A Multiobjectivised Memetic Algorithm for the Frequency Assignment Problem

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Abstract*—This work presents a set of approaches used to deal with the Frequency Assignment Problem (FAP), which is one of the key issues in the design of Global System for Mobile Communications (GSM) networks. The used formulation of the FAP is focused on aspects which are relevant for real-world GSM networks. The best up to date frequency plans for the considered version of the FAP had been obtained by using parallel memetic algorithms. However, such approaches suffer from premature convergence with some real world instances. Multiobjectivisation is a technique which transforms a mono-objective optimisation problem into a multi-objective one with the aim of avoiding stagnation. A Multiobjectivised Memetic Algorithm, based on the well-known Non-Dominated Sorting Genetic Algorithm II (NSGA-II) together with its required operators, is presented in this paper. Several multiobjectivised schemes, based on the addition of an artificial objective, are analysed. They have been combined with a novel crossover operator. Computational results obtained for two different real-world instances of the FAP demonstrate the validity of the proposed model. The new model provides benefits in terms of solution quality, and in terms of time saving. The previously known best frequency plans for both tested real-world networks have been improved.

I. INTRODUCTION

The *Frequency Assignment Problem* (FAP) is a well-known NP-complete combinatorial optimisation problem of great importance in the radio-communication area. The FAP arises as one of the crucial issues in the design of *Global System for Mobile Communications* (GSM) networks [1]. This problem is also known as *Automatic Frequency Planning* (AFP) and *Channel Assignment Problem* (CAP). The set of applications of the FAP leads to many different mathematical and engineering models, but all of them share two common features:

- A set of antennas must be assigned frequencies such that data transmissions between the two end points of each connection are possible.
- Depending on the frequencies assigned to the antennas, they may interfere to one another, resulting in quality loss of signal.

This work is focused in the FAP which arises in the design of GSM networks. In such a case, the available frequency band is slotted into channels which have to be allocated to the elementary transceivers (TRXs) installed in the base stations of the network. In GSM, the FAP is a hard design task because the usable radio spectrum is very scarce and frequencies have to be reused throughout the network, and consequently, some inevitable degree of interference will occur. The goal of the designer is to minimise the interferences of the network, i.e., minimise the quality loss of signal. In this work, the formulation proposed in [2] is used. It takes full advantage of realistic and accurate interference information from real-world GSM networks.

Regarding to the approaches designed to deal with FAP formulations, some exact proposals are seen to exist [3]. However, they are not feasible when tackling large instances of the problem [4], so several heuristic and meta-heuristics methods [5] have also been proposed to deal with the FAP. *Memetic Algorithms* (MAs) [6], [7] have been widely applied to this problem [8], [9], [10]. They are a synergy of a population-based approach with separate individual learning or local improvement procedures for problem search. There are two different basic forms of individual learning [11]. On the one hand, *Lamarckian learning* forces the genotype to reflect the result of improvement in the learning by placing the locally improved individual back into the population to compete for reproduction. On the other hand, *Baldwinian learning* only alters the objectives of the individuals and the improved genotype is not encoded back into the population. Both kinds of approaches have been successfully applied [12].

To our knowledge, the best sequential approach for the here considered FAP formulation is given in [13]. However, the best up to date results have been obtained by means of parallel strategies [14], [15]. In [13] a comparison of population-based and trajectory-based meta-heuristics was included. It revealed the good performance of a memetic algorithm with increasing population size. The algorithm is a modified version of a (1 + 1) Evolutionary Algorithm (EA), combined with a local search specifically designed to deal with this version of the FAP. The variation phase is based on a tailor-made mutation operator. This algorithm achieves high quality solutions for different instances. However, in some cases premature convergence could happen.

Several methods to face local optima stagnation have been designed [16]. Some of the simplest techniques rely on performing a restart of the approach when stagnation is detected. In other cases, a component which inserts randomness or noise in the search is used. Maintaining some memory, in order to avoid exploring the same zones several times, is also a typical approach. Finally, population-based strategies try to maintain the diversity of a solution set. By recombining such a set of solutions, more areas of the decision space can be explored.

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The term *multiobjectivisation* was introduced in [17] to refer to the reformulation of originally mono-objective problems as multi-objective ones. Multiobjectivisation changes the fitness landscape, so it can be useful to avoid local optima [18], and consequently, to make easier the resolution of the problem. However, it can also produce a harder problem [19]. There are two different ways of multiobjectivising a problem. The first one is based on a decomposition of the original objective, while the second one is based on adding new objective functions. The addition of alternative functions can be performed by considering problem-dependent or problem-independent information.

