Self-Controlling Dominance Area of Solutions in Evolutionary Many-Objective Optimization

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Abstract. Controlling dominance area of solutions (CDAS) relaxes the concepts of Pareto dominance with an user-defined parameter S. This method enhances the search performance of dominance-based MOEA in many-objective optimization problems (MaOPs). However, to bring out desirable search performance, we have to experimentally find out S that controls dominance area appropriately. Also, there is a tendency to deteriorate the diversity of solutions obtained by CDAS when we decrease S from 0.5. To solve these problems, in this work, we propose a modification of CDAS called self-controlling dominance area of solutions (S-CDAS). In S-CDAS, the algorithm self-controls dominance area for each solution without the need of an external parameter. S-CDAS considers convergence and diversity and realizes a fine grained ranking that is different from conventional CDAS. In this work, we use many-objective 0/1 knapsack problems with $m = 4 \sim 10$ objectives to verify the search performance of the proposed method. Simulation results show that S-CDAS achieves well-balanced search performance on both convergence and diversity compared to conventional NSGA-II, CDAS, IBEA$_{\epsilon+}$ and MSOPS.

1 Introduction

The research interest of the multi-objective evolutionary algorithm (MOEA) [1] community has rapidly shifted to develop effective algorithms for many-objective optimization problems (MaOPs) because more objective functions should be considered and optimized in recent complex applications. However, in general, Pareto dominance-based MOEAs such as NSGA-II [2] and SPEA2 [3] noticeably deteriorate their search performance as we increase the number of objectives to more than 4 [4,5]. This is because these MOEAs meet difficulty to rank solutions in the population, i.e., most of the solutions become non-dominated and the same rank is assigned to them, which seriously spoils proper selection pressure required in the evolution process.

To overcome this problem and induce more fine-grained ranking of solutions in MaOPs, recently some selection methods such as indicator, aggregating function

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and extending Pareto dominance based approaches have been proposed and verified its search performance in MaOPs. Indicator-based evolutionary algorithm (IBEA) [6] introduces fine grained ranking of solutions by calculating fitness value based on some indicators which measure the degree of superiority for each solution in the population. Multiple single objective Pareto sampling (MSOPS) aggregates fitness vector with multiple weight vectors, and reflects the ranking of solutions calculated for each weight vector in parent selection [4]. Compared with conventional Pareto dominance based NSGA-II, superiority of IBEA and MSOPS has been reported on continuous many objective optimization problems [9]. CDAS [7] relaxes the concepts of Pareto dominance by controlling dominance area of solutions using an user-defined parameter S to induce appropriate selection pressure in MOEA. CDAS shows better search performance in MaOP than NSGA-II due to convergence by the effects of fine grained ranking of solutions using $S < 0.5$ [8]. However, to bring out desirable search performance, we have to experimentally find out (select) S that controls dominance area appropriately. Also, there is a tendency to deteriorate diversity of obtained solutions by CDAS when we decrease S from 0.5.

In this work, we focus on CDAS [7] and propose a modification of CDAS, which is called self-controlling dominance area of solutions (S-CDAS), to solve the aforementioned problems and achieve well-balanced search performance between convergence and diversity toward optimal Pareto front on MaOPs. When calculating dominance relation among solutions, S-CDAS self-controls dominance area for each solution without using an external parameter, while the conventional CDAS controls using a same parameter S for all solutions. Due to this self-control of dominance area, S-CDAS realizes different fine grained ranking from conventional CDAS by considering that extreme solutions for each objective functions are never dominated by other solutions in the calculation of dominance area. In this work, we verify the search performance of the proposed method in many-objective 0/1 knapsack problems with $m = \{4, 6, 8, 10\}$ objectives by comparing with NSGA-II, IBEA$_{\epsilon+}$ [6], MSOPS [4] and CDAS [7].

