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A Method for Distributing Reference Points Uniformly along the Pareto Front of DTLZ Test Functions in Many-Objective Evolutionary Optimization

Yifan Li, Hailin Liu, Kan Xie and Xiuli Yu

***Abstract*—Testing and assessing the performance of proposed algorithms in a right way is a very important part in evolutionary optimization. If the number of optimization objectives is more than three, it's commonly referred as many-objective optimization problem(MaOP). Despite the fact there exists many test functions and assessment metrics, all of which are designed for testing and assessing the performance of evolutionary algorithms, the test functions DTLZ proposed by Deb(2005) and the assessment metrics Inverted Generational Distance(IGD) proposed by Bosman and Thierens(2003) are both widely recognized and used in most of evolutionary optimizations, especially in many-objective evolutionary optimization. When using IGD as a assessment tool, the vital task is to distribute reference points uniformly along the Pareto front of test functions. To overcome it, however, is really difficult with the number of objectives increase. For dealing with such difficulty, this paper introduce an approach called Inverse Transform Technique(ITT) suggested by Fang and Wang(1999) and successfully distributes reference points uniformly along the Pareto front of DTLZ test functions. In the final, three representative evolutionary algorithms: NSGA-II, MOEA/D and MOEA/D-M2M are selected to test their performance in dealing with many-objective optimization. As a result, extensive experiments and analysis are conducted on four DTLZ test functions from five to ten objectives.**

I. INTRODUCTION

IN real world, it is a common phenomenon that an optimization problem is composed of many objectives and needed to be optimized simultaneously. If the number of optimization objectives is more than three, it's referred as many-objective optimization problem (MaOP). A generic MaOP can be stated as follow:

$$\begin{aligned} &min \quad F(x) = (f_1(x), ..., f_M(x)) \\ &subject \quad to: x \in \prod_{i=1}^{N}[a_i, b_i] \end{aligned} \tag{1}$$

where x is a decision vector with N dimensions, donated as $x = (x_1, x_2, ..., x_N)^T$. $\prod_{i=1}^{N}[a_i, b_i] \subseteq R^N$ is the problem decision space and for each $i \in \{1, .., N\}$ satisfies $-\infty < a_i < b_i < +\infty$. $F(x)$ is a objective vector that consists M objectives. Let $u = (u_1, u_2, ..., u_M)^T, v = (v_1, v_2, ..., v_M)^T$ are two vectors in objective space. u is said to dominate v, denoted by $u \prec v$, if and only if $u_i \leq v_i$ for each $i \in \{1, .., M\}$ and $u_j < v_j$ for at least one index $j \in \{1, .., M\}$. The point x^* is said as a Pareto-optimal point if there exists no point to dominate $F(x^*)$ in the objective space. The set of all Pareto-optimal points in decision space is called Pareto set (PS) while its corresponding set of objective vector is called Pareto front(PF). When referring to a MaOP, the objectives usually conflict with each other, which lead to the fact that one objective is optimized at the expense of deterioration in other objectives, so the solution for a MaOP is not just one single solution but a Pareto set. Nevertheless, in most cases, decision makers usually focus on the several alternative solutions but not all optimal solutions, from which they can select one or more as final decision. Therefore, in recent years, evolutionary algorithms(EAs) are widely used to solve MaOP.

Due to the successful application in science, economics, communication and engineering in real-life [3][6][11], EAs have attracted high attentions from evolutionary community and as a result, many effective evolutionary algorithms have been proposed. Specially, the proposed algorithms could be roughly divided into two categories.The first category is based on Pareto dominated relations such as NSGA-II, SPEA2 and PESA [2][8][9] and second category is based on decomposition approach such as MOEA/D and MOEA/D-M2M [10][19]. These algorithms are originally designed for dealing with two or three objectives problem and extensive experiments have show these algorithms are capable of good search abilities in two or three objectives optimization. With the further development, the evolutionary community gradually focus on the many-objective evolutionary algoritms that objective number is more than three. A lot of researches have suggested the above algorithms meet with obstacles, mainly from the inefficiency of selection operators, high computation cost, and difficulty in visualization of objective space [7][12][13][15]. Nevertheless, there no extensive conducted experiments to reveal the real search abilities of theses algorithm categories.

