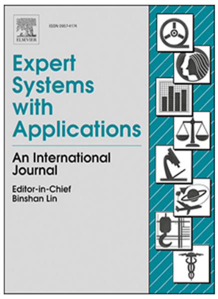
Journal Pre-proof



A novel direct measure of exploration and exploitation based on attraction basins

Jernej Jerebic, Marjan Mernik, Shih-Hsi Liu, Miha Ravber,
Mihael Baketarić, Luka Mernik, Matej Črepinšek

PII: S0957-4174(20)31036-8
DOI: https://doi.org/10.1016/j.eswa.2020.114353
Reference: ESWA 114353

To appear in: *Expert Systems With Applications*

Received date : 5 August 2020
Revised date : 17 November 2020
Accepted date : 18 November 2020

Please cite this article as: J. Jerebic, M. Mernik, S.-H. Liu et al., A novel direct measure of exploration and exploitation based on attraction basins. *Expert Systems With Applications* (2020), doi: https://doi.org/10.1016/j.eswa.2020.114353.

A Novel Direct Measure of Exploration and Exploitation Based on Attraction Basins

Jernej Jerebic[a,b] (jernej.jerebic@student.um.si), Marjan Mernik[a] (marjan.mernik@um.si), Shih-Hsi Liu[c] (shliu@CSUFresno.edu), Miha Ravber[a] (miha.ravber@um.si), Mihael Baketarić[a] (mihael.baketaric@student.um.si), Luka Mernik[d] (mernik.1@osu.edu), Matej Črepinšek[a] (matej.crepinsek@um.si)

[a] University of Maribor, Faculty of Electrical Engineering and Computer Science, Koroška cesta 46, 2000 Maribor, Slovenia

[b] Comtrade d.o.o., Letališka cesta 29b, 1000 Ljubljana, Slovenia

[c] California State University Fresno, Department of Computer Science, 2576 E San Ramon Dr, Fresno, CA 93740, USA

[d] The Ohio State University, Department of Mathematics, 100 Math Tower, 231 W 18th Ave, Columbus, OH 43210, USA

Corresponding Author:
Miha Ravber, PhD
University of Maribor, Faculty of Electrical Engineering and Computer Science
Koroška cesta 46, 2000 Maribor, Slovenia
Tel: (+386 2) 220 7462
Email: miha.ravber@um.si

A Novel Direct Measure of Exploration and Exploitation Based on Attraction Basins

Jernej Jerebic[a,b], Marjan Mernik[a], Shih-Hsi Liu[c], Miha Ravber[a,*], Mihael Baketarić[a], Luka Mernik[d], Matej Črepinšek[a]

[a] *University of Maribor, Faculty of Electrical Engineering and Computer Science, Koroška cesta 46, 2000 Maribor, Slovenia*
[b] *Comtrade d.o.o., Letališka cesta 29b, 1000 Ljubljana, Slovenia*
[c] *California State University Fresno, Department of Computer Science, 2576 E San Ramon Dr, Fresno, CA 93740, USA*
[d] *The Ohio State University, Department of Mathematics, 100 Math Tower, 231 W 18th Ave, Columbus, OH 43210, USA*

Abstract

Exploration, the process of visiting a new region in a search space, and exploitation, the process of searching in the neighborhood of previously visited regions, are two centerpieces of any metaheuristic algorithm. It is a common belief that good results can be obtained only if there is a good balance between exploration and exploitation. Hence, there is an urgent need to control the balance between exploration and exploitation in a direct manner. But, currently, direct measures of exploration and exploitation are almost non-existent, and researchers rely on indirect measures of exploration and exploitation, such as diversity, entropy, and fitness improvements. To remedy this situation, in this paper, a novel direct measure of exploration and exploitation is proposed that is based on attraction basins - parts of a search space where each part has its own point called an attractor, to which neighboring points tend to evolve. Each search point can be associated with a particular attraction basin. If a newly generated search point belongs to the same attraction basin as its parent then the search process is identified as exploitation, otherwise as exploration. In this paper, a new

*Corresponding author.
Email addresses: `jernej.jerebic@student.um.si` (Jernej Jerebic), `marjan.mernik@um.si` (Marjan Mernik), `shliu@CSUFresno.edu` (Shih-Hsi Liu), `miha.ravber@um.si` (Miha Ravber), `mihael.baketaric@student.um.si` (Mihael Baketarić), `mernik.1@osu.edu` (Luka Mernik), `matej.crepinsek@um.si` (Matej Črepinšek)

technique to compute attraction basins is presented, as well as a novel direct measure ($ExpBas$) of exploration and exploitation based on attraction basins. On the selected set of unimodal and multimodal optimization problems it is shown that the newly proposed direct measure of exploration and exploitation is more accurate than our previously proposed direct measure ($ExpDist$), as well as the common indirect measure based on diversity (Diversity).

Keywords: Metaheuristics, Exploration and Exploitation, Diversity, Attraction Basins

1. Introduction

It has been acknowledged for a long time that achieving a balance between exploration and exploitation in Metaheuristics is of primary importance (Eiben & Schippers, 1998; Črepinšek et al., 2013; Xu & Zhang, 2014). However, how to measure exploration and exploitation directly has been an open problem, and a common belief is that clear identification of exploration and exploitation is not possible (Zhang et al., 2019). To balance exploration and exploitation properly, a multiprocess-driven approach has been recommended over a single process-driven approach (Črepinšek et al., 2013; Liu et al., 2009). However, before controlling the balance between exploration and exploitation, we need to know how to measure exploration and exploitation. Until now, many indirect measures of exploration and exploitation have existed, which are based mainly on population diversity (Ollion & Doncieux, 2011; Gabor & Belzner, 2017) and fitness improvements (Tilahun, 2017; Tilahun & Tawhid, 2019). Although in our previous works (Črepinšek et al., 2013, 2011; Liu et al., 2013) we have already proposed a direct measure of exploration and exploitation, it was not accepted widely among researchers, since the threshold value that defines the boundary of the neighborhood of the closest neighbor has to be known in advance. Moreover, this threshold is problem dependent and, hence, difficult to set properly (Črepinšek et al., 2013). In this paper we are proposing a new and more accurate direct measure of exploration and exploitation, that is based on attraction basins (Locatelli & Wood, 2005; Chen et al., 2019) and does not require a threshold value that defines the boundary of the neighborhood.

In Črepinšek et al. (2013) exploration and exploitation have been defined as: "*Exploration is the process of visiting entirely new regions of a search space, whilst exploitation is the process of visiting those regions of a search space within the neighborhood of previously visited points.*" Both processes have been further formalized (see Section 2) and defined as follows. In the exploration phase, points which are outside of the current neighborhood of the closest neighbor are visited, and, conversely, the exploitation phase happens when a visited point is inside the current neighborhood of the closest neighbor. The problem with this definition is that it is very difficult to know where the boundary of the current neighborhood is. A similar definition of exploration and exploitation can be found in Morales-Castañeda et al. (2020): "*Exploration refers to the ability of a search algorithm to discover a diverse assortment of solutions, spread within different regions of the search space. On the other hand, exploitation emphasizes the idea of intensifying the search process overpromising regions of the solution space, with the aim of finding better solutions or improving the existing ones.*"