2 Dominance Based MOEA and Problems in MaOPs

NSGA-II [2] is one of the well-known MOEAs that use Pareto dominance, which robust performance has been verified on a wide range of MOP especially for two or three objective optimization problems. In parent selection process, NSGA-II first classifies solutions into several layers (fronts) based on non-dominance level. Then, it selects parent solutions from higher fronts until filling up the half size of entire population. When comparing solutions that belong to a same front, NSGA-II determines superiority of solutions based on crowding distance (CD) [2] which considers solution's distribution in objective space.

When analyzing NSGA-II in MaOP, most of the solutions become non-dominated (front $1 : \mathcal{F}_1$), and the number of solutions belonging to $\mathcal{F}_1$ exceeds the size of parent solutions $|\mathcal{P}_t|$ (half of the entire population) in early stage of the evolution. In such case, parent selection of NSGA-II becomes to rely on CD

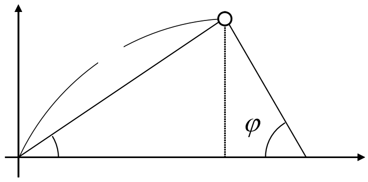


Fig. 1. Fitness modification to change the covered area of dominance

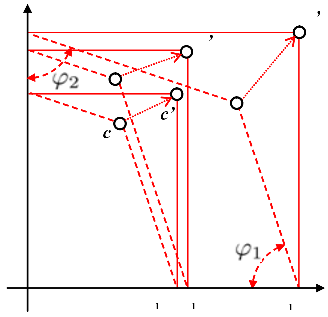


Fig. 2. Expanding dominance area by CDAS with $S < 0.5$

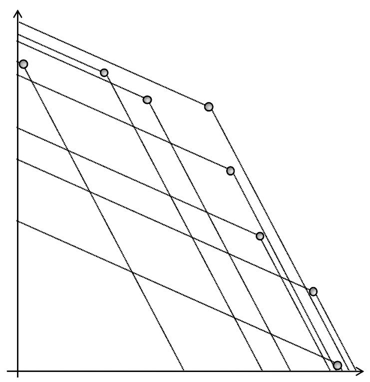
strongly. Consequently, the obtained POS are well-distributed in objective space, but convergence of POS towards the true POS is substantially deteriorated. To overcome this problem and enhance convergence of the obtained POS, it is necessity to improve selection pressure by discriminating non-dominated solutions using some effective manner.

3 Controlling Dominance Area of Solutions

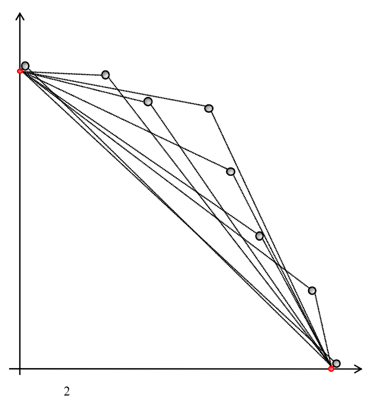
To induce appropriate selection pressure into Pareto dominance-based MOEAs, controlling dominance area of solutions (CDAS) [7] contracts or expands the dominance area of solutions before we calculate dominance relations among solutions. For a solutions $\boldsymbol{x}$ shown in **Fig.1**, CDAS modifies the fitness value for each objective function by changing a parameter S_i in the following equation

$$f_i'(\boldsymbol{x}) = \frac{r \cdot \sin(\omega_i + S_i \cdot \pi)}{\sin(S_i \cdot \pi)} \qquad (i = 1, 2, \cdots, m), \tag{1}$$

