# A Multi-objective Evolutionary Algorithm Based on an Enhanced Inverted Generational Distance Metric

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***Abstract*—As a pivotal component in multi-objective evolutionary algorithms (MOEAs), the environmental selection determines the quality of candidate solutions to survive at each generation. In practice, different environmental selection strategies can be based on different selection criteria, where the performance metrics (or indicators) are shown to be among the most effective ones. This paper proposes an MOEA whose environmental selection is based on an enhanced inverted generational distance metric that is able to detect noncontributing solutions (termed IGD-NS), thereby considerably accelerating the convergence of the evolutionary search. Experimental results on ZDT and DTLZ test suites demonstrate the competitive performance of the proposed MOEA/IGD-NS in comparison with some representative MOEAs.**

## I. Introduction

Many real-world optimization problems involve multiple conflicting objectives, known as multi-objective optimization problems (MOPs). Since the objectives are in conflict with each other, there does not exist one single solution that is able to optimize all objectives simultaneously; instead, a set of solutions, known as Pareto optimal solutions, can be obtained to present the trade-offs between different objectives. In the past two decades, evolutionary algorithms have shown its great advantages in solving MOPs [1], and correspondingly, a number of multi-objective evolutionary algorithms (MOEAs) have been developed for handling various MOPs, such as NSGA-II [2], SPEA2 [3], PESA-II [4], IBEA [5], MOEA/D [6] and IM-MOEA [7] for solving MOPs with two or three objectives, and HypE [8], GrEA [9], NSGA-III [10], Two_Arch2 [11] and KnEA [12] for solving MOPs with more than three objectives, a.k.a the many-objective optimization problems (MaOPs).

Most MOEAs consist of several basic components such as reproduction, mating selection and environmental selection. Among these components, the environmental selection plays a pivotal role as it determines the quality of the candidate solutions that are able to survive at each generation. Different environmental selection strategies are designed based on different selection criteria, e.g., Pareto dominance [2], [3], weighted aggregation [13], and performance metrics (or indicators) [5], [8]. Among various selection criteria, though the performance metrics were originally developed to evaluate the quality of solution set obtained by different algorithms, e.g., inverted generational distance (IGD) [14], generational distance (GD) [15] and hypervolume (HV) [16], they have also been successfully adopted as criteria for the environmental selection in some prevalent MOEAs such as the indicator-based evolutionary algorithm (IBEA) [5], the $S$ metric selection evolutionary multi-objective algorithm (SMS-EMOA) [17] and the hypervolume estimation algorithm (HypE) [8]. Following the line of existing performance metric based MOEAs, this paper proposes an enhanced inverted generational distance metric, IGD with noncontributing solution detection (IGD-NS), together with an MOEA based on it. The major contributions of this work can be summarized as follows:

(1) The proposed performance metric IGD-NS is not only able to evaluate the quality of a solution set in terms of both convergence and diversity, but also is able to distinguish the noncontributing solutions from each non-dominated front. It is believed that, compared to other solutions in the non-dominated solution set, the noncontributing solutions will hinder the evolution of a population towards the Pareto optimal front once they are selected as individuals of the population. Moreover, due to the high computational efficiency of the proposed metric, it is well suited to be used as a criterion for the environmental selection in MOEAs.

(2) An MOEA based on the proposed performance metric is developed, where the candidate solutions in the combined population at each generation are selected according to the metric values. To be specific, the selection procedure removes the candidate solutions with the minimal contributions to the metric values one by one until the population size is reached.

The rest of the paper is organized as follows. In Section II, we briefly review existing performance metrics together with some representative performance metric based MOEAs. The details of the proposed performance metric and the developed MOEA based on it are presented in Section III. In Section IV, the performance of the proposed MOEA is verified by comparing it with four representative MOEAs. Finally, the conclusion and future work are presented in Section V.

## II. Related Work

In this section, we first recall some existing performance metrics for evaluating the performance of MOEAs, and

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then present several representative performance metric based MOEAs.

### A. Existing Performance Metrics

There are a large number of performance metrics for evaluating the quality of solution set obtained by MOEAs, which can be roughly divided into the following two categories.

For the first category, priori knowledge about the Pareto optimal front is required in the metric calculations. Two representative metrics of this category are the inverted generational distance (IGD) [14] and the generational distance (GD) [15], both of which require a set of uniformly distributed reference points sampled from the Pareto optimal front as priori knowledge.

The definition of IGD is as follows [14]:

$$IGD(P, P^*) = \frac{\sum_{x \in P^*} \min_{y \in P} dis(x, y)}{|P^*|}, \tag{1}$$

where $P$ is the objective values of a set of non-dominated solutions obtained by any MOEA, $P^*$ is a set of uniformly distributed reference points sampled from the Pareto optimal front, and $dis(x, y)$ represents the Euclidean distance between points $x$ and $y$. IGD calculates an average minimum distance from each point in $P^*$ to those in $P$, which measures both convergence and diversity of solution set $P$. A smaller IGD value indicates a better convergence as well as diversity of $P$, and specially, if $IGD(P, P^*) = 0$, it means that $P^*$ is a subset of $P$.