The exploration and exploitation phases are the cornerstones of any meta-heuristic, and, hence, they are mentioned frequently in many papers (Zhang et al., 2019; Ishibuchi et al., 2003; Weber et al., 2009; Neri et al., 2011; Salgotra et al., 2018; Caraffini et al., 2019; Luan et al., 2019; Wei et al., 2019; Gupta et al., 2020), where new genetic operators were proposed. However, exploration and exploitation have often not been measured, and the reasoning is based only on the following assumption: If better results are obtained, then this must be due to a better balance between exploration and exploitation. In some other papers, contributions of specific genetic operators to exploration and exploitation are misunderstood. For example, the authors in Allawi et al. (2019) wrote: "*In any meta-heuristics optimization algorithm, there are three significant types of information exchange between a particular agent with other agents in the population. The first is called exploitation, which is a local search for the latest and the best solution found so far. The second is called exploration, which is a global* *search using another agent existing in the problem space. The third is called randomization, which is used rarely in some algorithms, or may not be used at all. The last procedure is similar to exploration, but a randomly-generated agent is used instead of an existing agent.*" Based on their conclusion, it seems that the authors of Allawi et al. (2019) mixed exploration and exploitation with particular individuals of the population. We argue that the main reason of the misinterpretation is that, despite the importance of exploration and exploitation in metaheuristics, both phases are only defined vaguely. To avoid this confusion a well established direct measure of exploration and exploitation is required, that is widely recognized by researchers working in the field of Metaheuristics. To complicate the issue further, the terms exploration and exploitation have also been used in Machine Learning. For example, in Active Learning and Reinforcement Learning they use the following definitions: "*In other words, we encounter a trade-off between minimizing the uncertainty of the target objective function, known as exploration and maximizing the underlying objective function given the available function estimates, which is known as exploitation*" (Elreedy et al., 2019).

For our newly proposed method on how to measure exploration and exploitation in metaheuristics directly, we relied on the works from Gonzalez-Fernandez & Chen (2015); Chen et al. (2019), where the notion of attraction basins has been associated with exploration and exploitation. An attraction basin is a part of a search region with a point called an attractor to which a system tends to evolve. A multi-modal search space can be decomposed into several attraction basins. Each attraction basin has it own single local optimum. Hence, each point in the attraction basin has a monotonic path to the attractor – the local optimum in this attraction basin. Whilst a uni-modal search space has only one attraction basin and only one attractor (Locatelli & Wood, 2005; Chen et al., 2019), using the concept of attraction basins seems like a more natural way to define exploration and exploitation. If a newly created individual (solution) belongs to the same attraction basin as its parent then the search process is in the exploitation phase. On the other hand, if a newly created individual belongs to a different attraction basin than its parent, then the search process is in the exploration phase. As such, identification of exploration/exploitation phases is now natural and easy. Previously, the common belief was (Zhang et al., 2019): "*It is, thus, generally difficult to predict if newly generated solutions fall into the exploration or exploitation zones.*"

The main contributions of this study are:

- A novel technique to compute attraction basins,
- A new direct measure (*ExpBas*) to compute exploration based on attraction basins,
- Comparison of the newly proposed measure (*ExpBas*) with the previously most used indirect measure for exploration and exploitation, that is based on diversity, showing that diversity is indeed a very rough measure for exploration and exploitation.

This paper is organized as follows. In Section 2, related work on different indirect measures for exploration and exploitation is reviewed briefly, as well as our previous direct measure (*ExpDist*). Section 3 describes attraction basins and how we can use attraction basins to measure exploration and exploitation. The extensive experimental part is given in Section 4, followed by a discussion in Section 5. Finally, the paper is concluded in Section 6.

2. Related Work

Indirect measures that take diversity into account have been proposed mainly to measure exploration and exploitation (Squillero & Tonda, 2016). If diversity is high, then individuals in a population are very different from each other. Hence, the algorithm must be in the phase of exploration. Conversely, if diversity is low, then individuals in a population are very similar and the algorithm must be in the phase of exploitation. Although it is simple and intuitive, it is only a rough approximation. The other problem is that diversity is defined at the level of population, while we are interested to know for each solution whether it is a result of exploration or exploitation. Furthermore, indirect measures of exploration and exploitation depend heavily on a good representation of a group of individuals and principles of locality. The distance in the problem space must be preserved and reflected in the space in which it is measured. Do small changes in the genotype produce small changes in the phenotype? Finally, there is no guarantee that two quite different individuals are not coming from the same attraction basin (see examples in Section 4, where it can be observed that attraction basins can be very large). The connection between diversity and exploration has traditionally been very strong. For example, in Ollion & Doncieux (2011) the authors wrote "*Exploration, in particular, is promoted by a specific diversity keeping mechanism generally relying on the genotype or on the fitness value*", and proposed a simple measure of exploration based on diversity in the behavioral space (phenotype). Yet another example of relating diversity to exploration-exploitation can be found in Gabor & Belzner (2017) where the authors wrote: "*Diversity thus plays a key role in adjusting the exploration-exploitation trade-off found in any kind of metaheuristics search algorithm.*" In Ursem (2002), diversity is measured based on the distance between individuals and the average individual (centroid). If diversity is low, then the process is in exploitation, and should switch to exploration by applying the mutation operator to increase diversity (Ursem, 2002). Otherwise, when diversity is high, the process is in exploration, and should switch to exploitation by applying crossover and selection operators (Ursem, 2002). Dimension-wise diversity has been used in Morales-Castañeda et al. (2020) to measure exploration and exploitation. The objective of this work was to find the optimal exploration and exploitation rates for different metaheuristics. Due to a rough diversity measure their conclusion was: "*In the majority of the 42 functions (multimodal, unimodal, hybrid and shifted) the balance that produced the best results was above 90% exploitation and less than 10% exploration.*"

Entropy, as a succinct measure of diversity, has been used to control exploration and exploitation in Liu et al. (2013): When entropy measure is high, representing high disorder of an evolution process, the process reduces exploration power by adjusting the evolutionary operators' rates lower. Conversely, as entropy measure is low, more exploration power is triggered by tuning the operator rates higher.

The authors in Tilahun (2017); Tilahun & Tawhid (2019) have proposed another indirect measure of exploration and exploitation by measuring fitness improvements, where a similar rationale has been applied to use diversity as a measure of exploration. If fitness improvements have indeed been achieved, then the algorithm must be in the phase of exploitation. The authors in the work Tilahun & Tawhid (2019) suggested to compare the fitness of the best individual x_b at iterations $(t-1)$ and t, in addition to the diversity measure. They proposed the Swarm Hyper-Heuristic Framework (SHH), where diversification (exploration) and intensification (exploitation) step lengths are controlled off-line by λ_{min} and λ_{max}. In Tilahun & Tawhid (2019) the following setting was used: $\lambda_{min} \approx \| \Sigma_{i=1}^{D}(\frac{x_{max,i}-x_{min,i}}{5}) \|$ and $\lambda_{max} \approx 2\lambda_{min}$. The authors acknowledged that step lengths should be set based on the size of the problem and problem dimensions. However, this assumption might hold only for regular problems, as in the Rastrigin function (see Section 4). In general, step lengths are not equal, even in the same search space (see different attraction basins for the Easom and Split Drop Wave functions in Section 4).

Yet another indirect measure of exploration and exploitation based on survival analysis has recently been proposed in Zhang et al. (2019). Observation that high-quality solutions survive longer than low-quality solutions can be used to determine whether the evolutionary process is in the phase of exploration or exploitation. The main rationale is that, when the average survival time of a population becomes longer, then the search must be in the exploitation phase. The authors applied the following survivability-based guidance mechanism (Zhang et al., 2019): If a solution has a high survival length indicator, then this solution is a promising one and further exploitation is required, otherwise the quality of the solution is unclear and exploration is preferred.

