Dynamic Crowding Distance—A New Diversity Maintenance Strategy for MOEAs

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Abstract

In multi-objective evolutionary algorithms (MOEAs), the diversity of Pareto front (PF) is significant. For good diversity can provide more reasonable choices to decision-makers. The diversity of PF includes the span and the uniformity. In this paper, we proposed a dynamic crowding distance (DCD) based diversity maintenance strategy (DMS) (DCD-DMS), in which individual's DCD are computed based on the difference degree between the crowding distances of different objectives. The proposed strategy computes individuals' DCD dynamically during the process of population maintenance. Through experiments on 9 test problems, the results demonstrate that DCD can improve diversity at a high level compared with two popular MOEAs: NSGA-II and ε-MOEA.

1. Introduction

Researches on multi-objective optimization problems (MOPs) have becoming more and more popular in many applied areas. As one kind of the most effective and popular methods for solving MOPs, multi-objective evolutionary algorithms (MOEAs) have made great contributions to the optimization of MOPs in the last 30 years. Some famous MOEAs were introduced during this period, such as: NSGA-II [1], SPEA-II [2], PAES [3], PESA-II [4], and so on.

Diversity [5, 6], Convergence [7] and robustness [8] of Pareto solutions are hot issues in the research of MOEAs. It is of great importance for Pareto front (PF) with good diversity. For a good diversity can give decision-makers more reasonable and efficient selections. Two aspects must be considered in the diversity, one is the span of PF, and the other is the uniformity. The diversity maintenance strategy (DMS) is often realized in the process of population maintenance, which use truncation operator to wipe off individuals when the number of non-dominated solutions exceeds population size. In this paper, we proposed a new DMS which based on dynamic crowding distance (DCD). In the experiments, we use DCD combined with NSGA-II (label as DCD), compared two popular MOEAs: NSGA-II and ε-MOEA, the results indicates that DCD can improve the diversity of PF at a great level.

2. Related Works

2.1 Problems Description

Let us consider the following MOP:

$$\text{Min } \vec{f}(X) = (f_1(X), f_2(X), \cdots, f_r(X)) \quad (1)$$

$$g_i(X) \geq 0; (i = 1, 2, ..., k) \quad (2)$$

$$h_i(X) = 0; (i = 1, 2, ..., l) \quad (3)$$

where $\vec{f}(X)$ is the objective vector, r is the dimension of objectives, (2) and (3) are equality-constraints and inequality-constraints, $X = (x_1, x_2, ..., x_n)$ is variable vector, n is the dimension of variables, $X \in \Omega$, $\Omega \subseteq R^n$,where Ω is the feasible space, then, $\vec{f}: \Omega \to \Pi$, $\Pi \subseteq R^r$, Π is the objective space.

Optimal solutions of MOPs are named as Pareto optimal solutions (Pareto solutions for short). The task of optimization of MOPs is to find Pareto solutions $X^* = (x_1^*, x_2^*, ..., x_n^*)$. In MOEAs, Pareto solutions of current population are often called non-dominated solutions and the Pareto solutions set is often called non-dominated set (NDS). In this paper, PF_{True} is the true Pareto front.

2.2 Some Works on DMS

DMS is an important job in the design of MOEAs. Some related works can be concluded from 4 aspects: (1) Niche: Cavicchio et al [9] suggested a niche technique based on pre-selection, Goldberg et al [10] presented niche techniques based on crowding mechanism and sharing mechanism. (2) Crowding density: some studies used crowding distance [1], similitude and influences of individuals to compute crowding density. (3) Hyper-grid: Hyper-grid is used to maintain diversity of population in [11]. (4) Clustering analysis: Paper [12] divided the population to several clustering. Some more detailed information about DMS, one is infer to [13].

3. DCD based Diversity Maintenance Strategy (DCD-DMS)

Most of MOEAs use population maintenance to wipe off individuals when the number of non-dominated solutions exceeds population size. The procedure of population maintenance is based on DMS.