The main contributions of the current work are the following: several mechanisms to multiobjectivise the FAP have been proposed, a novel non-destructive crossover operator has been defined, and both strategies have been integrated with the *Non-Dominated Sorting Genetic Algorithm II* [20] (NSGA-II). The novel strategy has been able to produce high quality results for two real-world instances of the FAP. In fact, to our knowledge, it has been able to produce the best known frequency plans for both instances.

The remaining of the paper is structured as follows: the mathematical formulation for the FAP is given in Section II. Section III is devoted to describe the applied optimisation scheme. Then, the experimental evaluation is described in Section IV. Finally, the conclusions and some lines of future work are given in Section V.

II. FAP: Formal Definition

Let $T = \{t_1, t_2, \ldots, t_n\}$ be a set of n elementary transceivers (TRX*s*), and let $F_i = \{f_{i1}, \ldots, f_{ik}\} \subset \mathbb{N}$ be the set of valid *frequencies* that can be assigned to a TRX $t_i \in T$, $i = 1, \ldots, n$. Note that k —the cardinality of F_i— is not necessarily the same for all the TRXs. Furthermore, let $S = \{s_1, s_2, \ldots, s_m\}$ be a set of given *sectors* (or cells) of cardinality m. Each TRX $t_i \in T$ is installed in exactly one of the m sectors. Henceforth we denote the sector in which a TRX t_i is installed by $s(t_i) \in S$. Finally, $M = \{(\mu_{ij}, \sigma_{ij})\}_{m\times m}$ is called the *interference matrix*. The two elements μ_{ij} and σ_{ij} of a matrix entry $M(i,j) = (\mu_{ij}, \sigma_{ij})$ are numerical values greater or equal than zero. μ_{ij} represents the mean and σ_{ij} the standard deviation of a Gaussian probability distribution describing the carrier-to-interference ratio (C/I) [21] when sectors i and j operate on a same frequency. The higher the mean value, the lower the interference and thus the better the communication quality. Note that the interference matrix is defined at sector (cell) level, because the TRXs installed in each sector all serve the same area.

A solution to the problem is obtained by assigning to each TRX $t_i \in T$ one of the frequencies from F_i. A solution (or frequency plan) is henceforth denoted by $p \in F_1 \times F_2 \times \cdots \times F_n$, where $p(t_i) \in F_i$ is the frequency assigned to TRX t_i. The objective is to find a solution p that minimises the following cost function:

$$C(p) = \sum_{t\in T} \sum_{u\in T, u\neq t} C_{\text{sig}}(p,t,u) \tag{1}$$

In order to define the function $C_{\text{sig}}(p,t,u)$, let s_t and s_u be the sectors in which the TRXs t and u are installed, that is, $s_t = s(t)$ and $s_u = s(u)$, respectively. Moreover, let $\mu_{s_t s_u}$ and $\sigma_{s_t s_u}$ be the two elements of the corresponding matrix entry $M(s_t, s_u)$ of the interference matrix with respect to sectors s_t and s_u. Then, $C_{\text{sig}}(p,t,u) =$

$$\begin{cases} K & \text{if } s_t = s_u, |p(t)-p(u)| < 2 \\ C_{\text{co}}(\mu_{s_t s_u}, \sigma_{s_t s_u}) & \text{if } s_t \neq s_u, \mu_{s_t s_u} > 0, |p(t)-p(u)| = 0 \\ C_{\text{adj}}(\mu_{s_t s_u}, \sigma_{s_t s_u}) & \text{if } s_t \neq s_u, \mu_{s_t s_u} > 0, |p(t)-p(u)| = 1 \\ 0 & \text{otherwise.} \end{cases} \tag{2}$$

