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Improved NSGA-II algorithm based on differential evolution mechanism

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Abstract: From simulation experiments of the multi-objective optimization control of wastewater treatment process (WWTP), it can be found that the number of obtained Pareto solutions is less using the normal non-dominated sorting genetic algorithm-II (NSGA-II) sometimes. To achieve a satisfactory optimal performance, an improved NSGA-II algorithm based on differential evolution mechanism is proposed in this paper. The simulation results show that the diversity of the solutions is enhanced and a better homogeneity of non-inferior solutions is kept using the proposed method.

Key Words: Non-dominated sorting genetic algorithm, differential evolution, multi-objective optimization, uniform, wastewater treatment

1 Introduction

Optimization control of the wastewater treatment process (WWTP) has a vital significance under the current background of lacking water resource and requiring green environment[1], and the optimal control of WWTP is a multi-objective optimization issue essentially, which has been a hot research problem recently[2-4].

The most important items of impacting the optimal performance lie in two aspects: one is the problem of establishing the multi-objective optimal model, and another is the problem of using an appropriate multi-objective solution method. For the latter, the non-dominated sorting genetic algorithm-II (NSGA-II) has been applied in the WWTP control and has been proved to be effective. Iqbal et al. realized the multi-objective optimization of an operating domestic wastewater treatment plant using binary coded elitist non-dominated sorting genetic algorithm[5], which enhances the plant performance without affecting the discharge effluent quality. Optimal design of activated sludge process (ASP) using multi-objective optimization was studied by Chen et al., where four indexes of percentage of effluent violation (PEV), overall cost index (OCI), total volume and total suspended solids were studied, and NSGA-II algorithm was used[6]. The results indicate that multi-objective optimization is a useful method for optimal design ASP. However, the emphasis of this paper is paid on the design parameters and normal NSGA-II algorithm is adopted without considering the control characteristic of the WWTP. Sweetapple et al. proposed a multi-objective control method for the WWTP[3],where the multi-objective evolutionary algorithm, NSGA-II, is used to derive sets of Pareto optimal operational and control parameter values for an activated sludge wastewater treatment plant, with objectives including minimization of greenhouse gas emissions, operational costs and effluent pollutant concentrations. However, the modeling problem and influence factors of the optimal algorithm have not been discussed.

In this paper, an improved NSGA-II algorithm based on differential evolution mechanism is proposed for enhancing the optimal performance of WWTP. The main improvement include two aspects: one is the initialization of the multi-objective optimization algorithm, where a reverse learning criterion is adopted; the other is the generation mode of new individual and the differential evolution mechanism is introduced. The simulation results demonstrate that the effectiveness of the proposed method.

2 NSGA-II algorithm

NSGA-II is proposed by Deb et al. [7], which is one of the most excellent multi-objective optimization evolution algorithms and has been employed widely[8-10]. The wastewater treatment process belongs to the slow time-varying nonlinear system, 15 minutes is taken as the basic sampling time, and the optimization period can reach up to the level of several hours. Therefore, it is appropriate that NSGA-II is introduced to solve the optimization problem of WWTP from the control real-time performance requirement[11].

The core of NSGA-II evolution algorithm lies in two aspects: fast non-dominated sorting for the individuals and elitist selection strategy. The fast non-dominated sorting is built on the indexes of non-dominated rank and crowding distance. The rank of the non-dominated is determined by the Pareto domination relationship among the optimal performance functions. Take a multi-objective optimal problem with two performance indexes as an example, Pareto domination is defined as follows: for the solution vector $\mathbf{x}_1$ and $\mathbf{x}_2$ in the feasible field, $\mathbf{x}_1$ is called Pareto domination or $\mathbf{x}_1$ dominates $\mathbf{x}_2$ (donate $x_1 \succ x_2$), when the following condition holds:

$$\begin{aligned} & f_i(\mathbf{x}_1) \le f_i(\mathbf{x}_2),\ \forall i \in (1,2) \wedge \\ & f_j(\mathbf{x}_1) \le f_j(\mathbf{x}_2),\ \exists j \in (1,2) \end{aligned} \tag{1}$$

Crowding-distance index of each individual is a representation of individual distribution uniformity in current population.

Evolution process of NSGA-II algorithm can be shown in Fig.1.