3.1 Crowding Distance

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As one of the most popular MOEA, NSGA-II use crowding distance (CD) based DMS (CD-DMS) in population maintenance. Individual's CD can be calculated as follows:

$$CD_i = \frac{1}{r}\sum_{k=1}^{r}\left|f_{i+1}^k - f_{i-1}^k\right| \quad (4)$$

where r is the dimension of objectives, f_i^k is the k^{th} objective of the i^{th} individual after ranking the population on one objective, formula (4) is not suit for boundary individuals. The CD of boundary individuals is given an infinite so that the boundary individuals can be copy to next population unconditionally. In this way, the span of population can be realized. Fig.1 shows individuals' CD of a 2-objective MOP.Black dots in the figure are non-dominated solutions, and white dots are dominated solutions. The CD of the i^{th} non-dominated individual is mean of rectangle's sides, which can be calculated follow the function (5).

$$CD_i = \frac{1}{2}[(f_{i+1}^1 - f_{i-1}^1) + (f_{i+1}^2 - f_{i-1}^2)]\text{; } (i = 1,\ldots,\lambda) \quad (5)$$

Suppose the population size is N, the number of non-dominated solutions in population is M, if $M>N$, then, MOEAs should use truncation operator to wipe off M-N individuals from NDS. CD-DMS calculate individuals'CD of NDS follow formula (4) and sort the NDS based on individuals' CD. Obviously, the CD-DMS is rough for it wipes off M-N individuals which have the lowest CD from NDS one time.

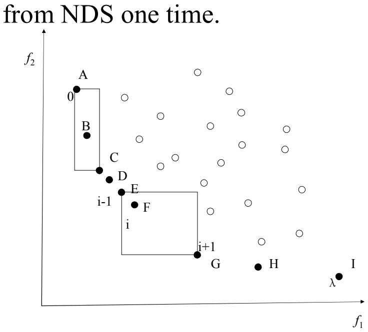


Fig.1. Crowding distance of individuals

3.2 The Proposal of DCD

From the analysis of CD discussed above, it is rough to use CD-DMS to maintain diversity of NDS. As shown in Fig.1, Two flaws of CD-DMS can be seen from Fig.1. (1) Due to CDs of individuals C, D and E are small, if CD-DMS is used to wipe off individuals in population maintenance, C, D and E will all be deleted. Therefore, there are not individuals existing in B and F. it is obvious that CD-DMS cannot obtain a NDS with good uniformity. (2) Some individuals, such as B, the CD of B is small because one side of the rectangle is short, although another side is long. However, the CD of F will be large because the length of one side almost equal to another side. If one individual must be deleted in B and F, CD-DMS will delete B and F will be retained. Nevertheless, diversity of B is better than F.

Apparently, individuals' CD are calculated only once during the process of population maintenance. In order to solve flaws of CD-DMS, we proposed a dynamic crowding distance (DCD) based DMS (DCD-DMS). The individuals' DCD are varying dynamically during the process of population maintenance. To solve flaw (1), DCD-DMS recalculate individuals' DCD after wipe off one individual from NDS during the process of population maintenance. To solve flaw (2), the i^{th} individual's DCD are calculated as follows:

$$DCD_i = \frac{CD_i}{\log(1/V_i)} \quad (6)$$

Where CD_i is calculated with formula (4), V_i is based on formula (7).

$$V_i = \frac{1}{r}\sum_{i=1}^{r}(\left|f_{i+1}^k - f_{i-1}^k\right| - CD_i)^2 \quad (7)$$

V_i is the variance of CDs of individuals which are neighbors of the i^{th} individual. V_i can give some information about the difference degree of CD in different objectives. For example, in the Fig.1, V_i of individual B is larger than F, and then DCD of B is larger than F. Therefore, if DCD-DMS is used in population maintenance, individuals similar to B in the NDS will have more chance to retain.

3.3 Design and Analysis of DCD

Suppose population size is N, the t^{th} generation NDS is $Q(t)$, the size of $Q(t)$ is M, if $M>N$, then use DCD-DMS to wipe off M-N individuals from NDS. The process of DCD-DMS is followed algorithm 1.

Algorithm 1. DCD-DMS

Step1. If $|Q(t)| <= N$, go to Step 5, else keep on;

Step2. Calculate all individuals' DCD in the $Q(t)$ based on formula (6);

Step3. Sort the NDS $Q(t)$ based on individuals' DCD;

Step4. Wipe off an individual which has the lowest DCD in the $Q(t)$;

Step5. If $|Q(t)| <= N$, stop population maintenance, else go to Step2 and keep on;

It can be seen from algorithm 1 that DCD-DMS has two important characters. (1) DCD-DMS only wipe off one individual every time and recalculate individuals' DCD after delete an individual from NDS. In this way, DCD-DMS can solve flaw (1) and obtain a PF with high uniformity. (2) The calculation of individuals' DCD is based on formula (6), which is very helpful because the difference degree of CD in different objectives is considered to maintain diversity of NDS.