Less frequently, population size has been used as an indirect factor to influence exploration and exploitation. In Piotrowski (2017), it is discussed how different population sizes impact the balance between exploration and exploration in Differential Evolution (DE). Although exploration and exploitation have not been measured, the common belief is that, with a larger population size the search space is explored better, as well as that in the beginning large exploratory steps prevail in DE, whilst, in the later stage, small steps become more frequent.

As can be noticed so far, indirect measures of exploration and exploitation prevail, where a degree of exploitation is measured by fitness improvements, and a degree of exploration is measured by population diversity or entropy. On the other hand, only a few direct measures of exploration and exploitation exist. In Črepinšek et al. (2013), we formalized the process of exploration and exploitation by Eqs. 1 and 2 using the concept of Similarity to the Closest Neighbor ($\mathcal{SCN}$).

$$\mathcal{SCN}(ind_{new}, P) > X \qquad \text{(exploration)} \tag{1}$$

$$\mathcal{SCN}(ind_{new}, P) \leq X \qquad \text{(exploitation)} \tag{2}$$

In Eqs. 1 and 2 the threshold variable X was introduced to delimit exploration from exploitation, and can be seen as identification of an individual's neighborhood (hypercube). $\mathcal{SCN}$ was defined as:

$$\mathcal{SCN}(ind_{new}, P) = d(ind_{new}, ind_{parent}), \text{ where } ind_{new} \ and \ ind_{parent} \in P \tag{3}$$

and $d(.,.)$ denotes the diversity/similarity measurements between two individuals at the genotype or at the phenotype levels within a population P. Examples of computing similarity between two individuals $\mathbf{x}$ and $\mathbf{y}$ are the Euclidean distance and Edit distance. Note that $\mathcal{SCN}$ can also be defined in several other ways, Črepinšek et al. (2013): As a similarity to the most similar individual within the whole population P, as a similarity to the most similar individual in the same niche, or as a similarity to the most similar individual throughout the history of populations. The problem of Eqs. 1 and 2 is that the threshold value X that defines the boundary of the neighborhood of the closest neighbor needs to be known in advance or to be approximated somehow. Furthermore, neighborhood has been modeled as a hypercube with the same length in all dimensions. As is shown in Section 4, this is not true, even for a 2D space.

In De Lorenzo et al. (2019) the authors proposed two visualization techniques for population movement and for the exploration-exploitation trade-off. In the latter case, direct measures of exploration and exploitation were developed, based on the work Črepinšek et al. (2013). Yet another visualization technique using recurrence plots for tracking exploration and exploitation has been used recently in Angus & Fister (2020). On the other hand, only few researches focus on theoretical estimation of exploitation and exploration. In Chen & He (2020) the authors evaluated exploration and exploitation of evolutionary algorithms by the probability to hit the promising region.

As can be noted, the threshold variable X was set in our previous work rather arbitrarily, and we were trying to eliminate X from Eqs. 1 and 2. In this manner, the works Gonzalez-Fernandez & Chen (2015); Chen et al. (2019) were important for relating attraction basins with the phases of exploration and exploitation. If the newly created individual belongs to the same attraction basin then this is exploitation. Otherwise, it is exploration. However, the works Gonzalez-Fernandez & Chen (2015); Chen et al. (2019) did not show how to compute attraction basins and how to measure exploration and exploitation. The authors Chen et al. (2019) rather concentrated on another important concept called "*failed exploration*". Namely, it is not only important to find an individual that belongs to a different attraction basin, but, such an individual needs to survive, and the identified attraction basin needs to be exploited further. The authors in Gonzalez-Fernandez & Chen (2015) proposed a new metaheuristic algorithm called Leaders and Followers (LaF) that can tackle a problem of "*failed exploration*" effectively. In our previous work Liu et al. (2013), we have also introduced similar measures (e.g., ExploreProgressiveness, ExploitProgressiveness) to measure the progressiveness of exploration and exploitation.

3. Method

3.1. Attraction basins

An attraction basin $A(x^*)$ is a part of a search space S with a local optimum x^* called an attractor, which can be accessed from any other point $y \in A(x^*)$ in this attraction basin simply by performing a local search (e.g., using the gradient approach) (Locatelli & Wood, 2005; Chen et al., 2019). In other words, the local optimum of a particular attraction basin can be obtained by performing exploitation. On the other hand, moving a search point from one attraction basin to another requires exploration (Chen et al., 2019). In such a manner, both phases, exploration and exploitation, can be identified easily. If a newly created search point (the red cross in Figure 1) belongs to the same attraction basin as its parent (the black dot on Figure 1), then the process is in the phase of exploitation. Otherwise, it is in exploration. Attraction basins can be colored arbitrarily for easier identification of their boundaries. It can be noticed in Figure 2 that there are two attraction basins for the function presented in Figure 1. The search points (cross and dot) on the left part of Figure 1 belong to the same attraction basin (colored as blue in Figure 2), whilst the search points (cross and dot) on the right part of Figure 1 belong to the different attraction basins (colored as gray and blue in Figure 2). Attractions basins have already been studied in the past. The difficultness of the problem to be solved has not only been related to the dimensionality and number of local optima, but also to the size and shape of the attraction basins (Pitzer et al., 2010), as well as to the distance between attraction basins of both global and local optima, and how large the attraction basin of the global optimum is (Caamaño et al., 2010).

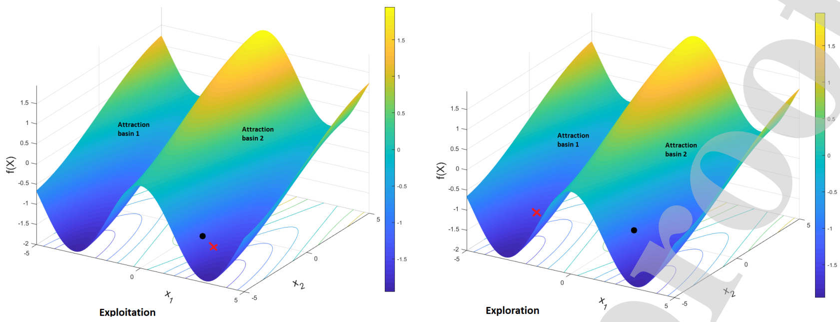


Figure 1: Exploitation and Exploration based on attraction basins.

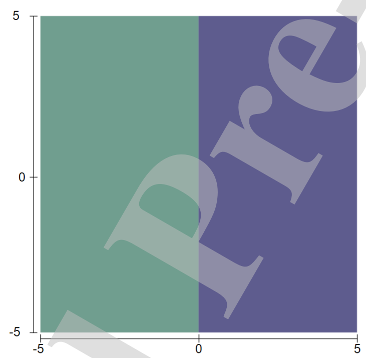


Figure 2: Attraction basins for the Simple 2D function shown in Figure 1.

We further illustrate attraction basins on Rastrigin's 2D function (Figures 3 and 4). Rastrigin's 2D function (Eq. 4, Figure 3) has a single global optimum at $x = (0, 0)$ with $f(x) = 0$. The local optima are distributed evenly over a regular grid with the size of 1 at the integer coordinates (-5 .. +5). Therefore, we expect attraction basins to be regular, symmetric and of similar sizes and shapes. Indeed, we calculated attraction basins that share the mentioned properties (see Figure 4). From Figure 4 it can be observed that there are 121 attraction basins $x_i \epsilon [-5.12, 5.12], n = 2$. The same number is obtained by calculating it with the formula 11^n (Gonzalez-Fernandez & Chen, 2015). With such regularities of attraction basins, our previous approach in Črepinšek et al. (2013); Liu et al. (2013) to measure exploration and exploitation directly is actually working quite well. However, note that such regularities of attraction basins are rare, and unlikely to be present in other optimization functions (see Section 4), especially on real-world optimization problems (Jesenik et al., 2018, 2020; Črepinšek et al., 2019b).

$$f(x) = 10d + \sum_{i=1}^{d}(x_i^2 - 10\cos(2\pi x_i)) \tag{4}$$
$$-5.12 \leq x_i \leq 5.12$$

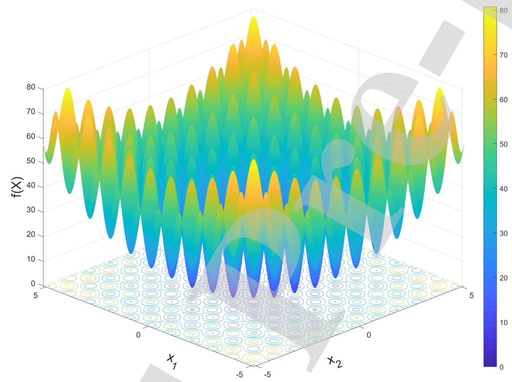


Figure 3: The Rastrigin function.