K is a very large constant ($K \gg 0$) defined by the network designer so as to make it undesirable allocating the same or adjacent frequencies to TRXs serving the same area. Furthermore, function $C_{\text{co}}(\mu,\sigma)$ is defined as follows:

$$C_{\text{co}}(\mu,\sigma) = 100\left(1.0 - Q\left(\frac{c_{\text{SH}} - \mu}{\sigma}\right)\right) \tag{3}$$

where

$$Q(z) = \int_z^\infty \frac{1}{\sqrt{2\pi}} e^{\frac{-x^2}{2}} dx \tag{4}$$

is the tail integral of a Gaussian probability distribution function with zero mean and unit variance, and c_{SH} is a minimum quality signalling threshold. Function Q is widely used in digital communication systems because it characterises the error probability performance of digital signals [22]. This means that $Q\left(\frac{c_{\text{SH}}-\mu}{\sigma}\right)$ is the probability of the C/I ratio being greater than c_{SH} and, therefore, $C_{\text{co}}(\mu_{s_t s_u}, \sigma_{s_t s_u})$ computes the probability of the C/I ratio in the serving area of sector s_t being below the quality threshold due to the interferences provoked by sector s_u. That is, if this probability is low, the C/I value in the sector s_t is not likely to be degraded by the interfering signal coming from sector s_u and thus the communication quality yielded is high. On the contrary, a high probability —and consequently a high cost— causes the C/I mostly to be below the minimum threshold c_{SH} and thus incurring in low quality communications.

As function Q has no closed form for the integral, it has to be evaluated numerically. For this purpose we use the complementary error function E:

$$Q(z) = \frac{1}{2} E\left(\frac{z}{\sqrt{2}}\right) \tag{5}$$

In [23], a numerical method is presented that allows the value of E to be computed with a fractional error smaller than $1.2 \cdot 10^{-7}$.

Analogously, function $C_{\text{adj}}(\mu,\sigma)$ is defined as

$$\begin{aligned} C_{\text{adj}}(\mu,\sigma) &= 100\left(1.0 - Q\left(\tfrac{c_{\text{SH}} - c_{\text{ACR}} - \mu}{\sigma}\right)\right) \\ &= 100\left(1.0 - \tfrac{1}{2}E\left(\tfrac{c_{\text{SH}} - c_{\text{ACR}} - \mu}{\sigma\sqrt{2}}\right)\right) \end{aligned} \tag{6}$$

Algorithm 1 MA Pseudocode

1: Generate an initial population
2: **while** (not stopping criterion) **do**
3: Evaluate all individuals in the population
4: Variation phase
5: Perform individual learning process in the population with a probability p_l
6: Select the individuals which survive to the next generation
7: **end while**

The only difference between functions C_{co} and C_{adj} is the additional constant $c_{\text{ACR}} > 0$ (*adjacent channel rejection*) in the definition of function C_{adj}. This hardware specific constant measures the receiver's ability to receive the wanted signal in the presence of an unwanted signal at an adjacent channel. Note that the effect of constant c_{ACR} is that $C_{\text{adj}}(\mu, \sigma) < C_{\text{co}}(\mu, \sigma)$. This makes sense, since using adjacent frequencies (channels) does not provoke such a strong interference as using the same frequencies.

The mathematical model is aimed at incorporating interference information directly imported from real world GSM frequency planning as currently conducted in the industry (and not generated in a computer by sampling random variables). Indeed, the computations carried out to obtain the cost values are motivated by real-world GSM networks since they are related to the computation of the BER (Bit Error Rate) performance of Gaussian Minimum Shift Keying (GSMK), the modulation scheme used for GSM [22].

III. Optimisation Scheme

A. Memetic Algorithms

Memetic algorithms [6], [7] are a synergy of a population-based approach with separate individual learning or local improvement procedures for problem search. These algorithms are also referred to in the literature as *Baldwinian Evolutionary Algorithms*, *Lamarckian Evolutionary Algorithms*, *Cultural Algorithms* or *Genetic Local Search*. MAs are of great value because they perform some orders of magnitude faster than traditional genetic algorithms for some problem domains [24]. These algorithms have been applied in the mono-objective field [25] and also in the multi-objective one [26].

Algorithm 1 shows a general pseudocode of a memetic strategy. The main difference, respect the corresponding original algorithm, is the addition of a learning process step (line 5). Usually, such a step makes use of problem domain information. Nevertheless, some general methods have also been used. The learning process step is applied with a probability p_l. In [27] the effect of the probability p_l is analysed for a set of multi-objective benchmark problems. In our case, the individuals obtained after the variation phase may have very low quality components, which can be easily avoided by the individual learning process. Moreover, an effort to optimise the learning process step has been performed. Therefore, we have no need to fix the value of p_l, i.e., the learning process is always executed.

