2	0.002	0.002	0.002	0.000	0.002	0.002	0.002	0.000	0.002	0.003	0.003	0.000	**0.002**	**0.002**	**0.002**	**0.000**
DTLZ3	0.002	0.002	0.002	0.000	0.002	0.002	0.002	0.000	0.002	0.003	0.002	0.000	**0.002**	**0.002**	**0.002**	**0.000**
DTLZ4	**0.002**	**0.002**	**0.002**	**0.000**	0.002	0.363	0.105	0.163	0.002	0.363	0.064	0.136	0.002	0.363	0.167	0.180
DTLZ5	0.002	0.002	0.002	0.000	0.002	0.002	0.002	0.000	0.002	0.003	0.003	0.000	**0.002**	**0.002**	**0.002**	**0.000**
DTLZ6	**0.002**	**0.002**	**0.002**	**0.000**	0.022	0.149	0.076	0.027	0.126	0.315	0.205	0.036	0.019	0.128	0.078	0.027
DTLZ7	0.003	0.003	0.003	0.000	0.003	0.003	0.003	0.000	0.002	0.003	0.003	0.000	**0.002**	**0.002**	**0.002**	**0.000**
UF1	**0.003**	**0.003**	**0.003**	**0.000**	0.004	0.004	0.004	0.000	0.005	0.006	0.006	0.000	0.003	0.005	0.004	0.001
UF2	0.004	0.007	0.005	0.001	**0.003**	**0.005**	**0.004**	**0.000**	0.008	0.010	0.010	0.000	0.004	0.006	0.005	0.001
UF3	**0.038**	**0.095**	**0.057**	**0.013**	0.141	0.237	0.180	0.022	0.052	0.127	0.084	0.020	0.119	0.210	0.183	0.021
UF4	0.020	0.024	0.022	0.001	0.024	0.031	0.026	0.001	0.027	0.039	0.033	0.003	**0.019**	**0.023**	**0.021**	**0.001**
UF5	**0.088**	**0.154**	**0.132**	**0.014**	0.079	0.593	0.265	0.120	**0.091**	**0.254**	**0.142**	**0.033**	0.079	0.521	0.215	0.131
UF6	**0.021**	**0.065**	**0.038**	**0.011**	0.066	0.529	0.380	0.108	0.037	0.542	0.193	0.114	0.064	0.432	0.266	0.103
UF7	**0.003**	**0.009**	**0.004**	**0.001**	**0.003**	**0.005**	**0.004**	**0.000**	0.007	0.008	0.007	0.000	0.003	0.242	0.046	0.082
Mean	0.016	0.029	0.021	0.003	0.026	0.105	0.062	0.023	0.027	0.095	0.051	0.019	0.026	0.107	0.060	0.027

Table 2: Statistical Tests and Deterioration Level of the IGD+ for problems with two objectives

	$\uparrow$	$\downarrow$	$\leftrightarrow$	**Score**	**Deterioration**
MOEA/D	23	34	12	-11	0.979
NSGA-II	11	49	9	-38	0.725
R2-EMOA	34	22	13	12	0.922
VSD-MOEA	51	14	4	37	0.036

each local optimal region is highlighted with a horizontal blue line, and the global optimal region is highlighted with a horizontal red line. The video shows that after just a few function evaluations, R2-EMOA has converged prematurely to the deceptive regions, contrarily to the VSD-MOEA, which is still exploring. At approximately 30% of the total number of function evaluations, VSD-MOEA has located three individuals in the global optimal region, and a few later (40% of function evaluations), the number of solutions in this region has significantly increased. Thereafter, at the 50% of all function evaluations, VSD-MOEA has located most of the individuals in the global optimal region and only some individuals are located in the local optimal regions. Thus, the results obtained by R2-EMOA are clearly outperformed by VSD-MOEA. Note that both schemes place a large number of points in the knee of the Pareto front, at the cost of leaving some holes near the boundaries. This is because both the $R2$ metric and the objective space density estimator used in VSD-MOEA prefer these regions. To underscore the superiority of VSD-MOEA, note that most of the solutions obtained by R2-EMOA are dominated by those located by VSD-MOEA. The main feature of VSD-MOEA, which is to delay convergence, is clearly shown in this video.