Meanwhile, testing and assessing the performance of proposed algorithms in a right way also attracts researchers attention in evolutionary community and many useful performance assessment metrics have been proposed one by one from the view of the convergence and diversity of solutions,

Yifan Li and Hailin Liu are with School of Applied Mathematics, Guangdong University of Technology, Guangdong Key Laboratory of IoT Information Technology, Guangzhou, China (Email: 2464290221@qq.com;hlliu@gdut.edu.cn).

Kan Xie is with School of Automation, Guangdong University of Technology, Guangdong Key Laboratory of IoT Information Technology, Guangzhou, China

Xiuli Yu is with School of Management,Guangdong University of Technology, Guangzhou, China(Email:24976796@qq.com)

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for instance, the mertic Generational Distance(GD) proposed by Van(1998), the metric Hypervolumn proposed by Zitzler E and Thiele Lsuggest(1999)[16][21]. Among all performance assessment metrics for the evolutionary algorithms, Inverted Generational Distance(IGD)[1], taking advantage on assessing the convergence and diversity of solutions, is a widely recognized and used performance assessment metric [18][20]. It works as equation (2),

$$IGD(P^*, P) = \frac{\sum_{v\epsilon P^*} d(v, P)}{|P^*|} \tag{2}$$

where P^* is a set of distributed uniformly reference points along the PF and P is an approximative solutions to the PF. $d(v, P)$ is the minimum Euclidean distance between v and the points in P. If reference points in P^* are distributed uniformly along the PF very well, $IGD(P^*, P)$ can well measure the convergence and diversity of P. To have a low value of IGD(P^*, P), the set P must be very close to the PF and can not miss any part of the whole. As a consequence, the reference points distributed uniformly along the PF is essential for IGD to assess algorithms performance. However, to realize it is no a easy task in many-objectives optimization. The frequent used approach is taking the points equally along each dimension in decision space and then combine them. This approach is rather simple and easy to use so that the people are easily unconscious of its inherent weakness. On the one hand, it's only suitable for the low dimension because reference points grow quickly with a exponential speed when the objective number increase. On the other hand, the points produced by this approach is not mean to the distributed uniformly points in objective space if the mapping between decision space and objective space is nonlinear. Let us consider the equations like equation (3).

$$\begin{cases} f_1(x) = cos(x_1)cos(x_2) \\ f_2(x) = cos(x_1)sin(x_2) \\ f_3(x) = sin(x_1) \end{cases} \tag{3}$$

we firstly take ten points equally in each dimension in decision space and then combine them. The next is compute their objective value vector (f_1, f_2, f_3) according to equation (3), which finally are plotted in Fig(1). Via Fig(1), it can be seen that the top of the PF is empty, which means that the points produced by this approach may miss some part of the PF and can't be on behalf of the real PF, so it illustrates clearly weakness of this approach.

Therefore, it's really meaningful to distribute reference points uniformly along the PF. The main work of this paper is to explain how to gain the uniform reference points along the PF on the DTLZ test functions. The reason why we choose DTLZ test function to research is for its widely use. When a researcher try to illustrate the performance of proposed evolutionary algorithm, he will select the DTLZ as the test function because the other researcher have done so. On the other hand, these test functions really able to test the selection operator performance of evolutionary algorithm. Moreover, it is for its optimal PF in a sense. For this reason, the research on uniform design along the PF on DTLZ test functions is meaningful and necessary for further research work in evolutionary community, because it will provide a correct approach to assess which would contribute to the evolutionary algorithms development.

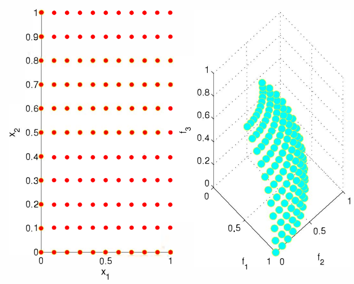

Fig. 1. Relation between decision space and objective space

The rest of this paper is organized as follows: Section II is mainly about the principle to take uniform reference points along the PF on DTLZ test functions via IIT approach. Section III is the experimental study, in this part,extensive experiment and analysis would be conducted for three state-of-the-art algorithms: NSAG-II, MOEA/D and MOEA/D-M2M. Finally, conclusion will be conducted in Section IV.