Two different MAs have been compared. The first one (*VarPopEA*) is derived from the mono-objective approach presented in [14]. It is a memetic algorithm which combines a modified evolutionary algorithm with a $(1+1)$ selection operator and a learning process specifically designed to face the FAP. The algorithm behaves as a trajectory-based algorithm when no stagnation is detected. However, it increases the population size in order to avoid local optima when necessary, behaving then as a population-based algorithm. The version used in this paper includes the usage of a crossover operator inside the variation phase. The second one is a modified version of the well-known Multi-Objective Evolutionary Algorithm (MOEA) NSGA-II. This version incorporates the learning process after the variation stage of the original algorithm. Individuals are encoded as arrays of integer values p, where $p(t)$ is the frequency assigned to TRX t.

B. Genetic Operators

A variation phase, consisting on applying mutation and crossover operators is performed on each generation of the memetic algorithm. The *Neighbour-based Mutation* (NM) operator, which obtained the best behaviour in [14], has been applied. First, a random TRX t_x is randomly reassigned. Then, the TRXs which interfere t_x, or are interfered by t_x are included in the list $affected$ and are mutated with a probability p_m. The previous step is repeated N times, but in following iterations the TRX is selected among those ones which are included in $affected$. By this way, this mutation operator is focused on one area of the network.

This strategy has been combined with a random crossover operator and with a new proposed directed one. This operators are applied with a probability p_c.

- *Uniform Crossover* (UX). For each gene, a random variable $p \in [0, 1]$ is generated. If $p < 0.5$, then the gene is inherited from the first parent. Otherwise, the gene is inherited from the second one.
- *Interference-based Crossover* (IX). A TRX t_x is randomly selected. Every gene associated to a TRX which interferes t_x or is interfered by t_x, including the gene which represents the own t_x, is inherited from the first parent. The remaining genes are inherited from the second parent.

C. Learning Process for the FAP

Generally, multi-objective memetic algorithms make use of a multi-objective learning process [28]. However, since in the current paper the FAP has been multiobjectivised, the learning process has only taken into account the original objective. The incorporated learning process is based on a mono-objective hill-climbing local search procedure. The application of local search methods allows admissible solutions to be achieved in relatively short times. This is a typical requirement within commercial tools, the context in which the FAP resides. Several local search methods for the FAP have been defined [29], [30]. The local search method proposed in [14] has improved the aforementioned strategies. Therefore, a local search based on it (Algorithm 2) has been used in the current work.

Algorithm 2 Pseudocode for the Local Search

```
1: Input: current solution S
2: nextSectors ← initialSectors
3: while (nextSectors != ∅) do
4:    currentSectors ← nextSectors
5:    nextSectors ← ∅
6:    while (currentSectors != ∅) do
7:       sec ← extract a random sector from currentSectors
8:       neighbour ← reassign frequencies of S in sector sec
9:       if (neighbour improves S) then
10:         S ← neighbour
11:         nextSectors += sectors interfered by sec
12:         nextSectors += sectors that interfere sec
13:      end if
14:   end while
15: end while
16: return S
```

The operation of the strategy is based on optimising the frequencies assignment to TRXs in a given sector, without modifying the remaining network assignments. The neighbours of a candidate solution are obtained by replacing the frequencies in the TRXs of each sector. The reassignment of frequencies within a sector is performed in the following way: first, the available frequencies for the sector are sorted by their involved cost. Then, two possibilities are considered, either assign the frequency with the lowest associated cost to a TRX that is allowed to use that frequency, or assign its two adjacent frequencies to two different TRXs (if they are allowed to use these frequencies). For each of the newly generated partial solutions the same process is repeated until all TRXs in the sector have been assigned a frequency. The complete solution with lowest associated cost is considered as the new neighbour, while the other ones are discarded.

The order in which neighbours are analysed is randomly determined. The set $initialSectors$ contains the sectors which are going to be initially checked (line 2). In the first generation, every neighbour is considered, i.e., the complete set of sectors is assigned to $initialSectors$. However, in order to reduce the computational cost of the local search, not every neighbour is taken into account in following generations. In fact, the only ones sectors which are considered ($initialSectors$) are those which have been modified by the mutation operator explained in Section III-B. Once $initialSectors$ is properly initialised, $currentSectors$ maintains the set of sectors which must be checked (line 4). For the generation of a new neighbour, a sector sec is randomly extracted from $currentSector$ (line 7) and its frequencies are reassigned as aforementioned (line 8). The local search moves to the first new generated neighbour that improves the current solution (lines 9–10), adding all the sectors that interfere or are interfered by sec to the set of the next sectors ($nextSectors$) to consider (lines 11–12). When $currentSectors$ set gets empty (line 6), sectors in $nextSectors$ are transferred to the current set (line 4) and $nextSectors$ set is cleared (line 5). The local search process stops when none of the neighbours improves the current solution (line 3).