4. Experiments and Datum Analysis

4.1 Test Problems and Parameters Settings

Test problems used for experiments are listed in table. 1.

Table.1. Test Problems

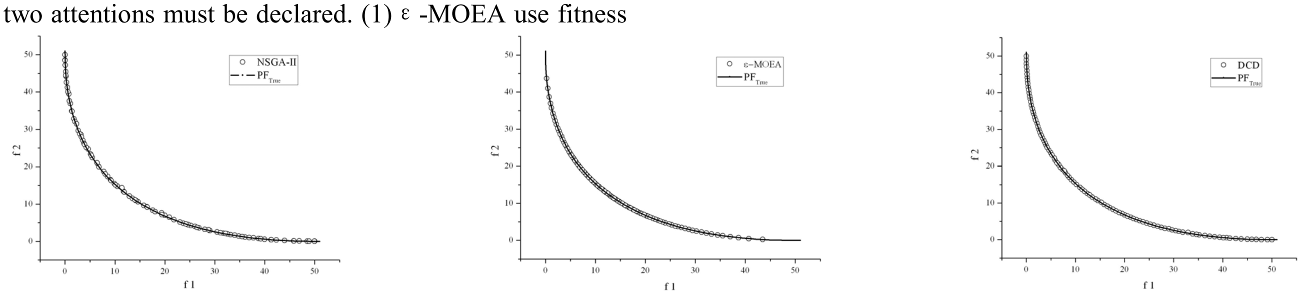
	MOPs	Expressions	Characters
F1	BNH1	$f_1(x,y)=x^2+y^2;\quad f_2(x,y)=(x-5)^2+(y-5)^2$	$-5\le x,y\le 10$
F2	POL	$f_1(x,y)=-[1+(A_1-B_1)^2+(A_2-B_2)^2];\quad f_2(x,y)=-[(x+3)^2+(y+1)^2]$ A_1=$0.5\sin 1-2\cos 1+\sin 2-1.5\cos 2$; A_2=$1.5\sin 1-\cos 1+2\sin 2-0.5\cos 2$ B_1=$0.5\sin x-2\cos x+\sin y-1.5\cos y$; $B_2=1.5\sin x-\cos x+2\sin y-0.5\cos y$	$-\pi\le x,y\le\pi$;convex; disconnected
F3	SCH1	$f_1(x)=x^2;\quad f_2(x)=(x-2)^2$	$-3\le x\le 3$; convex
F4	SCH2	$f_1(x)=\{-x, if\ x\le 1;\ -2+x, if\ 1<x\le 3;\ =4-x, if\ 3<x\le 4;\ =-4+x, if\ x\ge 4\ \}$ $f_2(x)=(x-5)^2$	$-5\le x\le 10$
F5	FON1	$f_1(x,y)=1-\exp(-(x-1)^2-(y+1)^2)$; $f_2(x,y)=1-\exp(-(x+1)^2-(y-1)^2)$	$-4\le x,y\le 4$; concave
F6	FON2	$f_1(\vec{x})=1-\exp(-\sum_{i=1}^{n}(x_i-\frac{1}{\sqrt{n}})^2)$; $f_2(\vec{x})=1-\exp(-\sum_{i=1}^{n}(x_i+\frac{1}{\sqrt{n}})^2)$	$n=3;-4\le x\le 4$; concave
F7	ZDT1	$f_1(x_1)=x_1$; $f_2(\vec{x})=g(1-\sqrt{(f_1/g)})$; $g(\vec{x})=1+9\sum_{i=2}^{m}x_i/(m-1)$	$n=30;0\le x_i\le 1$; convex
F8	ZDT2	$f_1(x_1)=x_1$; $f_2(\vec{x})=g(1-(f_1/g)^2)$; $g(\vec{x})=1+9\sum_{i=2}^{m}x_i/(m-1)$	$n=30;0\le x_i\le 1$; concave
F9	ZDT3	$f_1(x_1)=x_1$; $f_2(\vec{x})=g(1-\sqrt{f_1/g}-(f_1/g)\sin(10\pi f_1))$ $g(\vec{x})=1+9\sum_{i=2}^{m}x_i/(m-1)$	$n=30;0\le x\le 1$; convex; disconnected