II. The Uniform Design on DTLZ Test Functions

A. Introduction *for DTLZ Test Functions*

The DTLZ test functions constructed by Deb(2005) is a series of used frequently test functions for evaluating and comparing the performance of EAs, which also attributes to EAs improvement. There are totally nine functions problem in DTLZ test functions and each test function in DTLZ has its own characters in order to test specific ability of EAs. The DTLZ1 test function composed by M objectives is simple test problem because its optimal boundary is a linear hyperplane, which could be stated by eqution(4)

$$\begin{cases} min \quad f_1(x) = \frac{1}{2}x_1x_2...x_{M-1}[1 + g(X_M)] \\ min \quad f_2(x) = \frac{1}{2}x_1x_2...(1 - x_{M-1})[1 + g(X_M)] \\ \quad ... \\ min \quad f_{M-1}(x) = \frac{1}{2}x_1(1 - x_2)[1 + g(X_M)] \\ min \quad f_M(x) = \frac{1}{2}(1 - x_1)[1 + g(X_M)] \end{cases} \tag{4}$$

$$g(X_M) = 100\{| X_M | + \sum_{x_i \in X_M} (x_i - 0.5)^2 - cos[20\pi(x_i - 0.5)]\}$$

where X_M is the last $N - M + 1$ decision variables in the decision vetctor x and $| X_M |= N - M + 1$. For each i in $\{1, 2, .., n\}$, it satisfies $0 \leq x_i \leq 1$. And the objective optimal boundary is $\sum_{i=1}^{M} f_i = 0.5$.

The DTLZ2 test function has a similar form with the DTLZ1 and it could be stated by equation (5), whose optimal boundary is a hypersphere that satisfies function $\sum_{i=1}^{M} f_i^2 = 1$. As for the DLTZ3 and DTLZ4 test function, they are both constructed from the DTLZ2 test functions and consequently they has the same optimal boundary with DTLZ2 test function.

$$\begin{cases} minf_1(x) = cos(\frac{\pi}{2}x_1)...cos(\frac{\pi}{2}x_{M-1})[1+g(X_M)] \\ minf_2(x) = cos(\frac{\pi}{2}x_1)...sin(\frac{\pi}{2}x_{M-1})[1+g(X_M)] \\ \quad ... \\ minf_{M-1}(x) = cos(\frac{\pi}{2}x_1)sin(\frac{\pi}{2}x_2)[1+g(X_M)] \\ minf_M(x) = cos(\frac{\pi}{2}x_1)[1+g(X_M)] \end{cases} \tag{5}$$

$$g(X_M) = \sum_{x_i \in X_M} (x_i - 0.5)^2$$

The DTLZ6 and DTLZ7 are mainly designed for testing the convergence to a specified curve, which could be stated by equation (6). Though they also have similar form with the DLLZ2 test functionits objective optimal boundary is the curve across the whole objective space and its objective functions satisfies $\sum_{i=1}^{m} f_i^2 = 1$ if and only if all x_i in X_M are equal to 0.5.

$$\begin{cases} minf_1(x) = cos(\frac{\pi}{2}\theta_1)...cos(\frac{\pi}{2}\theta_{M-1})[1+g(X_M)] \\ minf_2(x) = cos(\frac{\pi}{2}\theta_1)...sin(\frac{\pi}{2}\theta_{M-1})[1+g(X_M)] \\ \quad ... \\ minf_{M-1}(x) = cos(\frac{\pi}{2}\theta_1)sin(\frac{\pi}{2}\theta_2)[1+g(X_M)] \\ minf_M(x) = cos(\frac{\pi}{2}\theta_1)[1+g(X_M)] \end{cases} \tag{6}$$