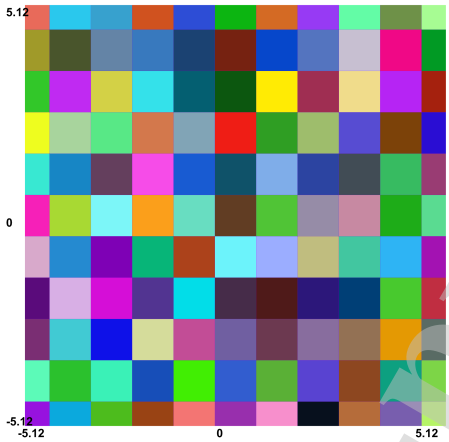


Figure 4: Attraction basins for the Rastrigin function.

Although the works Gonzalez-Fernandez & Chen (2015); Chen et al. (2019) showed how attraction basins can be related to exploration and exploitation, they did not show how to compute attraction basins and how to measure exploration and exploitation directly. Attraction basins are not trivial to define for every problem. Therefore, we used an approximation method, and calculated attraction basins for every function that was used in our benchmark (Section 4). First, we calculated the Heat Map (Einakian & Newman, 2019), where actual function values (fitness) were calculated on the entire search area by appropriate discretization of continuous functions. An example of such a calculation of Heat Map is visualized in Figure 5.

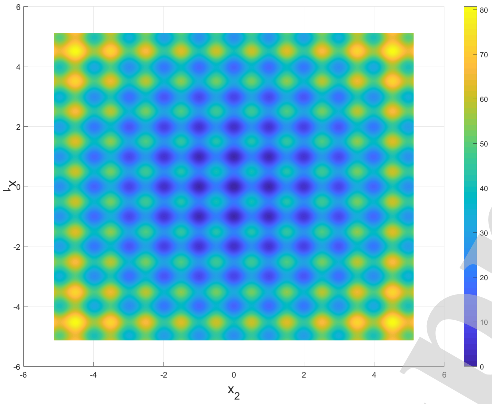


Figure 5: A Heat map example for the Rastrigin function.

Next, we marked all possible boundaries among attraction basins using the procedure in Algorithm 1, where we checked each point i if its fitness value was greater than its two closest neighbors from opposite sides. If it was, we marked it as a potential boundary point.

Input: Heat map

Output: Boundary/non-boundary marked map

for *each non edge* $point_{i,j}$ **do**

  if $point_{i-1,j} < point_{i,j}$ ***and*** $point_{i+1,j} < point_{i,j}$ **then**

    $point_{i,j} = markBoundaryPoint();$

  end

  else if $point_{i,j-1} < point_{i,j}$ ***and*** $point_{i,j+1} < point_{i,j}$ **then**

    $point_{i,j} = markBoundaryPoint();$

  end

  else if $point_{i-1,j-1} < point_{i,j}$ ***and*** $point_{i+1,j+1} < point_{i,j}$ **then**

    $point_{i,j} = markBoundaryPoint();$

  end

  else if $point_{i+1,j-1} < point_{i,j}$ ***and*** $point_{i-1,j+1} < point_{i,j}$ **then**

    $point_{i,j} = markBoundaryPoint();$

  end

end

Algorithm 1: Make boundaries algorithm.

Then, we used the simplest Boundary fill algorithm to mark each non-boundary point with the *id* of a corresponding attraction basin. Points with the same *id* are within a unique attraction basin. A pseudo code is presented in Algorithm 2, which has to be executed for each non-boundary point. The procedure takes as inputs three parameters: *Map* (Heat Map), *id* and *seedPoint*. Parameter *id* is a number which is common to all points that are part of the same attraction basin. Parameter *seedPoint* denotes a starting non-boundary point. The algorithm traverses all *seedPoint*'s neighbors till it hits a boundary point.

Input: Map, *id*, *seedPoint*

Output: Map - attraction basin

$points = initStack(seedPoint)$;

while *points_is_not_empty* **do**

  $current_{i,j} = pop(points)$;

  if $insideMapBounds(current_{i,j})$ **then**

    if $nonBoundaryPoint(current_{i,j})$ **then**

      $current_{i,j} = id$;

      $push(points, current_{i-1,j})$;

      $push(points, current_{i+1,j})$;

      $push(points, current_{i,j-1})$;

      $push(points, current_{i,j+1})$;

    end

  end

end

Algorithm 2: Boundary fill algorithm.

Finally, we have to remove (smooth) all false boundaries that were calculated by Algorithm 1. This is now possible, since we know the *ids* of attraction basins. The procedure is presented in Algorithm 3, where we have used several worth describing helper procedures: spanRight, spanLeft, spanDown, spanUp, spanDownRight, spanDownLeft, spanUpRight and spanUpLeft. Each helper procedure traverses points in some direction, stops after the first non-boundary point found and returns its indices. Then the algorithm checks if points from opposite sides are in the same attraction basin. If this is the case, the boundary point becomes part of the attraction basin.

Input: Attraction basins map

Output: Attraction basins with removed false boundaries

for *each non edge* $point_{i,j}$ **do**

  if $boundaryPoint(points_{i,j})$ **then**

    if $points_{spanLeft(points_{i,j})} == points_{spanRight(points_{i,j})})$ **then**

      $points_{i,j} = points_{spanLeft(points_{i,j})}$;

    end

    else if $points_{spanDown(points_{i,j})} == points_{spanUp(points_{i,j})})$ **then**

      $points_{i,j} = points_{spanDown(points_{i,j})})$;

    end

    else if

    $points_{spanDownLeft(points_{i,j})} == points_{spanUpRight(points_{i,j})})$ **then**

      $points_{i,j} = points_{spanDownLeft(points_{i,j})}$;

    end

    else if

    $points_{spanUpLeft(points_{i,j})} == points_{spanDownRight(points_{i,j})})$ **then**

      $points_{i,j} = points_{spanUpLeft(points_{i,j})}$;

    end

  end

end

Algorithm 3: Smooth false boundaries algorithm.

3.2. Ancestry trees

For calculating an amount of exploration we use the strategy of ancestry trees described in Črepinšek et al. (2013, 2011). The following information is stored in nodes of an ancestry tree: An individual's parent, its representation (genome), as well as how (e.g., by mutation, by crossover) and when (generation) the individual was created. As such, the ancestry tree describes the history of all individuals that were created during the evolution process. It records the influence of evolution operators and parent involvement for each individual. The population size determines the number of ancestry trees in an evolution process.