Table.2. Parameters settings

Population size(N)	Evolutionary generation(G)	Crossover probability (P_c)	Mutation probability(P_m)	Crossover distribution index(eta_c)	Mutation distribution index(eta_m)
100	200	0.9	0.1	10.0	10.0

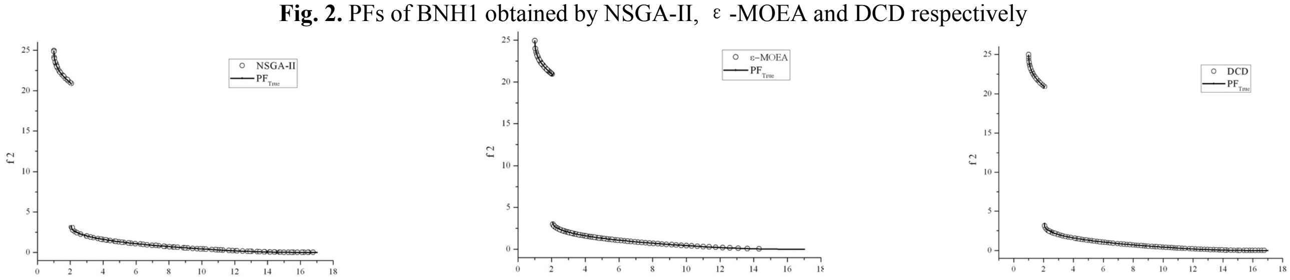
As one of the most popular MOEAs, NSGA-II use CD-DMS in population maintenance. In this paper, DCD-DMS is combined with NSGA-II, labeled as: DCD for short. Certainly, DCD can also be combined with other MOEAs. Another popular MOEA: ε-MOEA [14] is also used for comparison.

Table.2 shows the parameters settings. For ε-MOEA, two attentions must be declared. (1) ε-MOEA use fitness evaluation times instead of evolutionary generation, and fitness evaluation times = evolutionary generation × population size. Then, set fitness evaluation times as 20000 for ε-MOEA. (2) The value of ε is varied for different MOPs, so ε in this paper almost the same as paper [14].