D. Multiobjectivisations

Several multiobjectivised schemes have been explored. Most of them are problem-independent strategies, in which an artificial objective function is added to multiobjectivise the FAP. The first objective has been selected as the cost function of the FAP, while for the second one, an artificial function which tries to maximise the diversity has been used. One of the main challenges has been the selection of this artificial function. In fact, it has been demonstrated that the proper artificial function depends on the considered problem and even instance [31]. A comparison of a set of well-known schemes has been carried out. Moreover, two novel artificial objectives have also been tested.

Several options have been proposed to define the artificial objective [32]. Some schemes based on the usage of the Euclidean distance on the decision space have been analysed:

- DCN: Distance to the closest neighbour of the population.
- ADI: Average distance to all individuals of the population.
- DBI: Distance to the best individual of the population, i.e., the one with the lowest FAP fitness.

Also, the following ones have been taken into account:

- *Random*: A random value is assigned as the second objective to be minimised. Smaller random values may be assigned to some bad individuals which would get a chance to survive.
- *Inversion*: In this case, the optimisation direction of the original objective function is inverted and is used as the artificial objective. This approach decreases the selection pressure, so a large number of Pareto-optimal solutions could be included at each generation.

Finally, two novel variants of the DBI scheme have also been considered. They are based on the addition of a threshold which penalises those solutions that may have a poor quality. In DBI_TH1 a threshold is established over the FAP objective function. Thus, individuals that are not capable to achieve the fixed threshold are penalised by assigning a zero value to the second objective function. DBI_TH2 is similar to the previous mechanism, but the threshold is established over the distance.

Several novel problem-dependent multiobjectivisations were also tested. The best behaved one (*Dependent*) makes use of two objectives. The first one is the original cost function. For the second objective, the original FAP cost function is decomposed in two independent cost functions f_1 and f_2. The decomposition is performed in the following way. First, a table containing all possible interferences between each pair of TRXs is generated. Then, this table is ordered based on the cost of the appearance of each pair p. The resultant position of each p is denoted as i_p. The cost associated to each p is taken into account in the function f_n where $n = i_p \bmod 2$. Finally, f_1 is used as the second objective.

By multiobjectivising the FAP, selection pressure is decreased, so some low quality individuals could survive in the population. However, in the long term these individuals could help to avoid stagnation in local optima, so higher quality solutions could be obtained.

IV. Experimental Evaluation

In this section the experiments performed with the multiobjectivised memetic algorithm exposed in Section III are described. The obtained results are compared with the best up to date sequential approach. Tests have been run on a Debian GNU/Linux computer with four AMD ® Opteron ™ (model number 6164 HE) at 1.7 GHz and 64 Gb RAM. The compiler which has been used is *gcc 4.4.5*. Comparisons have been performed considering two US cities instances: Seattle and Denver. The Seattle instance has 970 TRXs and 15 different frequencies to be assigned. The Denver instance is larger. It is constituted by 2612 TRXs and 18 frequencies. In both cases, the constants used in the formal definition [2], have been set to $K = 100000$, $c_{SH} = 6\ dB$, and $c_{ACR} = 18\ dB$. These GSM networks are currently operating so finding their optimal planning is of great practical interest. It is important to remark that the data source to build the interference matrix based on the C/I probability distribution uses thousands of Mobile Measurement Reports (MMRs) [33] rather than propagation prediction models. The M matrix contains 59169 elements in the Seattle network, while it contains 20638 elements for the Denver instance.

Since we are dealing with stochastic algorithms, each execution has been repeated 30 times. Each experiment has been carried out for both instances. In order to provide the results with confidence, comparisons have been performed applying the following statistical analysis [34]. First, a *Shapiro-Wilk test* is performed in order to check whether the values of the results follow a normal (Gaussian) distribution or not. If so, the *Levene test* checks for the homogeneity of the variances. If samples have equal variance, an ANOVA *test* is done. Otherwise, a *Welch test* is performed. For non-Gaussian distributions, the non-parametric *Kruskal-Wallis* test is used to compare the medians of the algorithms. A confidence level of 95% is considered, which means that the differences are unlikely to have occurred by chance with a probability of 95%.

