Int. J. Fuzzy Syst.
https://doi.org/10.1007/s40815-020-00823-y

Control the Diversity of Population with Mutation Strategy and Fuzzy Inference System for Differential Evolution Algorithm

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Received: 3 January 2020 / Revised: 10 February 2020 / Accepted: 17 February 2020

Abstract This paper displays how to use fuzzy inference system (FIS) to control the individual uniform diversity for differential evolution algorithm (DE). DE solves nonlinear optimization problems, and a successful control mechanism for population diversity enhances the performance of DE. This study proposed a control mechanism that contains a novel mutation strategy and FIS because FIS is suitable for consecutive and hard classified inputs. The proposed control mechanism does not fix the target vector and controls the ratio of mutating toward the whole best individual by FIS. The FIS decides the *F* values for this novel mutation strategy. The experiments compared the winner of each evaluated functions among four uniform diversity goals (UDGs) with conventional strategies. From experimental results, the proposed method finds superior solutions to conventional mutation strategies at least 11 out of 15 evaluated functions in 10, 30, and 50 dimensions. Furthermore, not only the diversity curves confirm the control ability of FIS, but also different paths of convergence curves indicate the fast convergence and mitigation of evolutionary stagnation.

Keywords DE · Diversity control · Fuzzy inference system · Mutation strategy · Entropy

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1 Introduction

Evolutionary algorithms (EAs) solve nonlinear optimization problems. When studying EAs, how to balance exploitation and exploration during evolution is crucial. If an EA overly emphasizes exploitation, there is a high probability that it will fall into a local optimum or converge prematurely. By contrast, if an EA overly emphasizes exploration, its convergence time may become very long [1]. Numerous studies have focused on this tradeoff. Bosman and Thierens [2] proposed a framework to balance exploitation and exploration. Chen et al. [3] devised a selection strategy that included five rules to keep the genetic diversity for enhancing evolutionary programming. Segura et al. [4] define a distance metric for measuring diversity and considered the evolutionary stoppage criteria when setting their diversity goal.

The differential evolution (DE) algorithm [5], one of the EAs, also suffers from the problem of balancing exploitation and exploration. Studies have focused on ensuring diversity. Yu et al. [6] used the DE//best/1 mutation strategy, which achieves multiple local best individuals leading instead of global best leading, to increase diversity. Another approach is to apply different mutation strategies after dividing a population into different classes. Das et al. [7] handled three individual groups—DE, Brownian, and quantum individuals, and used a parameter to control the ratio of these three groups and the diversity of individuals. Yang et al. [8] manipulated entire individuals and presented a mechanism to eliminate the loss of diversity with each dimension. Due to these studies, this study believed that a suitable diversity control is crucial for balancing exploitation and exploration, then enhancing the performance of EAs such as DE.

Published online: 19 March 2020

The present study proposes a method that integrates conventional mutation strategies into a DE mutation strategy and uses a fuzzy inference system (FIS) to control the parameter for satisfying the diversity goal. The inputs of an inference system are consecutive numbers and hard to classify, so this study uses the fuzzy inference system. The experiments compared the proposed method with conventional mutation strategies. The experimental results demonstrate the diversity control ability of the proposed method and show that the proposed method has superior solutions to conventional strategies at least 11 out of 15 evaluated functions in 10, 30, and 50 dimensions. This method shows promise for balancing exploration and exploitation through diversity control. Furthermore, balancing exploitation and exploration enhanced DE even for single-objective optimization problems.

The remainder of this paper is organized as follows. Section 2 briefly introduces conventional DE mutation strategies, uniform diversity, and how to measure uniform diversity. Section 3 presents the proposed control mechanism, including the new mutation strategy and FIS. Section 4 describes the experiments and presents their results. Finally, Sect. 5 presents the conclusions of this study.

2 Background Knowledge

This section introduces the conventional DE mutation strategies, uniform diversity, and how to measure uniform diversity.

2.1 Conventional DE Mutation Strategies

The conventional DE mutation strategies introduced in this paper include DE/rand/2, DE/best/2, DE/rand-to-best/2, and DE/current-to-best/2. The expression of a common DE mutation strategy is shown below:

$$\mathbf{v}_i = \mathbf{x}_{\text{target}} + F \cdot (\mathbf{x}_{diff}), \quad (1)$$

where $\mathbf{v}_i$ is the ith mutant vector, $\mathbf{x}_{\text{target}}$ denotes the target vector, the F is a scaling factor, and $\mathbf{x}_{\text{diff}}$ is a differential vector. The target vector is the individual which is mutated to generate a mutant vector. The mutant mechanism adds the differential vector to the target vector after multiplying a scaling factor. The description template of a DE mutation strategy is “DE/how to generate target vector/the number of differential vectors.” For example, DE/rand/2 presents that “randomly selected an individual as target vector plus weighted two differential vectors.”

2.1.1 DE/rand/2

The expression of this mutation strategy is as follows:

$$\mathbf{v}_i = \mathbf{x}_{r1} + F \cdot (\mathbf{x}_{r2} - \mathbf{x}_{r3} + \mathbf{x}_{r4} - \mathbf{x}_{r5}), \quad (2)$$

where $\mathbf{x}_{rj}$ is the jth randomly selected individual, and each individual cannot be selected repeatedly. This mutation strategy is an explored-mutant strategy. This paper uses the Str1 for DE/rand/2.

2.1.2 DE/best/2

The expression of DE/best/2 is as follows:

$$\mathbf{v}_i = \mathbf{x}_{\text{best}} + F \cdot (\mathbf{x}_{r1} - \mathbf{x}_{r2} + \mathbf{x}_{r3} - \mathbf{x}_{r4}), \quad (3)$$

where $\mathbf{x}_{\text{best}}$ is the global best individual. This strategy is greedy since this mutation strategy is based on the global best (GB), which has the highest fitness value searched so far. This mutation strategy is an exploited-mutant strategy. The main drawback is that the GB maybe a local optimal. In this paper, we denote Str2 as DE/best/2 as follows.

2.1.3 DE/rand-to-best/2

The mutation strategy is similar to a weighted average of Str1 and Str2, and its expression is as follows:

$$\mathbf{v}_i = \mathbf{x}_{r1} + k \cdot (\mathbf{x}_{\text{best}} - \mathbf{x}_{r1}) + F \cdot (\mathbf{x}_{r2} - \mathbf{x}_{r3} + \mathbf{x}_{r4} - \mathbf{x}_{r5}), \quad (4)$$

where k is a parameter that represents the ratio of Str2. This paper denotes the DE/rand-to-best/2 as Str3 as follows.

2.1.4 DE/current-to-best/2

Recently, some nature-inspired algorithms [9] feature local search, and the DE/current-to-best/2 is also a local search mutation strategy. The expression of this mutation strategy is as follows:

$$\mathbf{v}_i = \mathbf{x}_i + F \cdot (\mathbf{x}_{\text{best}} - \mathbf{x}_i) + F \cdot (\mathbf{x}_{r1} - \mathbf{x}_{r2}), \quad (5)$$

where $\mathbf{x}_i$ is the ith individual in the population. This strategy has less ability for exploration than Str1. This paper denotes the DE/current-to-best/2 as Str4 as follows.