The definition of GD is similar to IGD, which can be described as follows [15]:

$$GD(P, P^*) = \frac{\sqrt{\sum_{y \in P} \min_{x \in P^*} dis(x, y)^2}}{|P|}. \tag{2}$$

It can be seen that GD calculates an average minimum distance from each point in $P$ to those in $P^*$, which measures the convergence of $P$, and a smaller GD value indicates a better convergence of $P$. However, GD is unable to evaluate the diversity of $P$. For example in the most extreme case, even if all the points in $P$ are converged to the same point in $P^*$, the minimal GD value, i.e., $GD(P, P^*) = 0$, can also be obtained.

It is worth noting that there is another recently proposed performance metric called $\Delta_p$, $p \geq 1$, which is based on the definitions of both IGD and GD [18]. The $\Delta_p$ is defined as follows:

$$\Delta_p(P, P^*) = \max(GD_p(P, P^*), IGD_p(P, P^*)) \tag{3}$$

where

$$GD_p(P, P^*) = (\frac{1}{|P|} \sum_{y \in P} \min_{x \in P^*} dis(x, y)^p)^{1/p},$$

$$IGD_p(P, P^*) = (\frac{1}{|P^*|} \sum_{x \in P^*} \min_{y \in P} dis(x, y)^p)^{1/p}.$$

The $\Delta_p$ adopts the averaged Hausdorff distance as a performance metric, which can be seen as a modified hybridization of IGD and GD. Since $\Delta_p$ has taken into consideration the average convergence of all the points in $P$, it is not only able to evaluate the convergence and diversity of the solution set, but is also able to distinguish the noncontributing solutions to some extent.

For the second category, there is no priori knowledge about the Pareto optimal front required in the metric calculations. The most representative metric of this type is the hypervolume (HV) indicator [16].

Given a solution set $P$, the HV value of $P$ is defined as the area covered by $P$ with respect to a set of predefined reference points $R$ in the objective space [19]:

$$HV(P, R) = \lambda(H(P, R)) \tag{4}$$

where

$$H(P, R) = \{z \in Z | \exists x \in P, \exists r \in R : f(x) \leq z \leq r\},$$

and $\lambda$ is the Lebesgue measure with

$$\lambda(H(P, R)) = \int_{\mathbb{R}^n} 1_{H(P,R)}(z) dz,$$

where $1_{H(P,R)}$ is the characteristic function of $H(P, R)$. As can be seen from the definition, the calculation of HV only requires a set of predefined reference points $R$. A higher HV value indicates the better convergence as well as diversity of the points in $P$. However, although the calculation of HV does not require any priori knowledge about the Pareto optimal front, the computational complexity considerably increases with the number of objectives [8].

The performance metrics mentioned above are all originally developed for evaluating the quality of a solution set obtained by MOEAs. Some of these metrics have also been further employed as the criteria for environmental selection in MOEAs. In the next subsection, we will briefly review some representative performance metric based MOEAs.

### B. Existing Performance Metric Based MOEAs

Among many others, the IBEA [5], the SMS-EMOA [17], the HypE [8] and the AS-MOEA [20] are four representative performance metric based MOEAs.

The IBEA [5] is a classical performance metric based MOEA, which was firstly developed by Eckart and Künzli in 2004. The main idea of IBEA is to first define the optimization goal in terms of a binary performance metric and then use it as the criterion for environmental selection in an MOEA. Since IBEA is a general framework, it can be embedded with any other performance metric as well. Experimental results on continuous and discrete benchmark problems demonstrate that IBEA outperforms NSGA-II and SPEA2.

The SMS-EMOA is another representative performance metric based MOEA proposed in [17], where the HV was originally adopted as the criterion for environmental selection. In SMS-EMOA, only one offspring was generated at each generation, and the solution with the least contribution to the HV value of the population is discarded. SMS-EMOA shows promising performance on MOPs with two or three objectives, but it is difficult to apply the algorithm for solving MOPs with more than three objectives due to the considerably increased time complexity of HV calculation.

The HypE, as another HV based MOEA, has been specifically tailored for solving MOPs with more than three objectives [8]. In HypE, a more efficient method has been suggested to reduce the computational cost of HV calculations. To be specific, instead of calculating the exact HV values, the Monte Carlo simulation has been adopted to estimate the approximate HV values. In the environmental selection of HypE, the population is first divided into several fronts with non-dominated sorting [21], [22], then the solutions on the last front are distinguished by their contributions to the HV values of the population.

The AS-MOEA [20] is developed on the basis of the performance metric $\Delta_1$. This algorithm applies $\Delta_1$ to distinguish non-dominated solutions according to their contributions to the $\Delta_1$ values. Note however, that the calculation of $\Delta_1$ requires a set of predefined reference points. Experimental results show that AS-MOEA is able to obtain solutions according to the predefined reference set.

Note that, very recently a GD based MOEA, termed GD-MOEA, was suggested in [23]. Since GD can not maintain the diversity in the population, a strategy that relied on Euclidean distance was used in GD-MOEA. A new version of GD-MOEA that incorporated a selection mechanism based on $\varepsilon$-dominance, called GDE-MOEA, was proposed in [24], which achieved better results than GD-MOEA on most of the considered test instances.

## III. The Proposed Performance Metric and Algorithm

This section first presents the proposed performance metric IGD-NS. Then, the details of the MOEA based on the proposed metric are given.

