Genetic Algorithm for Multi-objective Optimization Using GDEA

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Abstract. Recently, many genetic algorithms (GAs) have been developed as an approximate method to generate Pareto frontier (the set of Pareto optimal solutions) to multi-objective optimization problem. In multi-objective GAs, there are two important problems : how to assign a fitness for each individual, and how to make the diversified individuals. In order to overcome those problems, this paper suggests a new multi-objective GA using generalized data envelopment analysis (GDEA). Through numerical examples, the paper shows that the proposed method using GDEA can generate well-distributed as well as well-approximated Pareto frontiers with less number of function evaluations.

1 Introduction

Most decision making problems involve multiple and conflicting objectives, and are formulated as multi-objective optimization problems. There does not necessarily exist a solution that optimizes simultaneously all objectives, because the presence of conflicting objectives. Thus, the concept well known as Pareto optimal solution has been used. Usually, there are a lot of Pareto optimal solutions which are considered as candidates of a final solution to the decision making problem. It is an issue how a decision maker chooses her/his most preferable solution from the set of Pareto optimal solutions in the objective function space (i.e., Pareto frontier). In cases with two or three objective functions, if it does not take so much time to evaluate the value of each objective function, Pareto frontier can be depicted relatively easily. Seeing Pareto frontiers, we can grasp the trade-off relation among objectives totally. Therefore, it would be the best way to depict Pareto frontiers in cases with two or three objectives. In recent years, the research applying genetic algorithms (GAs) to generate Pareto frontiers has been extensively developed, and also has been observed to be useful for visualizing Pareto frontiers In this research, the important subjects are how fast individuals converge to Pareto frontier and how well-distributed they are on the whole Pareto frontier. To this end, many contrivances have been reported for gene operators and fitness function [1,2,3,4,5,6,7,8]. Most

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conventional algorithms are adopting Pareto optimality-based ranking method which is the way by the number of dominant individuals, although the rank does not reflect the "distance" itself between each individual and Pareto frontier. Therefore, we have suggested several multi-objective GAs using generalized data envelopment analysis (GDEA) to generate Pareto frontier, in short, GDEA methods [10], [12]. The characteristic of GDEA methods is in measuring the degree how far each individual is from Pareto frontier by solving some linear programming problem [11]. As a result, we have observed through several applications that GDEA methods can provide much closer Pareto frontier to the real one with less number of generations. In this paper, we propose a new method of crossover using GDEA in order to generate well-distributed Pareto frontier. In addition, we show that the proposed method can provide much well-distributed Pareto frontier through the comparison with the results by several methods.

2 Multi-objective Genetic Algorithm Using GDEA

Multi-objective optimization problems are formulated as follows:

$$\begin{aligned} \underset{\boldsymbol{x}}{\text{minimize}} \quad & \boldsymbol{f}(\boldsymbol{x}) = (f_1(\boldsymbol{x}), \ldots, f_m(\boldsymbol{x}))^T \\ \text{subject to} \quad & \boldsymbol{x} \in X = \{\, \boldsymbol{x} \in \mathbb{R}^n \mid g_j(\boldsymbol{x}) \leqq 0,\ j = 1, \ldots, l \,\}, \end{aligned} \tag{MOP}$$

where $\boldsymbol{x} = (x_1, \ldots, x_n)^T$ is a vector of design variable and X is the set of all feasible solutions.

Generally, unlike traditional optimization problems with a single objective function, there does not always exist an optimal solution that minimizes all objective functions $f_i(\boldsymbol{x}),\ i = 1, \ldots, m$, simultaneously does not necessarily exist in the problem (MOP). Based on Pareto domination relation, Pareto optimal solution is introduced, and there may be many Pareto optimal solutions. Pareto frontier is the set of them in the objective function space. (See Fig. 1.)