An example of an ancestry tree is shown in Figure 6 (Left), where it is quite obvious that fitter individuals, those who are selected for the next generations, contribute to the growth of the ancestry tree. However, only descendant relation and operators were captured in an ancestry tree. But, information on whether an individual is an outcome of exploration or exploitation is not captured in an ancestry tree yet. Hence, we add information about similarity to its parent ($\mathcal{SCN}$) to nodes of the ancestry tree. Whenever $\mathcal{SCN}$ is above threshold X, which delimits exploration from exploitation, the ancestry tree has to be split (Črepinšek et al., 2013, 2011). By using a splitting process, the ancestry tree splits into several sub-trees, called exploitation trees (see an example in Figure 6 (Middle)), since all nodes, except the root nodes of sub-trees, are obtained by exploitation of neighborhood regions (similarity to its parent node is lower than the threshold value X, and therefore, should be in the vicinity of a parent solution).

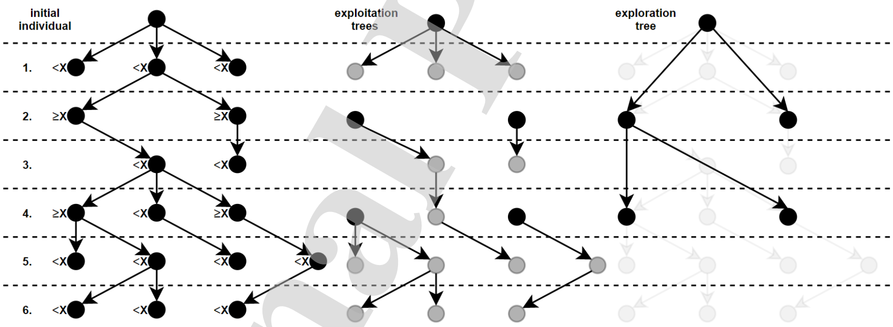


Figure 6: An ancestry tree after six generations (Left), exploitation trees (Middle), an exploration tree (Right).

The root nodes (black nodes) in Figure 6 (Middle) were obtained by exploration forming an exploration tree Figure 6 (Right), while all other nodes (gray nodes) Figure 6 (Middle), were obtained by exploitation. The exploration ratio can be defined as the percentage of nodes obtained by exploration (black nodes) and all nodes in all exploitation trees. By using trees as a representation, we were able to define in Črepinšek et al. (2011) several other useful measures, such as the progressiveness of exploitation and exploration. The exploration ratio using our previous approach based on threshold value X is called in this paper *ExpDist*, while *exploreRatio* was used in our previous work Črepinšek et al. (2013, 2011); Liu et al. (2013) ($exploitRatio = 1 - exploreRatio$), where a complete procedure was described in greater detail. The main challenge was how to determine X (a threshold value to delimit exploration from exploitation) to split an ancestry tree into sub-trees. Originally, threshold value X was determined by the trial and error approach, and there was no exact method to calculate it. Therefore, in this paper, we proposed to measure exploration and exploitation based on attraction basins. The main differences from our previous approach (Črepinšek et al., 2013, 2011; Liu et al., 2013) are:

- An ancestry tree simply splits if a node (individual) does not belong to the same attraction basin as its parent node (parent). Previously, splitting depended on threshold value X. The main problem of how to compute attraction basins is presented in this Section.
- Since on ancestry trees only a single parent can be recorded, the rule which node becomes a parent in an ancestry tree needs to be changed when multiple parents are involved (e.g., crossover). Previously, the most similar parent ($\mathcal{SCN}$) was selected, whilst in the proposed approach, a parent becomes the first individual among contributed individuals (parents) that belong to the same attraction basin as a newly created individual (if existing).

Note, that to compute the newly proposed measure of exploration (*ExpBas*) that is based on attraction basins, the tree representation in the form of ancestry is not necessary. We need to know only if a newly generated solution belongs to the same attraction basin as its parent. The tree representation is needed to compute other features, such as the progressiveness of exploration and exploitation. But, this is out of the scope of this paper. This subsection is written solely to understand our previous direct measure of exploration (*ExpDist*) better.

4. Experiments

In the experiments, a newly proposed direct measure based on attraction basins (ExpBas) was compared to our previous direct measures of exploration and exploitation that are based on threshold value X ($ExpDist$) (Črepinšek et al., 2013, 2011; Liu et al., 2013) and to diversity (Diversity) as an indirect measure of exploration and exploitation. Diversity is measured as the average distance of individuals from the centroid and is defined as follows (Salehinejad et al., 2014; Wang et al., 2017; Hussain et al., 2018; Albunian et al., 2020):

$$Diversity = \frac{1}{N_p} \sum_{i=1}^{N_p} \sum_{d=1}^{D} |(x_{i,d} - x_d^c)| \tag{5}$$

where the centroid x^c is defined as:

$$x_d^c = \frac{1}{N_p} \sum_{i=1}^{N_p} x_{i,d}, \qquad \forall d \in \{1, ..., D\} \tag{6}$$

Diversity is computed as the average distance from the centroid x^c, where N_p is the population size, $x_{i,j}$ is the d-th dimension (out of D dimensions) of the i-th solution.

The Pearson correlation coefficient ρ has been used to measure linear correlation between different exploration measures:

$$\rho(X,Y) = \frac{Cov(X,Y)}{\sigma(X)\sigma(Y)} \tag{7}$$

where $\sigma(X)$ and $\sigma(Y)$ are the Standard Deviation of variables X and Y, respectively, and $Cov(X,Y)$ is the covariance of the two variables. If the covariance of X and Y ($Cov(X,Y)$) is larger than 0, then variables X and Y are correlated positively. If the $Cov(X,Y)$ is equal to 0, then the variables X and Y are uncorrelated. Otherwise, the variables X and Y are correlated negatively. The guidelines for the interpretation of a correlation coefficient are as follows:

- uncorrelated: $0 \leq |\rho| \leq 0.09$
- weak correlation: $0.1 \leq |\rho| \leq 0.3$

- medium correlation: $0.3 < |\rho| \leq 0.5$
- strong correlation: $0.5 < |\rho| \leq 1.0$

For comparison purposes, several algorithms were included: Random Walk (RWSi) (Spitzer, 2013), Hill Climbing (Russell & Norvig, 2002), JADE (Zhang & Sanderson, 2009), DE (Storn & Price, 1997), jDElscop (Brest & Maučec, 2011), TLBO (Venkata Rao et al., 2012; Čepinšek et al., 2012; Črepinšek et al., 2016). The setting of control parameters in the applied algorithms are presented in Table 1.

Algorithm	Control parameters
DE-best-1-bin	F=0.5, Cr=0.9
JADE	p=0.05, c=0.1
jDElscop	none
TLBO	keep=0
Hill Climbing	discretization = 4000
ExpDist	threshold X = 0.5
ExpBas	discretization = 4000

Table 1: Control parameters' setting.

Note that, in this study, we were not primarily interested in comparing the performance of the selected algorithms, which has been done intensively in the research community. The aim was to introduce a new direct measure for exploration and exploitation, and to compare it to our previous approach in Črepinšek et al. (2013); Liu et al. (2013) and to a common indirect measure for exploration and exploitation based on diversity (Salehinejad et al., 2014). Hence, default settings of control parameters were used in the selected algorithms. Nevertheless, the optimum found (Fitness) by selected algorithms are presented in the results, so readers were informed about which algorithms performed well.

4.1. 2D functions

This Section shows the experimental results of several 2D functions: Rastrigin, Schwefel, Easom, Sphere, Ackley, Griewank, Holder Table, Shifted Rotated Rastrigin, Split Drop Wave, and Rotated Hybrid Composition function 1. 2D functions were selected because attraction basins for 2D functions are easier to visualize and interpret. The experiment was executed with population size $N_p = \{10, 20, 50\}$ and maximum fitness evaluations $maxFEs = \{2000, 5000, 10000\}$. Hence, for each 2D function, nine different configurations exist with 50 independent runs. The obtained conclusions were similar, hence only the results for $N_p = 20$ and $MaxFEs = 10000$ are reported in the continuation.