The individual selection can be described as follows: firstly, take the tth generation evolution as an example, the whole individuals come from two parts: Parent individuals (P_t) and Offspring individuals (Q_t) from the last generation; secondly, calculate the non-dominated rank F and crowding distance index for each individual in the new population of (P_t, Q_t).

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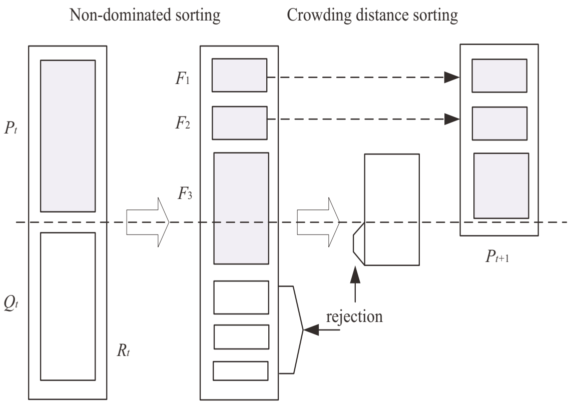


Fig. 1: Evolution process of NSGA-II algorithm[7]

According to the rank of each individual (small grade is priority), the excellent individual is selected into the elitist population. For these individuals with the same rank, the crowding distance index is used and the individual with a big crowding distance value has the priority. Then, the evolution for the tth generation is finished until the size of population is up to N, and they constitutes the fresh parent population P_{t+1}.

3 Optimization problem of WWTP

The problem of establishing multi-objective optimization model of WWTP has been an open topic, and is also a bottleneck for the WWTP control. In [12], a neural network online modeling method is proposed to construct the optimization model[12], where the function relationship between optimal set-point values and optimal performances is established. When considering the dissolved oxygen concentration and nitrate level as the optimized variables (they are the most important controlled parameters in the WWTP[12]), and the energy consumption and the effluent index as optimal performances. We can construct the following multi-objective optimization model,

$$\begin{cases} \min F(\mathbf{x}) = \{f_{EC}(\mathbf{x}), f_{EQ}(\mathbf{x})\} \\ s.t. \quad \mathbf{x} \in S \end{cases} \tag{2}$$

where $\mathbf{x}(k) = [x_1(k), x_2(k)]$ is the optimal vector; $f_{AE}(\mathbf{x})$ is the function that reflects the mapping between the energy consumption and optimal variables, and $f_{EQ}(\mathbf{x})$ is the function that reflects the mapping between the effluent quality and optimal variables. S is the constraint set for the optimal variable and can be demonstrated by the following formula,

$$S = \{\mathbf{x} \in R^2 \,|\, g_j(\mathbf{x}) - C_j \le 0, j = 1,\ldots,5, x_i^l \le x_i \le x_i^u, i = 1,2\} \tag{3}$$

where x_1^l and x_1^u are the lower limit value and the upper limit value of optimal variables. $g_j(\mathbf{x})$ reflects the function relationship between the optimal variables and the effluent parameters; C_j is the upper limit value of the effluent parameters. The effluent total nitrate N_{tot} and S_{NH} (two key effluent parameters) is considered in[12], and the multi-objective optimal control is studied based on the international benchmark simulation model no.1(BSM1).

A Pareto set can be observed when the NSGA-II is operated in the experiment process. However, the Pareto set is different from each other because of the dynamic characteristic of the WWTP. The typical Pareto set obtained during an optimization process can be demonstrated in Fig.2.

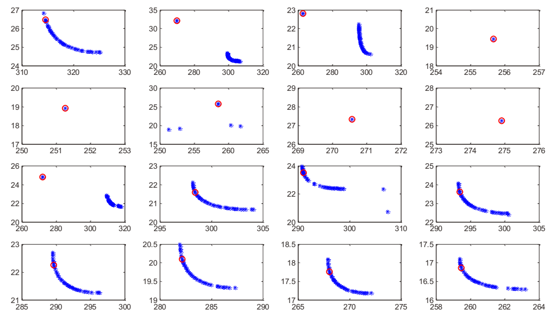


Fig.2: Typical Pareto set obtained during the optimization process

From the figures, it can be seen clearly that the number of obtaining Pareto solutions maybe less in some cases when solving the optimization problem of WWTP using the normal NSGA-II. Therefore, it is the motivation of our studies for improving the NSGA-II algorithm, where the diversity and better homogeneity of the solutions is expected.