2.2 Uniform Diversity and its Measurements

There are two types of diversity: extended diversity (ED) and uniform diversity (UD) [10]. The broad distribution range represents high ED; the narrow distribution range is low ED. However, the ED does not consider the distribution inside each dimension. For example, there are two sets: **A** = {0, 0 , 0, 9, 9, 9} and **B** = {1, 2 ,3, 4, 5, 6}. According to the measurement of ED, the ED of **A** is larger than the ED of **B**. Nevertheless, set **A** only has two types of elements and set **B** has six different types of elements. Studying diversity should consider not only extent but also uniformity. The uniform distribution is high UD and offers comprehensive information. The concentrative distribution that is low UD offers less information.

Burke et al. [11] discussed many diversity measures, including entropy. A typical measurement of distribution is variance and standard deviation. The computation of variance is as follows:

$$\mathrm{Var}(\mathbf{y}) = \frac{\sum_{m=1}^{\mathrm{Num}} (y_m - \bar{y})^2}{\mathrm{Num}} \tag{6}$$

where $\mathbf{y}$ is the universal set of data y, y_m is the mth data, Num denotes the total number of data, and $\bar{y}$ is the average value of all y. Zaharie [1, 12] uses the variance to measure population diversity. Zaharie deduces that the expected value of the variance of the trial individual is related to F, CR, and total dimension, D. The expression of the expected value of the variance of the trial individual is as follows:

$$\mathrm{E}[\mathrm{Var}(\mathbf{u})] = (2 \times F^2 \times \mathrm{CR} - \frac{2 \times \mathrm{CR}}{D} + \frac{\mathrm{CR}^2}{D} + 1) \times \mathrm{Var}(\mathbf{x}) \tag{7}$$

From (7), the user can tune the variance of a trial individual by tuning F, CR values. Nevertheless, the variance is not the best way to measure UD. For instance, there are another two sets, **C** = {− 28, − 21, 0, 21, 28} and **D** = {− 35, 0, 0, 0, 35}. The set **C** and set **D** have the same variance, but the set **C** offers more information than set **D**. Another set **E** = {− 70, 0, 0, 0, 70} has larger variance value than set **D** but offers almost the same information. They both contain three types of elements. From the above instances, the variance is not suitable for measuring UD.

In Information Theory, Shannon entropy is the measurement of information. Shannon [13] introduces entropy to information theory in 1948. Some studies [14, 15] use entropy to compute population diversity.

The original expression of Shannon entropy is as follows:

$$\mathrm{Entro} = -\sum_{m=1}^{\mathrm{Num}} p_m \times \log_b(p_m) \tag{8}$$

where Entro denotes the Shannon entropy, p_m is the probability of mth data, and b is the radix. The typical value of b is 2, natural number e, or 10. The entropy (uses 10 as radix) of these sets, **A**, **B**, **C**, **D**, and **E** is 0.3010, 0.7781, 0.6990, 0.4127, and 0.4127, respectively. The entropy indeed measures the distribution of data than the variance.

3 The Proposed Method

This section introduces the proposed mutation strategy and how to control diversity with FIS.

3.1 Diversity Measurement of Individuals

This study divides the distribution range of the current population in each dimension into equal population-sized portions; then, the proposed method calculates the probability distribution of each dimension. The expression of the probability is as follows:

$$p_{d,m} = \frac{\mathrm{NumInd}_{d,m}}{\mathrm{PS}} \tag{9}$$

where $p_{d,m}$ is the probability of the mth portion of the distribution range in the dth dimension; $\mathrm{NumInd}_{d,m}$ represents the number of individuals that belong to the mth portion of the distribution range in the dth dimension. After obtaining the probability distribution, this study modifies Shannon entropy as follows:

$$D = -\frac{1}{N} \times \sum_{d=1}^{N} \sum_{m=1}^{PS} (p_{d,m} \times \log_{10} p_{d,m}) \tag{10}$$

where D is the uniform diversity value, N is the dimension of the population, and PS is the population size. This modified Shannon entropy value is considered equivalent to the uniform diversity level of the individuals.

3.2 Proposed Mutation Strategy

The expression of proposed novel mutation strategy, DE/target-to-best/2, in this study is as follows:

$$\mathbf{v}_i = \mathbf{x}_{\mathrm{target}} + F \times (\mathbf{WB} - \mathbf{x}_{\mathrm{target}}) + (1 - F) \times (\mathbf{x}_{\mathrm{r1}} - \mathbf{x}_{\mathrm{r2}} + \mathbf{x}_{\mathrm{r3}} - \mathbf{x}_{\mathrm{r4}}) \tag{11}$$

where $\boldsymbol{x}_{\mathrm{target}}$ is the target vector. The proposed mechanism selects the target vector from among individuals randomly or the current individual directly. ***WB*** represents the whole best individual which is instead of the global best individual to speed up the convergence in [16]. Hereafter, Str5 refers to DE/target-to-best/2. The decision method of the target vector is as follows:

$$\mathbf{x}_{\mathrm{target}} = \begin{cases} \mathbf{x}_i, & \text{if } D_{\mathrm{goal},g} \le D_g \\ \mathbf{x}_{\mathrm{rand}}, & \text{others} \end{cases} \tag{12}$$

where D_g is the diversity of individuals in the gth generation, $\boldsymbol{x}_i$ is the ith individual in the parent when generating the ith mutant individual, and $\boldsymbol{x}_{\text{rand}}$ is the randomly selected individual that cannot equal to $\boldsymbol{x}_i$. The idea behind this decision is that when diversity is insufficient, the randomly selected individual is used as the target vector for further exploration to increase diversity. When diversity is sufficient, this control mechanism uses a local search. Using the current individual as a target vector may keep the diversity, and using a randomly selected individual as a target vector may change the diversity.

There are two differential vectors in the Str5. The first one is used to move close to the ***WB***, as shown in Fig. 1. The second one is used to move away from the ***WB***. The FIS outputs directly the F value, which represents the degree of individuals mutating toward the ***WB***. This mutation strategy needs one individual or point to be the reference point but not limited to ***WB***. This study regards the ***WB*** as a promising individual, so the considerable F value makes this mutation is an exploitative mutation strategy and enhances the ability to evolve optimal solution; otherwise, it is an explorative mutation strategy.

3.3 Fuzzy Inference System Design

This study used FIS to infer F value because the inputs of the inference system are consecutive and hard to classify. The first input is the normalized Euclidean distance between the ***WB*** and an individual, and the expression of it is as follows:

$$Dis_{\text{nor},i} = \begin{cases} 0, & \text{if } Dis_{\max} - Dis_{\min} = 0 \\ \dfrac{\text{Dis}_i - \text{Dis}_{\min}}{Dis_{\max} - Dis_{\min}}, & \text{others} \end{cases} \tag{13}$$

where $Dis_{\text{nor},i}$ is the normalized Dis_i, Dis_i is the Euclidean distance between the ***WB*** and ith individual, $Dis_{\max}$ is the maximum value of Dis_i, and $Dis_{\min}$ is the minimum value of Dis_i.