4.1.1. Rastrigin function

The Rastrigin 2D function (Eq. 4, Figure 3), $F_{min}(0, 0) = 0$. has the property that its optima are integers. Its attraction basins are computed easily. Hence, this function serves as the main test to apply our algorithm for finding attraction basins (Figure 4).

Algorithm	Fitness	ExpBas	ExpDist	Diversity
RWSi	0.8517 ±0.5041	1.0000 ±0.0000	1.0000 ±0.0000	5.1173 ±0.0291
Hill Climbing	5.9112 ±3.3128	0.0041 ±0.0060	0.0020 ±0.0000	4.9672 ±0.3875
JADE	0.0000 ±0.0000	0.0575 ±0.0084	0.0601 ±0.0090	0.2259 ±0.0448
DE-best-1-bin	0.4776 ±0.7033	0.0183 ±0.0043	0.0204 ±0.0049	0.0728 ±0.0256
jDElscop	0.0199 ±0.1407	0.0572 ±0.0117	0.0598 ±0.0120	0.2294 ±0.0514
TLBO	0.0199 ±0.1407	0.0663 ±0.0197	0.0716 ±0.0217	0.3982 ±0.1056

Table 2: The results of the Rastrigin function.

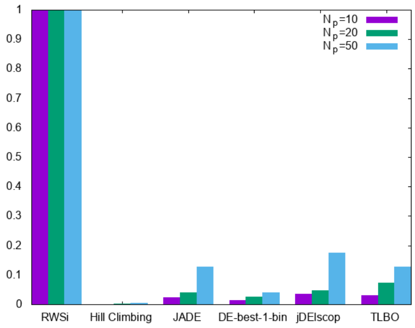


Figure 7: ExpBas for different N_p.

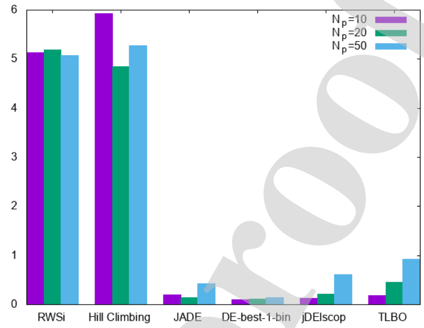


Figure 8: Diversity for different N_p.

Algorithm	ExpBas vs. ExpDist	ExpBas vs. Diversity	ExpDist vs. Diversity
Hill Climbing	-1.0000	-0.1297	0.0000
JADE	-0.9831	0.7587	0.8237
DE-best-1-bin	-0.9810	0.7361	0.7453
jDElscop	-0.8970	0.8693	0.9016
TLBO	-0.9885	0.8905	0.8775

Table 3: Pearson correlation coefficient for the Rastrigin function.

In Table 2, we can observe how different algorithms behaved in finding the optimal solution. From the results we can notice that there is a small difference between the newly proposed method (*ExpBas*) based on attraction basins and our previous approach (*ExpDist*) (Črepinšek et al., 2013; Liu et al., 2013). This is mainly because of the regularities of attraction basins of the Rastrigin function (see Figure 4). A small difference between *ExpBas* and *ExpDist* is also shown by the Pearson correlation coefficient ρ (Table 3), which indicates a strong correlation between *ExpBas* and *ExpDist*. On the other hand, ρ between *ExpBas*/*ExpDist* and *Diversity* is small (-0.1297), indicating a weak linear correlation, or even 0, on the Hill Climbing algorithm indicating that there is no linear correlation between *ExpBas*/*ExpDist* and *Diversity*. From Table 2 it can be concluded that bigger diversity indicates higher exploration for all algorithms except for Hill Climbing. The computed diversity of Hill Climbing in Table 2 shows that diversity was almost as high as for RWSi, but the amount of exploration of Hill Climbing was much less than RWSi. This fact indicates that diversity is not a very precise measure for exploration. As mentioned before, diversity in our case is measured as the average distance of individuals from the centroid (i.e., Eq. 5). A high diversity value implies that many individuals might be far away from the centroid individual. *ExpDist* is the formula associated with the number of splits determined by a hyper-cube-like threshold X. A low *ExpDist* value implies that low exploration occurs within the specified threshold. Since attraction basins of the Rastrigin function are regular and threshold X was tailored to this problem, the results of *ExpDist* are in good correlation with *ExpBas*. Figure 7 shows that exploration is increased when population size is increased. Hence, the belief that exploration is increased when dealing with a bigger population is supported by our experiment. From Table 2 it can be also observed that JADE and jDElscop applied similar amounts of exploration on this problem, whilst the amount of exploration of DE-best-1-bin was smaller, and the amount of exploration of TLBO was bigger than JADE and jDElscop. Figures 7 and 8 together show again that diversity was not a good measure of exploration for Hill Climbing.

4.1.2. Schwefel function

The attraction basins of the Schwefel 2D function (Eq. 8, Figure 9), $F_{min}(x^*) = 0$ at $x^* = (420.968746, 420.968746)$, which are computed from a Heat Map (Figure 10), are presented in Figure 11. It can be observed from Figure 11 that the sizes of the attraction basins are no longer regular. Hence, the amount of exploration computed by our previous approach (Črepinšek et al., 2013; Liu et al., 2013) was no longer accurate (Table 4) due to the irregularity of the attraction basins. For example, *ExpDist* is almost four times bigger than *ExpBas* on JADE (Table 4). Pearson's correlation coefficient ρ (Table 5) indicates only weak linear correlation to TLBO (-0.1418).

Diversity is again only a rough measure for exploration for JADE, DE-best-1-

bin, jDElscop, and TLBO. Whilst exploration measures *ExpBas* and *Diversity*, they are uncorrelated to Hill Climbing, and only weakly correlated to TLBO. Figures 12 and 13 show another example that *Diversity* on Hill Climbing is not a good measure for exploration. Figure 12 exhibits another interesting point. It shows clearly that exploration of TLBO and DE-best-1-bin is much higher, and lower respectively, than exploration in JADE and jDElscop on this problem.

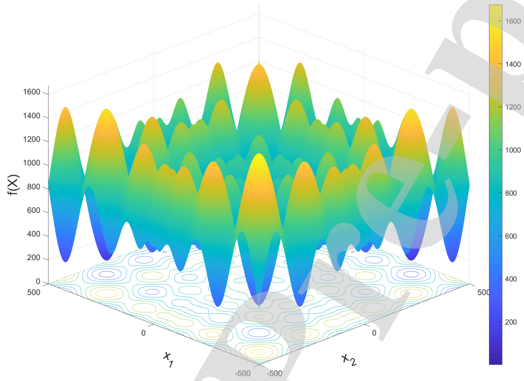


Figure 9: The Schwefel function.

$$f(\mathbf{x}) = 418.9829d - \sum_{i=1}^{d} x_i \sin(\sqrt{|x_i|}) \tag{8}$$
$$-500 \le x_i \le 500$$

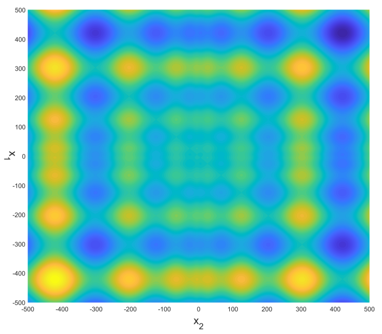


Figure 10: The heat map of the Schwefel function.