4 Improved NSGA-II based on DE mechanism

From the evolution process of NSGA-II, it can be seen that the generation mechanism of new individual and non-dominated sorting criterion of each individual have a powerful influence on the algorithm performance. Non-dominated sorting criterion of individuals has been proven an effective norm. Therefore, our studies are considerate from the generation mechanism of new individual, and an improved NSGA-II algorithm is proposed, which is based on the crossover and mutation operation of the differential evolution (DE) for improving the diversity and homogeneity of Pareto solution.

Similar to the NSGA-II, DE also belongs to the evolution algorithm and the most prominent merit is simple and efficient[13]. The crossover and mutation operation are the core of DE algorithm, which has the same combination point with our current studied problem. In this paper, we introduce the crossover and mutation mechanism of DE to enhance the diversity of new individual generation. Simultaneously, a reverse learning criterion is employed to initiate the population for improving the searching speed of NSGA-II.

4.1 Crossover and mutation in DE algorithm

Different from the genetic algorithm(GA), the mutation operator is conducted primarily and then crossover operation of new individual generation is done in DE algorithm. The mutation and crossover operator can be described as follows.

Mutation operation is executed according to (4),

$$V_{i,G} = X_{r1,G} + F(X_{r2,G} - X_{r3,G}) \tag{4}$$

where $X_{r1,G}, X_{r2,G}$ and $X_{r3,G}$ are three individuals randomly selected from the population; r_1, r_2 and r_3 are generated randomly and different from each other; G is the current evolution generation. The ith mutation vector can be expressed as $V_{i,G} = (v_{i,G}^1, v_{i,G}^2, \ldots, v_{i,G}^D)$, $F \in [0,1]$ is the extension or contraction factor, which can control the step of explore direction, and a bigger value can enhance the mutation capability but decrease convergence speed and a smaller value can reduce the diversity of population and is prone to trap into local optimization. In this paper, an adaptive mode is adopted for the factor F, namely, $F=((F_{\max}-F_{\min})\frac{k}{G}+F_{\min})\cdot y_{\text{logistic}}(k)$, $F_{\max}$ and $F_{\min}$ are the maximum and minimum value of the factor F, k is the current generation.

Crossover operation is executed according to (5),

$$x_{i,G}^{j}=\begin{cases} v_{i,G}^{j}, & if\ rand_{i,j}[0,1]\le C_r \text{ or } j=j_{rand} \\ x_{i,G}^{j}, & \text{otherwise} \end{cases} \tag{5}$$

where $rand_{i,j}[0,1]$ is a random value with normal distribution within the scope of [0,1]; C_r is the crossover probability factor，a bigger C_r value can increase the mutation level and a smaller one can promote the exploration; $j_{rand}\in[1,D]$ is a random positive integer, which can guarantee that at least one dimension in the new generated vector is different from the original vector.

4.2 Reverse learning criterion

According to [13], using the reverse learning norm to initialize the population can enhance the searching speed of evolution algorithm [13]. The main idea can be described as follows: firstly, the uniform distribution style is adopted in the searching scope to randomly generate N individuals $X_{i,0}^{'}=\{x_{i,0}^{j'}\}$, and then utilize the reverse learning criterion (6) to generate corresponding N individuals $X_{i,0}^{"}=\{x_{i,0}^{j"}\}$.

$$X_{i,0}^{"}=\{x_{i,0}^{j"}=x_i^{j,l}+x_i^{j,u}-x_{i,0}^{j'}\} \tag{6}$$

Calculate each performance function value(also called fitness value) of $2N$ initial individuals; the rank sorting is operated according to the Pareto non-dominated criterion, and N individuals of the $2N$ individuals are selected to constitute the initial parent population $P_0=(X_{1,0},X_{2,0},\ldots,X_{N,0})$.

4.3 Realization of improved NSGA-II algorithm

Denote D as the dimension of searching scope, N as the population size, and M as the evolution generation. The *Gth* individual $X_{i,G}$ in population can be expressed by $X_{i,G}=\{x_{i,G}^{j}\}$, where $1\le i\le N$, $1\le j\le D$, $x_{i,G}^{j}$ is a real-value within the scope of $[x_i^{j,l},x_i^{j,u}]$ and can be changed; $x_i^{j,l}$ and $x_i^{j,u}$ are the lower limit and the upper limit value of variables.