where $\varphi_i = S_i \cdot \pi$, r is the norm of $\boldsymbol{f}(\boldsymbol{x}) = (f_1(\boldsymbol{x}), f_2(\boldsymbol{x}), \cdots, f_m(\boldsymbol{x}))$, $f_i(\boldsymbol{x})$ is the fitness value in the i-th objective, and ω_i is the declination angle between $\boldsymbol{f}(\boldsymbol{x})$ and $f_i(\boldsymbol{x})$. In [7], a same parameter S is used for all fitness functions f_i $(i = 1, 2, \cdots, m)$. When $S < 0.5$, the i-th fitness value $f_i(\boldsymbol{x})$ is increased to $f_i'(\boldsymbol{x}) > f_i(\boldsymbol{x})$. On the other hand, when $S > 0.5$, $f_i(\boldsymbol{x})$ is decreased to $f_i'(\boldsymbol{x}) < f_i(\boldsymbol{x})$. When $S = 0.5$, $f_i'(\boldsymbol{x}) = f_i(\boldsymbol{x})$ which is equivalent to conventional dominance. **Fig.2** shows an example in the case of expanding dominance area with $S < 0.5$. In the case of conventional dominance, the solution $\boldsymbol{a}$ dominates $\boldsymbol{c}$, but $\boldsymbol{a}$ and $\boldsymbol{b}$, and $\boldsymbol{b}$ and $\boldsymbol{c}$ do not dominate each other. If we modify fitness value with $S < 0.5$ the dominance area of solutions $\boldsymbol{a}'$, $\boldsymbol{b}'$ and $\boldsymbol{c}'$ is expanded from the original one of $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{c}$. This causes that $\boldsymbol{a}'$ dominates $\boldsymbol{b}'$ and $\boldsymbol{c}'$, and $\boldsymbol{b}'$ dominates $\boldsymbol{c}'$. That is, expansion of dominance area by smaller $S < 0.5$



(a) Conventional CDAS



(b) Proposed S-CDAS

Fig. 3. Difference of front classification between CDAS [7] and proposed S-CDAS

works to produce a more fine grained ranking of solutions and would strengthen selection.

In case of solving MaOP, CDAS shows better search performance than conventional NSGA-II when we set $S < 0.5$. The hypervolume value for obtained non-dominated solutions is larger than NSGA-II due to convergence improvement by the effects of fine grained ranking of solutions using the optimum parameter S^* [8]. However, as a side effect, the diversity of obtained solutions deteriorates when we decrease S from 0.5. Also, to bring out desirable performance of CDAS, we have to find out appropriate parameter S experimentally.

4 Proposed Method

4.1 Motivation

To solve aforementioned problems in the conventional CDAS and achieve well-balanced search performance between convergence and diversity toward optimal Pareto front on MaOPs, the proposed S-CDAS reclassifies solutions in each front classified by NSGA-II to realize fine-grained ranking that is different from CDAS.

Fig.3 shows that difference of front classification between CDAS [7] and S-CDAS, where all solutions area included in $\mathcal{F}_1$ by the classification of NSGA-II. When we classify these solutions by the conventional CDAS, three fronts $\mathcal{F}'_1, \mathcal{F}'_2$ and $\mathcal{F}'_3$ are obtained as shown in **Fig.3** (a). In this example, since the angles of dominance area specified by all solutions are the same, only well-converged solutions distributed in a limited central region of Pareto front are included in $\mathcal{F}'_1$. Accordingly, extreme solutions having the maximum fitness values are dominated by these solutions. Thus, it becomes difficult to maintain diversity in the population. On the other hand, when we classify solutions by S-CDAS, different three fronts $\mathcal{F}''_1, \mathcal{F}''_2$ and $\mathcal{F}''_3$ are obtained as shown in **Fig.3** (b), where

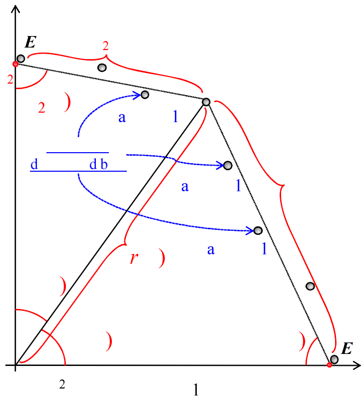


Fig. 4. Reclassification of solutions in $\mathcal{F}_1$ by the proposed algorithm

the angle of dominance area specified by each solution is different. Also, S-CDAS always guarantees the inclusion of extreme solutions in $\mathcal{F}_1''$. In other words, in the proposed method, not only highly converged solutions but also widely distributed ones are classified into higher front.