### A. The Proposed Performance Metric IGD-NS

In the calculation of IGD, it is often observed that some non-dominated solutions are always ignored due to the fact that they are not the nearest neighbor of any reference point uniformly selected from the Pareto optimal front for calculating IGD. This means that these solutions in non-dominated solution set do not have any contribution to the IGD value of the set, thereby being less important than other non-dominated solutions in the set for approximating the Pareto optimal front. For this reason, we called these solutions as noncontributing solutions in non-dominated solution set. Specifically, a noncontributing solution is defined as follows.

Solution $y'$ is considered to be a noncontributing solution in non-dominated solution set $P$ for given $P^*$ if the following condition is satisfied:

$$\nexists x \in P^* : dis(x, y') = \min_{y \in P} dis(x, y) \tag{5}$$

where $P^*$ is a set of reference points that are uniformly sampled on the Pareto optimal front. It can be learnt from the above equation that, the noncontributing solutions are the solutions which are not the nearest neighbor of any point in $P^*$.

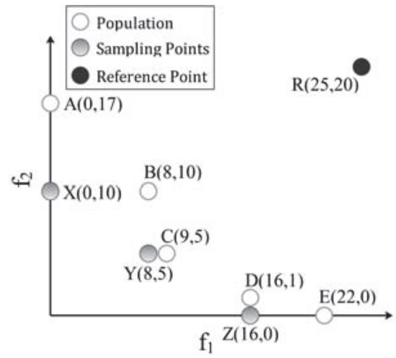


Fig. 1. An example of a 2-objective minimization problem. The population contains five solutions $A$, $B$, $C$, $D$ and $E$. $X$, $Y$ and $Z$ are points in a reference set for the calculations of IGD, GD, $\Delta_1$ and IGD-NS, and $R$ is a reference point for the calculation of HV. Four out of the five solutions are to be selected for the next generations.

By taking noncontributing solutions into considerations, the proposed performance metric, namely IGD with noncontributing solution detection (IGD-NS), is defined as follows:

$$\texttt{IGD-NS}(P, P^*) = \sum_{x \in P^*} \min_{y \in P} dis(x, y) + \sum_{y' \in P'} \min_{x \in P^*} dis(x, y') \tag{6}$$

where $P'$ is the set of noncontributing solutions in population $P$ as defined by (5). The first term in (6) is similar to IGD, which controls the diversity and convergence of $P$; while the second term in (6) is the sum of the minimum distance from each noncontributing solution to the points in $P^*$, which minimizes the number of noncontributing solutions in $P'$. As a result, a small (good) IGD-NS metric value can be obtained if and only if the following two conditions are satisfied: first, the population has good convergence as well as diversity; second, the population contains as few noncontributing solutions as possible.

In the following subsection, an illustrative example is given to show the advantage of IGD-NS over some other representative performance metrics.

### B. Discussions

As shown in Fig. 1, let us consider a minimization problem with two objectives, where the Pareto front is the line defined by $5x + 8y = 80$ in the first quadrant, and $X(0, 10)$, $Y(8, 5)$ and $Z(16, 0)$ are three points sampled on it. The current population contains five solutions, i.e. $A(0, 17)$, $B(8, 10)$, $C(9, 5)$, $D(16, 1)$ and $E(22, 0)$, four of which are to be selected for next generation. In order to eliminate one solution from the five solutions, one of the five metrics (IGD, GD, $\Delta_1$, HV and IGD-NS) is adopted to detect the worst one. In the calculations of the metrics, $X$, $Y$ and $Z$ are used as the reference set for IGD, GD, $\Delta_1$ and IGD-NS, and $R(25, 20)$ is used as the reference point for HV. Table I lists the IGD, GD, $\Delta_1$, HV and IGD-NS values of the population after eliminating each solution. Note that, for IGD, GD, $\Delta_1$ and

TABLE I
THE METRIC VALUES OF IGD, GD, $\Delta_1$, HV AND IGD-NS OF THE POPULATION AFTER DELETING EACH SOLUTION.

| Metrics | Eliminated solutions | | | | | Worst |
|---|---|---|---|---|---|---|
| | A | B | C | D | E | |
| IGD | 10/3 | 9/3 | 13/3 | 14/3 | 9/3 | B or E |
| GD | $\sqrt{63}$ | $\sqrt{87}$ | $\sqrt{111}$ | $\sqrt{111}$ | $\sqrt{76}$ | A |
| $\Delta_1$ | 10/3 | 15/4 | 19/4 | 19/4 | 14/4 | A |
| HV | 289 | 306 | 278 | 289 | 310 | E |
| IGD-NS | 16 | 15 | 19 | 19 | 14 | E |

**Algorithm 1** General Framework of MOEA/IGD-NS

**Input:** $N$ (population size), $NA$ (archive size)
**Output:** $P$ (final population)

```
1: P ← RandomInitialize(N)
2: A ← UpdateArchive(P, NA)
3: while termination criterion not fulfilled do
4:     P ← P ∪ Variation(P, N)
5:     A ← UpdateArchive(A ∪ P, NA)
6:     P ← EnvironmentalSelection(P, A, N)
7: end while
8: return P
```

IGD-NS, a smaller metric value indicates a better quality of the population, while in the case of HV, it is the larger the better.