$$\theta_i = \frac{\pi}{4(1+g(X_M))}[1+2g(X_M)x_i], (i = 2, 3, ..., M-1)$$

$$g(X_M) = \sum_{x_i \in X_M} (x_i - 0.5)^2$$

B. The uniform design principle for the PF of DTLZ test functions

The DTLZ test function is a important test series for MaOP, not only for it's easy to remember and use, but also for it can test the various abilities of proposed algorithm. Notice the fact that the optimal boundary of the DTLZ test functions can be divided into three class: the hyperplane, the hypersphere and the hypercurve. Obviously, it's comparatively easy to take distributed uniformly points along the hypercurve, so the main work of this paper mainly focus on taking distributed reference points uniformly on the hyperplane and the hypersphere of the DTLZ test functions. Since the optimal boundary of the DTLZ test functions in objective space is mainly composed by the hyperplane or the hypersphere and the decision space is constructed by the hypercube, the problem can be transformed into taking uniform reference points in decision space and then let these reference points distribute uniformly along the optimal boundary of the DTLZ test functions. To avoid the case that the total number of reference points increase rapidly with a exponential speed when the objective number increase, here we adopt the uniform design method in statistics to take the uniform reference points in decision space. The statistics has developed many mature uniform design approaches in hypercube and we choose a pretty classical approach called good lattice method [14] to take reference points in decision space, it can be stated by equation (7),

$$\begin{cases} q_{ki} \equiv mod(kh_i, n) \\ x_{ki} = \dfrac{2q_{ki}-1}{2n} \end{cases} \tag{7}$$

where n is the total points number, x_{ki} is the coordinate of the kth point in ith dimension, h_i is a positive integer and satisfies simultaneously $h_i < n, (n, h_i) = 1, h_i \neq h_j$. Here, an approach called Inverse Transform Technique(ITT)

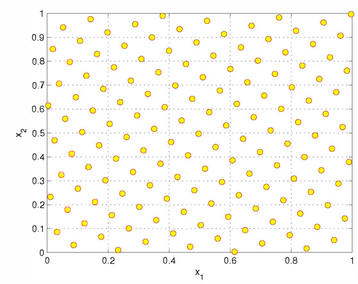

Fig. 2. Uniform design in decision space with 2 variables

suggested by Fang and Wang(1999) [4][17] will be introduce which is capable of taking uniform reference points along the PF. In order to fully use this approach, the next part mainly introduces the principle of ITT.

Assuming a random vector $F = (f_1, f_2, ..., f_s)^T$ distributes uniformly on area D, so the $p.d.f$ of F is:

$$p(F) = \begin{cases} \frac{1}{v(D)} & F \in D \\ 0 & F \notin D \end{cases} \tag{8}$$

Now, consider the transformation as follow:

$$f_i = f_i(x_1, x_2, ..., x_t), i = 1, 2, ..., M, t \leq M \tag{9}$$

then we have:

$$v(D) = \int_{D^*} J(F \to x)dx \tag{10}$$

where $v(D)$ is the volume of F, D^* is the value area of X, and the density function of $X = (x_1, x_2, ..., x_t)$ is:

$$\frac{1}{v(D)} J(F \to x) \tag{11}$$

if the equation (11) can be expressed as the product of all t density functions like equtation (12)

$$\frac{1}{v(D)} J(F \to x) = \prod_{j=1}^{t} p_j(x_j) \tag{12}$$

then we can make conclusion that:

1) $x_1, x_2, ..., x_t$ are independent variables.
2) the density function of x_j is $p_j(x_j), j = 1, 2, ..., t$.

From the equation (12), it can be extended, when a random vector could be divided into the product of several density function with one variable, these density function must be independent if the random vector's distribution is uniform. That is to say, if there exists the number of t independent density functions with only one variable and all of them satisfy the transformation given by the equation (12), the random vector is uniform. Therefore, according to the principle introduced above, it's not impossible to transform the distributed uniformly points in the decision space along the PF in the objective space. The computational process as follow:

1) Considering the mapping relation between decision variables x_j and objective variables $f_i, j = 1, 2, ..t; i = 1, 2, ...M$
2) Computing the cumulative distribution function $P_j(x_j)$ of every density function $p_j(x_j)$
3) Computing the inverse function $P_j(x_j)$ and let $x_j = P_j^{-1}(x_j)$
4) According to the x_j given by 3) and the mapping relation given by 1), compute the objective value

Based on the computational process above, Wang and Fang give the equation for the uniform design of hyperplane. it is showed in equation (13) and the final result could be seen in Fig (3).

