A New Evolutionary Algorithm for Solving Many-Objective Optimization Problems

Xiufen Zou, Yu Chen, Minzhong Liu, and Lishan Kang

***Abstract*—In this paper, we focus on the study of evolutionary algorithms for solving multiobjective optimization problems with a large number of objectives. First, a comparative study of a newly developed dynamical multiobjective evolutionary algorithm (DMOEA) and some modern algorithms, such as the indicator-based evolutionary algorithm, multiple single objective Pareto sampling, and nondominated sorting genetic algorithm II, is presented by employing the convergence metric and relative hypervolume metric. For three scalable test problems (namely, DTLZ1, DTLZ2, and DTLZ6), which represent some of the most difficult problems studied in the literature, the DMOEA shows good performance in both converging to the true Pareto-optimal front and maintaining a widely distributed set of solutions. Second, a new definition of optimality (namely, L-optimality) is proposed in this paper, which not only takes into account the number of improved objective values but also considers the values of improved objective functions if all objectives have the same importance. We prove that L-optimal solutions are subsets of Pareto-optimal solutions. Finally, the new algorithm based on L-optimality (namely, MDMOEA) is developed, and simulation and comparative results indicate that well-distributed L-optimal solutions can be obtained by utilizing the MDMOEA but cannot be achieved by applying L-optimality to make *a posteriori* selection within the huge Pareto nondominated solutions. We can conclude that our new algorithm is suitable to tackle many-objective problems.**

***Index Terms*—Evolutionary algorithm, many-objective optimization, Pareto optimality, performance assessment.**

I. Introduction

MOST real-world problems in the field of science, engineering, and business management are multiobjective optimization problems (MOPs), and multiple objectives usually conflict with each other. During the last 20 years, evolutionary computation techniques have been successfully used to solve MOPs [1]–[3]. However, many multiobjective evolutionary algorithms (MOEAs) are widely established and well developed for problems with two or three objectives. The state-of-the-art multiobjective optimization algorithms, such as the nondominated sorting genetic algorithm II (NSGAII) or strength Pareto evolutionary algorithm 2 (SPEA2), have been shown to be not suited to solve optimization problems with more than three objectives [4]–[6], which have been termed many-objective problems by Farina and Amato [7]. Because a large number of objectives lead to further difficulties with respect to computation, visualization, and decision making, in recent years, almost all hot issues in the design of MOEAs have been related to the handling of many-objective optimization problems, and some approaches for improving the scalability of MOEAs to many-objective optimization problems have been proposed [8]. Multiple single-objective Pareto sampling (MSOPS) and MSOPSII [9], [10] used an aggregation method and a ranking scheme regarding multiple weight or target vectors, but for high dimensions, these techniques reach their limits because either it is hard to determine good weights or the fitness of the optimal solution is not known in advance. Indicator-based evolutionary algorithms (IBEAs) [11] and hypervolume-based search algorithms as the S-metric selection evolutionary multiobjective optimization algorithm (SMS-EMOA) [12] were reported to outperform established algorithms such as NSGAII and SPEA2 for multiobjective problems with three to six objectives [4], but the time of computing the hypervolume in most of the hypervolume-based algorithms exponentially grows with the number of objectives. Therefore, Brockhoff and Zitzler developed objective reduction techniques to improve hypervolume-based MOEAs [13]. Other dimensionality reduction methods such as the principal-component-analysis-based method (C-PCA-NSGAII and VU-PCA-NSGAII) [14] and the relation-based method [15] were proposed to cope with high-dimensional objective spaces.

Another problem is that solutions with many objectives are usually nondominated with each other by using the Pareto dominance relation. This means a very low selection pressure toward the Pareto front in Pareto-dominance-based evolutionary multiobjective optimization [16]. To increase the selection pressure toward the Pareto front, some modifications of the Pareto dominance relation and rank definition have been suggested [17], [18]. In [19] and [20], an alternative relation, called *Favour* or *ε-Preferred*, was proposed, and experiments had shown that *Preferred* clearly outperformed relation *Dominates*, but it is possible to have cycles in the relation graph of the elements of a search space. A fuzzy definition of optimality (k-optimality) was presented; however, this definition was only used as *a posteriori* selection criteria within the huge Pareto-optimal front [21], [22].

The aim of this paper is twofold. First, in order to converge to the Pareto-optimal solutions with a wide diversity among the solutions, a dynamical MOEA based on the principle

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of thermodynamics (namely, DMOEA) is presented to solve many-objective problems and compared with some state-of-the-art algorithms, such as the IBEA, MSOPS, and NSGAII. Second, in order to provide reasonable solutions for decision making in real-world problems, a new definition of optimality for handling many-objective problems is proposed. One MOEA based on new selection schemes is suggested, and numerical experiments are conducted.

This paper is structured as follows. In Section II, a proposed MOEA (a full description of which is presented in [23]) is shortly summarized. In Section III, we define a new concept, which is the L-dominance, and the corresponding L-optimal set as well as the new selection strategy L-ranking. Some properties and related theorems are obtained. One MOEA based on new selection schemes for dealing with many-objective problems (namely, MDMOEA) is suggested. Simulation results are presented to demonstrate the behavior of the new schemes and to highlight the important differences to the existing approaches in Section IV. Finally, some conclusions are addressed in Section V.

II. DMOEA Converging to Pareto-Optimal Solutions

Without loss of generality, we consider the following MOP and assume that all objectives are to be minimized:

$$\begin{aligned} &\underset{X\in S}{\text{Minimize}} \quad G(X) = \{g_1(X), g_2(X), \ldots, g_M(X)\} \\ &\text{subject to} \quad h_i(X) \leq 0, \qquad i = 1, 2, \ldots, l \end{aligned} \tag{1}$$

where S is the feasible region of decision variables, M is the number of objectives (generally, $M > 2$), and l is the number of constrained conditions.

A. Fitness Assignment Strategy

The DMOEA, i.e., the method based on the principle of the minimal free energy in thermodynamics, was first described by Zou *et al.* [23]. The DMOEA adopted an aggregating function that combined ranking with entropy and density. The DMOEA focused on nondominated solutions and simultaneously maintained diversity in the solutions that converted the original multiple objectives into a new fitness to a solution. The fitness was composed of three parts, i.e.,

$$fitness(i) = R(i) - TS(i) - d(i) \tag{2}$$

where $fitness(i)$ denotes the fitness of the individual i in the population.