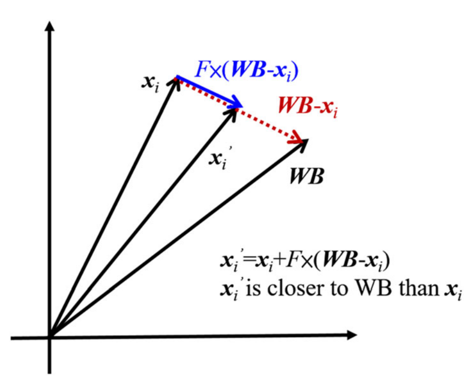


Fig. 1 Illustration of the first differential part in Str5 is used to move close to the ***WB***

The second input is normalized diversity error (Err_{div}). The diversity error is the difference between the current diversity (D_g) and the diversity goal (D_{goal}). Here, the sigmoid function is used to normalize diversity error. The expression of the normalized diversity error is as follows:

$$Err_{\text{div}} = \frac{1}{1+e^{-\frac{D_g - D_{\text{goal},g}}{D_{\text{goal},g}}\times\beta}} \tag{14}$$

where the β denotes steepness of sigmoid function and $D_{\text{goal},g}$ is the diversity goal in gth generation. The setting of β value is

$$\beta = \left(-\ln\left(\frac{z_2}{1-z_2}\right)\right) \times \frac{1}{z_1} \tag{15}$$

where z_1 is the input of sigmoid function; z_2 is the output value of the sigmoid function and is also the input of FIS. In this paper, z_2 is the Err_{div}. Setting β as 5.1734 in this study makes the sigmoid input value can map to the expected fuzzy set (z_1 is 0.5, and z_2 is 0.93) as Fig. 2. The input values should be normalized because the order of magnitude of these inputs varies during the evolutionary procedure.

Most membership functions of the inputs are trapezoidal, as illustrated in Fig. 3. The first and second inputs have three and five fuzzy sets, respectively. For $Dis_{\text{nor},i}$, the fuzzy sets are {Close, Medium, Far}. For Err_{div}, the fuzzy sets are {Negative large, Negative small, Fit, Positive small, Positive large}.

The output of FIS is used as the F value directly. Table 1 shows the fuzzy rule base. This study devises singleton-valued fuzzy rules to simplify the FIS operation. This study sets each value through trial and error. If the individuals require low diversity, the FIS gives F a positive value, which guides the individual to the ***WB***; by contrast, if the individuals require high diversity, a negative F value guides the individual far from the ***WB***. The distribution range of the F value is [− 0.5, 0.9]. Figure 4 is the control surface of FIS, and this paper obtained the control surface by the brute force method.

3.4 The Flow of DE with Uniform Diversity Control

Figure 5 shows the pseudocode of DE with uniform diversity control. First, the algorithm initializes the population and CR values and calculates D_{ini}. Each generation begins by computing the diversity of individuals and $D_{\text{goal},g}$. The $D_{\text{goal},g}$ can be decided by users. Subsection 4.1 introduces four uniform diversity goals (UDGs) used in this study. Subsequently, Err_{div} and $Dis_{\text{nor},i}$ are calculated. By using these two inputs, the proposed method operates the FIS to select the F value then decides $\boldsymbol{x}_{\text{target}}$ by the difference between the current diversity and the UDG.

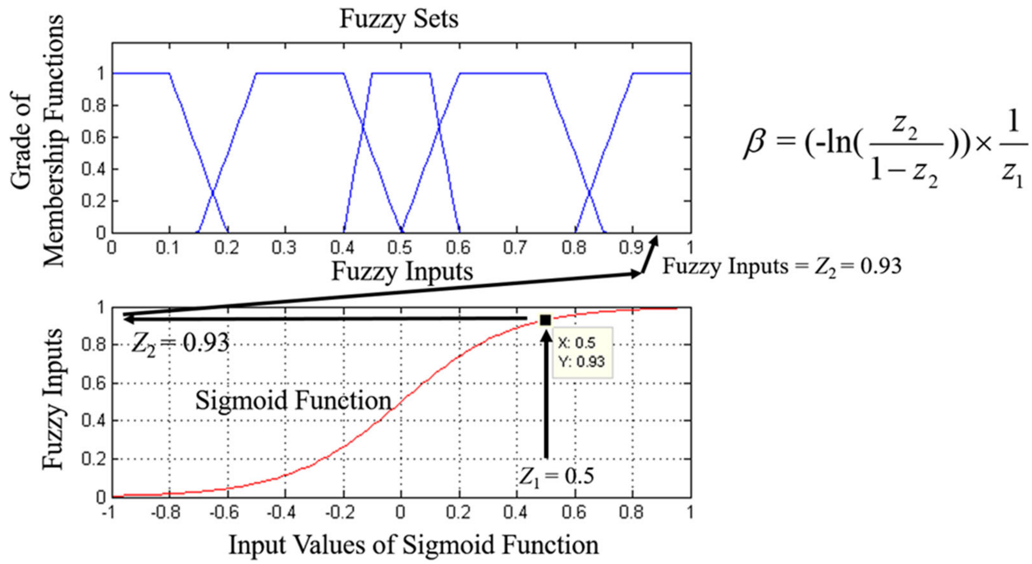


Fig. 2 The relationship between sigmoid function and fuzzy sets

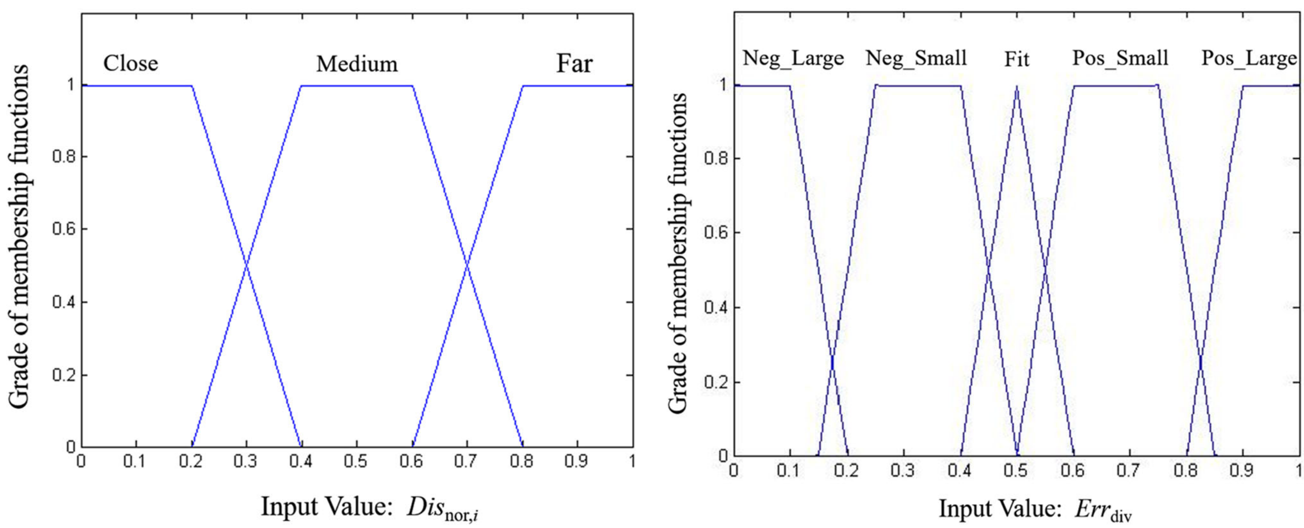


Fig. 3 Membership functions of fuzzy inputs: $Dis_{\mathrm{nor},i}$ (left) and Err_{div} (right)