In the first experiment, 24 different configurations of the multiobjectivised memetic algorithm have been tested. They have been constituted by combining the eight multiobjectivised proposed schemes with three different variation schemes. In the first variation scheme, the UX operator with $p_c = 1$ has been used. In the second one, the IX operator with $p_c = 1$ has been applied. Finally, for the last variation scheme, the crossover operator has been disabled, i.e., $p_c = 0$. Every variation schemes have used the next parameterisation for the Neighbour-based Mutation operator: $p_m = 0.9$ and $N = 7$. Each configuration has been executed during 4 hours.

Figure 1 shows, for the Seattle instance, the evolution of the cost function average values for the different multiobjectivised schemes. The same information is shown in Figure 2 for the Denver instance. They are compared with VarPopEA, which was the best up to date sequential strategy. VarPopEA has also been executed with the three aforementioned variation schemes. Results are shown with the best behaved variation scheme for each approach. Since the *Reverse* multiobjectivisation has obtained very low quality results, it has not been taken into account. Table I shows whether the row configuration is statistically better (↑), not different (↔), or worse(↓), than the corresponding column configuration, in 4 hours of execution for the Seattle instance. Table II shows the same information for the Denver instance. For the Seattle instance, three multiobjectivised schemes have obtained better average cost values than VarPopEA. Moreover, two of them are statistically better than VarPopEA, showing the benefits of multiobjectivised techniques. For the Denver instance, the best average results have been obtained by VarPopEA-IX. However, differences between it and the best three multiobjectivised approaches are not statistically significant.

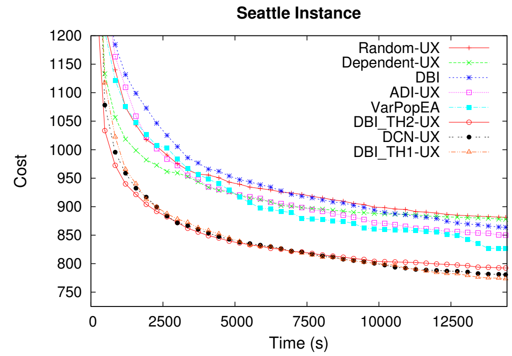

Figure 1. Interference evolution for the Seattle best configurations

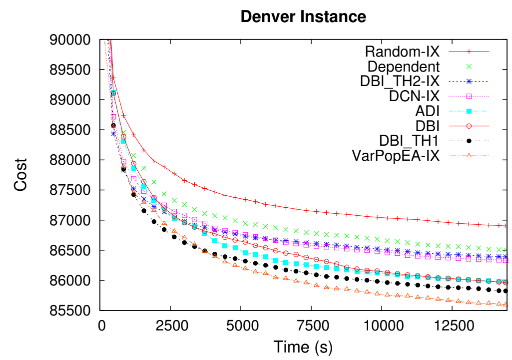

Figure 2. Interference evolution for the Denver best configurations

Taking into account the best multiobjectivised approach, Table III shows, for the Seattle instance, a statistical comparison among the three tested variation schemes. The same information is shown in Table IV for the Denver instance For the Seattle instance, no significant differences have been detected among different variation schemes. In the case of the Denver instance, the IX operator is statistically better than the UX operator, but not statistically different from the variation scheme with no crossover operator. Moreover, taking into consideration average cost values, results of four schemes have been improved by using the IX operator. Therefore, a deeper analysis with other instances should be performed to better explore the IX benefits.

Table I
STATISTICAL COMPARISON OF CONFIGURATIONS FOR THE SEATTLE INSTANCE

	DBI_TH1-UX	DCN-UX	DBI_TH2-UX	VarPopEA	ADI-UX	DBI	Dependent-UX	Random-UX
DBI_TH1-UX	↔	↔	↔	↑	↑	↑	↑	↑
DCN-UX	↔	↔	↔	↑	↑	↑	↑	↑
DBI_TH2-UX	↔	↔	↔	↔	↑	↑	↑	↑
VarPopEA	↓	↓	↔	↔	↑	↑	↔	↑
ADI-UX	↓	↓	↓	↓	↔	↔	↔	↔
DBI	↓	↓	↓	↓	↔	↔	↔	↔
Dependent-UX	↓	↓	↓	↔	↔	↔	↔	↔
Random-UX	↓	↓	↓	↓	↔	↔	↔	↔