The first part $R(i)$ is the Pareto rank value of the individual i, which is equal to the number of solutions n_i that dominate solution i [24]. The Pareto rank values can be computed as follows:

$$R(i) = |\Omega_i| \tag{3}$$

where $\Omega_i = \{X_j | X_j \succ_p X_i, 1 \leq j \leq N, j \neq i\}$.

The second part $S(i) = -p_T(i) \log p_T(i)$, where $T > 0$ is the analog of temperature. $p_T(i) = (1/Z) \exp(-R(i)/T)$ is the analog of the Gibbs distribution, $Z = \sum_{i=1}^{N} \exp(-R(i)/T)$ is called the partition function, and N is the population size.

The third part $d(i)$ is the crowding distance, which is computed by using a density estimation technique that is described in Deb *et al.*'s NSGAII [25].

In the proposed algorithm, the new fitness values of all individuals are sorted in increasing order, and the individuals whose fitness values are highest are recorded, which are called "worst individuals"; however, the rank values that are equal to zero are considered as "best individuals." Concepts of best and worst individuals in the proposed algorithm are different from those in conventional MOEAs. Moreover, only the worst individuals are eliminated through selections in each step of the DMOEA, which can ensure the good diversity of population.

B. Selection Criterion

We attempted to employ the Metropolis criterion of simulated annealing algorithm [26] and the crowding distance to guide the select process, i.e.,

1) if $R(X_{\text{new}}) < R(X_{\text{worse}})$, then $X_{\text{worst}} := X_{\text{new}}$;
2) if $R(X_{\text{new}}) = R(X_{\text{worse}})$ and $d(X_{\text{new}}) > d(X_{\text{worse}})$, then $X_{\text{worst}} := X_{\text{new}}$;
3) else, if $\exp((R(X_{\text{worst}}) - R(X_{\text{new}}))/T) > random(0,1)$, then $X_{\text{worst}} := X_{\text{new}}$.

Note that $R(X_{\text{worst}})$ and $R(X_{\text{new}})$ are the Pareto rank value of the worst individuals and the new individuals, respectively.

Algorithm 1 (DMOEA)

1) Randomly generate the initial population $P(0)$ and set generation index $t = 0$.

2) REPEAT

a) Calculate the Pareto rank values $\{R_1(t), \ldots, R_N(t)\}$ of all individuals in $P(t)$ according to (3), where N is the population size.

b) Evaluate the fitness of each individual in $P(t)$ according to (2), sort them in increasing order, and record the worst individuals.

c) Save the individuals whose Pareto rank values are equal to zero to form the Pareto front.

d) Apply genetic operations to generate new individuals.

e) Select new individuals based on the selection criterion in this section and derive the new population $P(t+1)$.

f) $t := t + 1$.

UNTIL the stopping criteria are fulfilled.

3) Output the Pareto front.

III. MDMOEA Converging to L-Optimal Solutions

A. Normalization of Objectives

When all objectives have the same importance, we will consider the normalization of all objective functions. Suppose $g_{i,\max}$ and $g_{i,\min}$ are the maximum and minimum values of the objective function $g_i(X)$ in the feasible set S, let

$$f_i(X) = (g_i(X) - g_{i,\min}) / (g_{i,\max} - g_{i,\min}), \quad i = 1, 2, \ldots, M. \tag{4}$$

The original problem is converted to the following normalized MOP (NMOP):

$$\begin{aligned} &\underset{X\in S}{\text{Minimize}} \quad F(X) = \{f_1(X), f_2(X), \ldots, f_M(X)\} \\ &\text{subject to} \quad h_i(X) \le 0, \qquad i = 1, 2, \ldots, l \end{aligned} \tag{5}$$

where the values of all new objectives are mapped into a new range, i.e., normalization to the interval [0, 1].

The p-norm of one solution X, which is the distance from any point $F(X)$ to the ideal objective vector (the origin) for the NMOP (5), is defined as follows:

$$\|F(X)\|_p = \left(\sum_{i=1}^{M} (f_i(X))^p\right)^{1/p} \quad (\text{usually } p = 1, 2, \infty).$$

B. Taking Into Account the Number of Improved Objectives

When the Pareto optimality definition is considered, two solutions are equivalent if at least in one objective the first solution is strictly better than the second one and at least in one objective the second one is strictly better than the first one (or if they are equal in all the objectives). Indeed, a more general definition, which is able to cope with a wider variety of problems, should take into account the number of objectives where the first candidate solution is better than the second one and vice versa. Let us first introduce the following functions, which were presented in [21]:

$$B_t(X_1, X_2) = |\{i \in \mathbb{N} | i \le M \wedge f_i(X_1) < f_i(X_2)\}|$$

$$E_q(X_1, X_2) = |\{i \in \mathbb{N} | i \le M \wedge f_i(X_1) = f_i(X_2)\}|$$

$$W_s(X_1, X_2) = |\{i \in \mathbb{N} | i \le M \wedge f_i(X_1) > f_i(X_2)\}|.$$

For each pair of solutions, $B_t(X_1, X_2)$, $E_q(X_1, X_2)$, and $W_s(X_1, X_2)$ count the number of objectives for which solution X_1 is better than, equivalent to, and worse than solution X_2, respectively. For simplicity, we will instead use B_t, E_q, and W_s, respectively. It is clear that

$$B_t + E_q + W_s = M, \qquad 0 < B_t, \quad E_q, \quad W_s < M.$$

C. Pareto Optimality and L-Optimality

For the convenience of comparison, we recall here the well-known definition of Pareto dominance, Pareto-optimal set, and Pareto-optimal front in an MOP.

Definition 1 (Pareto Dominance Relation) [27]: Let X_1, $X_2 \in S$. A solution X_1 is said to Pareto-dominate the other solution X_2, denoted $X_1 \succ_p X_2$, iff

1) $\forall i \in \{1, \ldots, M\}$, $f_i(X_1) \le f_i(X_2)$;
2) $\exists j \in \{1, \ldots, M\}$, $f_j(X_1) < f_j(X_2)$.

Based on the concept of Pareto dominance, the Pareto optimality and Pareto-optimal set can be defined as follows.

Definition 2 (Pareto Optimality): $X^* \in S$ is said to be Pareto-optimal if there is no $X \in S$ such that X Pareto-dominates X^*.