Table 1 Fuzzy rule base

	$Dis_{\mathrm{std},i}$		
$Err_{\mathrm{std},i}$	Close	Medium	Far
Positive large	0.7	0.8	0.9
Positive small	0.5	0.6	0.7
Fit	0	0.2	0.4
Negative small	− 0.3	− 0.2	− 0.1
Negative large	− 0.5	− 0.4	− 0.3

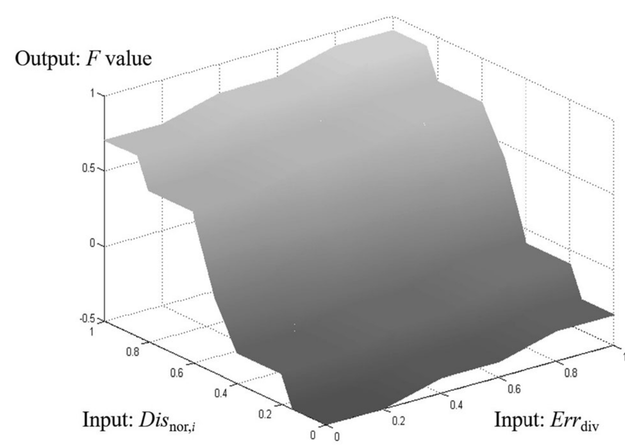


Fig. 4 Control surface of FIS

```
/* Initial Parameters Setting */
Set parameters:
        Population Size (PS), Dimension (N), Max Function Evaluated Times (FEs),
        Crossover Rate (CR), Search Range;
Initialize population: randomly generate the individuals inside the initial ranges;
Get the objective value of each individual and decide the WB;
Calculate the initial diversity of individuals, D_ini , by (9) and (10);
FE = 0;
WHILE (FE < Max FEs)
      Compute the diversity of individuals, D_g , in this generation as (9) and (10);
      Get the diversity goal of individuals, D_goal,g;
      Calculate normalized error of diversity, Err_div by (14) and (15);
      /* Mutation Stage */
      FOR i = 1 to PS
            Get the normalized distance Dis_nor,i , through (13);
            Operate the FIS to get the an F value for ith individual;
            Do a mutation operation by strategy as (11) and (12);
      END FOR
      /* Crossover Stage */
      FOR i = 1 to PS
          FOR d = 1 to N
              Calculate the trial individual;
          END FOR
      END FOR
       /* Get the BCoS */
       Compute the CoS;
       Decide whether to update BCoS;
       FE = FE+1;
       /* Selection Stage */
       FOR i = 1 to PS
           Decide individuals in next generation;
       END FOR
       Decide the GB;
       /* Choose the WB */
       Decide the WB;
FE = FE + PS;
END WHILE
```

Fig. 5 Pseudo code of the DE with uniform diversity control

Finally, the algorithm executes the mutation, crossover, the whole best individual generation [16], and selection operations.

4 Experiments

This section displays the experimental results of the proposed control mechanism. Subsection 4.1 illustrates the experimental settings. Subsection 4.2 compared conventional mutation strategies and Str5. Finally, subsection 4.3 discusses the experimental results.

4.1 Experimental Settings

Table 2 lists the experimental settings in this paper. This study uses single-objective functions as evaluated functions, and these 15 evaluated functions are the benchmark for a competition held in IEEE Congress on Evolutionary Computation 2015 (CEC2015). This benchmark includes unimodal functions (f1 and f2), multimodal functions (f3–f5), hybrid functions (f6–f8), and composition functions (f9–f15). Liang et al. [17] suggest the benchmark for evaluating the ability to tune the parameters of an algorithm. The benchmark is suitable for this study because the proposed method is adaptive parameters for each evolutionary environment. For comparison of different algorithms, two criteria, mean and standard deviation, are used to evaluate the numerical results for each evaluated function. The proposed control mechanism needs the UDG, and users define it. This study designs four UDGs as Fig. 6 in experiments. Table 3 displays the descriptions of each UDG. The experiments set CR as 0.6 and set K as 0.5. That is, the target vector for Str3 is the average of $\boldsymbol{WB}$ and a randomly selected individual. Moreover, the K value kept the same during evolution. FIS decides F values. The compared mutation strategies contain Str5 with one of four UDGs, Str1, Str2 $\boldsymbol{WB}$, Str3 $\boldsymbol{WB}$, and Str4 $\boldsymbol{WB}$. The entropy value is the highest when there happens to be one individual in each portion. This study divides the distribution range of the current population in each dimension into equal population-sized portions. Therefore, the highest entropy value is 1.681 because the population size is 48 in this paper.

Table 2 Experimental settings

	Str1, Str2 $\boldsymbol{WB}$, Str4 $\boldsymbol{WB}$	Str3 $\boldsymbol{WB}$	The proposed mutation strategy
Population size	48		
Benchmark functions	Problem definitions and evaluation criteria for the CEC 2015 competition on learning-based real-parameter single-objective optimization		
Max FEs	10,000 * N		
Independent runs	51		
Dimensions (N)	10/30/50		
Crossover rate	0.6		
Mutant factor (F)	0.6		The distribution range is [− 0.5, 0.9] and decided by FIS
Mutant factor (K)	–	0.5	–

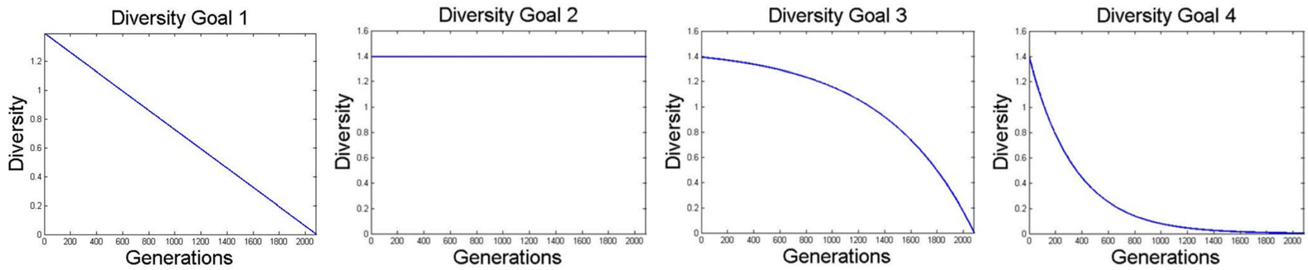


Fig. 6 Four uniform diversity goals

Table 3 The four UDGs used in this study

Uniform diversity goals	Expression	Illustration of parameters
Goal1	$D_{\text{goal},g} = D_{\text{ini}} \times \left(1 - \frac{g}{\text{GEN}}\right)$	D_{ini}, the initial UD of the population; GEN, the total number of generations
Goal2	$D_{\text{goal},g} = D_{\text{ini}}$	No parameters
Goal3	$D_{\text{goal}} = D_{\text{ini}} - \frac{e^{\left(\alpha\times\frac{g}{GEN}\right)}}{e^{\alpha}-1} \times D_{\text{ini}}$	α is curvature of the natural logarithm, and this study sets α as three here
Goal4	$D_{\text{goal}} = D_{\text{ini}} \times e^{\left(\gamma\times\frac{-g}{\text{GEN}}\right)}$	γ is curvature of the natural logarithm, and this study sets γ as six here

4.2 Experimental Results

This study offers the UD curves to verify the diversity control ability of the proposed method and regards the UD curves that met the UDGs as superior control ability. Figure 7 shows UD curves of diversity goal, DE with original selection stage and DE with modified selection stage for f1, f5, f6, f8, and f10. From Fig. 7, the UD values were met the UDGs for most functions except for f5.