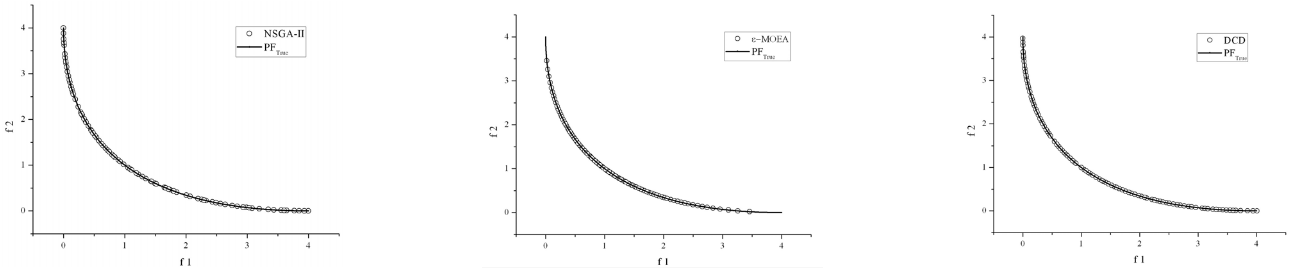
(a) NSGA-II (b) ε-MOEA (c) DCD

Fig. 2. PFs of BNH1 obtained by NSGA-II, ε-MOEA and DCD respectively



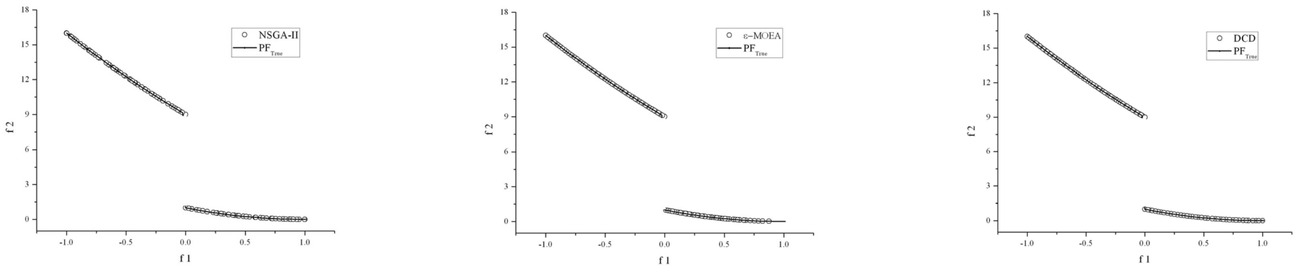
(a) NSGA-II (b) ε-MOEA (c) DCD

Fig. 3. PFs of POL obtained by NSGA-II, ε-MOEA and DCD respectively



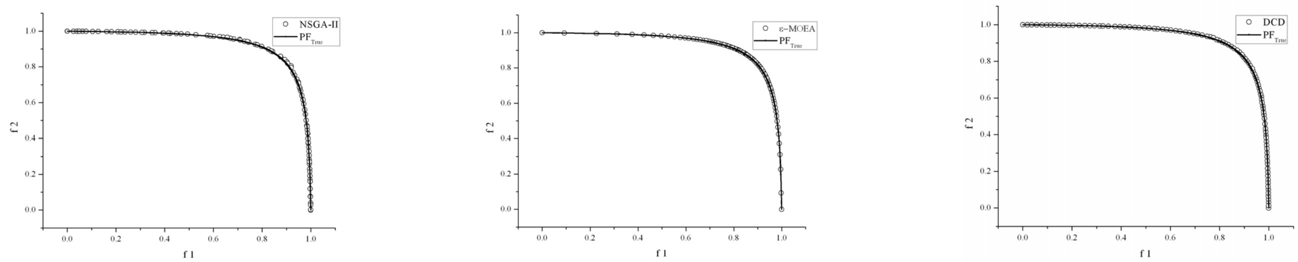
(a) NSGA-II (b) ε -MOEA (c) DCD

Fig. 4. PFs of SCH1 obtained by NSGA-II, ε -MOEA and DCD respectively



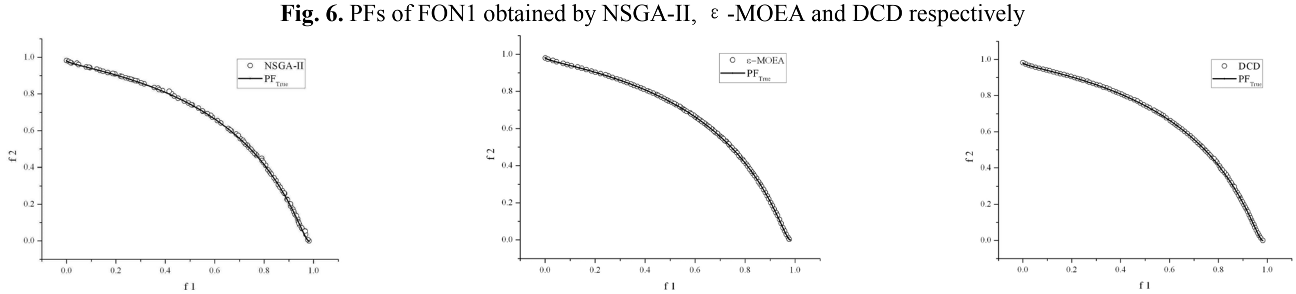
(a) NSGA-II (b) ε -MOEA (c) DCD

Fig. 5. PFs of SCH2 obtained by NSGA-II, ε -MOEA and DCD respectively



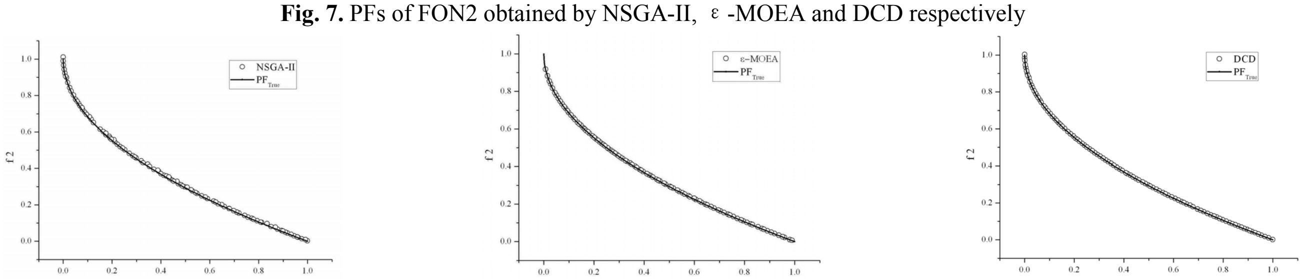
(a) NSGA-II (b) ε -MOEA (c) DCD

Fig. 6. PFs of FON1 obtained by NSGA-II, ε -MOEA and DCD respectively



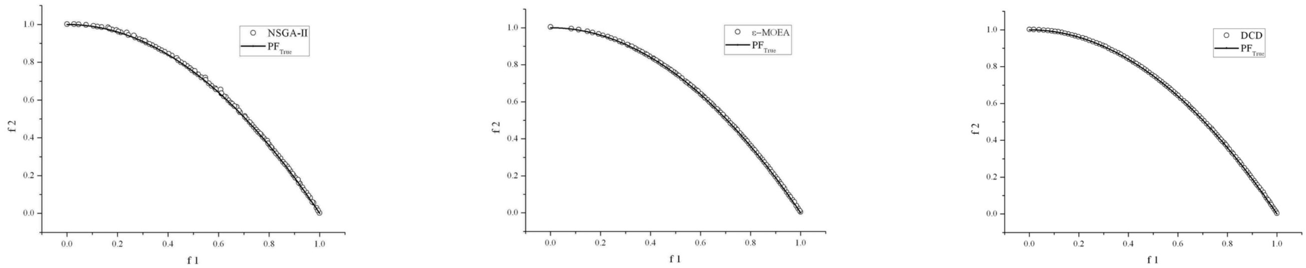
(a) NSGA-II (b) ε -MOEA (c) DCD

Fig. 7. PFs of FON2 obtained by NSGA-II, ε -MOEA and DCD respectively



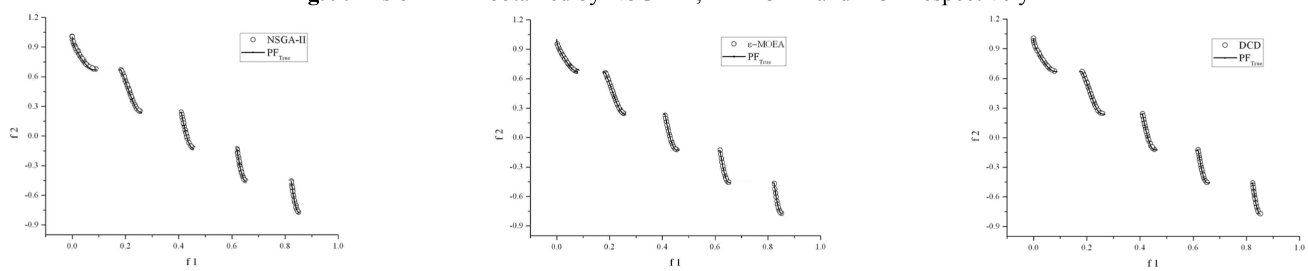
(a) NSGA-II (b) ε -MOEA (c) DCD

Fig. 8. PFs of ZDT1 obtained by NSGA-II, ε -MOEA and DCD respectively



(a) NSGA-II (b) ε -MOEA (c) DCD

Fig. 9. PFs of ZDT2 obtained by NSGA-II, ε -MOEA and DCD respectively



(a) NSGA-II (b) ε -MOEA (c) DCD

Fig. 10. PFs of ZDT3 obtained by NSGA-II, ε -MOEA and DCD respectively

4.2 Results Representation