4.2 Algorithm of S-CDAS

S-CDAS reclassifies the solutions in each front $\mathcal{F}_j$ $(j = 1, 2, \cdots)$ which is classified by NSGA-II using the following procedure. **Fig.4** shows the illustration of the reclassification by the proposed algorithm.

***Step 1*:** Move the origin to $\boldsymbol{O} = (O_1, O_2, \cdots, O_m)$ in objective space. In this work, we set $O_i = f_i^{min} - \delta$ $(i = 1, 2, \cdots, m)$, where f_i^{min} is the minimum value of the i-th objective function in $\mathcal{F}_j$ and δ is a tiny constant value.

***Step 2*:** Create a set of landmark vectors $\mathcal{L} = \{\boldsymbol{p}_1, \boldsymbol{p}_2, \cdots, \boldsymbol{p}_m\}$, where $\boldsymbol{p}_i = (O_1, O_2, \cdots, f_i^{max} - \delta, \cdots, O_m)$, and f_i^{max} denotes the maximum value of the i-th objective function, which is derived from the extreme solution $\boldsymbol{E}_i$ in $\mathcal{F}_j$.

***Step 3*:** Repeat the following calculation for all solutions in $\mathcal{F}_j$.

***Step 3-1*:** For a single solution $\boldsymbol{x}$, calculate $\boldsymbol{\varphi}(\boldsymbol{x}) = (\varphi_1(\boldsymbol{x}), \varphi_2(\boldsymbol{x}), \cdots, \varphi_m(\boldsymbol{x}))$ by Eq.(2) derived from sine theorem. Here, $\varphi_i(\boldsymbol{x})$ is the angle determined by the solution $\boldsymbol{x}$ and the landmark vector $\boldsymbol{p}_i$ in the i-th objective function. $\boldsymbol{\varphi}(\boldsymbol{x})$ determines the individual dominance area of $\boldsymbol{x}$, which does never dominate extreme solutions $\boldsymbol{E}_i$ $(i = 1, 2, \cdots, m)$ having the maximum fitness value for each objective function.

$$\varphi_i(\boldsymbol{x}) = \sin^{-1}\left\{\frac{r(\boldsymbol{x}) \cdot \sin(\omega_i(\boldsymbol{x}))}{l_i(\boldsymbol{x})}\right\} \qquad (i = 1, 2, \cdots, m), \tag{2}$$

where $l_i(\boldsymbol{x})$ is Euclidean distance between the solution $\boldsymbol{x}$ and the landmark vector $\boldsymbol{p}_i$.

***Step 3-2*:** Modify fitness values of all other solutions $\boldsymbol{y} \in \mathcal{F}_j$ by the following equation

$$f_i^{'}(\boldsymbol{y}) = \frac{r(\boldsymbol{y}) \cdot \sin(\omega_i(\boldsymbol{y}) + \varphi_i(\boldsymbol{x}))}{\sin(\varphi_i(\boldsymbol{x}))} \quad (i = 1, 2, \cdots, m). \tag{3}$$

***Step 3-3*:** Check dominance relations between the solution $\boldsymbol{x}$ and all other solutions $\boldsymbol{y} \in \mathcal{F}_j$. If a solution $\boldsymbol{y} \in \mathcal{F}_j$ is dominated by $\boldsymbol{x}$, the counter ($rank$) of $\boldsymbol{y}$ is incremented.

***Step 4*:** Finally, reclassify all the solutions in $\mathcal{F}_j$ based on the accumulated $rank$ values, i.e., smaller rank corresponds to higher front, and larger rank corresponds to lower front. When multiple solutions have a same $rank$ value, they are included in the same front.

In the proposed method, reclassification procedure is performed for each non-dominated front $\mathcal{F}_j$ $(j = 1, 2, \cdots)$ obtained by NSGA-II. That is, the proposed method makes front distribution fine-grained, but superiority of each solution is never overturned by S-CDAS. In other words, superiority of solutions in fronts obtained by NSGA-II is maintained even after the reclassification by S-CDAS.