Definition 3 (Pareto-Optimal Set and Front): We call a *Pareto-optimal set* S_p and a *Pareto-optimal front* F_p the set of Pareto-optimal solutions in the design variable domain and the objective function domain, respectively.

Now, we are able to give a new definition of dominance, namely, L-dominance and L-optimality.

Definition 4 (L-Dominance): Let X_1, $X_2 \in S$. A solution X_1 is said to L-dominate the other solution X_2, denoted $X_1 \succ_L X_2$, iff

1) $B_t - W_s = L > 0$;
2) $\|F(X_1)\|_p < \|F(X_2)\|_p$ (for a certain p).

The first condition in Definition 4 means that the majority of objectives are superior, and the second condition implies that the sum of all objective values is improved and can be used to control the values of inferior objectives.

Definition 5 (L-Optimality): $X^* \in S$ is said to be L-optimal if there is no $X \in S$ such that X L-dominates X^*.

Definition 6 (L-Optimal Set and Front): We call an *L-optimal set* S_L and an *L-optimal front* F_L the set of L-optimal solutions in the design variable domain and the objective function domain, respectively.

With the aforementioned definition of the L-dominance relation, three properties are obtained.

1) The L-dominance relation is not reflexive since any solution X does not L-dominate itself according to Definition 4.
2) The L-dominance relation is not symmetric because X_1 L-dominates X_2 does not imply that X_2 L-dominates X_1.
3) The L-dominance relation is not transitive since X_1 L-dominates X_2 and X_2 L-dominates X_3 does not imply that X_1 L-dominates X_3, but implies that X_3 does not L-dominate X_1. Thus, we say that the L-dominance relation is *semitransitive*.

Therefore, we define a new partial ordering relation. The following two theorems give the relationship between L-dominance and classical Pareto dominance.

Theorem 1: If a solution X_1 Pareto-dominates the other solution X_2, then X_1 L-dominates X_2.

Proof: Since X_1 Pareto-dominates X_2, from Definition 1, we have

1) $\forall i \in \{1, \ldots, M\}$, $f_i(X_1) \le f_i(X_2)$;
2) $\exists j \in \{1, \ldots, M\}$, $f_j(X_1) < f_j(X_2)$.

Then, we easily obtain

1) $B_t > 0, W_s = 0$, so $B_t > W_s$;
2) $(\sum_{i=1}^{M}(f_i(X_1))^2)^{1/2} < (\sum_{i=1}^{M}(f_i(X_2))^2)^{1/2}$.

From Definition 4, we have that X_1 L-dominates X_2. ■

Theorem 2: If $X^* \in S$ is L-optimal, then X^* is Pareto-optimal. That is, $S_L \subseteq S_p$.

Proof: Suppose X^* is not Pareto-optimal, we obtain $\exists X \in S$ such that X Pareto-dominates X^*; therefore, from Theorem 1, we have that X L-dominates X^*, which is inconsistent to $X^* \in S$ being L-optimal.

Therefore, we can obtain the conclusion. ■

From Theorems 1 and 2, we have the following.

1) Pareto-dominate implies L-dominate, but L-dominate does not imply Pareto-dominate. It means that we can save more solutions by this new selection strategy and can maintain a better diversity of solutions.
2) $S_L \subseteq S_p$, i.e., an L-optimal set is a subset of Pareto-optimal solutions. It means that we can choose reasonable solutions from a Pareto-optimal set.

When the number of objectives M is equal to two, it is easily obtained that $S_L = S_p$, which means that our definition is equivalent to the Pareto-optimal definition. Hence, the classical Pareto-optimal definition will be a special case of our new definition.

D. MDMOEA Based on L-Optimality

The extended definitions of L-optimality can be applied to MOPs at two levels: 1) a new selection of nondominated solutions obtained via the L-dominance relation and 2) development of new algorithms directly converging toward the L-optimal front.

The L-rank value of the individual i, denoted $LR_i(t)$, which is equal to the number of solutions n_i that L-dominate solution i, can be computed as follows:

$$LR_i(t) = |\Omega_i|$$

where $\Omega_i = \{X_j | X_j \succ_L X_i, 1 \leq j \leq N, j \neq i\}$. Accordingly, the fitness of the individual i in the population can be obtained by substituting the Pareto rank values with L-rank values in (2), i.e.,

$$fitness(i) = LR(i) - T \cdot LS(i) - d(i) \qquad (6)$$

where $fitness(i)$ denotes the fitness of the individual i in the population.

Based on the aforementioned new theoretic frames, we propose an MOEA for solving many-objective problems, which is denoted MDMOEA, by substituting the Pareto rank values with the L-rank values in Algorithm 1 (DMOEA).

Algorithm 2 (MDMOEA)

1) Randomly generate the initial population $P(0)$ and set generation index $t = 0$.

2) REPEAT

a) Calculate the L-rank values $\{LR_1(t), \ldots, LR_N(t)\}$ of all individuals in $P(t)$, where N is the population size.

b) Evaluate the fitness of each individual in $P(t)$ according to (6), sort them in ascending order, and record the worst individuals.

c) Save the individuals whose L-rank values are equal to zero to form the L-front.

d) Apply genetic operations to generate new individuals.

e) Select new individuals based on the new selection criterion via the L-dominance relation and derive the new population $P(t+1)$.

f) $t := t + 1$.

UNTIL the stopping criteria are fulfilled

3) Output the L-front.

IV. Numerical Experiments and Results

A. Comparison of the DMOEA and the Other Modern MOEAs

A lot of scalable test problems were proposed to test the efficacy of a new MOEA in handling problems with more than two objectives [28]. Here, three scalable test problems DTLZ1, DTLZ2, and DTLZ6 are considered, where two test problems DTLZ1 and DTLZ2 were used to compare different algorithms for handling many-objective problems by Wagner *et al.* [4], and test problem DTLZ6 is one of the most difficult problems suggested in the literature [28]. There are $n = M + k - 1$ decision variables in these problems, where M is the number of objectives, and k specifies the distance to the Pareto front. According to Deb *et al.* [28] and Wagner *et al.* [4], $k = 5$ is used in DTLZ1, whereas $k = 10$ is used in DTLZ2 and DTLZ6. The Pareto front of DTLZ1 is a linear hyperplane $f_1 + f_2 + \cdots + f_M = 0.5$. DTLZ2 features a Pareto front that corresponds to the positive part of the unit hypersphere $(f_1^2 + f_2^2 + \cdots + f_M^2 = 1)$. The Pareto front of DTLZ6 is $f_1^2 + f_2^2 + \cdots + f_M^2 = 1$ and $f_2 = f_1 \in [0,\ 2^{(M-2)/2}]$ when $M = 3$, or $f_{M-1} = \sqrt{2} f_{M-2} = \cdots = 2^{(M-3)/2} f_2 = 2^{(M-3)/2} f_1$ when $M > 3$. The domain of all decision variables is [0, 1]. The Pareto set of three test functions corresponds to $x_M, \ldots, x_N = 0.5$ with arbitrary values for $x_1, \ldots, x_{M-1}$.