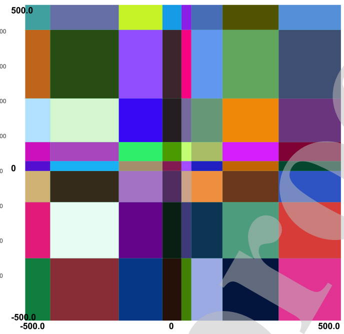


Figure 11: Attraction basins for the Schwefel function.

Algorithm	Fitness	ExpBas	ExpDist	Diversity
RWSi	7.7212 ±6.4954	1.0000 ±0.0000	1.0000 ±0.0000	500.2794 ±3.7241
Hill Climbing	266.4859 ±121.5960	0.0028 ±0.0030	0.1881 ±0.0320	488.6567 ±43.1112
JADE	23.6877 ±58.6118	0.0255 ±0.0093	0.0965 ±0.0152	47.5569 ±30.0672
DE-best-1-bin	85.6611 ±88.8029	0.0080 ±0.0028	0.0290 ±0.0059	8.6218 ±4.6996
jDElscop	9.4751 ±32.4577	0.0783 ±0.0087	0.1353 ±0.0136	97.4250 ±23.4284
TLBO	26.0565 ±55.0348	0.3086 ±0.0275	0.5251 ±0.0868	252.6841 ±91.2893

Table 4: The results of the Schwefel function.

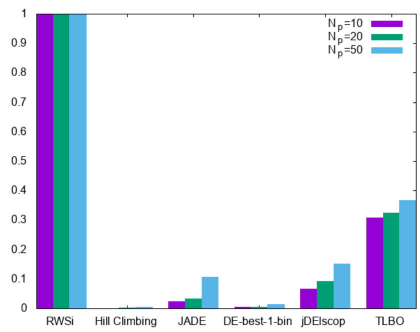


Figure 12: ExpBas for different N_p.

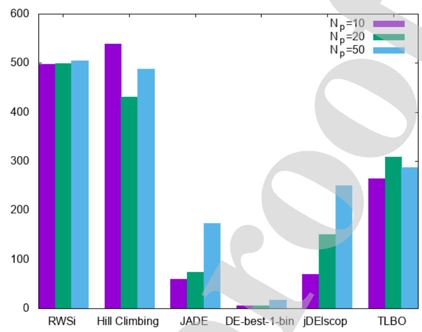


Figure 13: Diversity for different N_p.

Algorithm	ExpBas vs. ExpDist	ExpBas vs. Diversity	ExpDist vs. Diversity
Hill Climbing	0.1749	-0.0406	0.1354
JADE	-0.8810	0.8885	0.8648
DE-best-1-bin	-0.7359	0.6912	0.4955
jDElscop	-0.8981	0.8821	0.8638
TLBO	-0.1418	0.3029	0.8843

Table 5: Pearson correlation coefficients for the Schwefel function.

4.1.3. Easom function

The attraction basins of the Easom 2D function (Eq. 9, Figure 14), ($F_{min}(\pi, \pi) = -1$), which were computed from the Heat Map (Figure 15), are presented in Figure 16. The Easom 2D function has a large plateau, which can be identified easily in Figures 14 - 16. This plateau belongs to only one large attraction basin. This is the reason why our previous approach to measure exploration (*ExpDist*) (Črepinšek et al., 2013; Liu et al., 2013) is not well aligned with our new and more accurate measure of exploration (*ExpBas*). Our previous measure *ExpDist* is up to three times bigger than *ExpBas* (Table 6). This is also exposed in Pearson correlation coefficient ρ, which shows weak (DE-best-1-bin) and medium (TLBO) correlation, and even uncorrelation (JADE) between *ExpBas* and *ExpDist*. Similarly, correlation between *ExpBas* and *Diversity* was weak (DE-best-1-bin, jDElscop), medium (JADE), or even uncorrelated (TLBO, Hill Climbing) (Table 7). Indication that diversity is not an accurate measure for exploration is also shown on Figures 17 and 18.

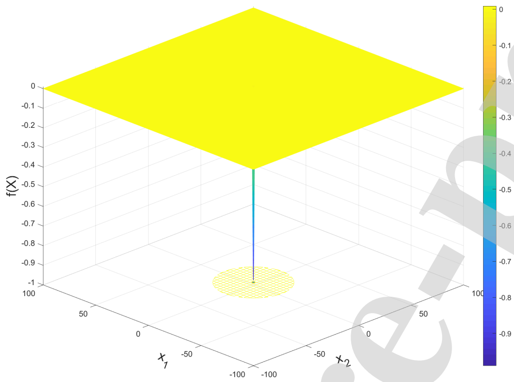


Figure 14: The Easom function.

$$f(x) = -\cos(x_1)\cos(x_2)\exp\left(-(x-\pi)^2 - (x_2-\pi)^2\right) \tag{9}$$

$$-100 \leq x_i \leq 100$$

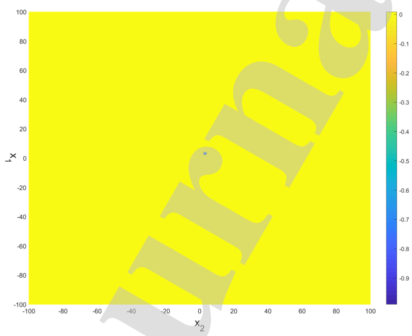


Figure 15: The heat map of the Easom function.

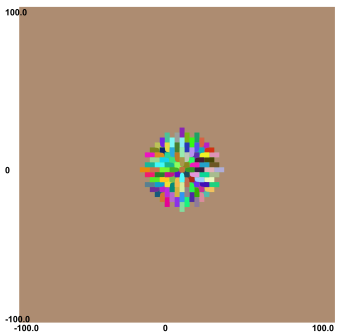


Figure 16: Attraction basins for the Easom function.

Algorithm	Fitness	ExpBas	ExpDist	Diversity
RWSi	-0.1809	1.0000	1.0000	99.8960
	±0.2687	±0.0000	±0.0000	±0.5404
Hill Climbing	-0.0200	0.0025	0.0057	96.0517
	±0.1414	±0.0025	±0.0049	±9.1713
JADE	-1.0000	0.0398	0.1271	7.9450
	±0.0000	±0.0063	±0.0207	±2.7384
DE-best-1-bin	-1.0000	0.0153	0.0421	2.7841
	±0.0000	±0.0024	±0.0136	±3.1682
jDElscop	-1.0000	0.0797	0.1523	6.9160
	±0.0000	±0.0059	±0.0201	±2.4821
TLBO	-1.0000	0.2090	0.4049	6.6957
	±0.0000	±0.0180	±0.0180	±1.4661

Table 6: The results of the Easom function.

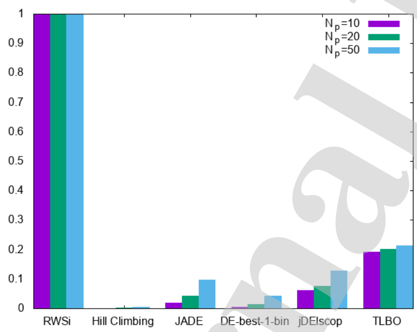


Figure 17: ExpBas for different N_p.

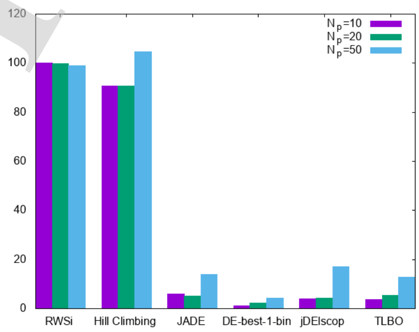


Figure 18: Diversity for different N_p.