After the reclassification of all fronts, the proposed algorithm selects parent solutions $\mathcal{P}_t$ from higher fronts until filling up the half size of entire population similar to NSGA-II [2]. Also, to create offspring solutions, the crowded tournament selection is applied [2].

5 Experimental Results and Discussion

5.1 Preparation

In this work, we verify the search performance of the proposed method in many-objective 0/1 knapsack problems [10] by comparing with NSGA-II [2], IBEA$_{\epsilon+}$ [6], MSOPS [4] and CDAS [7]. We generate problems with $m = \{4, 6, 8, 10\}$ objectives, $n = 100$ items, and feasibility ratio $\phi = 0.5$. For all algorithms, we adopt two-point crossover with a crossover rate $P_c = 1.0$, and apply bit-flipping mutation with a mutation rate $P_m = 1/n$. In the following experiments, we show the average performance with 30 runs, each of which spent $T = 2,000$ generations. Population size is set to $N = 200$ (size of parent and offspring population are $|P_t| = |Q_t| = 100$). In IBEA$_{\epsilon+}$, scaling parameter k is set to 0.05 similar to [6]. Also, in MSOPS, we use $W = 100$ uniformly distributed weight vectors [9], which maximizes *Hypervolume* (HV) [11] in the experiments.

In this work, to evaluate search performance of MOEA we use HV, which measures the m-dimensional volume of the region enclosed by the obtained non-dominated solutions and a dominated reference point in objective space. Here we use $\boldsymbol{r} = (0, 0, \cdots, 0)$ as the reference point. Obtained POS showing a higher value of hypervolume can be considered as a better set of solutions from both convergence and diversity viewpoints. To calculate the hypervolume, we use the improved dimension-sweep algorithm proposed by Fonseca et al. [12], which significantly reduces computational time especially for large m. To provide additional information separately on convergence and diversity of the obtained POS, in this work we also use *Norm* [13] and *Maximum Spread* (MS) [11], respectively. Higher value of $Norm$ generally means higher convergence to true POS. On the other hand, Higher MS indicates better diversity in POS which can be approximated widely spread Pareto front.

5.2 Performance Comparison with Conventional CDAS

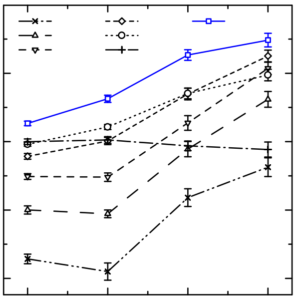
First, we observe the search performance of CDAS [7] and the proposed S-CDAS as we vary the number of objectives. **Fig.5** (a) $\sim$ (c) show results on HV as a combined metric of convergence and diversity, $Norm$ as a measure of convergence, and MS as a measure of diversity. The vertical bars, overlaying the markers, represent 95% confidence intervals. For CDAS, we plot multiple results when we vary the parameter S in the range $[0.25, 0.50]$. In these figures, all the plots are normalized by the results of NSGA-II [2].

From **Fig.5** (b) and (c), the conventional CDAS increases $Norm$ while decreases MS as we decrease S from 0.5. Obviously, there is a trade-off between convergence and diversity in the solutions obtained by CDAS. Therefore, only when we select well-balanced S between convergence and diversity, we can achieve higher HV values as shown in **Fig.5** (a), where the optimal parameters of CDAS to maximize HV are $S^* = \{0.50, 0.45, 0.45, 0.40\}$ for $m = \{4, 6, 8, 10\}$ objectives, respectively. On the other hand, both $Norm$ and MS achieved by S-CDAS are higher than CDAS with $S = 0.45$. Consequently, as shown in **Fig.5** (a) the value of HV achieved by S-CDAS are higher than CDAS with any S for all $m = \{4, 6, 8, 10\}$ objectives problems.