For performance assessment, convergence [29] and hypervolume measures [30] are considered. The convergence measure describes the average distance of the approximation to the Pareto front in objective space. Due to the special structure of the employed Pareto fronts, the Euclidean distance to the nearest optimal solution, which is similar to the study of Wagner *et al.* [4], is analytically determined without using a reference set. The hypervolume metric determines the size of the dominated hypervolume in objective space bounded by a reference point r. Such a metric can provide a qualitative measure not only of convergence but also of diversity. The maximal hypervolume value is reached by the Pareto front. According to previous studies [4], the reference points $r = 0.7^M$ for DTLZ1 and $r = 1.1^M$ for DTLZ2 and DTLZ6 were used, and the metric values are normalized by calculating the fraction of the analytical optimal value. Note that points that do not dominate the reference point are discarded for metric calculation.

In our studies, five different MOEAs are compared: DMOEA, IBEA, MSOPS1, MSOPS2, and NSGAII. The source codes of four algorithms, namely, IBEA, MSOPS1, MSOPS2, and NSGAII, are downloaded from the Program for International Student Assessment website (www.tik.ee.ethz.ch/pisa/), and all algorithms have been implemented on a PC with a Pentium IV processor, running at 1.6 GHz and with 256-MB RAM. All parameters in the IBEA, MSOPS, and NSGAII are according to the studies of Deb *et al.* [29] and Wagner *et al.* [4]. Thirty thousand function evaluations are accomplished, and the population size $N = 100$ is chosen. The specific parameter T in the DMOEA is set to 10 000. For each MOEA on each test function, ten runs are performed. When the number of objectives is three, four, five, or six, we only calculate the results with the DMOEA since the other four algorithms have been implemented by Wagner *et al.* [4]. In cases of seven,

TABLE I
CONVERGENCE MEASURE FOR FIVE ALGORITHMS

Obj.	Algorithm	DTLZ1		DTLZ2		DTLZ6	
		mean	Std.dev	mean	Std.dev	mean	Std.dev
3	DMOEA	0.000168	5.70933e-9	0.000382	2.59455e-8	0.002845	4.16963e-7
4	DMOEA	0.000436	1.86588e-8	0.000462	3.69088e-8	0.025722	0.000101
5	DMOEA	0.000353	3.46519e-9	0.000554	8.57961e-8	0.034645	0.000307
6	DMOEA	0.000705	1.51685e-7	0.001611	1.34866e-6	0.052835	0.000765
7	DMOEA	0.000943	9.75516e-7	0.001318	8.84272e-7	0.010952	0.000161
	IBEA	0.001208	5.86721e-6	0.000533	2.32702e-8	0.036965	1.7322e-5
	MSOP 1 200	0.080465	0.000892	0.003143	6.46929e-7	0.962027	0.000722
	MSOP 2 200	0.104909	0.006762	0.006838	5.18994e-6	0.890586	0.000321
	NSGAII	18.2258	0.723624	0.22638	4.00722e-5	1.01560	3.58739e-5
8	DMOEA	0.000580	7.91967e-86.53	0.001474	1.89847e-6	0.013174	9.71513e-5
	IBEA	0.001329	044e-6	0.000751	4.87352e-9	0.043231	3.4545e-6
	MSOP 1 200	0.085996	0.001880	0.005631	7.07295e-6	0.947361	0.000294
	MSOP 2 200	0.065893	0.0015157	0.011437	7.36562e-6	0.916568	0.000246
	NSGAII	16.9174	0.293263	0.235362	3.48321e-5	1.024840	7.94254e-5
9	DMOEA	0.000593	2.05488e-7	0.002257	7.12376e-6	0.011635	5.10013e-5
	IBEA	0.001989	2.76568e-5	0.000689	1.97591e-8	0.053858	9.98077e-6
	MSOP 1 200	0.112301	0.00543349	0.004655	2.22367e-6	0.947949	0.000604
	MSOP 2 200	0.091654	0.00103721	0.007463	2.96853e-6	0.915081	8.37971e-5
	NSGAII	16.4009	0.088511	0.236024	2.46679e-5	1.030750	9.04042e-5

TABLE II
RELATIVE HYPERVOLUME FOR FIVE ALGORITHMS

Obj.	Algorithm	DTLZ1		DTLZ2		DTLZ6	
		mean	Std.dev	mean	Std.dev	mean	Std.dev
3	DMOEA	0.976766	1.3179e-6	0.891491	2.85012e-5	0.951048	4.25909e-5
4	DMOEA	0.979937	4.53185e-7	0.822677	0.000138	0.931013	0.000165
5	DMOEA	0.989379	3.2361e-7	0.802704	9.17072e-5	0.915242	0.000109
6	DMOEA	0.995041	3.16389e-8	0.809959	0.000215	0.894835	0.000432
7	DMOEA	0.997702	1.07082e-8	0.825988	0.000153	0.969376	1.87946e-5
	IBEA	0.913461	0.000995	0.899216	1.79887e-6	0.475326	0.000378
	MSOP 1 200	0.183300	0.026411	0.742508	0.000820	0	0
	MSOP 2 200	0.453798	0.188529	0.753655	0.000582	0	0
	NSGAII	0	0	0	0	0	0
8	DMOEA	0.998938	5.67927e-90.00	0.843450	8.41141e-5	0.962984	5.83982e-5
	IBEA	0.906695	0757	0.911692	3.03884e-6	0.340051	0.000172
	MSOP 1 200	0.252789	0.116741	0.653482	0.001779	0	0
	MSOP 2 200	0.415275	0.238582	0.672910	0.001556	0	0
	NSGAII	0	0	0	0	0	0
9	DMOEA	0.999494	1.73787e-9	0.851401	4.24662e-5	0.957184	5.81189e-5
	IBEA	0.790622	0.0795929	0.927348	3.80372e-6	0.261867	0.000587
	MSOP 1 200	0.073239	0.00654673	0.617534	0.00367766	0	0
	MSOP 2 200	0.280636	0.157066	0.736118	0.00086739	0	0
	NSGAII	0	0	0	0	0	0