Table II
STATISTICAL COMPARISON OF CONFIGURATIONS FOR THE DENVER INSTANCE

	VarPopEA-IX	DBI_TH1	DBI	ADI	DCN-IX	DBI_TH2-IX	Dependent	Random-IX
VarPopEA-IX	↔	↔	↔	↔	↑	↑	↑	↑
DBI_TH1	↔	↔	↔	↔	↑	↑	↑	↑
DBI	↔	↔	↔	↔	↔	↔	↑	↑
ADI	↔	↔	↔	↔	↑	↔	↑	↑
DCN-IX	↓	↓	↔	↓	↔	↔	↔	↑
DBI_TH2-IX	↓	↓	↔	↔	↔	↔	↔	↑
Dependent	↓	↓	↓	↓	↔	↔	↔	↔
Random-IX	↓	↓	↓	↓	↓	↓	↔	↔

Table III
STATISTICAL COMPARISON OF VARIATION SCHEMES FOR SEATTLE

	UX	No Crossover	IX
UX	↔	↔	↔
No Crossover	↔	↔	↔
IX	↔	↔	↔

Table IV
STATISTICAL COMPARISON OF VARIATION SCHEMES FOR DENVER

	No Crossover	IX	UX
No Crossover	↔	↔	↑
IX	↔	↔	↑
UX	↓	↓	↔

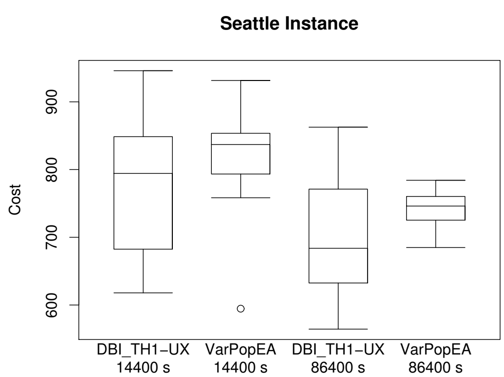

Figure 3. Boxplots for the Seattle best configurations

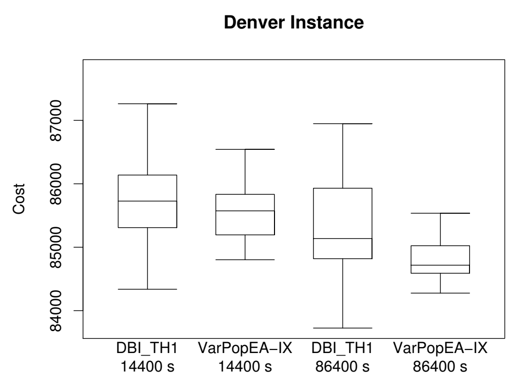

Figure 4. Boxplots for the Denver best configurations

In order to check the behaviour of the multiobjectivised approaches in the long term, a second experiment has been carried out. Specifically, the best multiobjectivised scheme and VarPopEA, with its best variation scheme, have been executed using a stopping criterion of 24 hours. Figure 3 shows the boxplot of the cost values achieved in 4 and 24 hours for the Seattle instance. For both values of the stopping criterion, the multiobjectivised approach is statistically better than VarPopEA. Figure 4 shows the same information for Denver. Considering 4 hours for the stopping criterion, both models are similar. In fact, there are no statistical differences. In the case of 24 hours, most of the VarPopEA-IX executions are better than the DBI_TH1 executions. However, some DBI_TH1 executions have been able to deal better with local optima. In fact, DBI_TH1 has been able to obtain the best solutions.

Previous experiments have compared different multi-objective and mono-objective configurations in terms of the quality achieved at fixed times. By the use of *run-length distributions* [35], the run-time behaviour can be deeper analysed. Run-length distributions show the relation between success ratios and time. Success ratio is defined as the probability of achieving a certain quality level. In order to establish a