We used NSGA-II , ε -MOEA and DCD to test MOPs listed in table 1 under settings of table 2. Fig.2-10 show the PFs and PF_{True}.

It can be seen from Fig.2-10 that the span and uniformity of PFs obtained by DCD are much better than NSGA-II and ε -MOEA. (1) For all test problems, PFs obtained by NSGA-II and ε -MOEA have a worse uniformity. For example, the individuals in some parts of the PFs are too crowded, some parts are sparseness, and individuals in some parts are missed. (2) For some of test problems, such as: BNH1, POL, SCH1 and ZDT1. The borders of PFs obtained by ε -MOEA are missed.

4.3 Performance Comparisons

In the above paragraphs, a qualitative analysis is given for diversity of PFs obtained by NSGA-II , ε -MOEA and DCD. In order to give a quantitative analysis, we introduced two performance metrics here.

(1) Spacing (SP): SP is proposed by Schott [15]. SP is one of the most popular metric for diversity. SP is shown in (8).

$$SP=\sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(\bar{d}-d_i)^2} \tag{8}$$

where $d_i=\min_j\{\sum_{k=1}^{r}\left|f_k^i-f_k^j\right|\}$, ($i,j$ = 1,2,…n), $\bar{d}$ is the mean of d, f_k^i is the k^{th} objective of the i^{th} individual, n is the number of points which are uniformly distributed in true PF. Smaller SP means better diversity of PF.