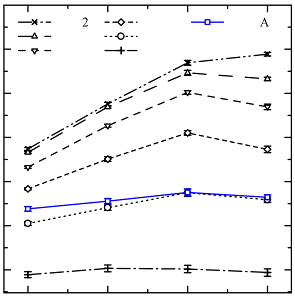
5.3 Performance Comparison with Conventional MOEAs

Second, we compare the search performance of the proposed method (S-CDAS) with conventional MOEAs: NSGA-II, CDAS, IBEA$_{\epsilon+}$ [6] and MSOPS [4] as we vary the number of objectives. **Fig.6** (a) $\sim$ (c) show results on HV, $Norm$ and MS. For CDAS, we plot only results using the optimal parameter S^* that maximizes HV. Similar to **Fig.5**, all the plots are normalized by the results of NSGA-II.

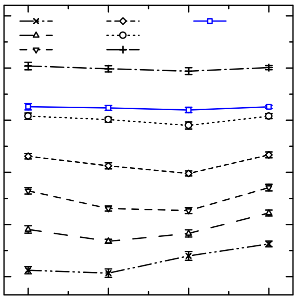
NSGA-II achieves the highest MS for $m = \{6, 8, 10\}$ objectives while $Norm$ remains the minimum. That is, NSGA-II can obtain well-distributed POS, but its convergence is poor in MaOP. Consequently, the values of HV are also the lowest for $m = \{6, 8, 10\}$ objectives. IBEA$_{\epsilon+}$ achieves the highest convergence, but its diversity is the minimum. MSOPS achieves balanced search on both convergence and diversity, and consequently it achieves the highest HV among conventional MOEAs compared in **Fig.6**. On the other hand, S-CDAS achieves higher $Norm$ and MS than conventional NSGA-II. Although $Norm$ achieved by S-CDAS are lower than IBEA, MSOPS and CDAS in $m > 8$ objectives, S-CDAS achieves



(a) *HV*

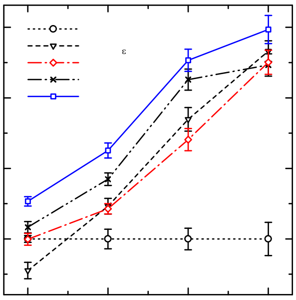


(b) *Norm*

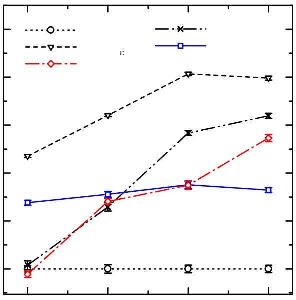


(c) *MS*

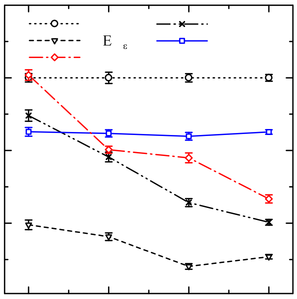
Fig. 5. Performance comparison with conventional CDAS [7]



(a) *HV*



(b) *Norm*



(c) *MS*

Fig. 6. Performance comparison with conventional MOEAs

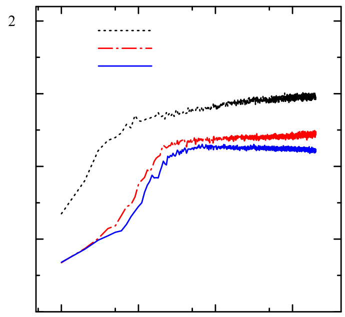


Fig. 7. Transition of front distribution over generation ($m = 8$)

high MS next to NSGA-II. Consequently, S-CDAS achieves the highest HV among all MOEAs compared in **Fig.6** for $m = \{4, 6, 8, 10\}$ objectives problems due to well-balanced search between convergence and diversity.

5.4 Algorithm's Behavior of NSGA-II, CDAS and S-CDAS

Here we observe the transition of solutions search by NSGA-II, CDAS($S^* = 0.45$) and S-CDAS in $m = 8$ objectives problem. Fist, we show the transition of the number of solutions belonging to the top front for each algorithm over generation in **Fig.7**. Among three methods, the ratio that the top front of NSGA-II ($\mathcal{F}_1$) occupied in the population is noticeably larger than CDAS and S-CDAS. This means more difficult in NSGA-II to discriminate solutions to obtain enough selection pressure. Consequently, the convergence of POS obtained by NSGA-II noticeably deteriorate especially for large m as shown in **Fig.6 (b)**. On the other hand, we can see that CDAS and S-CDAS considerably reduce the ratio of the top front ($\mathcal{F}_1'$ and $\mathcal{F}_1''$) and enhance selection pressure by using extended dominance area. Consequently, CDAS and S-CDAS improve the convergence of the obtained POS compared to NSGA-II as shown in **Fig.6 (b)**.