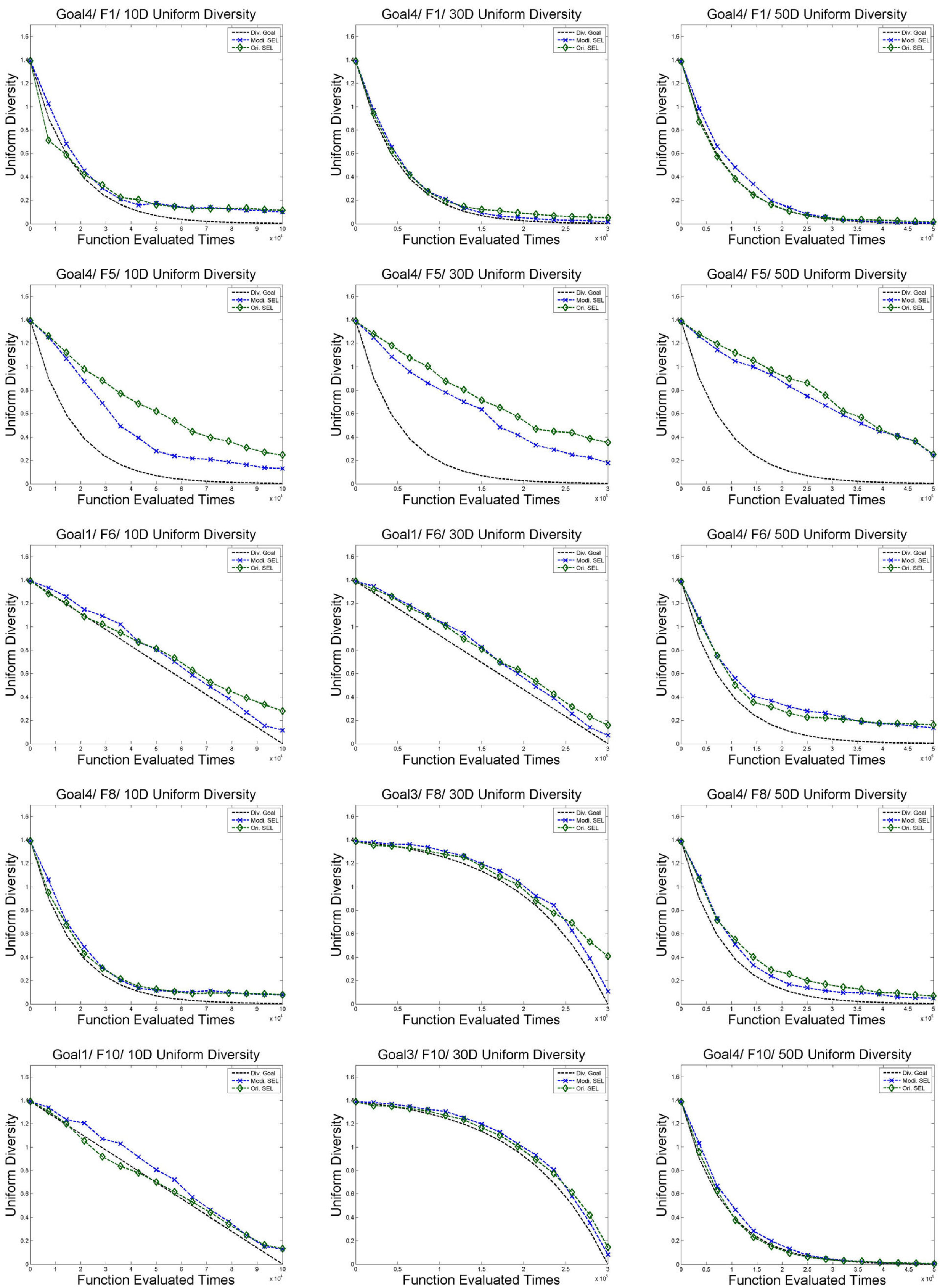
Numerical results reveal the performance, and the low mean and standard deviation values are superior ones. Tables 4, 5, and 6 list the numerical results of the proposed method and conventional mutation strategies. The “Goal” is the UDG used by Str5, and the best result is colored blue with bold font. The UDGs that used in these experiments came from the best results among four UDGs for each evaluated function. The Str5 almost won all evaluated functions except for f3 and some composition functions. The Str5 evolved superior individuals and had better evolutionary results than conventional mutation strategies.

The convergence curves show the evolutionary procedure. The convergence curves reveal the convergent speed and whether evolutionary stagnation exists or not. Figure 8 shows convergence curves of different mutation strategies for f1, f5, f6, f8, and f10. The convergence curves can divide into two types: The first type is Str5 that always had fast convergent speed (ex: f8 in 10 dimensions, f5 in 30 dimensions) and the second type is Str5 that had slow convergent speed in the early evolutionary duration and fast convergent speed over the conventional ones at the end of evolutionary duration (f1 in 10 dimensions and f10 in 30 dimensions). The Str5 mitigated the evolutionary stagnation (f5 in 50 dimensions) and also increased convergent speeds (f1 in 30 dimensions).

The UD curves show the UD changes in the evolutionary procedure. Figure 9 shows the UD curves of different mutation strategies. For conventional mutation strategies, the highest UD curve was Str1; the lowest was Str2 ***WB***; Str3 ***WB*** and Str4 ***WB*** were in the middle. The UD curves of Str5 were almost below those of conventional mutation strategies at the end of the evolutionary duration, and the initial diversity is high.

5 Discussion

From UD curves, the trend of conventional mutation strategies is almost the same. Str1 is an explorative strategy, and it expects to evolve superior individuals by increasing more possibilities of evolution. For Str1, it was natural to have a great UD. Str2 ***WB*** regards the ***WB*** as a promising individual, emphatically evolving ***WB*** and is an exploitative strategy. For Str2 ***WB***, it is natural to have low UDs. The UD curves of Str3 ***WB*** and Str4 ***WB*** are in the middle of UDs of Str1 and Str2 ***WB*** because of their characteristics. Str3 ***WB*** using a K value and Str4 ***WB*** using an F value decide the proportion of Str1 and Str2 ***WB***, so they are natural to have middle UDs. Although Str2 ***WB*** also emphatically evolved ***WB***, the Str2 ***WB*** already


Goal4/ F1/ 10D Uniform Diversity
Goal4/ F1/ 30D Uniform Diversity
Goal4/ F1/ 50D Uniform Diversity
Goal4/ F5/ 10D Uniform Diversity
Goal4/ F5/ 30D Uniform Diversity
Goal4/ F5/ 50D Uniform Diversity
Goal1/ F6/ 10D Uniform Diversity
Goal1/ F6/ 30D Uniform Diversity
Goal4/ F6/ 50D Uniform Diversity
Goal4/ F8/ 10D Uniform Diversity
Goal3/ F8/ 30D Uniform Diversity
Goal4/ F8/ 50D Uniform Diversity
Goal1/ F10/ 10D Uniform Diversity
Goal3/ F10/ 30D Uniform Diversity
Goal4/ F10/ 50D Uniform Diversity
Uniform Diversity
Function Evaluated Times
Div. Goal
Modi. SEL
Ori. SEL

◀**Fig. 7** Uniform diversity curves of diversity goal, DE with original selection and DE with modified selection stage for f1, f5, f6, f8, and f10

emphatically evolved ***WB*** in the early evolutionary duration, and individuals may stick in local optima.