(2) Inverted Generational Distance (IGD): this metric is designed for both convergence and diversity. IGD is shown in (9).

$$IGD=\frac{\sum_{v\in P^*}d(v,P)}{|P^*|} \tag{9}$$

where P^* is a set of uniformly distributed points in true PF, P is the NDS obtained by MOEAs, $d(v,P)$ is the minimum Euclidean distance between v and the points in P. if $|P^*|$ is large enough to represent PF_{True} very well, IGD could measure both the diversity and convergence of P in a sense. To have a low value of IGD, P must be very close to the PF_{True} and cannot miss any part of true PF. In our experiments, we select $|P^*|$ as 500. IGD has been recently used by other researchers [16,17], as its name suggests, it is an inverted variation of the widely-used generational distance (GD) performance metric [18]. The GD can only represent the average distance of the points in NDS to the PF_{True}, but can not effectively measure the diversity of the NDS.

For all MOPs in Table 1, we run NSGA-II , ε -MOEA and DCD 20 times. Table 3 shows mean and standard deviation (σ) of SP. Bold numbers in table 3 are minimums. Obviously, DCD can improve diversity of PF highly compared with NSGA-II and ε -MOEA. Table 4 shows mean and standard deviation (σ) of IGD for part of MOPs. Other MOPs did not provide true PF_{True}, so IGD is unavailable according to formula (9). It can be seen from table 4 that most of mean IGDs obtained by DCD are samller than NSGA-II and ε -MOEA, which means that DCD can improve convergence at the same time of the improvement of diversity.

Table.3. Mean SP and Standard deviation SP (σ)
(20 runs of NSGA-II, ε-MOEA and DCD on 9 MOPs)

MOEAs	MOPs	BNH1	POL	SCH1	SCH2	FON1	FON2	ZDT1	ZDT2	ZDT3
NSGA-II	SP (mean)	0.433369	0.106804	0.036418	0.056339	0.008257	0.008194	0.006029	0.006159	0.007135
	SP(σ)	0.040546	0.005073	0.002240	0.006367	0.000506	0.000768	0.000531	0.000809	0.000776
ε-MOEA	SP (mean)	0.518945	0.146025	0.038069	0.049350	0.022753	0.004864	0.005904	0.008615	0.011164
	SP(σ)	0.011123	0.006371	0.003464	0.049350	0.000399	0.000255	0.000280	0.000881	0.000516
DCD	SP (mean)	**0.140977**	**0.049755**	**0.010715**	**0.040399**	**0.002862**	**0.002719**	**0.002999**	**0.003462**	**0.004665**
	SP(σ)	0.018390	0.005979	0.000974	0.001771	0.000330	0.000241	0.000286	0.000473	0.001557

Table.4. Mean IGD and Standard deviation IGD (σ)
(20 runs of NSGA-II, ε-MOEA and DCD on 4 MOPs)

MOEAs	MOPs	SCH1	FON2	ZDT1	ZDT2
NSGA-II	IGD (mean)	0.022413	0.431218	0.006205	0.006360
	IGD(σ)	0.000767	0.001281	0.000314	0.000303
ε-MOEA	IGD (mean)	0.054191	0.432850	**0.004635**	0.006120
	IGD(σ)	0.000777	0.000156	0.000152	0.000332
DCD	IGD (mean)	**0.016766**	**0.430345**	0.005456	**0.005919**
	IGD(σ)	0.000072	0.000520	0.000204	0.000385

5. Conclusions

In this paper, we proposed a dynamic crowding distance based diversity maintenance strategy (DCD-DMS). The DCD-DMS has two characters: (1) DCD-DMS calculate individuals' DCD dynamically, that is, DCD-DMS recalculate individuals' DCD after delete an individual from NDS during the process of population maintenance. Hence, DCD-DMS can avoid phenomena that some parts of the PF are too crowded and some parts are sparseness, and then DCD-DMS can improve uniformity of PF. (2) The difference degree of CD in different objectives is considered in the design of DCD. Through experiments on 9 test problems, compared with NSGA-II and ε-MOEA, the results demonstrate that DCD can improve diversity of PF at a high level and convergence of PF is also be improved a little.

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