The realization step of improved NSGA-II algorithm can be concluded as follows:

Step1: Let G=1, obtain the initial parent population $P_0=(X_{1,0},X_{2,0},\ldots,X_{N,0})$ based on the reverse learning criterion;

Step2: select the (N/2)parent elite individuals based on the tournament selection method from the N parent individuals;

Step3: for parent elite individuals, do crossover and mutation operation based on the DE mechanism and generate N offspring individuals;

Step4: combine N parent individuals and generated N offspring individuals, calculate the performance function values of the $2N$ large-scale individuals, obtain the sorting rank and crowding distance indexes and conduct the non-dominated sorting;

Step5: select N excellent individuals as the next generation parent individuals based on the non-dominated sorting rank, crowding distance index and tournament selection method;

Step6: let G=G+1 , if evolution G reaches the maximum M, the evolution process is over, and a group of Pareto solutions is provided for the current multi-objective optimal problem; or turn back to Step2 and enter into the next generation evolution.

5 Simulation experiment studies

To verify the optimal performance of the improved NSGA-II algorithm, two classic multi-objective optimization cases with constraints are selected as the simulation studies, which include the CONSTR multi-objective optimization problem and SRN multi-objective optimization problem. These cases have the similar characteristic with the multi-objective optimization problem of WWTP control to be solved.

Case 1: CONSTR multi-objective optimization problem

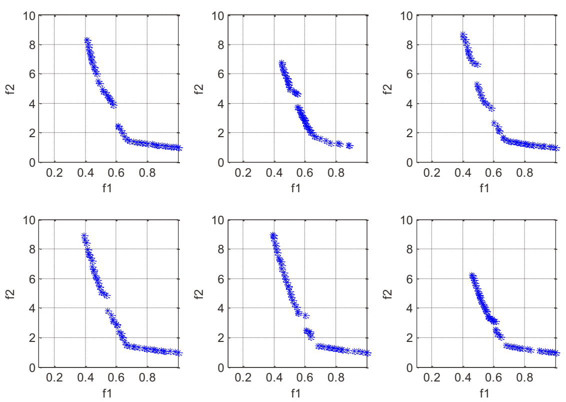
Objective functions are expressed by (7) and the constraint conditions are described by (8) for CONSTR problem.

$$\min\{f_1(\mathbf{x}), f_2(\mathbf{x})\}$$

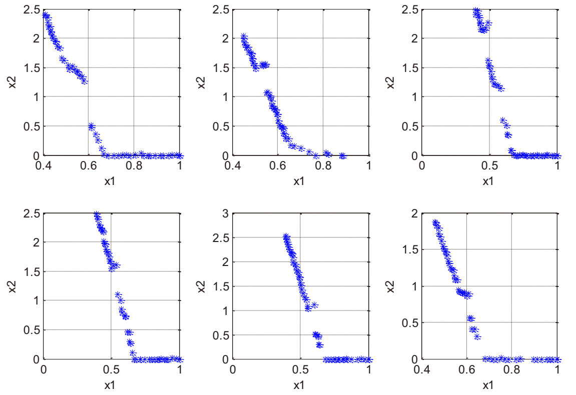
$$\begin{cases} f_1(\mathbf{x})=x_1 \\ f_2(\mathbf{x})=(1+x_2)x_1 \end{cases} \tag{7}$$

$$\begin{cases} g_1(\mathbf{x})=x_2+9x_1\ge 6 \\ g_2(\mathbf{x})=-x_2+9x_1\ge 1 \end{cases} \tag{8}$$

where the scope of optimal variables is $x_1\in[0.1,\ 1.0]$ and $x_2\in[0,\ 5.0]$, respectively. The parameters in the simulation experiments are set as follows: population size M is 50, the generation epoch G is 30, searching dimension D is 2; the crossover and mutation parameters in NSGA-II algorithm adopt the standard value[7]; in the improved NSGA-II algorithm, the crossover factor C_r is taken as 0.9, $F_{\max}$=1.5, $F_{\min}$=0.5, y_logistic(1)=0.1. For the CONSTR optimal problem, Pareto front obtained using NSGA-II and improved NSGA-II (continuously run 6 times) are shown in Fig.3 and Fig.4.