Next, we focus on the transition of extreme solutions $\boldsymbol{E}_i$ $(i = 1, 2, \cdots, m)$, each of which has maximum value for i-th objective function. **Fig.8** shows the transition of averaged maximum objective value in the top front given by $\bar{f}^{max} = \frac{1}{m} \cdot \sum_{i=1}^{m} f_i(\boldsymbol{E}_i)$. In CDAS($S^* = 0.45$), the possibility that extreme solutions $\boldsymbol{E}_i$ $(i = 1, 2, \cdots, m)$ are dominated and dismissed in parent selection is high as shown in **Fig.3 (a)**. Consequently, $\bar{f}^{max}$ obtained by CDAS becomes lower than NSGA-II over generations. On the other hand, S-CDAS achieves high $\bar{f}^{max}$ which is comparative with NSGA-II, since it has a mechanism to keep extreme solutions $\boldsymbol{E}_i$ $(i = 1, 2, \cdots, m)$ in $\mathcal{F}_1''$. S-CDAS achieves better diversity for the obtained POS than conventional CDAS as shown in **Fig.6 (c)**, although S-CDAS reduces the size of the top front $\mathcal{F}_1''$ similar to conventional CDAS.

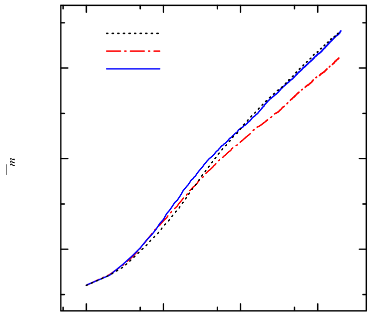


Fig. 8. Transition of average maximum objective value over generation ($m = 8$)

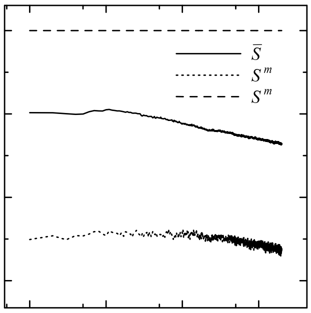


Fig. 9. Transition of $\bar{S}$, S^{max} and S^{min} in S-CDAS over generation ($m = 8$)

Finally, we observe the transition of dominance area which is self-controlled by S-CDAS for each solutions. For all solutions in top front $\mathcal{F}_1''$, we calculate the average angle $\bar{\varphi}$ from each solution's $\varphi_i (i = 1, 2, \cdots, m)$ which determines the dominance area, and transform $\bar{\varphi}$ to $\bar{S}$ by $\bar{\varphi}/\pi$. **Fig.9** shows the transition of $\bar{S}$ as well as the maximum S^{max} and the minimum S^{min} over generation. From this figure, $\bar{S}$ is gradually decreasing as we spend more generations. That is, S-CDAS gradually expands dominance area of solutions and strengthen selection pressure while constructing variety of dominance areas in the range $[S^{min}, S^{max}]$ for each solution. Since S^{max} is 0.5 throughout the entire evolution, S-CDAS achieves fine-grained ranking than NSGA-II similar to CDAS while keeping higher $\bar{f}^{max}$ than CDAS similar to NSGA-II.