The UD curves of Str5 were lower than those of conventional mutation strategies toward the end of the evolutionary duration. Because UD of Str5 met UDGs, it decreased UDs and emphatically evolved ***WB*** toward the end of the evolutionary duration. Besides that, the trend of the UD curves of Str5 was different from those of conventional mutation strategies for most evaluated functions. This situation made the proposed UD control mechanism evolve a better individual and got lower objective value than conventional mutation strategies. The Str5 is both explorative and exploitative, and it combines most of the

Table 4 Numerical results of comparing different mutation strategies in 10 dimensions

	Str1		Str2WB		Str3WB		Str4WB		Str5		
	Mean	Std.	Mean	Std.	Mean	Std.	Mean	Std.	Mean	Std.	Goal
f1	1.23E+05	3.70E+04	5.82E+03	4.87E+03	8.52E+03	4.30E+03	7.50E+03	3.32E+03	**1.04E+00**	1.50E+00	4
f2	3.07E+02	2.27E+02	**5.56E−06**	5.16E−06	8.68E−05	7.23E−05	5.28E−05	4.07E−05	4.03E−03	1.35E−02	4
f3	2.03E+01	5.68E−02	2.02E+01	4.08E−01	2.03E+01	6.41E−02	2.02E+01	1.86E−01	**1.83E+01**	6.00E+00	1
f4	2.25E+01	4.02E+00	2.11E+01	3.92E+00	2.08E+01	2.61E+00	2.05E+01	3.10E+00	**3.78E+00**	1.76E+00	4
f5	9.62E+02	1.66E+02	8.86E+02	1.55E+02	9.37E+02	1.16E+02	8.68E+02	1.28E+02	**2.18E+02**	2.21E+02	4
f6	8.35E+01	2.16E+01	5.11E+01	4.44E+01	4.14E+01	1.12E+01	4.61E+01	2.58E+01	**1.60E+01**	7.90E+00	1
f7	1.10E+00	2.51E−01	6.57E−01	2.75E−01	5.59E−01	1.75E−01	7.72E−01	2.21E−01	**4.39E−01**	3.48E−01	1
f8	6.52E+00	2.12E+00	2.46E+00	1.02E+01	7.86E−01	4.52E−01	2.08E+00	8.55E−01	**6.42E−01**	3.14E−01	4
f9	1.00E+02	4.41E−02	1.00E+02	3.07E−02	1.00E+02	**2.64E−02**	1.00E+02	3.38E−02	1.00E+02	3.72E−02	4
f10	2.29E+02	2.91E+00	2.19E+02	4.92E+00	2.18E+02	1.18E+00	2.20E+02	1.72E+00	**2.17E+02**	1.03E+00	1
f11	2.02E+02	1.37E+02	2.35E+02	1.28E+02	2.41E+02	1.17E+02	1.82E+02	1.44E+02	**3.47E+01**	8.72E+01	3
f12	1.03E+02	4.24E−01	1.02E+02	3.97E−01	1.03E+02	3.08E−01	1.03E+02	3.44E−01	1.02E+02	3.81E−01	4
f13	3.31E+01	1.28E+00	3.10E+01	1.92E+00	3.11E+01	1.53E+00	3.11E+01	1.14E+00	**2.88E+01**	1.15E+00	1
f14	3.19E+03	9.20E+02	4.86E+03	2.03E+03	5.44E+03	1.75E+03	4.30E+03	2.24E+03	**2.75E+03**	6.96E+02	3
f15	1.00E+02	**0.00E+00**	1.00E+02	1.17E−14	1.00E+02	1.25E−14	1.00E+02	1.35E−14	1.00E+02	5.66E−05	4

Table 5 Numerical results of comparing different mutation strategies in 30 dimensions

	Str1		Str2WB		Str3WB		Str4WB		Str5		
	Mean	Std.	Mean	Std.	Mean	Std.	Mean	Std.	Mean	Std.	Goal
f1	3.74E+07	9.35E+06	7.43E+06	2.32E+06	1.02E+07	2.64E+06	7.79E+06	1.18E+06	**1.63E+05**	1.56E+05	4
f2	1.50E+07	3.76E+06	6.50E+03	1.29E+04	1.34E+05	1.49E+05	2.15E+05	8.87E+04	**1.07E+03**	1.21E+03	4
f3	2.09E+01	4.57E−02	2.09E+01	**3.48E−02**	2.09E+01	5.01E−02	2.09E+01	5.31E−02	2.09E+01	6.33E−02	1
f4	1.66E+02	1.44E+01	1.66E+02	3.02E+01	1.59E+02	1.32E+01	1.58E+02	1.85E+01	**2.74E+01**	1.17E+01	4
f5	6.75E+03	2.88E+02	6.75E+03	2.32E+02	6.73E+03	3.11E+02	6.63E+03	3.07E+02	**3.44E+03**	2.10E+03	4
f6	2.27E+06	6.92E+05	2.36E+05	1.46E+05	3.44E+05	1.23E+05	3.01E+05	1.02E+05	**1.09E+04**	7.66E+03	1
f7	1.47E+01	7.52E−01	8.01E+00	9.23E−01	9.33E+00	2.34E+00	1.13E+01	1.38E+00	**6.25E+00**	1.13E+00	4
f8	8.94E+04	2.64E+04	3.02E+03	4.23E+02	3.52E+03	5.09E+02	3.37E+03	4.60E+02	**1.62E+03**	9.92E+02	3
f9	1.04E+02	1.91E−01	1.21E+02	6.38E+01	1.12E+02	4.71E+01	1.03E+02	1.73E−01	**1.02E+02**	1.96E−01	4
f10	1.41E+05	4.77E+04	4.47E+03	1.02E+03	6.51E+03	1.30E+03	5.35E+03	9.99E+02	**1.78E+03**	7.76E+02	3
f11	7.67E+02	1.28E+02	4.10E+02	4.39E+01	3.98E+02	2.33E+01	4.21E+02	8.03E+01	**3.31E+02**	4.87E+01	3
f12	1.09E+02	8.15E−01	1.08E+02	7.08E−01	1.08E+02	4.44E−01	1.07E+02	5.04E−01	**1.05E+02**	5.86E−01	4
f13	1.25E+02	2.03E+00	1.21E+02	3.44E+00	1.21E+02	2.52E+00	1.23E+02	2.26E+00	**1.09E+02**	4.34E+00	4
f14	3.38E+04	5.62E+02	3.22E+04	9.40E+02	3.23E+04	9.95E+02	**3.15E+04**	8.14E+02	3.22E+04	9.15E+02	4
f15	1.00E+02	1.83E−03	1.00E+02	**0.00E+00**	1.00E+02	**0.00E+00**	1.00E+02	**0.00E+00**	1.00E+02	8.46E−04	4

Table 6 Numerical results of comparing different mutation strategies in 50 dimensions