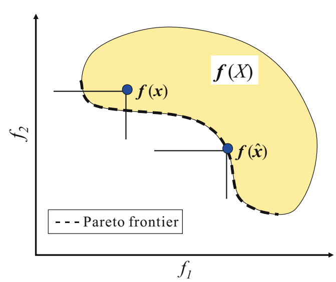


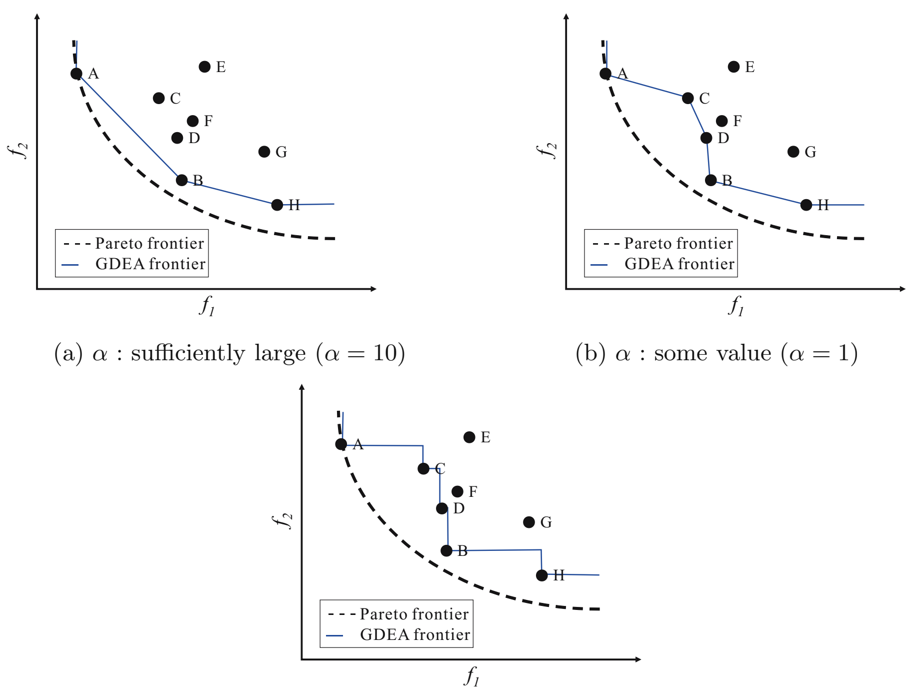
Fig. 1. Pareto frontier in the objective function space

Definition 1 (Pareto optimal solution). *A point $\hat{\boldsymbol{x}} \in X$ is said to be Pareto optimal if there exists no $\boldsymbol{x} \in X$ such that $f_i(\boldsymbol{x}) \leqq f_i(\hat{\boldsymbol{x}})$, $\forall\ i = 1, \ldots, m$ and $\boldsymbol{f}(\boldsymbol{x}) \neq \boldsymbol{f}(\hat{\boldsymbol{x}})$.*

For assessing a fitness for each individual $\boldsymbol{x}^o$, $o = 1, \ldots, p$ (p : the number of population), we suggested GDEA method of fitness evaluation given by an optimal value to the following linear programming problem [10]:

$$
\begin{aligned}
\underset{\Delta, \nu_i}{\text{maximize}} \quad & \Delta && \text{(GDEA)} \\
\text{subject to} \quad & \Delta \leqq \tilde{d}_j - \alpha \sum_{i=1}^{m} \nu_i (f_i(\boldsymbol{x}^o) - f_i(\boldsymbol{x}^j)), \ \ j = 1, \ldots, p, \\
& \sum_{i=1}^{m} \nu_i = 1, \\
& \nu_i \geqq \varepsilon, \ \ i = 1, \ldots, m,
\end{aligned}
$$

where ε is a sufficiently small number, and $\tilde{d}_j$, $j = 1, \ldots, p$, is the value of multiplying $\max\limits_{i=1,\ldots,m} \left(-f_i(\boldsymbol{x}^o) + f_i(\boldsymbol{x}^j)\right)$ by its corresponding weight, for example,



(a) α : sufficiently large ($\alpha = 10$)

(b) α : some value ($\alpha = 1$)

(c) α : sufficiently small ($\alpha = 10^{-7}$)

Fig. 2. GDEA frontiers by varying the parameter α

$\tilde{d}_j = 2\nu_1$ if $-\boldsymbol{f}(\boldsymbol{x}^o) + \boldsymbol{f}(\boldsymbol{x}^j) = (2, -1)$. α is a value of monotonically decreasing with respect to the number of generation. The parameter α decides so-called GDEA frontier as shown in Fig. 2, and the optimal value Δ^* means the degree how far an individual $\boldsymbol{x}^o$ is from GDEA frontier in the objective space. By adjusting the parameter α, we have observed that GDEA method can generate well-approximated Pareto optimal solutions with small number of generations.