6 Conclusions

In this work, we have proposed a modification of CDAS [7] called S-CDAS, which self-controls dominance area for each solution without the need of an external parameter. S-CDAS realizes fine-grained ranking that always guarantees the inclusion of extreme solution in top front, which is different from conventional CDAS. Through performance verification using many-objective 0/1 knapsack problems with $m = 4 \sim 10$ objectives, we have shown that S-CDAS achieves well-balanced search performance on both convergence and diversity compared to NSGA-II, IBEA$_{\epsilon+}$, MSOPS and conventional CDAS with optimal parameter S^*. Also, we have observed the transition of solutions search to see algorithm's behavior in detail.

As future works, we are planning to verify the search performance of the proposed algorithm when we vary the size of solution space and the feasibility ratio. Also, we want to apply our method to many-objective continuous optimization problems.

References

1. Deb, K.: Multi-Objective Optimization using Evolutionary Algorithms. John Wiley & Sons, Chichester (2001)
2. Deb, K., Agrawal, S., Pratap, A., Meyarivan, T.: A Fast Elitist Non-Dominated Sorting Genetic Algorithm for Multi-Objective Optimization: NSGA-II. KanGAL report 200001 (2000)
3. Zitzler, E., Laumanns, M., Thiele, L.: SPEA2: Improving the Strength Pareto Evolutionary Algorithm. TIK-Report, No.103 (2001)
4. Hughes, E.J.: Evolutionary Many-Objective Optimisation: Many Once or One Many? In: Proc. IEEE Congress on Evolutionary Computation (CEC 2005), pp. 222–227 (September 2005)
5. Aguirre, H., Tanaka, K.: Working Principles, Behavior, and Performance of MOEAs on MNK-Landscapes. European Journal of Operational Research 181(3), 1670–1690 (2007)
6. Zitzler, E., Kunzili, S.: Indicator-Based Selection in Multiobjective Search. In: Yao, X., Burke, E.K., Lozano, J.A., Smith, J., Merelo-Guervós, J.J., Bullinaria, J.A., Rowe, J.E., Tiňo, P., Kabán, A., Schwefel, H.-P. (eds.) PPSN 2004. LNCS, vol. 3242, pp. 832–842. Springer, Heidelberg (2004)
7. Sato, H., Aguirre, H., Tanaka, K.: Controlling Dominance Area of Solutions and Its Impact on the Performance of MOEAs. In: Obayashi, S., Deb, K., Poloni, C., Hiroyasu, T., Murata, T. (eds.) EMO 2007. LNCS, vol. 4403, pp. 5–20. Springer, Heidelberg (2007)
8. Sato, H., Aguirre, H., Tanaka, K.: Effect of Controlling Dominance Area of Solutions in MOEAs on Convex Problems with Many Objectives. In: Proc. 7th Intl. Conf. on Optimization: Techniques and Applications (ICOTA7), in CD-ROM (2007)
9. Wagner, T., Beume, N., Naujoks, B.: Pareto-, Aggregation-, and Indicator-Based Methods in Many-Objective Optimization. In: Obayashi, S., Deb, K., Poloni, C., Hiroyasu, T., Murata, T. (eds.) EMO 2007. LNCS, vol. 4403, pp. 742–756. Springer, Heidelberg (2007)
10. Zitzler, E., Thiele, L.: Multiobjective optimization using evolutionary algorithms – a comparative case study. In: Eiben, A.E., Bäck, T., Schoenauer, M., Schwefel, H.-P. (eds.) PPSN 1998. LNCS, vol. 1498, pp. 292–304. Springer, Heidelberg (1998)
11. Zitzler, E.: Evolutionary Algorithms for Multiobjective Optimization: Methods and Applications, PhD thesis, Swiss Federal Institute of Technology, Zurich (1999)
12. Fonseca, C., Paquete, L., López-Ibáñez, M.: An Improved Dimension-sweep Algorithm for the Hypervolume Indicator. In: Proc. 2006 IEEE Congress on Evolutionary Computation, pp. 1157–1163 (2006)
13. Sato, M., Aguirre, H., Tanaka, K.: Effects of δ-Similar Elimination and Controlled Elitism in the NSGA-II Multiobjective Evolutionary Algorithm. In: Proc. IEEE Congress on Evolutionary Computation (CEC 2006), pp. 3980–3398 (2006)