	Str1		Str2WB		Str3WB		Str4WB		Str5		
	Mean	Std.	Mean	Std.	Mean	Std.	Mean	Std.	Mean	Std.	Goal
f1	2.06E+08	4.51E+07	4.92E+07	1.14E+07	7.67E+07	1.45E+07	6.67E+07	1.33E+07	**1.71E+06**	8.88E+05	4
f2	1.63E+08	6.97E+07	8.54E+03	3.02E+04	1.45E+06	1.14E+06	1.64E+06	7.00E+05	**1.05E+04**	1.10E+04	4
f3	2.11E+01	**4.36E−02**	2.11E+01	3.18E−02	2.11E+01	2.77E−02	2.11E+01	4.26E−02	2.11E+01	4.58E−02	3
f4	3.29E+02	2.04E+01	3.18E+02	2.99E+01	3.17E+02	2.52E+01	3.24E+02	2.88E+01	**5.69E+01**	5.10E+01	1
f5	1.30E+04	3.45E+02	1.28E+04	3.82E+02	1.28E+04	3.72E+02	1.28E+04	3.43E+02	**6.34E+03**	3.48E+03	4
f6	1.48E+07	3.25E+06	4.00E+06	1.09E+06	4.73E+06	1.23E+06	4.78E+06	1.44E+06	**1.52E+05**	7.03E+04	4
f7	5.21E+01	1.68E+00	4.56E+01	6.87E+00	4.21E+01	1.11E+00	4.45E+01	6.52E+00	**4.07E+01**	4.89E−01	1
f8	4.33E+06	1.12E+06	1.49E+06	4.22E+05	2.45E+06	6.23E+05	1.83E+06	5.02E+05	**8.09E+04**	4.68E+04	4
f9	1.09E+02	7.55E−01	1.30E+02	8.78E+01	1.06E+02	2.28E−01	1.06E+02	2.26E−01	**1.04E+02**	2.29E−01	4
f10	2.20E+05	7.79E+04	1.02E+04	2.04E+03	1.59E+04	3.29E+03	1.33E+04	2.81E+03	**1.69E+03**	2.96E+02	4
f11	1.31E+03	9.44E+01	**4.18E+02**	5.09E+01	4.30E+02	7.10E+01	6.11E+02	1.92E+02	4.61E+02	2.59E+01	4
f12	1.14E+02	1.12E+00	1.11E+02	5.62E−01	1.11E+02	4.67E−01	1.11E+02	7.11E−01	**1.08E+02**	6.33E−01	4
f13	2.26E+02	2.63E+00	2.20E+02	3.32E+00	2.21E+02	2.81E+00	2.21E+02	4.08E+00	**2.08E+02**	5.32E+00	4
f14	6.17E+04	4.88E+03	6.32E+04	8.51E+03	6.20E+04	5.44E+03	**6.00E+04**	4.89E+03	6.17E+04	5.10E+03	3
f15	1.22E+02	1.59E+00	1.00E+02	**4.20E−14**	1.00E+02	4.26E−14	1.00E+02	4.26E−14	1.00E+02	4.22E−03	4

conventional mutation strategies through the control of the F value and by randomly or currently selecting an individual as the target vector. Table 7 shows the relationship between conventional mutation strategies and Str5. Consider an extreme example: for exploration, set the F value to zero in (11), and the randomly selected individual as the target vector. In this case, (11) becomes DE/rand/2 with $F = 1$. Users often use DE/rand/2 for exploration. Furthermore, consider an extreme example for exploitation: $F = 1$. In this case, the mutant individual is the ***WB***.

These UD curves did not meet the UDGs exactly in Fig. 7. Zaharie [12] expresses that the selection stage decreases the variance of the population, so this study suspects the selection stage is the main reason why the UD did not meet UDGs. This study modified the selection stage and recorded the UD to verify the influence of selection. The modified selection stage closes the selection stage as the measured UD less than UDGs. However, the modified selection kept the mechanism used to decide whether to update GB. The blue cross-marked curves in Fig. 7 are the UD curves for the modified selection stage. Compared with the original selection stage (green diamond-marked curves) and modified selection stage, the UD curves of the modified selection stage were more closed to UDG than the UD curves of the original selection stage, no matter the UD is over or below UDGs. However, f5 is the exception. No matter whether the modified selection stage or not, the UD curves were higher than UDGs curves. These results indicated the f5 needs high UD to have superior evolutionary results. The selection stage keeps the evolutionary results, so this study does not remove or modify it at will.

6 Conclusions

This paper proposed a mechanism for diversity control of individuals with a new mutation strategy and a fuzzy inference system (FIS) for a differential evolution (DE) algorithm. When studying evolutionary computation, the balance between exploitation and exploration is crucial. An emphasis on exploitation may lead to premature convergence. By contrast, an emphasis on exploration may lead to slow convergence and make it challenging to find the optimal solution. The proposed method combines conventional mutation strategies into one mutation strategy and handles the entropy to measuring uniform diversity. This new mutation strategy does not fix the target vector, and it controls the degree of mutation toward the whole best individual (***WB***) using the FIS. By doing so, it can balance exploitation and exploration by making the diversity of individuals fit the diversity goal in each generation.

Two factors are considered in this FIS: normalized diversity error and normalized distance between the ***WB*** and each individual. The FIS is a suitable inference system in this study because these two factors are consecutive numbers and hard to classify. The fuzzy rule base has singleton-valued fuzzy rules to simplify this control mechanism, and the design principle is that if the diversity

