R2-EMOA: Focused Multiobjective Search Using R2-Indicator-Based Selection

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R2-EMOA: Focused Multiobjective Search Using R2-Indicator-Based Selection[*]

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Abstract. An indicator-based evolutionary multiobjective optimization algorithm (EMOA) is introduced which incorporates the contribution to the unary R2-indicator as the secondary selection criterion. First experiments indicate that the R2-EMOA accurately approximates the Pareto front of the considered continuous multiobjective optimization problems. Furthermore, decision makers' preferences can be included by adjusting the weight vector distributions of the indicator which results in a focused search behavior.

Keywords: Multiobjective Optimization, Performance Assessment, EMOA, R2-Indicator, Indicator-Based Selection, Preferences

1 Introduction

Throughout this paper, we consider multiobjective optimization problems consisting of d objectives Y_j and objective functions $f_j : \mathbb{R}^n \to \mathbb{R}$ with $1 \leq j \leq d$. In the context of performance assessment of multiobjective optimizers, the (binary) R-indicator family was introduced by Hansen and Jaszkiewicz [5]. It is based on a set of utility functions. In total, three different variants were proposed which differ in the way the utilities are evaluated and combined – the ratio of one set being better than the other (R1), the mean difference in utilities (R2), or the mean relative difference in utilities (R3). In particular, the second variant R2 is one of the most recommended performance indicators [8] together with the hypervolume (HV, [9]) which directly measures the dominated objective hypervolume bounded by a reference point dominated by all solutions. Recently, we defined an equivalent unary version of this R2 indicator [3]. In case the standard weighted Tchebycheff utility function with ideal point $\mathbf{i}$ is used, it is defined as

$$\mathrm{R2}(A, \Lambda, \mathbf{i}) = \frac{1}{|\Lambda|} \sum_{\lambda \in \Lambda} \min_{a \in A} \left\{ \max_{j \in \{1, \ldots, d\}} \{\lambda_j |\mathbf{i}_j - a_j|\} \right\}$$

for a solution set A and a given set of weight vectors $\lambda = (\lambda_1, \ldots, \lambda_d) \in \Lambda$.

Theoretical and experimental comparisons to the HV for $d = 2$ revealed that, contrarily to common assumptions, the R2 indicator even has a stronger bias towards the center of the Pareto front than the HV [3]. Furthermore, it could be proven that for $d = 2$ the optimal placement of a point w.r.t. the R2-indicator solely depends on its two nearest neighbors and a subset of Λ. In [6], the influence of the R2-indicator parametrization on the optimal distribution of μ points on the true Pareto front (PF) regarding R2 was investigated. It was shown that this distribution heavily depends on the position of the ideal point, as well as on the domain and distribution of the weight vectors. Thus, preferences of the decision maker can be reflected by a specifically parametrized R2-indicator. In [1] a similar approach relying on linear utility functions was used to identify knees of Pareto fronts. In this paper, we will investigate whether the approximated optimal distributions of μ points regarding R2 based on different preference articulations [6] can be accurately reproduced by a greedy R2-EMOA.

2   R2-EMOA

The proposed R2-EMOA implements a steady state strategy based on the contribution to the unary R2-indicator (see Alg. 1).

1: draw multiset P with μ elements $\in \mathbb{R}^n$ at random
2: **repeat**
3:     generate offspring $z \in \mathbb{R}^n$ from P by variation
4:     $P = P \cup \{z\}$
5:     non-dominated sorting:
        build ranking $R_1, \ldots, R_h$ from P
6:     $\forall x \in R_h : r(x) = R2(P \setminus \{x\}; \Lambda; \mathbf{i})$
7:     $x^* = \mathsf{argmin}\{r(x) : x \in R_h\}$
8:     $P = P \setminus \{x^*\}$
9: **until** stopping criterion fulfilled

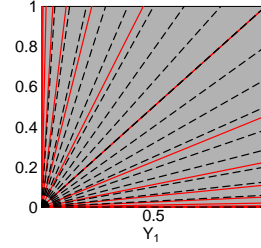Algorithm 1: Pseudo code of the R2-EMOA.

Fig. 1: $|\Lambda| = 19$ for $\gamma = 1$ (black dashed line) and $\gamma = 4$ (red solid line).

3   Experiments

Experiments were conducted to empirically show that the evolutionary procedure (selection pressure, variation) of the R2-EMOA is adequate to accurately approximate the R2-optimal distributions. This cannot be directly assumed, as the greedy strategy of the EMOA which only changes single solutions could be
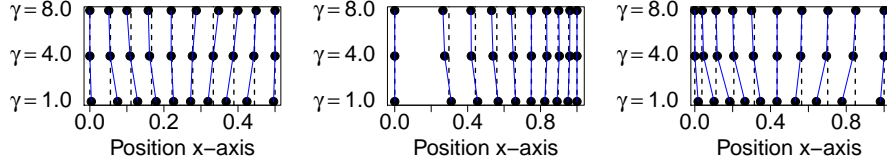
Fig. 2: Results of best R2-EMOA runs for increasing γ on DTLZ1 (left), DTLZ2 (middle) and ZDT1 (right). The movement of the x-axis positions for $\gamma \in \{1, 4, 8\}$ is shown. The optimal distributions regarding HV are reflected by dashed vertical lines.

stuck in local optima of the objective functions or in suboptimal distributions[4].