According to the definition in (5), solutions $B$ and $E$ are identified as noncontributing solutions in the proposed IGD-NS metric, since solutions $A$, $C$ and $D$ are in correspondence with reference points $X$, $Y$ and $Z$, respectively. There are several observations that can be made from the results in Table I. First, if GD or $\Delta_1$ is employed as the selection criterion, solution $A$, which has the worst metric value, will be eliminated from the population. However, since solution $A$ is the only solution around reference point $X$, eliminating it will lead to a poor diversity. Second, if IGD is employed, either solution $B$ or $E$ will be eliminated. Hence, solution $E$ can possibly not be distinguished by IGD, despite that it has worse convergence than solution $B$ according to the distances to the Pareto optimal front. On the contrary, the proposed performance metric IGD-NS (as well as HV) is able to detect and eliminate solution $E$, which is the noncontributing solution with the worst convergence.

In addition to the observations above, some empirical comparisons between IGD-NS, IGD, GD and $\Delta_1$ will be carried out in Section IV-A, which further confirms the advantage of IGD-NS over the other three performance metrics. It is worth noting that, although HV shows similar performance as IGD-NS, its computational complexity is much higher.

In the following subsection, the details of the IGD-NS metric based MOEA will be given.

### C. The IGD-NS Metric Based MOEA

The framework of the IGD-NS metric based MOEA, denoted as MOEA/IGD-NS hereafter, is shown in Algorithm 1.

**Algorithm 2** $UpdateArchive(P, NA)$

**Input:** $P$ (current population), $NA$ (archive size)
**Output:** $A$ (new archive)

```
1: NDS ← NondominatedSolution(P)
2: A ← ExtremeSolution(NDS)
3: if |A| > NA then
4:     Randomly delete |A| − NA solutions from A
5: else
6:     while |A| < NA and |A| < |NDS| do
7:         p ← arg max_{p∈NDS\A} min_{q∈A} arccosine(p, q)
8:         A ← A ∪ {p}
9:     end while
10: end if
11: return A
```

The main idea is to iteratively eliminate the candidate solution that has the least contribution to the IGD-NS metric value in the population one by one until the number of remaining solutions reaches the predefined population size. On the other hand, since the Pareto optimal front is not known a priori, it is not possible to sample points on it as references. Instead, an external archive, which stores a set of non-dominated solutions with the best convergence and diversity found so far, is used as the reference set for the IGD-NS calculations.

As shown in Algorithm 1, to begin with, population $P$ is randomly initialized with a number of $N$ candidate solutions (Line 1, Algorithm 1). Then $P$ is used to update the external archive $A$ (Line 2, Algorithm 1), where the details can be found in Algorithm 2.

To update the external archive $A$, all the non-dominated solutions in $P$ are first copied to $NDS$ (Line 1, Algorithm 2). Then the archive $A$ is filled with the extreme solutions[1] in $NDS$ (Line 2, Algorithm 2). Afterwards, if the size of $A$ is larger than the archive size $NA$, a number of $NA - |A|$ solutions are randomly eliminated (Line 4, Algorithm 2); otherwise, $A$ is filled up by solution $p$ one by one until $|A| = NA$ is reached (Line 7–8, Algorithm 2), where solution $p$ is the one in $NDS \backslash A$ which has the maximum value of $\min_{q \in A} arccosine(p, q)$, with $arccosine(p, q)$ indicating the the acute angle between $p$ and $q$. Such an archive update strategy guarantees that the solutions maintained in $A$ are always non-dominated solutions with good diversity.

After the update of archive $A$, the evolutionary process repeats until the termination condition is met (Line 3, Algorithm 1). Firstly, a number of $N$ offspring solutions are generated and merged into the parent population $P$ (Line 4, Algorithm 1). Then archive $A$ is updated with $A \cup P$ using the strategy as described above (Line 5, Algorithm 1). Afterwards, the environmental selection is performed on the combined population, where the pseudocode of environmental selection can be found in Algorithm 3.

The environmental selection starts with the non-dominated

[1]Extreme solutions refer to the solutions that have the maximum values on at least one objective.

**Algorithm 3** $EnvirnonmentalSelection(P, A, N)$

**Input:** $P$ (combined population), $A$ (archive), $N$ (population size)

**Output:** $P$ (population for next generation)

1: $F \leftarrow NondominatedSort(P)$
2: $P \leftarrow F_1 \cup F_2 \cup \ldots \cup F_k$ /*$k$ is the maximal value such that $|F_1 \cup F_2 \cup \ldots \cup F_k| < N$*/
3: **while** $|F_{k+1}| > N - |P|$ **do**
4: Find $y'$ in $F_{k+1}$ by Equation (7)
5: $F_{k+1} \leftarrow F_{k+1} \setminus \{y'\}$
6: **end while**
7: $P \leftarrow P \cup F_{k+1}$
8: **return** $P$

sorting, where the combined population $P$ is divided into several non-dominated fronts $F_1, F_2, \ldots,$ (Line 1, Algorithm 3). In order to improve the efficiency of the non-dominated sorting, the efficient non-dominated sorting (ENS) [21] is employed here. Afterwards, the solutions in the first $k$ fronts are selected, where $k$ is the maximum value satisfying $|F_1 \cup F_2 \cup \ldots \cup F_k| < N$ (Line 2, Algorithm 3). Then, the solutions in $F_{k+1}$ is eliminated one by one until $|F_{k+1}| = N - |P|$ is reached (Line 3–6, Algorithm 3), where the solution to be eliminated each time, denoted as $y'$, satisfies:

$$y' = \underset{y \in F_{k+1}}{\operatorname{argmin}} \, \texttt{IGD-NS}(F_{k+1} \backslash \{y\}, A). \tag{7}$$

Finally, all the solutions remained in $F_{k+1}$ are add to $P$ (Line 7, Algorithm 3).