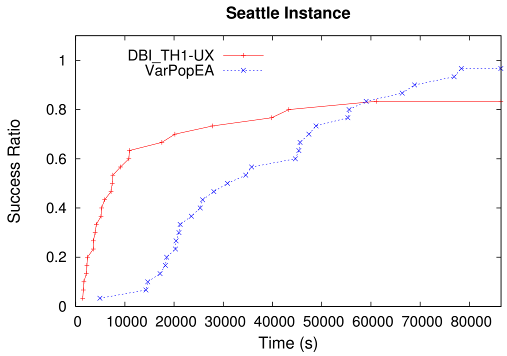

Figure 5. Run-length distribution for the Seattle instance

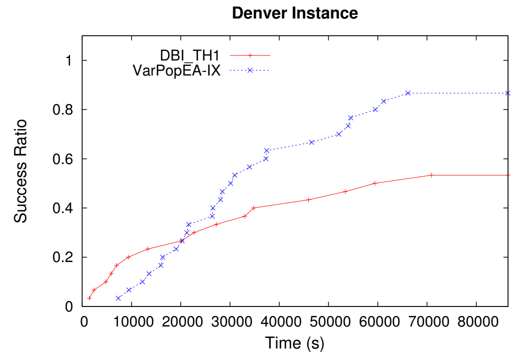

Figure 6. Run-length distribution for the Denver instance

high enough quality level, it has been fixed as the average cost obtained in 8 hours of execution of VarPopEA with its best variation scheme for each instance. Figure 5 shows the run-length distribution for the Seattle instance. The 50% of executions of DBI_TH1-UX has achieved the fixed quality level in less than 7440 seconds. In the case of VarPopEA, the required time is 30840 seconds. Thus, the speedup factor is 4.14. By contrast, the success ratio of VarPopEA is higher than the success ratio of DBI_TH1-UX, considering 24 hours of execution. The run-length distribution for the Denver instance is depicted in Figure 6. In this case, considering the 50% of the executions, VarPopEA-IX is 1.97 times faster than DBI_TH1. Moreover, considering the results for 24 hours, VarPopEA-IX success ratio is higher than the DBI_TH1 success ratio. However, DBI_TH1 has been able to obtain higher success ratios than VarPopEA-IX, when times lower than 5.5 hours are considered. Therefore, for the considered quality level, the most promising model depends on time constraints.

As stated in the paper, the analysed GSM networks are currently operating so finding their optimal planning is of great practical interest. To our knowledge, the best frequency plans for the considered instances were reported in [14], [36]. The multiobjectivised approach has improved such frequency plans. In the case of the Seattle instance the interference cost has been decreased from 654.53 to 564.35. The interference cost has been decreased from 83991.30 to 83725.60 for the Denver instance. The best variant of VarPopEA has obtained the values 594.60 and 84276.80.

V. Conclusions and Future Work

This paper has presented a set of approaches used to deal with the FAP. Relevant aspects of real-world GSM networks have been considered in the mathematical formulation of the FAP. Both, problem-dependent and problem-independent multiobjectivised techniques have been applied. They have been tested with NSGA-II, one of the most successful MOEAs. Experiments have been carried out with several variation schemes. One of them makes use of the novel IX operator. These variation schemes have also been incorporated in VarPopEA, which was the best up to date sequential method for the considered version of the FAP. Two real world instances have been tackled with the new designed approaches. Experimental results have demonstrated the validity of the proposals. For the Seattle instance, three multiobjectivised schemes have obtained better results than VarPopEA. For the Denver instance, most of the VarPopEA-IX executions have been better than the multiobjectivised ones. However, some multiobjectivised executions have been able to deal better with local optima. In fact, for both instances, the DBI_TH1 multiobjectivisation with its best variation scheme have obtained the best solutions. The experimental evaluation has not demonstrated the IX operator superiority. For the Seattle instance, none variation scheme have been statistically different from the IX-based variation scheme. In the case of the Denver instance, it has reported statistically better results than the UX-based one. However, the IX-based variation scheme has not been statistically different from the variation scheme with no crossover for this instance. Therefore, a deeper analysis with other instances should be performed, in order to demonstrate the contribution of this operator. The run-time analysis clearly shows the contribution of multiobjectivised techniques for the Seattle instance. In the case of the Denver instance, DBI_TH1 obtains higher success ratios than VarPopEA-IX when short periods of time are considered. However, VarPopEA-IX achieves higher success ratios than DBI_TH1 in the long term. Therefore, the most promising model depends on time constraints. Finally, it is worth mentioning that the best known up to date frequency plans for both analysed instances have been improved by the multiobjectivised approach.

Future work will be focused in the analysis of other multiobjectivised schemes. The usage of other problem-dependent and problem-independent multiobjectivisations should be analysed. Also, in order to reduce the time required to attain high quality solutions, parallel MOEAs should be tested. Since the appropriate multiobjectivisation could depend on the instance which is being solved, the hybridisation of multiobjectivisation and hyper-heuristics seems a promising approach. Thus, the selection of the multiobjectivised method which must be used could be performed in an automatic way. In order to draw more conclusions about the contribution of the IX operator, other instances should be tested.

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