eight, and nine objectives, the mean and standard deviation of the convergence measure and relative hypervolume for five algorithms are calculated. The mean and standard deviation values of the convergence and relative hypervolume metrics obtained by using the five algorithms are listed in Tables I and II, respectively. In Tables I and II, we observe that the DMOEA reaches the best convergence and hypervolume metrics of all considered algorithms for problems DTLZ1 and DTLZ6 on seven, eight, and nine objectives. The DMOEA is outperformed by the IBEA only for problem DTLZ2, but the DMOEA shows significantly better performance than the other algorithms MSOPS1, MSOPS2, and NSGAII. In all cases, the standard deviation values of the convergence and relative hypervolume metrics with the DMOEA are small, which indicates that the DMOEA is robust with increasing number of objectives.

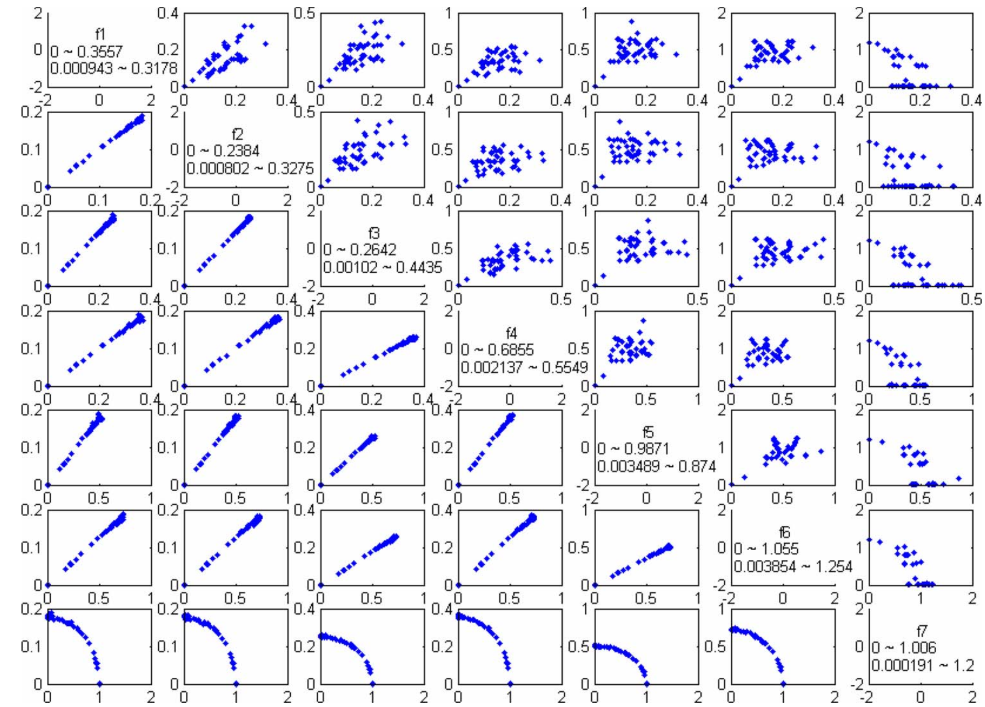

Fig. 1. Lower diagonal plots are for the DMOEA Pareto front, whereas upper diagonal plots are for the IBEA Pareto front on test problem DTLZ6 with seven objectives. Compare the (i, j) plot (DMOEA procedure with $i > j$) with the (j, i) plot (IBEA).

Surprisingly, for two test problems DTLZ1 and DTLZ6, the DMOEA reaches better relative hypervolume values in higher dimensions of objectives than in lower dimensions of objectives, which shows that our aggregated fitness expression is suitable to deal with the high-dimensional problems.

The studies of Wagner *et al.* [4] showed that the MSOPS obtains very promising results for problems DTLZ1 and DTLZ2 with three to six objectives, but in our simulation experiments, the MSOPS rapidly decreases in quality with increasing dimension of objective space. Particularly for problem DTLZ6 with seven, eight, and nine objectives, no relative hypervolume of both MSOPS variants is measured because no point dominating the reference point is achieved (Table II). From the results of both tables, we further confirm the observations of Hughes [5] and Wagner *et al.* [4] that the NSGAII does not converge to the Pareto-optimal set.

For illustration, we show only one of the ten runs of the DMOEA and IBEA on problem DTLZ6 with seven objectives in Fig. 1 by using the scatter-plot matrix method of comparing solutions, which is similar to that in [1] and [25]. The upper diagonal portion shows the (f_i, f_j) $(i < j)$ front of the IBEA between f_i and f_j, whereas the lower diagonal portion shows the (f_j, f_i) $(i < j)$ front of the DMOEA between f_j and f_i. For example, the plot at the first row and third column has its horizontal axis as f_1 and its vertical axis as f_3 for the IBEA, whereas the plot at the third row and first column has its horizontal axis as f_3 and its vertical axis as f_1 for the DMOEA. The diagonal boxes show labels and ranges used for each axis, where the DMOEA range is shown in the first row, and the IBEA range is shown in the second row. The DMOEA Pareto front in the lower diagonal portion is very clear, but only approximate patterns are obtained by using the IBEA. We observe that the DMOEA not only can achieve better convergence than the IBEA but also can obtain a uniformly spread distribution of solutions over the whole Pareto front.

B. Comparison of the L-Optimal Front Obtained by Using the DMOEA and MDMOEA

In order to test L-optimality definitions, we calculate the L-optimal front under the following two cases:

1) to select the L-optimal front within the huge Pareto-optimal front obtained via the DMOEA;
2) to directly obtain the L-optimal front via the MDMOEA.

In all test problems, we also use the scatter-plot matrix method to depict the solutions. In Figs. 2–7, the upper diagonal portion shows the (f_i, f_j) $(i < j)$ front between f_i and f_j, whereas the lower diagonal portion shows the (f_j, f_i) $(i < j)$ front between f_j and f_i.