It can be seen from the procedure of Algorithm 3 that, the framework of the proposed environmental selection strategy is similar to that in NSGA-II, where the first selection is still dominance based, while in the secondary selection, the propose IGD-NS metric is employed to distinguish the solutions in the last non-dominated front. As a consequence, the solutions which have the least contribution to the IGD-NS metric value will be eliminated, according to the definition in (7).

## IV. Experimental Results

In this section, firstly, IGD-NS is compared with IGD, GD and $\Delta_1$ under the same framework of MOEA/IGD-NS. Then the proposed MOEA/IGD-NS is compared with NSGA-II [2], MOEA/D [6], HypE [8] and IBEA [5]. The empirical comparisons are conducted on the ZDT [25] and DTLZ [26] test suites. The details of the experimental settings are given as follows.

*Population sizing and termination condition:* Due to the fact that the population size of MOEA/D equals to the number of uniformly weight vectors, it is fixed to 100 on test instances with 2 and 105 on those with 3 objectives. The population size of NSGA-II, HypE, IBEA and MOEA/IGD-NS is set to the same as that of MOEA/D on all test instances. The maximum number of iterations is adopted as the termination condition, namely, 300 for ZDT1–ZDT6, 500 for DTLZ1 and DTLZ3, and 200 for DTLZ2, DTLZ4–DTLZ7.

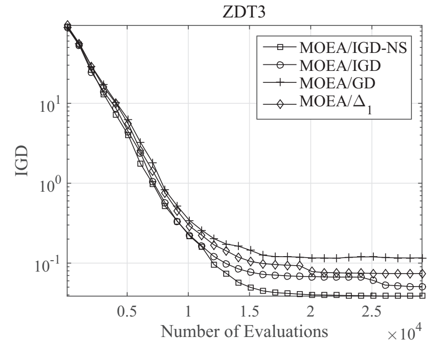


Fig. 2. The convergence profiles of IGD obtained by MOEA/IGD-NS, MOEA/IGD, MOEA/GD and MOEA/$\Delta_1$ on ZDT3 problem.

*Crossover and mutation:* The one-point crossover and bit-wise mutation are applied in ZDT5, and the simulated binary crossover (SBX) [27] and polynomial mutation [28] are applied in all the other test problems. The probabilities of crossover and mutation are set to 1.0 and $1/D$ ($D$ denotes the number of decision variables), respectively. Both the distribution indexes of SBX and polynomial mutation are set to 20.

*Settings of problems and algorithms:* The parameter settings of ZDT and DTLZ are taken from [25] and [26], respectively. The parameters of MOEA/D are set to the recommended values in MOEA/D and the Tchebycheff approach is employed as the aggregation function [6]. The parameter setting of HypE is taken from [12]. The fitness scaling factor $\kappa$ in IBEA is set to 0.05. The archive size $NA$ in MOEA/IGD-NS is set to 200 for test problems with 2 objectives, and 500 for test problems with more than 2 objectives.

*Quality evaluating:* The performance metrics of IGD and HV are applied for evaluating the results. About 5,000 uniformly distributed points are sampled from the Pareto optimal front of each problem for the calculation of IGD. Before calculating the HV value, each objective of the test problems has been normalized between the interval $[0, 1]$, and the reference points $(1, 1)$ and $(1, 1, 1)$ are used for 2-objective and 3-objective test problems, respectively. 30 independent runs are performed for each test instance, and the mean and standard deviation of the metric values are recorded. Note that for a fair comparison, the final population is used for calculating the performance metric value of MOEA/IGD-NS and the archive is only used in the calculations of IGD-NS.

### A. Comparisons Between IGD-NS and Other Performance Metrics

In this experiment, the proposed IGD-NS is substituted with the performance metrics of IGD, GD and $\Delta_1$ in the framework of MOEA/IGD-NS, respectively. Then these three algorithms, namely MOEA/IGD, MOEA/GD and MOEA/$\Delta_1$, are compared with the original MOEA/IGD-NS on a typical

TABLE II

IGD Metric Values Obtained by the Five Algorithms on ZDT and DTLZ Test Suites, Where the Best Mean for Each Test Problem is Highlighted with a Gray Background.