$$\begin{cases} f_{ki} = x_{k1}^{\frac{1}{M}} \prod_{j=2}^{i} x_{kj}^{\frac{1}{M-j+1}} (1 - x_{k,i+1}^{\frac{1}{M-i}}), i = 1, ..., M-1 \\ f_{kM} = x_{k1}^{\frac{1}{M}} \prod_{j=2}^{M} x_{kj}^{\frac{1}{M-j+1}}, k = 1, 2, ..., n \end{cases} \tag{13}$$

As for the optimal boundary is hypersphere, the computa-

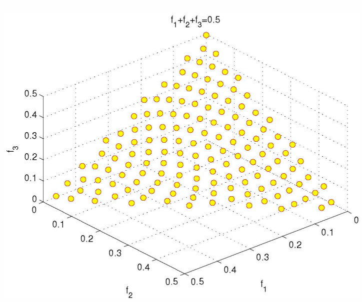

Fig. 3. Uniform Design for hyperplane with 3 dimensions

tional equations is as follow:
if $M = 2m$, then

$$\begin{cases} f_{k,2l-1} = d_{k1} cos(2\pi x_{k,m+l-1}) & l = 1, .., m \\ f_{k,2l} = d_{k1} cos(2\pi x_{k,m+l-1}) & k = 1, ...n \end{cases} \tag{14}$$

if $M = 2m + 3$, then

$$\begin{cases} f_{k1} = d_{k1}(1 - 2x_{km}) \\ f_{k2} = d_{k1}\sqrt{x_{km}(1 - 2x_{km})} cos(2\pi x_{k,m+1}) \\ f_{k3} = d_{k1}\sqrt{x_{km}(1 - 2x_{km})} sin(2\pi x_{k,m+1}) \\ f_{k,2l} = d_{k1} cos(2\pi x_{k,2l}) \\ f_{k,2l+1} = d_{k1} cos(2\pi x_{k,2l}) \\ l = 2, ..., m \end{cases} \tag{15}$$

where $d_{kl} = \sqrt{g_{kl} - g_{k,l-1}}$ and let $g_{km} = 1, g_{k0} = 0, k = 1, ..., n$ and $g_{ki} = g_{k,j+1} x_{kj}^{\frac{1}{j}}, j = m-1, m-2, .., 1$. Through this approach, the uniform points along the PF of hypersphere can be seen in Fig (4).

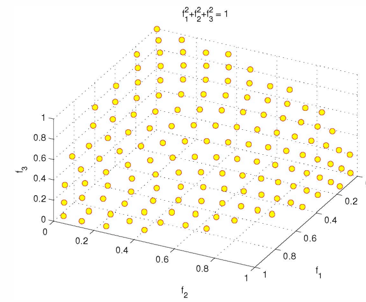

Fig. 4. Uniform Design for hypersphere with 3 dimensions

III. Experimental Study

This part is mainly to assess the performance for several state-of-the-art EAs in dealing with many-objective optimization problem, and here the algorithm NSGA-II, MOEA/D and MOEA/D-M2M are chosen. All of them can represent the state-of-art evolutionary development [5]. In addition, the experimental test for these algorithms remain on two or three objectives while do little on many-objective optimization problem, which does not reflect these algorithm performance in dealing with many-objective problems.

NSGA-II [2] is one of successful EAs based on domination-based fitness assignment. Through fast non-dominated sorting approach, the population is divided into different level and then the crowding distance of each individual is computed in the same level. When comparing the two individuals, the level is considered firstly and the crowding distance would be considered secondly in the case that the individual are incomparable by the non-domination level.

MOEA/D with Tchebycheff approach [19] is one of representative decomposition-based algorithms, which has well good search abilities. By using decomposition approach, a MaOP is decomposed into a number of scalar subproblems and optimized simultaneously. Specially, each subproblem is optimized only using information from its several neighboring subproblems, which decrease greatly the computational burden.

MOEA/D-M2M [10] is a recent EAs based on decomposition, it divides the optimization problem into some subproblems and each subproblem has its own population so that the subproblems can be optimized simultaneously in a collaborative way, which play a important role in the population diversity.