1) Test Problem 1 (SPH-3 [31]):

$$\min f_1(\vec{x}) = (x_1 - 1)^2 + \sum_{i=2}^{n} x_i^2$$

$$\min f_2(\vec{x}) = x_1^2 + (x_2 - 1)^2 + \sum_{i=3}^{n} x_i^2$$

$$\min f_3(\vec{x}) = x_1^2 + x_2^2 + (x_3 - 1)^2 + \sum_{i=4}^{n} x_i^2$$

where $n = 100$, and $x_i \in [-1000, 1000]$. The range of optimal solutions is $x_1, x_2, x_3 \in [0, 1]$, $x_1 + x_2 + x_3 = 1$, $x_i = 0$, $i = 4, \ldots, n$.

For this problem, in Fig. 2, the star shows the Pareto front via the DMOEA, and the triangle represents the L-front

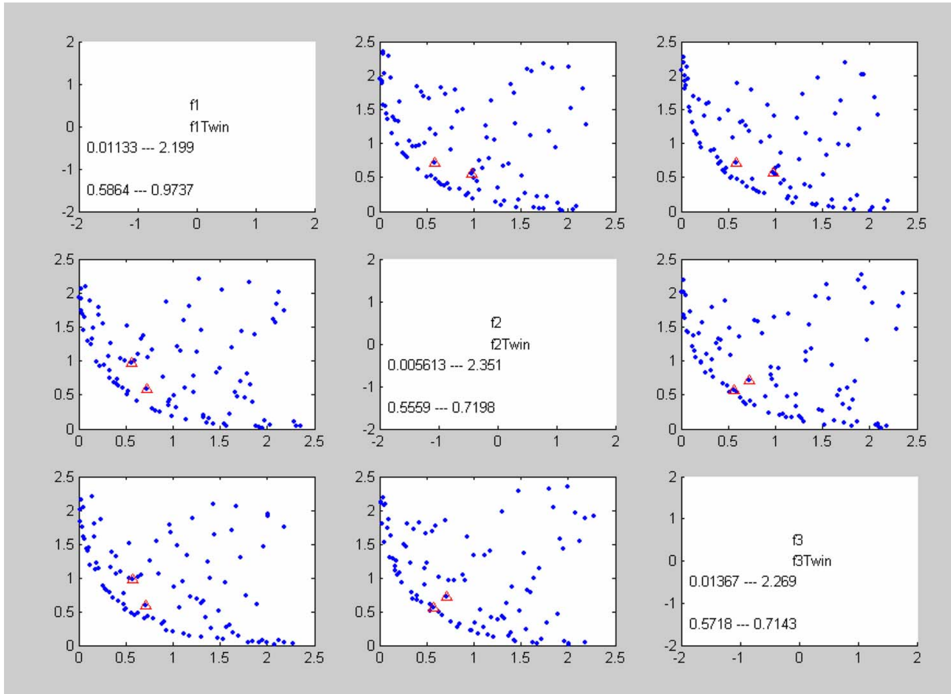

Fig. 2. Pareto front (star) and L-front (triangle) via the DMOEA for SPH-3.

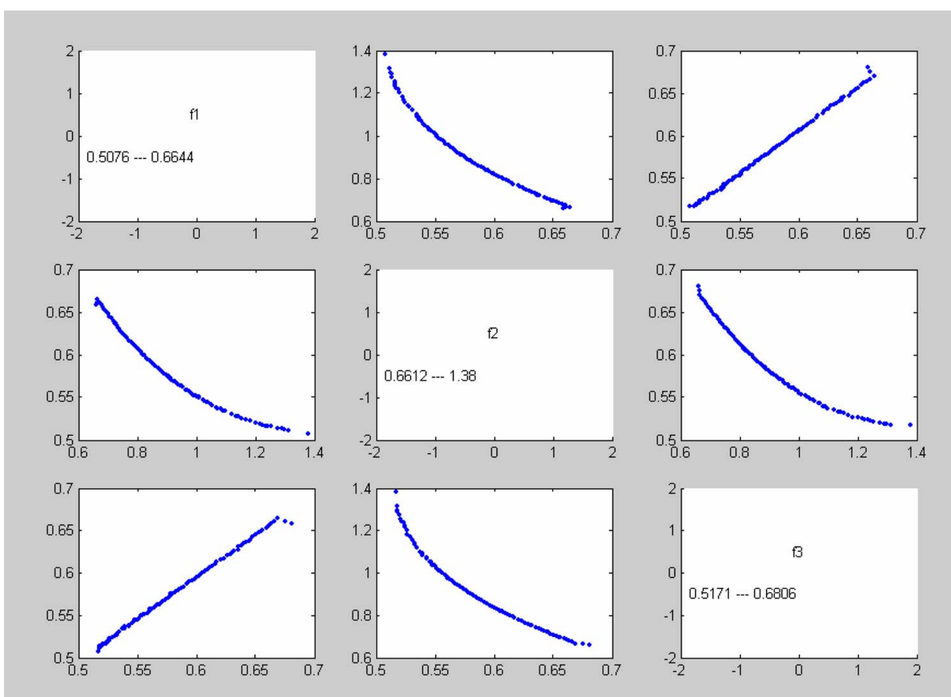

Fig. 3. L-front for SPH-3 via the MDMOEA.

selected from the Pareto front. Fig. 3 shows the L-front via the MDMOEA. Comparing Fig. 2 with Fig. 3, we can see that the good-distribution L-front is obtained with the MDMOEA, but only a little L-front is obtained with the DMOEA.