| Problem | Obj. | NSGA-II | MOEA/D | HypE | IBEA | MOEA/IGD-NS |
|---|---|---|---|---|---|---|
| ZDT1 | 2 | 4.6345e-3(1.85e-4)− | 4.9935e-3(2.44e-3)− | 3.7223e-3(2.40e-5)+ | 4.5117e-3(1.58e-4)− | 4.0441e-3(4.66e-5) |
| ZDT2 | | 4.7483e-3(1.70e-4)− | 4.6502e-3(1.27e-3)≈ | 5.8180e-3(5.65e-3)− | 9.3737e-3(8.19e-4)− | 4.1288e-3(3.72e-5) |
| ZDT3 | | 5.8624e-2(4.55e-2)− | 2.2904e-2(2.10e-2)≈ | 3.6597e-2(4.46e-2)− | 8.3026e-2(6.97e-2)− | 3.4327e-2(3.51e-2) |
| ZDT4 | | 5.3257e-3(6.88e-4)− | 7.9198e-3(1.96e-3)− | 4.1860e-3(6.48e-4)+ | 8.4851e-2(8.91e-2)− | 5.0074e-3(9.11e-4) |
| ZDT5 | | 4.6910e-1(7.07e-2)≈ | 1.7603e+0(4.19e-1)− | 4.3335e-1(1.26e-1)≈ | 2.0146e+0(6.42e-1)− | 4.3060e-1(1.17e-1) |
| ZDT6 | | 4.4802e-2(8.58e-5)+ | 4.4542e-2(8.90e-5)+ | 4.4387e-2(1.20e-5)+ | 4.5812e-2(7.74e-5)+ | 4.5890e-2(6.10e-4) |
| DTLZ1 | 3 | 2.6976e-2(1.01e-3)− | 2.8439e-2(8.79e-5)− | 9.8189e-2(1.42e-2)− | 1.6663e-1(2.57e-2)− | 1.9222e-2(5.43e-4) |
| DTLZ2 | | 6.8753e-2(2.17e-3)− | 6.8407e-2(4.76e-4)− | 1.5771e-1(1.95e-2)− | 8.0996e-2(2.92e-3)− | 5.0494e-2(3.42e-4) |
| DTLZ3 | | 7.3948e-2(1.20e-2)− | 7.0255e-2(4.04e-3)− | 2.7724e-1(2.15e-2)− | 4.7830e-1(4.08e-3)− | 5.5467e-2(5.05e-3) |
| DTLZ4 | | 9.6676e-2(1.60e-1)+ | 4.8002e-1(3.35e-1)− | 3.8046e-1(2.98e-1)− | 1.0866e-1(1.58e-1)− | 1.1629e-1(1.69e-1) |
| DTLZ5 | | 5.7080e-3(2.64e-4)− | 1.1857e-2(3.80e-5)− | 1.6353e-2(1.84e-3)− | 1.3296e-2(1.02e-3)− | 4.0879e-3(4.42e-5) |
| DTLZ6 | | 5.1418e-3(1.53e-4)− | 1.1822e-2(9.58e-5)− | 2.3459e-1(7.01e-2)− | 2.2459e-2(4.22e-3)− | 3.9456e-3(2.83e-5) |
| DTLZ7 | | 7.6885e-2(3.34e-3)− | 2.5349e-1(8.44e-2)− | 4.3127e-1(2.70e-1)− | 9.1084e-2(7.59e-2)− | 6.7154e-2(5.45e-2) |
| +/ − / ≈ | | 2/10/1 | 1/10/2 | 3/9/1 | 1/12/0 | |

'+', '−' and '≈' indicate that the result is significantly better, significantly worse and statistically similar to that obtained by MOEA/IGD-NS, respectively.

TABLE III

HV Metric Values Obtained by the Five Algorithms on ZDT and DTLZ Test Suites, Where the Best Mean for Each Test Problem is Highlighted with a Gray Background.