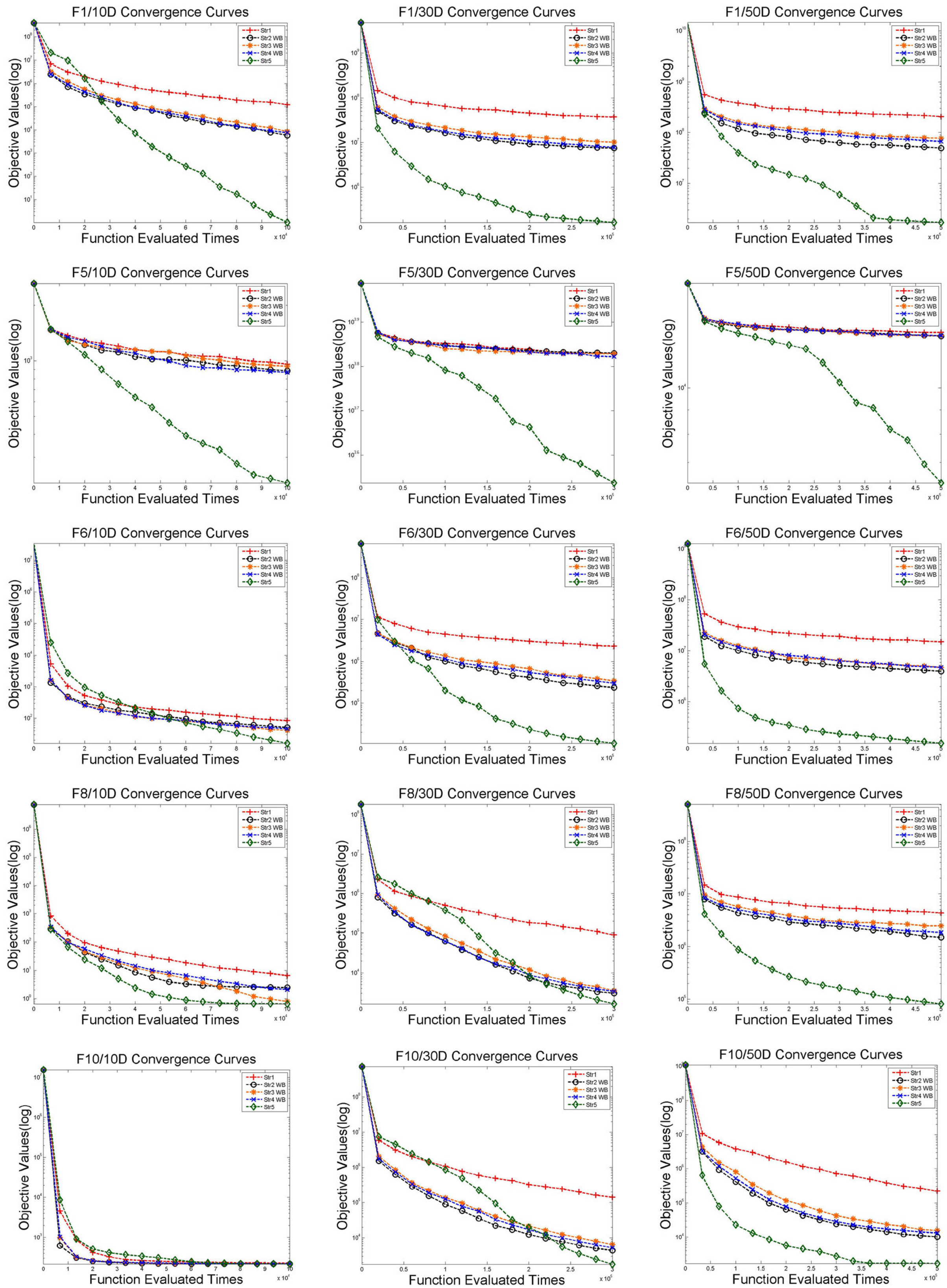


Fig. 8 Convergence curves of different mutation strategies for f1, f5, f6, f8, and f10

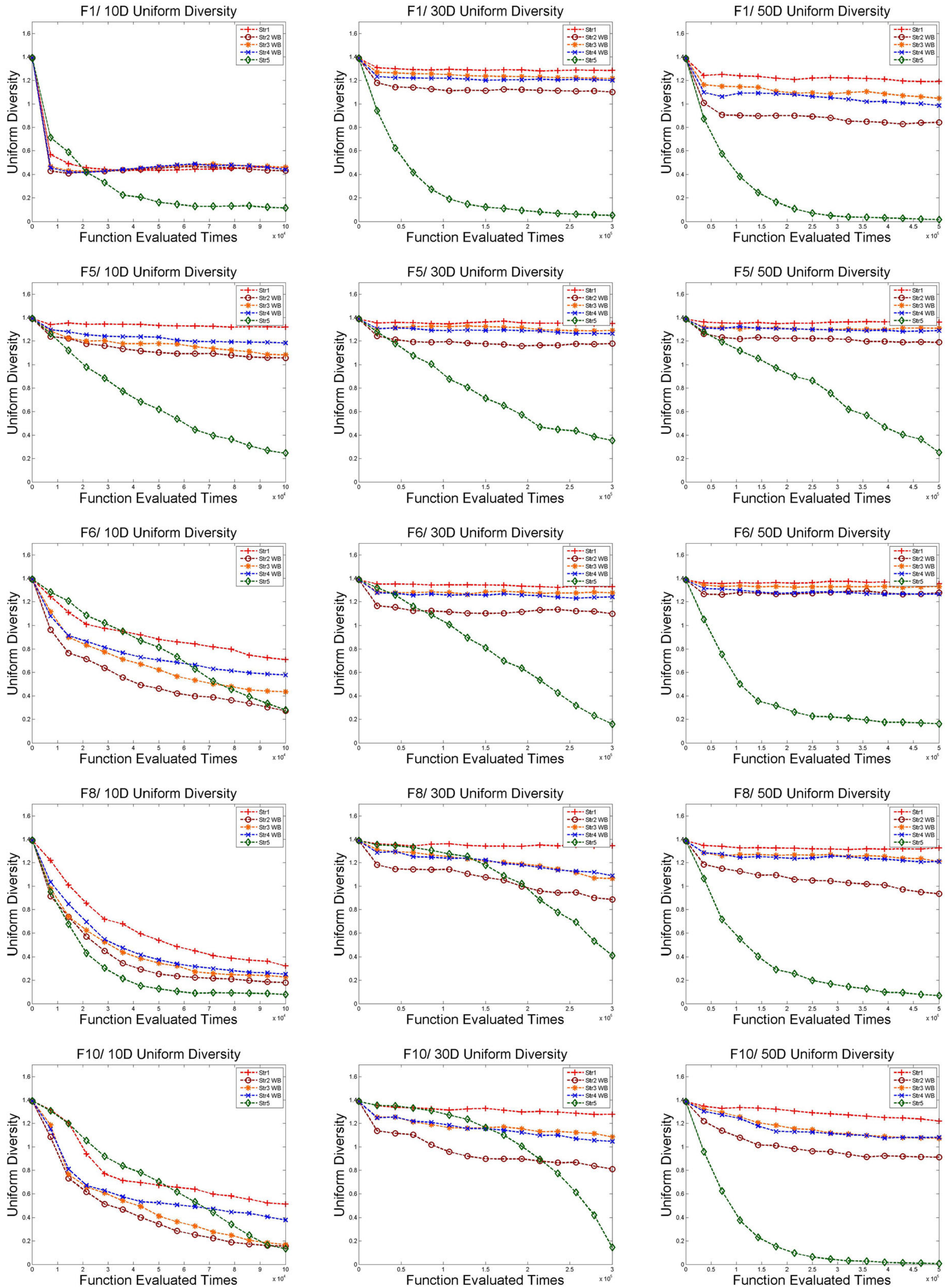


Fig. 9 Uniform diversity curves of different mutation strategies for f1, f5, f6, f8, and f10

Table 7 The relationship between Str5 and conventional ones

Conventional strategies	The setting of Str5
DE/rand/2, $F = 1$	$\mathbf{x}_{base} = \mathbf{x}_{rand}$, $F = 0$
DE/best/2, $F = 0.1$	$\mathbf{x}_{base} = \mathbf{x}_{rand}$, $F = 0.9$
DE/current-to-best/2, $F = 0.5$	$\mathbf{x}_{base} = \mathbf{x}_{i}$, $F = 0.5$
DE/rand-to-best/2, $K = 0.5$, $F = 0.5$	$\mathbf{x}_{base} = \mathbf{x}_{rand}$, $F = 0.5$

is higher than the diversity goal, a considerable F value is assigned to move the individual closer to the $\boldsymbol{WB}$.

Combining the proposed mutation strategy with the FIS offers a promising approach for diversity control of individuals for DE as well as population-based stochastic search algorithms. The experimental results show that the proposed strategy has superior performance at least 11 out of 15 evaluated functions in 10, 30, and 50 dimensions compared with the conventional strategies. The convergence curves show that the proposed method keeps the diversity of individuals high (i.e., individuals converged slowly) in preliminary generations and low (i.e., individuals converged fast) at the end of the evolutionary procedure. The diversity curves indicated the high diversity control ability of the proposed method. The selection stage and an unsuitable diversity goal may cause failed diversity control and evaluated functions with inferior performance, respectively. An exciting future research topic could be how to set a suitable diversity goal during the evolutionary procedure.

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