2) Test Problem 2 (DTLZ2): For DTLZ2, when $M = 5$, the Pareto front and L-front via the DMOEA are depicted in Fig. 4, whereas the L-front with the MDMOEA is shown in Fig. 5. Comparing Fig. 4 with Fig. 5, we can also observe that the good-distribution L-front is obtained via the MDMOEA, but only a little L-front is obtained via the DMOEA. From the numerical simulation, although M-dimensional $(M > 5)$, the L-optimal front can easily be obtained, i.e., $f_2 = \cdots = f_{M-1} = 0$ and $f_1^2 + f_M^2 = 1$ $(f_1, f_M \in [0, 1])$. These results indicate that new algorithms based on the L-dominance definition can provide the decision making well-distributed L-optimal solutions, whereas the MOEAs based on the Pareto dominance

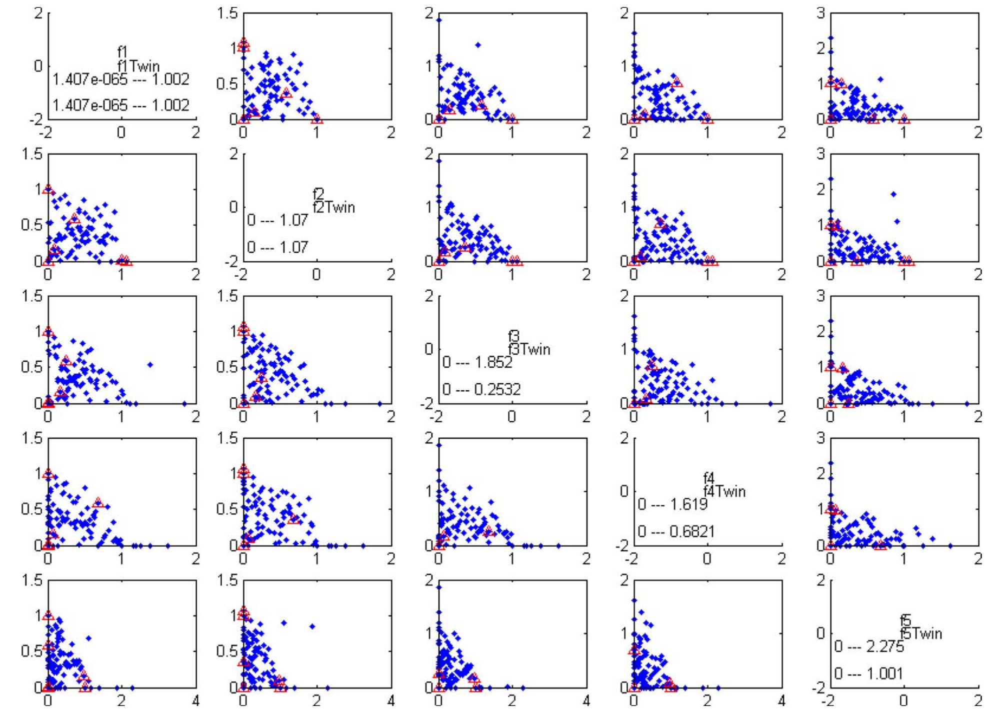

Fig. 4. Pareto front (star) and L-front (triangle) via the DMOEA on test problem DTLZ2 with five objectives.

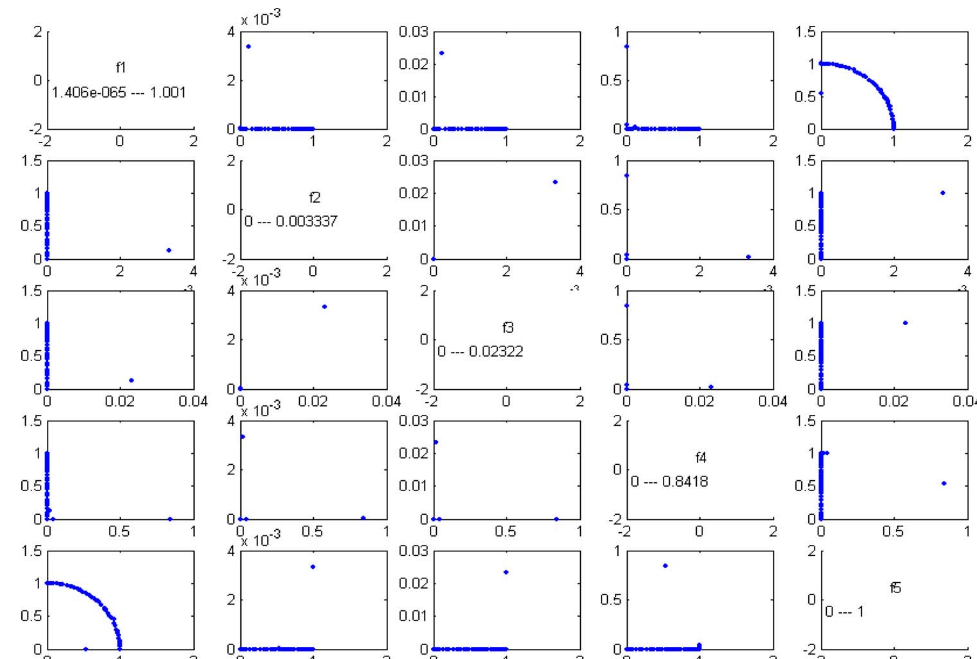

Fig. 5. L-front via the MDMOEA on test problem DTLZ2 with five objectives.

definition can only provide the decision making less L-optimal solutions, although huge Pareto-optimal solutions are available particularly when the number of objectives is quite high.

3) Test Problem 3 (DTLZ8): For DTLZ8, when $M = 5$, the Pareto front and L-front via the DMOEA are depicted in Fig. 6, whereas the L-front via the MDMOEA is shown in Fig. 7. Comparing Fig. 6 with Fig. 7, we can also observe that the good-distribution L-front is obtained via the MDMOEA, but only a little L-front is obtained via the DMOEA.

V. Conclusion and Discussions

Real-world optimization problems often have a large number of objectives. Hence, there is a clear requirement to develop new methods and new concepts for handling many-objective problems. In this paper, we have presented a DMOEA that adopted an aggregating function by combining ranking with entropy and density and have conducted numerical experiments for three test problems (DTLZ1, DTLZ2, and DTLZ6) with three to nine objectives. Moreover, a comparative study