Furthermore, in the paper, we consider the dual problem (GDEA$_D$) to the primal problem (GDEA) as follows:

$$\begin{aligned} \underset{\omega,\lambda_j,s_i}{\text{minimize}} \quad & \omega - \varepsilon \sum_{i=1}^{m} s_i \qquad\qquad (\text{GDEA}_D) \\ \text{subject to} \quad & \sum_{j=1}^{p} \left\{ \alpha \left(-f_i(\boldsymbol{x}^o) + f_i(\boldsymbol{x}^j) \right) + d_{ij} \right\} \lambda_j - \omega + s_i = 0, \ \ i = 1, \ldots, m, \\ & \sum_{j=1}^{p} \lambda_j = 1, \ \lambda_j \geqq 0, \ \ j = 1, \ldots, p, \\ & s_i \geqq 0, \ \ i = 1, \ldots, m, \end{aligned}$$

where d_{ij} is a component of a matrix $\left[-\boldsymbol{f}(\boldsymbol{x}^o) + \boldsymbol{f}(\boldsymbol{x}^1), \cdots, -\boldsymbol{f}(\boldsymbol{x}^o) + \boldsymbol{f}(\boldsymbol{x}^p) \right]^T$ replaced by 0, except for the maximal component in each column.

Let ω^*, $\left(\lambda_1^*, \ldots, \lambda_p^*\right)$ and $(s_1^*, \ldots, s_m^*)$ be the optimal solution to the problem (GDEA$_D$) for an individual $\boldsymbol{x}^o$. Then, as well known from the duality theory of linear programming problem, ω^* has the same meaning with Δ^* in the primal problem (GDEA). $(s_1^*, \ldots, s_m^*)$ represents the slackness which can distinguish easily individuals to be weak Pareto optimal. $\left(\lambda_1^*, \ldots, \lambda_p^*\right)$ represents a domination relation between an individual $\boldsymbol{x}^o$ and another individuals. That is, if λ_j^* is positive for some $j \neq o$, $\boldsymbol{x}^o$ is dominated by $\boldsymbol{x}^j$ which may be regarded as a reference individual. Making efficient use of the reference individual, we suggest that the new offspring is generated by the parents with same reference individuals in order to keep the diversity of individuals. As is shown in Fig. 3,

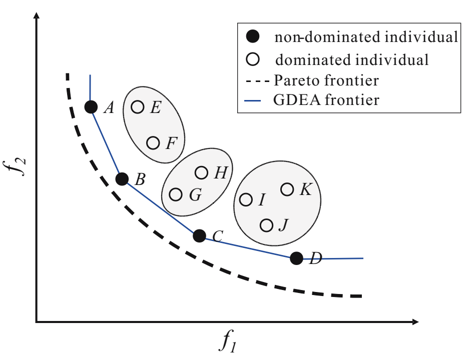


Fig. 3. Crossover by the proposed method

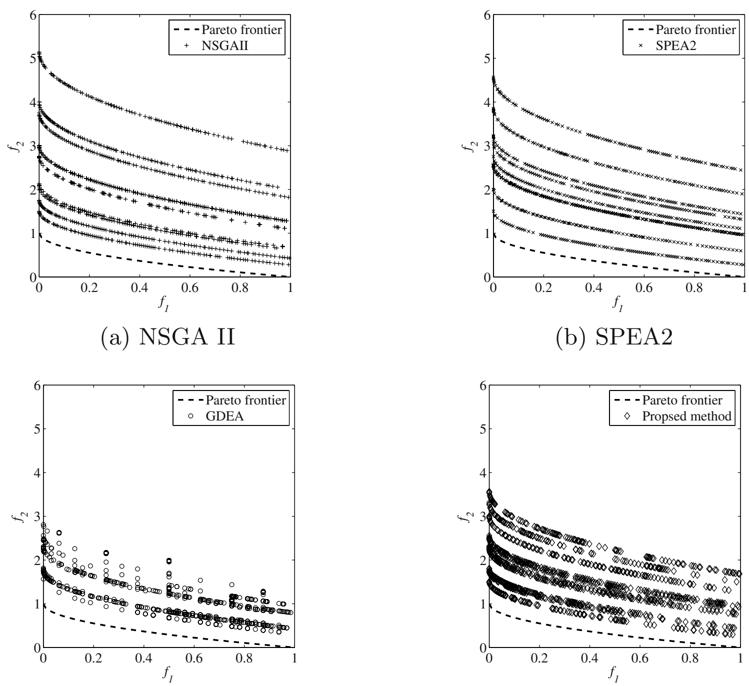
for instance, the reference individuals of E and F are A and B, and the children are generated by the parents E and F. This means that E and F are evolved toward GDEA frontier of between A and B. By divide the population into several sub-populations, Pareto frontier is generated piecewise. Consequently, the proposed method can not only generate well-distributed Pareto frontier, but also converge much faster and more effectively to the real Pareto frontier than the conventional algorithms.