| Problem | Obj. | NSGA-II | MOEA/D | HypE | IBEA | MOEA/IGD-NS |
|---|---|---|---|---|---|---|
| ZDT1 | 2 | 6.6054e-1(2.84e-4)− | 6.6035e-1(1.34e-3)≈ | 6.6187e-1(5.95e-5)+ | 6.6122e-1(1.56e-4)+ | 6.6085e-1(3.29e-4) |
| ZDT2 | | 3.2734e-1(2.71e-4)+ | 3.2698e-1(1.90e-3)≈ | 3.2752e-1(4.75e-3)+ | 3.2661e-1(3.40e-4)− | 3.2705e-1(2.37e-3) |
| ZDT3 | | 8.7484e-1(4.41e-2)≈ | 8.9602e-1(2.89e-2)≈ | 8.9149e-1(3.95e-2)+ | 8.5399e-1(8.26e-2)− | 8.8151e-1(6.06e-2) |
| ZDT4 | | 6.5804e-1(1.57e-3)≈ | 6.5365e-1(3.09e-3)− | 6.6011e-1(1.46e-3)+ | 6.1052e-1(4.75e-2)− | 6.5837e-1(2.05e-3) |
| ZDT5 | | 7.9668e-1(1.18e-2)≈ | 7.6169e-1(1.34e-2)− | 8.0183e-1(1.46e-2)≈ | 7.9038e-1(1.00e-2)− | 8.0320e-1(1.36e-2) |
| ZDT6 | | 3.2143e-1(3.56e-4)+ | 3.2174e-1(3.80e-4)+ | 3.2244e-1(1.88e-4)+ | 3.2056e-1(1.03e-4)− | 3.2112e-1(4.84e-4) |
| DTLZ1 | 3 | 7.6290e-1(5.68e-3)− | 7.4522e-1(7.79e-4)− | 5.8028e-1(4.50e-2)− | 3.0054e-1( 8.37e-2)− | 7.7283e-1(3.96e-3) |
| DTLZ2 | | 3.7763e-1(3.63e-3)− | 3.7982e-1(1.48e-3)− | 3.6228e-1(1.35e-2)− | 4.1097e-1(1.47e-3)+ | 4.0195e-1(1.93e-3) |
| DTLZ3 | | 3.5596e-1(2.72e-2)− | 3.6810e-1(8.78e-3)− | 9.9858e-2(2.31e-2)− | 8.3501e-5(4.57e-4)− | 3.8107e-1(1.60e-2) |
| DTLZ4 | | 3.6794e-1(6.97e-2)− | 2.1307e-1(1.47e-1)− | 2.6854e-1(1.36e-1)− | 3.9728e-1(7.50e-2)+ | 3.7997e-1(6.76e-2) |
| DTLZ5 | | 9.2211e-2(2.20e-4)− | 8.8992e-2(2.35e-5)− | 8.8182e-2(6.68e-4)− | 9.1575e-2(2.74e-4)− | 9.3095e-2(8.57e-5) |
| DTLZ6 | | 9.2952e-2(1.49e-4)− | 8.9022e-2(3.44e-5)− | 8.3446e-3(8.97e-3)− | 8.8971e-2(1.42e-3)− | 9.3425e-2(4.33e-5) |
| DTLZ7 | | 1.5619e-1(2.21e-3)≈ | 1.1958e-1(9.37e-3)− | 1.1764e-1(9.99e-3)− | 1.6662e-1(6.68e-3)+ | 1.5656e-1( 3.74e-3) |
| +/ − / ≈ | | 2/7/4 | 1/9/3 | 5/7/1 | 4/9/0 | |

'+', '−' and '≈' indicate that the result is significantly better, significantly worse and statistically similar to that obtained by MOEA/IGD-NS, respectively.

MOP, ZDT3, which is a 2-objective MOP with a Pareto optimal front containing five discontinues regions.

The convergence profiles of IGD values obtained by the four algorithms on ZDT3 are plotted in Fig. 2. It can be seen from Fig. 2 that MOEA/IGD-NS performs better than the other algorithms on ZDT3. This is mainly due to the fact that IGD-NS is not only capable of detecting the solutions with the worst convergence in the population, but is also able to keep a satisfactory diversity in the population, which confirms the conclusions in Section III-B. The MOEA/IGD-NS has a similar performance with those using GD, IGD and $\Delta$ in the early stages of evolution, since the combined population consisting of parent and offspring populations contains less non-dominated solutions than the population size. The result in Fig. 2 indicates that the proposed IGD-NS is a promising environmental selection strategy in MOEAs, especially for the populations that consist of only one or two fronts.

### B. Comparisons Between MOEA/IGD-NS and Other MOEAs

The mean and standard deviation of the IGD and HV values obtained by NSGA-II, MOEA/D, HypE, IBEA and the proposed MOEA/IGD-NS on ZDT and DTLZ test suites are

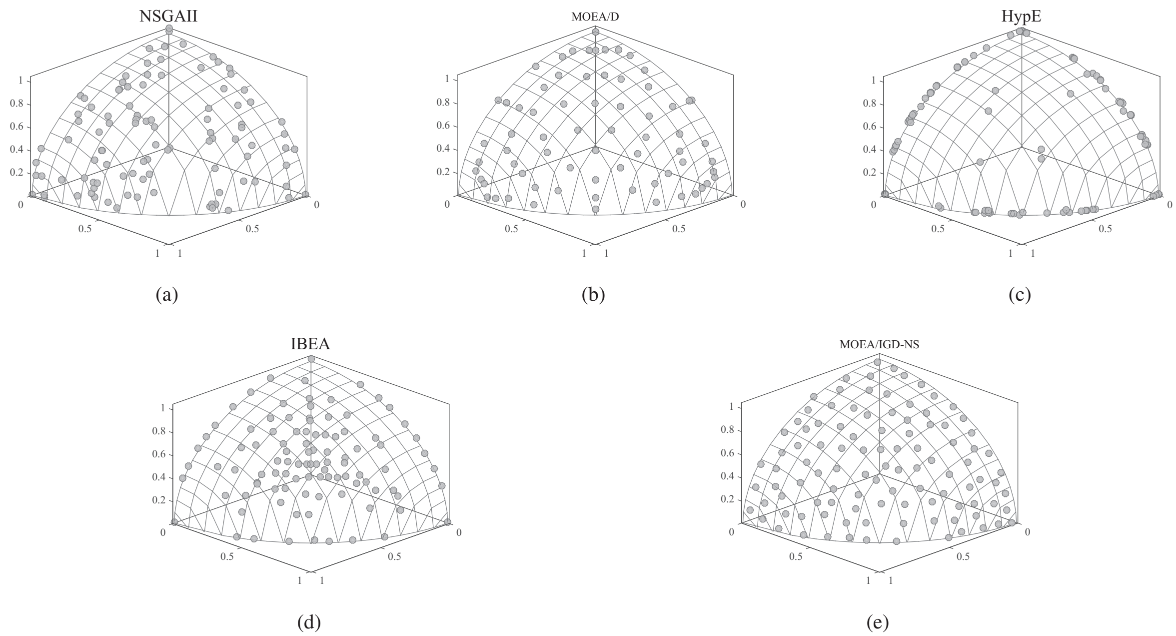


Fig. 3. The final population in the run associated with the lowest IGD metric values obtained by the five algorithms on 3-objective DTLZ2 problem.