For the experiments, three bi-objective test functions with different problem characteristics were selected: ZDT1 (convex PF, n=30) [7], DTLZ1 (linear PF, n=6), and DTLZ2 (concave PF, n=11) [4]. On each function, ten independent runs were conducted using simulated binary crossover (SBX) and polynomial mutation ($p_c = 0.9$, $p_m = 1/n$, $\eta_c = 15$, $\eta_m = 20$), 150.000 function evaluations (FE), ideal point $\mathbf{i} = (0,0)'$, and 501 weight vectors. A population size of μ=10 was chosen in order to allow a clear visualization of the results and the comparison to the reference distributions of [6].

The influence of restricted weight vector domains and altered weight vector distributions on the outcome of the R2-EMOA results is considered. Therefore, Alg. 1 of [6] was used to generate weight vector distributions with increasing focus on the extremes of the weight vector domain (see Fig. 1). This is reflected by an increased value of γ while $\gamma = 1$ corresponds to equally distributed weight vectors in $[0,1]^2$. The R2-EMOA is able to accurately approximate the optimal distributions. With increasing γ, the points tend to drift towards the extremes of the front (Fig. 2) which is perfectly in line with the results of [6]. Individually for each problem, distributions close to the optimal ones regarding HV can be obtained for a specific choice of γ.

Moreover, the first component of the weight vector domain was restricted to one or two intervals within $[0,1]$. From [6] it is known that in this setting the optimal solutions regarding R2 lie within the target cone defined by the two outmost weight vectors of the interval(s). This is reflected by the respective R2-EMOA results (Fig. 3).

Fig. 4 relates the final R2 values of all experiments to the approximated optimal 10-distributions regarding R2 [6]. It can be observed that the variance of the R2-EMOA results is small. Sometimes even slightly better approximations of the optimal distributions are obtained than in [6]. This is rather surprising as these reference solutions were determined based on a global optimization on the front. The evolutionary mechanism and the greedy selection seem to provide efficient heuristics for the considered class of problems.

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[4] For the HV indicator, it has been, for example, theoretically proven that such a greedy strategy cannot always find a solution set with optimal HV value [10, 2].
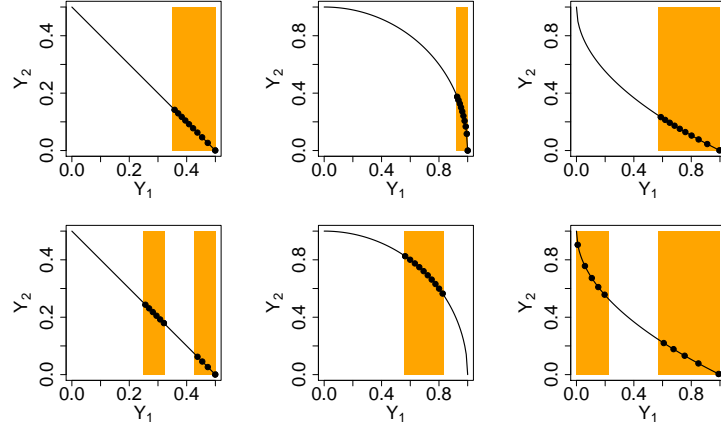
Fig. 3: Results of the best R2-EMOA runs (black dots) with restricted weight vector domains for DTLZ1 (left), DTLZ2 (middle) and ZDT1 (right). The areas within the intersections with the true PF (solid line) are highlighted.
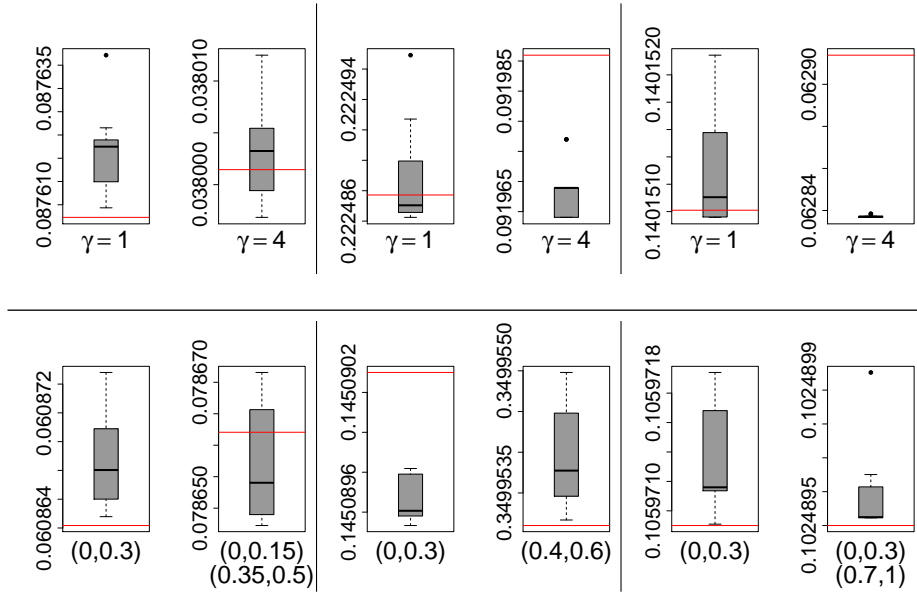

Fig. 4: Boxplots of R2 values at final R2-EMOA generation for DTLZ1 (left), DTLZ2 (middle) and ZDT1 (right) for altered weight distributions with parameter γ (top) or restricted weight space (bottom) corresponding to Fig. 3. The R2 value of the approximated optimal 10-distribution of R2 in [6] is visualized by a red horizontal line.

4   Conclusions and Outlook

First experiments show very promising results of the R2-EMOA regarding solution quality and the possibility of incorporating preferences of the decision maker. In future studies, the R2-EMOA will be theoretically and empirically compared to other EMOA optimizing the R2-indicator, such as MOEA/D and MSOPS. Furthermore, theoretical derivations of optimal distributions of μ solutions regarding R2 are needed.

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