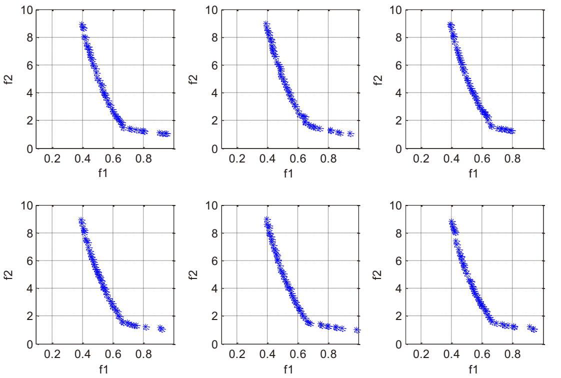


(a) Space distribution of objective functions

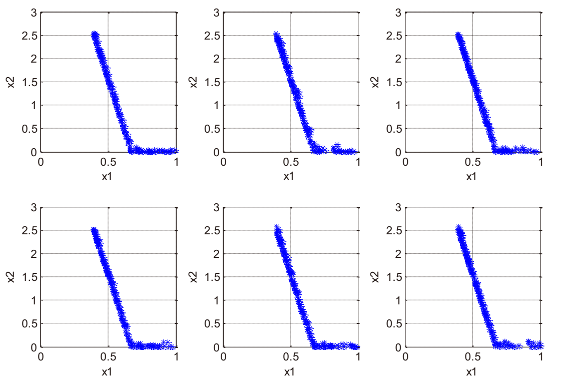


(b) Space distribution of the solutions

Fig. 3: Pareto front of CONSTR optimization problem using NSGA-II



(a) Space distribution of objective functions



(b) Space distribution of the solutions

Fig. 4: Pareto front of CONSTR optimization problem using the improved NSGA-II.

Table 1: Average SP value under 30 independent runs(CONSTR)

Method	SP index			Running time/s
	Best value	Average	Worst value	
NSGA-II	0.055	0.085	0.124	0.331
INSGA-II	0.037	0.071	0.098	0.205

From the simulation results, we can see that under the same simulation environment, compared with the standard NSGA-II, the improved NSGA-II (INSGA-II) can obtain much more uniform Pareto solutions, and uniform distribution of the objective functions space and the solution space is improved greatly. The SP index and running time under the NSGA-II and the improved NSGA-II method are provided in Table 1.

Analyzing the values in the Table 1, it can be obtained that a much better distribution of Pareto solutions is achieved using the improved NSGA-II, and the best value and the average value of the SP index is also prior to NSGA-II algorithm. For the running time, no distinct advance appears in two methods. However, a better distribution of Pareto solutions is popular for the optimization problem of WWTP. We set the same evolution generation in the experiments, and it can be found that the improved NSGA-II algorithm can achieve a satisfactory Pareto solutions with much less iterations.

Case 2: SRN multi-objective optimization problem

Objective functions are expressed by (9) and the constraint conditions are described by (10) for SRN problem.

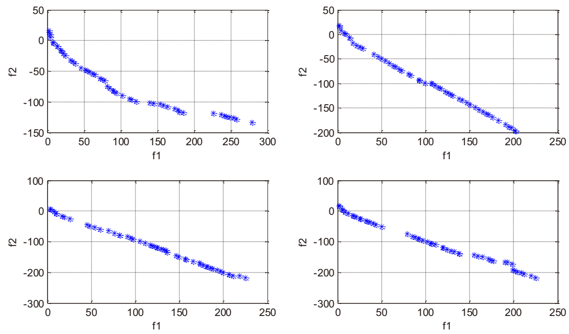
$$\min\{f_1(\mathbf{x}), f_2(\mathbf{x})\}$$

$$\begin{cases} f_1(\mathbf{x}) = (x_1-2)^2 + (x_2-1)^2 + 2 \\ f_2(\mathbf{x}) = 9x_1 - (x_2-1)^2 \end{cases} \tag{9}$$