given in Tables II and III, respectively. The Wilcoxon rank sum test is adopted at a significance level of 0.05, where the symbols '+', '−' and '≈' indicate that the result is significantly better, significantly worse and statistically similar to that obtained by MOEA/IGD-NS, respectively.

As can be observed from the tables, MOEA/IGD-NS has obtained the best result on 8 out of 13 instances in terms of IGD values. In detail, for the ZDT test suite, the performance of MOEA/IGD-NS is better than the other algorithms on ZDT2 and ZDT5, while MOEA/D shows the best performance on ZDT3, and HypE shows the best performance on ZDT1, ZDT4 and ZDT6. For the DTLZ test suites with 3 objectives, MOEA/IGD-NS shows the best performance on all the test instances except DTLZ4. It seems that the performance of HypE and IBEA will increase when the HV metric is adopted, however, it is not difficult to find that the proposed MOEA/IGD-NS achieves a competitive performance.

For further observations, the final populations in the runs associated with the smallest IGD values obtained by each algorithm on 3-objective DTLZ2 and 3-objective DTLZ7 are plotted in Figs. 3 and 4, respectively. It can be seen that the diversity of the population obtained by MOEA/IGD-NS is significantly better than the populations obtained by the other three algorithms.

## V. Conclusion and Future Work

In this paper, an enhanced IGD metric, termed IGD-NS, for evaluating the population on multi-objective optimization problems is proposed. To obtain a good IGD-NS metric value, a solution set should not only have good convergence and diversity, but also contain as few noncontributing solutions as possible. Then the proposed IGD-NS metric is integrated into the environmental selection of a new MOEA, known as the MOEA/IGD-NS. The experimental results indicate that MOEA/IGD-NS performs better than several representative MOEAs on the ZDT and DTLZ test suites.

However, the current version of MOEA/IGD-NS may not work well on MOPs with more than three objectives, as it will become very difficult to maintain an external archive with good convergence as well as diversity if the number of objectives is large. Therefore, an important topic for further research is to extend the proposed MOEA/IGD-NS for many-objective optimization.

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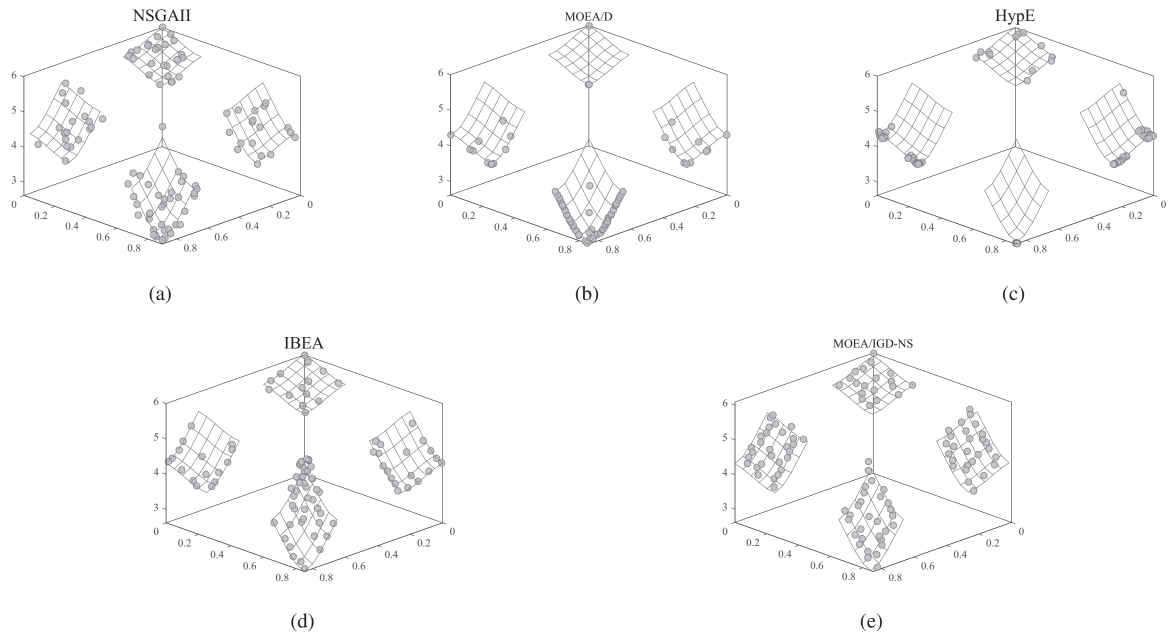


Fig. 4. The final population in the run associated with the lowest IGD metric values obtained by the five algorithms on 3-objective DTLZ7 problem.

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