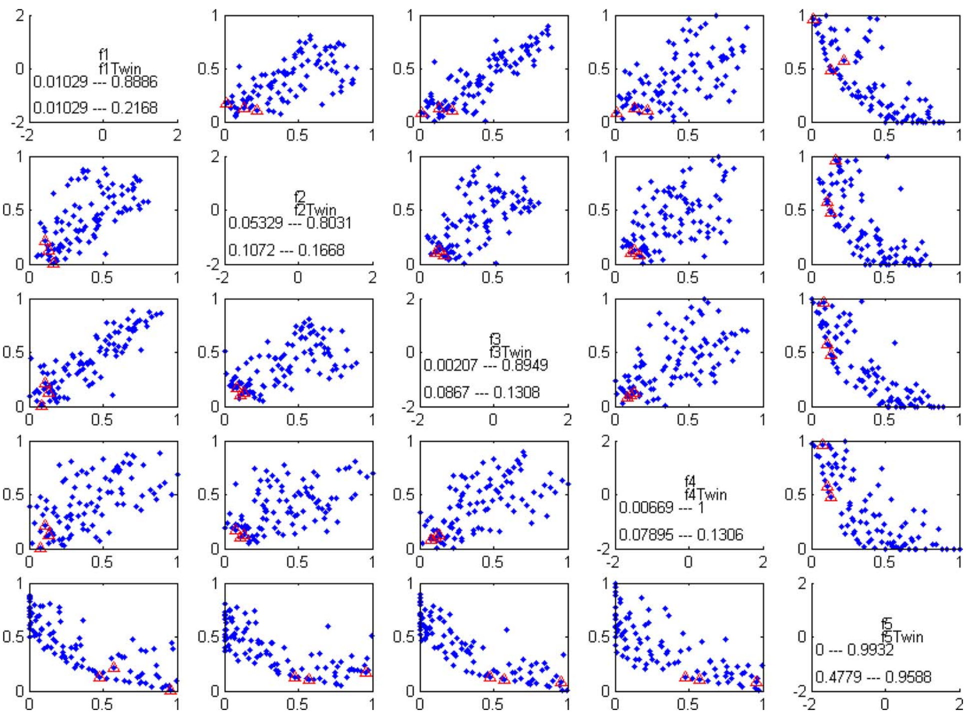

Fig. 6. Pareto front (star) and L-front (triangle) via the DMOEA on test problem DTLZ8 with five objectives.

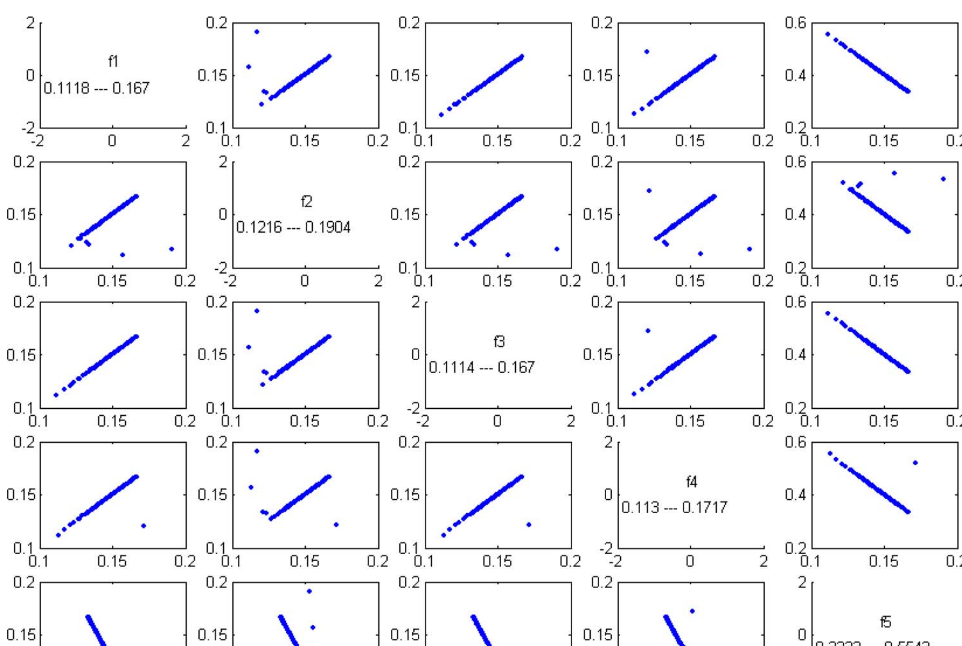

Fig. 7. L-front via the MDMOEA on test problem DTLZ8 with five objectives.

among the proposed method (DMOEA) and the other existing state-of-the-art algorithms (IBEA, MSOPS1, MSOPS2, and NSGAII) has been presented. For two different performance measures, the proposed method DMOEA has been shown to generate significantly better results on two test problems DTLZ1 and DTLZ6 compared to other four approaches, and to outperform the MSOPS1, MSOPS2, and NSGAII on test problem DTLZ2. In this paper, we have limited the performance assessment to less than ten-objective dimensions because the calculation of the hypervolume metric needs exponential time in the number of objectives. In addition, for this reason, we have not compared the DMOEA with the SMS-EMOA, although the SMS-EMOA was shown to have good performance for two test problems DTLZ1 and DTLZ2 when the number of objectives is from three to six in the experimental studies of Wagner *et al.* [4]. The performance assessment of MOEAs for solving optimization problems with more than nine objectives is a challenging issue in future research.

Another important aspect of this paper is that we propose L-dominance as a solution concept for evolutionary multiobjective optimization when many-objective problems are being considered. The new algorithm MDMOEA based on the L-dominance definition has been implemented and compared with the DMOEA. Simulations and comparative experiments have indicated that the new algorithm MDMOEA can converge to the true L-optimal front and maintain a widely distributed set of solutions. Although we have proven that L-optimal solutions are subsets of Pareto-optimal solutions, L-optimal solutions cannot be obtained only by choosing from Pareto-optimal solutions, which utilize MOEAs based on the Pareto dominance concept.

In our new definitions, we assumed that all objectives are equally important. When the number of objectives is large and not all objectives are equally important, the unimportant objectives could be deleted from the optimization problems. One purpose for normalizing all objectives is to obtain a nondimensionalized optimization problem since different objectives possibly have different measure units, and numerical experiments have shown that the units of objectives have effects on the results, even if the same algorithms were used. We also assumed that the maximum and minimum values for each objective are available. However, if the maximum and/or minimum values of some objectives are not available, we can estimate their approximate values by employing single-objective optimization techniques because it is not necessary to have the exact maximum and/or minimum objective values in our new definitions and new algorithms. In addition to these methods, the approach of weighted objectives can also be considered. Accordingly, it is important to determine good weights. Another future work is to prove the convergence and estimate the convergence rate of the MOEA based on the L-optimal definition.

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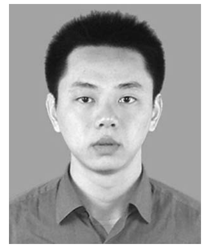

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