3 Numerical Examples

In this section, we illustrate the effectiveness of the proposed method through the following examples [9]:

$$\underset{\boldsymbol{x}}{\text{minimize}} \qquad f_1(\boldsymbol{x}) = x_1 \tag{ZDT4}$$

$$\underset{\boldsymbol{x}}{\text{minimize}} \qquad f_2(\boldsymbol{x}) = g(\boldsymbol{x}) \times \left(1 - \sqrt{\frac{f_1(\boldsymbol{x})}{g(\boldsymbol{x})}}\right)$$



(a) NSGA II
(b) SPEA2
(c) GDEA Method
(d) Proposed Method

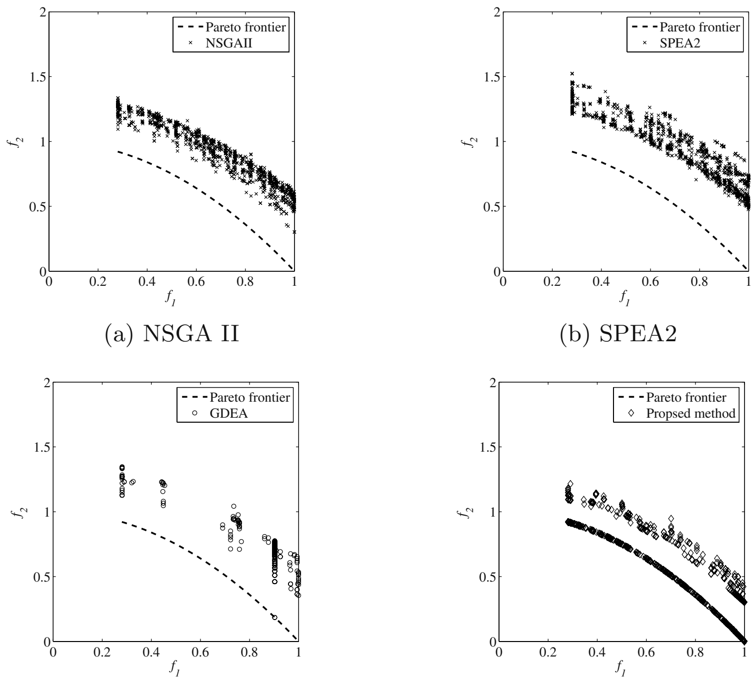
Fig. 4. Results for ZDT4

$$
\begin{aligned}
\text{subject to} \quad & g(\boldsymbol{x}) = 1 + 10(N-1) + \sum_{i=2}^{N} \left(x_i^2 - 10\cos(4\pi x_i) \right), \\
& x_1 \in [0,1],\ x_i \in [-5,5],\ i = 1,2,\ldots,N.
\end{aligned}
$$

$$
\begin{aligned}
\underset{\boldsymbol{x}}{\text{minimize}} \quad & f_1(\boldsymbol{x}) = 1 - \exp(-4x_1)\sin^6(6\pi x_1) \qquad \text{(ZDT6)} \\
\underset{\boldsymbol{x}}{\text{minimize}} \quad & f_2(\boldsymbol{x}) = g(\boldsymbol{x}) \times \left(1 - \left(\frac{f_1(\boldsymbol{x})}{g(\boldsymbol{x})} \right)^2 \right) \\
\text{subject to} \quad & g(\boldsymbol{x}) = 1 + 9 \left(\frac{\sum_{i=2}^{N} x_i}{N-1} \right)^{0.25}, \\
& x_i \in [0,1],\ i = 1,\ldots,N.
\end{aligned}
$$