In the experimental study, the test functions DTLZ1 to DTLZ4 with 30 decision variables and 5 to 10 objectives are chosen to test the performance for the chosen algorithms above. The evolutionary parameters for each algorithm are the same setting: the size of population N is 300, the evolutionary generations Gen is 6000. In addition, assessment metric IGD is executed in every 200 times. Due to the essential difference of the algorithm themselves, their own parameters setting as follow:

1) MOEA/D: a uniformly spread of N weight vectors which is correspondence to a subproblem. The neighborhood number of each subproblem or weights is 30.
2) MOEA/D-M2M: The optimization problem is divided into 20 subproblems and the individual will be allocated automatically into subproblem region according to the distance between the individual and the weight.

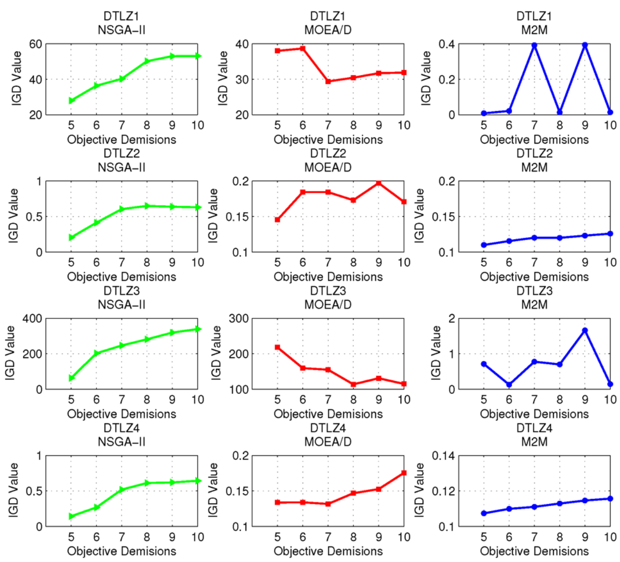

Fig. 5. The result of the algorithm

In order to reduce random effect from evolutionary operator, the experiment for each algorithm has been conducted 10 times and the final result is the average of these 10 experiments, which is showed in the Fig (5). Through Fig (5), some interesting result can be found. Firstly, with the increase of the objective number, the performance of NSGA-II is getting worse from DTLZ1 to DTLZ4 test function, that is to say, NSGA-II meets with great difficulty in dealing with many-objective optimization problem. However, as for MOEA/D and MOEA/D-M2M, it can be noticed that MOEA/D and MOEA/D-M2M both get worse on DTLZ2 and DLTZ4 test function in each objective dimension, but such situation does not appear on DTLZ1 and DTLZ3 test function. In addition, viewing from IGD valueMOEA/D-M2M performs better than NSGA-II and MOEA/D. How to explain such interesting results above? Now, let us see the evolutionary process of every algorithm in Fig (6) and Fig (7). From Fig (6), almost in every dimension, MOEA/D-M2M performs better than MOEA/D and NSGA-II on DTLZ1 and DTLZ3 test function. However, on DTLZ2 and DTLZ4 test function, it has similar performance between NSGA-II and MOEA/D as showed in Fig (7).

Next, this paper will explain the experimental results from three aspects. The dimension of objective space would be