2 Comparison against State-of-the-art MOEAs in long-term executions

This section presents the results obtained by VSD-MOEA and state-of-the-art schemes in terms of IGD+(Ishibuchi et al., 2015). Specifically, we present the results for the long-term executions, meaning the stopping criterion was set to 2.5×10^7 function evaluations. The structure of the tables is the same as in the main document. Thus, the only

Table 3: Statistical Tests and Deterioration Level of the IGD+ for problems with three objectives

	↑	↓	↔	Score	Deterioration
MOEA/D	15	37	5	-22	0.787
NSGA-II	6	46	5	-40	1.214
R2-EMOA	35	16	6	19	0.669
VSD-MOEA	49	6	2	43	0.039

modification is that instead of using the hypervolume, IGD+ is adopted in this case.

Table 1 shows the IGD+ values obtained for the benchmark functions with two objectives. Specifically, the minimum, maximum, mean and standard deviation of the IGD+ values is provided for each method and test problem adopted. The last row shows the results considering all the test problems. For each test problem, the data for the method that yielded the lowest mean is shown in **boldface**. Additionally, all the methods that were not statistically inferior to the method that yielded the lowest mean are shown in **boldface**. From here on, the methods shown in **boldface** for a given problem are referred to as the winning methods. Based on the number of functions where each method is in the group of winning methods for the cases with two objectives, the best methods are VSD-MOEA and R2-EMOA with 13 and 8 wins, respectively. Thus, VSD-MOEA is the most competitive method in terms of this metric. More impressive is the fact that the mean IGD+ attained by VSD-MOEA, when all the problems are considered simultaneously, is much lower than that attained by R2-EMOA. In fact, the total means of R2-EMOA (0.060), NSGA-II (0.051) and MOEA/D (0.062) are quite similar. In contrast, VSD-MOEA yielded a much lower value (0.021). When the data is inspected carefully, it is clear that in the cases where VSD-MOEA loses, the difference with respect to the best method is not very large. For instance, the difference between the mean IGD+ attained by VSD-MOEA and by the best method was never larger than 0.05. However, all the other methods exhibited a deterioration greater than 0.05 in several cases. Specifically, it happened in 7, 5 and 8 problems for MOEA/D, NSGA-II and R2-EMOA, respectively. This means that even if VSD-MOEA loses in some cases, its deterioration is always small, exhibiting a much more robust behavior than any other method. Exactly the same situation appeared when analyzing the data in terms of hypervolume.

In order to better clarify these findings, pair-wise statistical tests were done between each method tested in each test problem. Table 2 shows the results, with the same meaning as in the main document. The calculated data confirms that although VSD-MOEA loses in some cases, the overall numbers of wins and losses favor VSD-MOEA. More importantly, the total deterioration is considerably lower in the case of VSD-MOEA, confirming that when VSD-MOEA loses, the deterioration is not very high.

Tables 3 and 4 show the same information for the problems with three objectives. In this case, the superiority of VSD-MOEA is even clearer. Taking into account the mean of all the test problems, VSD-MOEA again yielded a much lower mean IGD+ than the other methods. Specifically, VSD-MOEA obtained a value of 0.059, whereas the second-ranked algorithm (R2-EMOA) obtained a value of 0.093. Once again, the difference between the mean IGD+ obtained by VSD-MOEA and by the best method was never greater than 0.05. However, all the other methods exhibited a deterioration greater than 0.05 in several cases. In particular, this happened in 5, 8 and 7 problems for MOEA/D, NSGA-II and R2-EMOA, respectively. Moreover, VSD-MOEA is considerably superior than the other methods not only in terms of total deterioration, but also in terms of total wins and losses. VSD-MOEA was in the group of winning methods for 14 out of 19