In the above problems, $N = 10$, and both the true Pareto frontiers are formed with $g(\boldsymbol{x}) = 1$. Under the following parameters, we simulate 10 times with random initial population, and show the results in Fig. 4 and Fig. 5.

generation : 100 (ZDT4), 120 (ZDT6), population size : 100
crossover rate : 1.0, mutation rate : 0.05



(a) NSGA II
(b) SPEA2
(c) GDEA Method
(d) Proposed Method

Fig. 5. Results for ZDT6

The problem (ZDT4) is used the test for the ability to deal with multimodality, because of containing many local Pareto frontiers. The problem (ZDT6) has the feature that the Pareto optimal solutions are non-uniformly distributed on the true Pareto frontier. As is seen from the computational results of the figures, the proposed method gives the results that the obtained solutions are more widely distributed and closer to the real Pareto frontiers, comparing the results by two conventional NSGAII and SPEA2.

4 Concluding Remarks

In many practical engineering problems, we have black-box objective functions whose forms are not explicitly known in terms of design variables. The values of objective functions for each design variable can be given by sampled real/computational experiments, for example, structural analysis, fluid mechanical analysis, thermodynamic analysis, and so on. Usually, these analyses are considerably expensive, and take too much computation time. Also, we do not know when to stop the computation in advance, and the computation is terminated relatively early by the given computation time and cost limitation. Under this circumstance, it is an important issue to generate well-approximated solution with less function evaluations (= the size of population $\times$ the number of generations) as possible. From this point of view and the experimental results, it can be concluded that the proposed method using GDEA has the desirable performance.

References

1. M. Arakawa, H. Nakayama, I. Hagiwara, H. Yamakawa, Multiobjective Optimization Using Adaptive Range Genetic Algorithms with Data Envelopment Analysis, *A Collection of Technical Papers on 7th Symposium on Multidisciplinary Analysis and Optimization (TP98-4970), AIAA*, Vol.3, pp.2074–2082, 1998
2. C.A. Coello Coello, D.A. Van Veldhuizen, G.B. Lamont, Evolutionary Algorithms for Solving Multi-Objective Problems, *Kluwer Academic Publishers*, 2002
3. K. Deb, Multi-Objective Optimization using Evolutionary Algorithms, *John & Wiley Sons, Ltd.*, 2001
4. K. Deb, S. Agarwal, A. Pratap, T. Meyarivan, A Fast Elitist Non-dominated Sorting Genetic Algorithm for Multi-objective Optimisation: NSGA-II, *Proceedings of 6th International Conference on Parallel Problem Solving from Nature*, pp.849–858, 2000
5. C.M. Fonseca, P.J. Fleming, Genetic Algorithms for Multi-objective Optimization: Formulation, Discussion and Generalization", *Proceedings of the Fifth International Conference on Genetic Algorithms*, pp.416–426, 1993
6. D.E. Goldberg, Genetic Algorithms in Search, Optimization and Machine Learning, Massachusetts : Addison-Wesley, Inc., 1989
7. J.D. Schaffer, Multiple Objective Optimization with Vector Evaluated Genetic Algorithms", *Proceedings of the First International Conference on Genetic Algorithms*, pp. 93–100, 1985

8. N. Srinivas, K. Deb, Multi-Objective Function Optimization using Non-Dominated Sorting Genetic Algorithms, Evolutionary Computation, Vol.3, pp.221–248, 1995
9. E. Zitzler, K. Deb, L. Thiele, Comparison of Multiobjective Evolutionary Algorithms: Empirical Result, Evolutionary Computation, Vol.8, No.2, pp. 173–195, 2000
10. Y.B. Yun, H. Nakayama, T. Tanino, M. Arakawa, Generation of efficient frontiers in multi-objective optimization problems by generalized data envelopment analysis, European Journal of Operational Research, Vol.129, No.3, pp.586–595, 2001
11. Y.B. Yun, H. Nakayama, T. Tanino, A generalized model for data envelopment analysis, European Journal of Operational Research, Vol.157, No.1, pp.87–105, 2004
12. Y.B. Yun, H. Nakayama, M. Arakawa, Multiple criteria decision making with generalized DEA and an aspiration level method, European Journal of Operational Research, Vol.158, No.1, pp.697–706, 2004