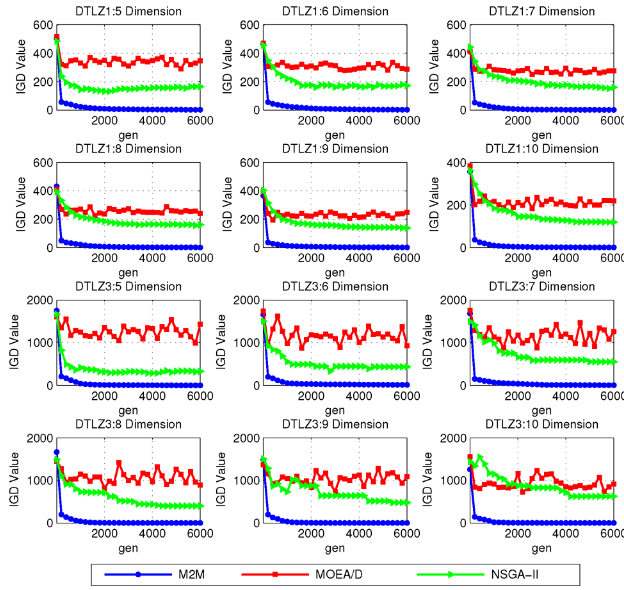

Fig. 6. Evolutionary Process of the algorithms

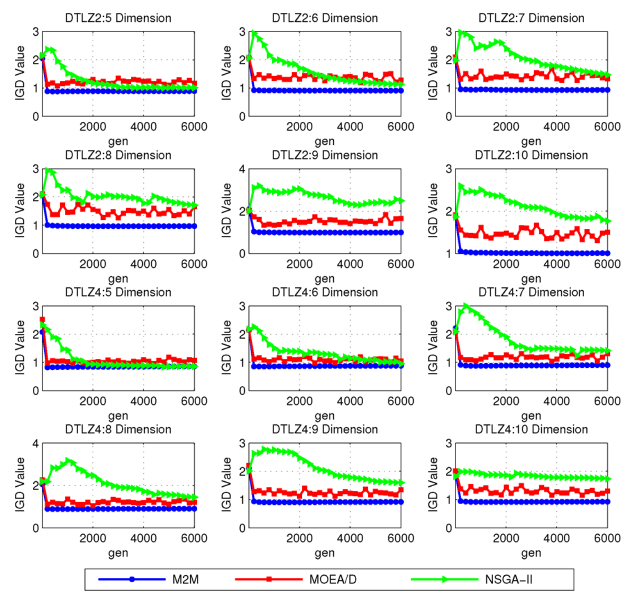

Fig. 7. Evolutionary Process of the algorithms

considered firstly. With the increase of the objective number the proportion of non-dominated solutions in the population increase, so the algorithm based on Pareto dominated relation, NSGA-II, is unable to select the correct solutions so as to provide selection pressure towards the PF. The algorithm MOEA/D and MOEA/D-M2M rely greatly on the a set weights which must spread uniformly in the objective space, but with the number of objectives problem increase, the weights produced by Lattice approach is unable to represent the whole objective space, which also lead to the deterioration both in convergence and diversity. Secondly, the construction of DTLZ test function play a important role on the algorithm performance. DTLZ1 and DTLZ3 has potential to test the convergence performance of the algorithm. Generally, the search region of population is often far away the PF, so affected by the evolutionary operator, all of three algorithm will converge toward the PF, which makes IGD value goes down. For the DTLZ2 and DTLZ4 test function, its main purpose is to test the diversity performance of the algorithm. And the research region of populations has been near by the PF, in this time, the population diversity decide IGD value. Due to a set of weights spread uniformly on the objective space, so the population diversity of MOEA/D-M2M and MOEA/D gain the great support. In contrary, the domination-based algorithm NSGA-II will perform worse than MOEA/D-M2M and MOEA/D. Thirdly, the update mechanism also have great effect on the algorithm performance, especially,which often determine the performance of the algorithm. In NSAG-II, the offspring is produced by championship approach according to the level and the crowding distance. For MOEA/D, it update its offspring via the neighborhood solution of the subproblem. As for the MOEA/D-M2M, when each subproblem update its solutions, it will compare the offspring solution and parent solution and chose the best one according to its corresponding weight. As a result, we could see MOEA/D-M2M performance better than NSGA-II and MOEA/D especially in DTLZ1 and DTLZ3.

IV. Conclusions

IGD is a representative performance assessment metric, which assess the algorithm performance by computing the average value of the minimum distance between each uniform reference point along the PF and the attained solutions. Due to it can assess the convergence and the diversity of the algorithm simultaneously, it is widely used in testing and assessing algorithms performance. However, when using IGD to assess a algorithm, it is necessary to distribute the reference points uniformly along the PF, which is often difficult in many-objective optimization problem. Through used the approach ITT suggested by Wang and Fang(1999), this paper achieve to distribute reference points uniformly along the PF on DTLZ test functions. Finally, extensive experiment and analysis have been executed for several the state-of-art EAs: NSGA-II, MOEA/D/D and MOEA/D-M2M. Through the experimental result, we would find MOEA/D-M2M performance better than NSAG-II and MOEA/D in some DTLZ test functions.

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