Table 4: Summary of the IGD+ results attained for problems with three objectives

	VSD-MOEA				MOEA/D				NSGA-II				R2-EMOA			
	Min	Max	Mean	Std	Min	Max	Mean	Std	Min	Max	Mean	Std	Min	Max	Mean	Std
WFG1	**0.049**	**0.070**	**0.058**	**0.006**	0.080	0.100	0.090	0.005	0.142	0.179	0.160	0.010	0.058	0.098	0.079	0.010
WFG2	**0.031**	**0.048**	**0.037**	**0.004**	0.057	0.068	0.063	0.002	0.073	0.133	0.097	0.014	0.102	0.104	0.103	0.000
WFG3	0.033	0.033	0.033	0.000	0.023	0.023	0.023	0.000	0.031	0.061	0.039	0.005	**0.022**	**0.023**	**0.022**	**0.000**
WFG4	**0.090**	**0.094**	**0.093**	**0.001**	0.127	0.127	0.127	0.000	0.121	0.144	0.132	0.005	0.095	0.098	0.097	0.001
WFG5	**0.140**	**0.150**	**0.146**	**0.003**	0.177	0.184	0.181	0.002	0.160	0.186	0.170	0.005	0.147	0.158	0.153	0.003
WFG6	0.156	0.173	0.166	0.005	0.155	0.205	0.175	0.012	0.159	0.196	0.177	0.009	**0.122**	**0.151**	**0.140**	**0.007**
WFG7	**0.092**	**0.094**	**0.094**	**0.001**	0.127	0.127	0.127	0.000	0.113	0.138	0.123	0.007	0.094	0.102	0.097	0.001
WFG8	**0.099**	**0.154**	**0.109**	**0.015**	0.189	0.194	0.192	0.001	0.244	0.274	0.256	0.008	0.161	0.166	0.163	0.001
WFG9	**0.099**	**0.210**	**0.118**	**0.036**	0.130	0.240	0.154	0.036	0.138	0.246	0.224	0.025	0.099	0.211	0.119	0.037
DTLZ1	0.014	0.014	0.014	0.000	0.014	0.014	0.014	0.000	0.017	0.020	0.018	0.001	**0.013**	**0.014**	**0.014**	**0.000**
DTLZ2	0.024	0.025	0.024	0.000	0.027	0.027	0.027	0.000	0.030	0.036	0.032	0.001	**0.023**	**0.024**	**0.023**	**0.000**
DTLZ3	0.024	0.025	0.024	0.000	0.027	0.027	0.027	0.000	0.027	0.032	0.030	0.001	**0.023**	**0.023**	**0.023**	**0.000**
DTLZ4	**0.024**	**0.025**	**0.024**	**0.000**	0.027	0.595	0.092	0.181	0.028	0.036	0.032	0.001	0.023	0.595	0.190	0.225
DTLZ5	**0.002**	**0.002**	**0.002**	**0.000**	0.003	0.003	0.003	0.000	0.003	0.003	0.003	0.000	0.002	0.002	0.002	0.000
DTLZ6	**0.002**	**0.002**	**0.002**	**0.000**	0.022	0.163	0.087	0.032	0.126	0.224	0.187	0.027	0.003	0.136	0.069	0.033
DTLZ7	**0.027**	**0.029**	**0.028**	**0.000**	0.045	0.045	0.045	0.000	0.038	0.052	0.044	0.003	0.060	0.087	0.079	0.008
UF8	**0.025**	**0.034**	**0.029**	**0.002**	0.048	0.365	0.069	0.051	0.093	0.220	0.178	0.031	0.027	0.159	0.033	0.022
UF9	**0.022**	**0.028**	**0.024**	**0.001**	0.041	0.151	0.086	0.049	0.106	0.314	0.139	0.049	0.025	0.137	0.094	0.053
UF10	**0.070**	**0.187**	**0.103**	**0.026**	0.163	0.565	0.294	0.125	0.198	0.658	0.261	0.080	0.159	0.553	0.257	0.131
Mean	0.054	0.074	0.059	0.005	0.078	0.170	0.099	0.026	0.097	0.166	0.121	0.015	0.066	0.150	0.093	0.028

test problems, whereas the second best-ranked algorithm (R2-EMOA) was in the group of winning methods for only 5 test problems. These conclusions are again quite similar to those drawn for the hypervolume in the main document.

References

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