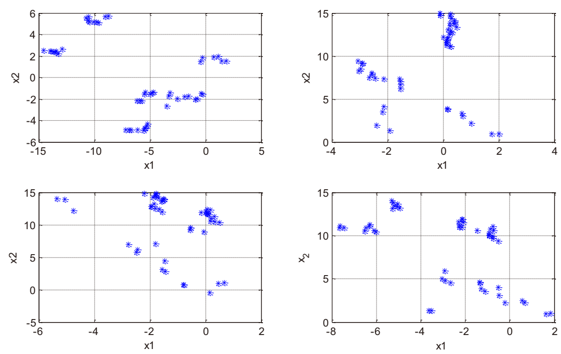
$$\begin{cases} g_1(\mathbf{x}) = x_1^2 + x_2^2 \le 225 \\ g_2(\mathbf{x}) = x_1 - 3x_2 \le 10 \end{cases} \tag{10}$$

where the scope of optimal variables is $x_1 \in [-20,\ 20]$ and $x_2 \in [-20,\ 20]$.

The parameters in the simulation experiments are set the same with the Case 1. For the SRN optimal problem, Pareto front obtained using NSGA-II and improved NSGA-II (continuously run 6 times) are shown in Fig.5 and Fig.6.

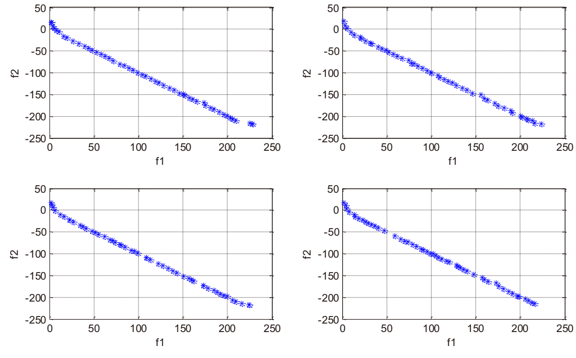


(a) Space distribution of objective functions

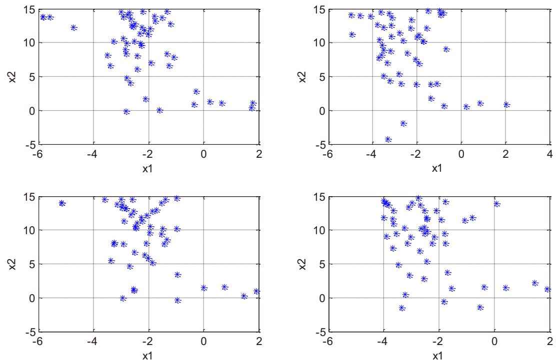


(b) Space distribution of the solutions

Fig. 5: Pareto front of SRN optimization problem using NSGA-II



(a) Space distribution of objective functions



(b) Space distribution of the solutions

Fig. 6: Pareto front of CONSTR optimization problem using the improved NSGA-II

From the simulation results of Fig.5 and Fig.6, we can see that compared with the standard NSGA-II, the improved NSGA-II can obtain much more uniform Pareto solutions, and uniform distribution of the objective functions space and the solution space is improved greatly for the CONSTR optimization problem. The SP index and running time under NSGA-II and the improved NSGA-II (INSGA-II) method are provided in Table 2.

Table 2: Average SP value under 30 independent runs(SRN)

Method	SP index			Running time/s
	Best value	Average	Worst value	
NSGA-II	1.96	3.58	8.85	0.295
INSGA-II	1.56	2.73	3.91	0.189

From the performance index demonstrated by Table 2, a much better distribution of solutions under the improved NSGA-II is achieved and the best value and the average value of SP index is also prior to NSGA-II. It means that we can obtain much better stability by evaluating the SP index based on the improved NSGA-II algorithm.

6 Conclusion

An improved NSGA-II algorithm is proposed in this paper, which is based on the differential evolution mechanism and the idea of crossover and mutation operation is adopted. Moreover, a reverse learning criterion is introduced to speed the convergence of the evolution algorithm. Two simulation cases verifies the efficiency of the improved NSGA-II algorithm, where a much better distribution of solutions is achieved and the SP index is also prior to NSGA-II. Considering the similarity to be solved in WWTP optimization control with the current experiment studies, it provides a better foundation of research for next optimization problem to be solved. Therefore, in the following studies, the improved NSGA-II will be applied to the optimization control of WWTP.

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