Algorithm	ExpBas vs. ExpDist	ExpBas vs. Diversity	ExpDist vs. Diversity
Hill Climbing	-0.4017	0.0561	0.0318
JADE	-0.0588	-0.3148	0.8825
DE-best-1-bin	-0.2367	0.1016	0.9638
jDElscop	-0.5542	0.2155	0.9148
TLBO	-0.4354	-0.0276	0.5722

Table 7: Pearson correlation coefficients for the Easom function.

4.1.4. Sphere function

The only unimodal function in our experiment was the Sphere 2D function (Eq. 10, Figure 19), $F_{min}(0,0) = 0$. Attraction basins were computed from a Heat Map (Figure 20). Indeed, there was only one attraction basin (Figure 21). Our previous approach to measure exploration, that was based on distance among solutions (ExpDist), was clearly inaccurate (Table 8) exhibiting even more than 20 times bigger exploration by *ExpDist* than by *ExpBas*. Apparently, *ExpDist* exaggerates exploration because of a predetermined threshold that does not reflect the problem domain accurately. Instead, *ExpBas* does. Since Sphere function is unimodal with only one attraction basin, only the solutions from initial population are classified as exploration. Hence, the amount of exploration is the same for Hill Climbing, JADE, DE-best-1-bin, jDElscop, and TLBO (Table 8). But, the amount of *Diversity* is very different, clearly indicating that diversity is not a good measure for exploration. Pearson correlation coefficient ρ exhibits uncorrelation between *ExpBas* and *Diversity* (Table 9).

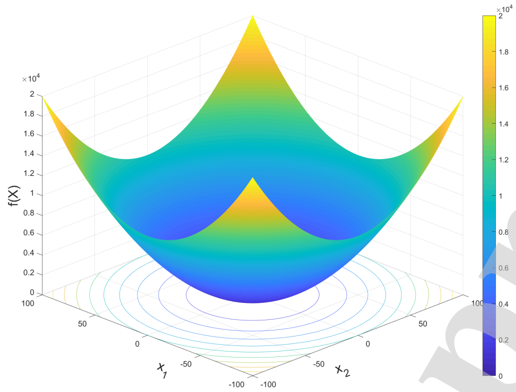


Figure 19: The Sphere function.

$$f(x) = \sum_{i=1}^{d} x_i^2 \tag{10}$$

$$-100 \leq x_i \leq 100$$

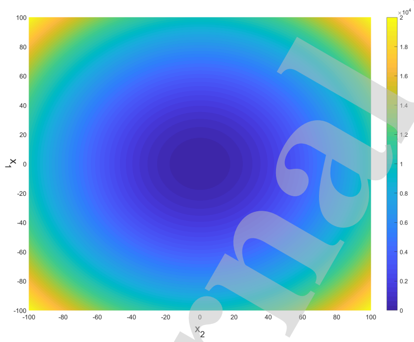


Figure 20: Sphere heat map.

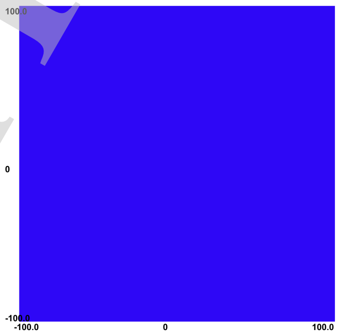


Figure 21: Attraction basin for the Sphere function.

Algorithm	Fitness	ExpBas	ExpDist	Diversity
RWSi	2.5602 $\pm$2.0671	1.0000 $\pm$0.0000	1.0000 $\pm$0.0000	100.0252 $\pm$0.6291
Hill Climbing	457.3662 $\pm$523.3280	0.0020 $\pm$0.0000	0.2002 $\pm$0.0302	93.8682 $\pm$9.9305
JADE	0.0000 $\pm$0.0000	0.0020 $\pm$0.0000	0.0450 $\pm$0.0034	1.4385 $\pm$0.1744
DE-best-1-bin	0.0000 $\pm$0.0000	0.0020 $\pm$0.0000	0.0196 $\pm$0.0016	0.6892 $\pm$0.1521
jDElscop	0.0000 $\pm$0.0000	0.0020 $\pm$0.0000	0.0429 $\pm$0.0043	1.5676 $\pm$0.2422
TLBO	0.0000 $\pm$0.0000	0.0020 $\pm$0.0000	0.0381 $\pm$0.0044	1.5753 $\pm$0.2399

Table 8: The results of the Sphere function.

Algorithm	ExpBas vs. ExpDist	ExpBas vs. Diversity	ExpDist vs. Diversity
Hill Climbing	-1.0000	0.0000	0.1096
JADE	-1.0000	0.0000	0.6230
DE-best-1-bin	-1.0000	0.0000	0.5169
jDElscop	-1.0000	-0.0000	0.5784
TLBO	-1.0000	0.0000	0.6062

Table 9: Pearson correlation coefficients for the Sphere function.

4.1.5. Ackley function

The attraction basins of the Ackley 2D function (Eq. 11, Figure 22), (F_{min}(0, 0) = 0), which were computed from a Heat Map (Figure 23), are presented in Figure 24. It can be noticed that the attraction basins are similar in shape and size. This is a reason that our previous approach to measure exploration, *ExpDist*, gave comparable results to our newly and more accurate measure

ExpBas (Table 10). Indeed, Pearson correlation coefficient ρ exhibits very strong correlation between *ExpBas* and *ExpDist* on JADE, DE-best-1-bin, jDElscop, and TLBO (Table 11), while correlation on Hill Climbing is weak on all three exploration measures (Table 11 and Figure 26). Yet another example that *Diversity* is not a good measure of exploration is shown in Table 10, where exploration in JADE ($ExpBas = 0.0344$) is bigger than on TLBO ($ExpBas = 0.0319$), yet diversity in JADE is smaller ($Diversity = 0.4972$) than in TLBO ($Diversity = 0.6018$). Figure 25 shows that the amount of exploration in JADE, jDElscop, and TLBO is remarkably similar for this problem.

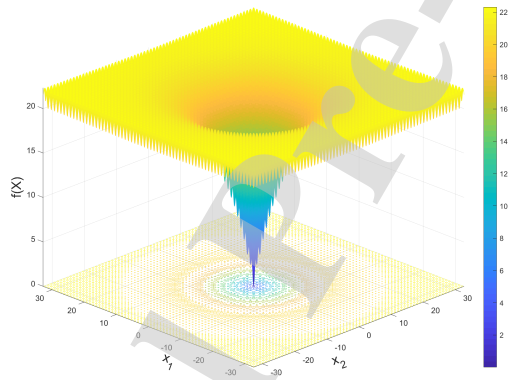


Figure 22: The Ackley function.

$$f(x) = -20 \exp\left(-0.2\sqrt{\frac{1}{d}\sum_{i=1}^{d} x_i^2}\right) - \exp\left(\frac{1}{d}\sum_{i=1}^{d}\cos(2\pi x_i)\right) + 20 + e \quad (11)$$

$$-32 \leq x_i \leq 32$$

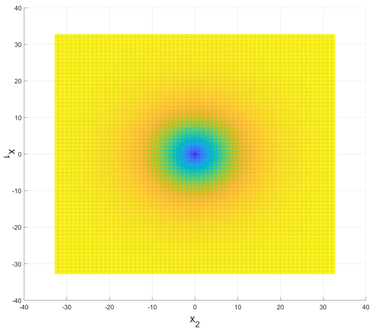


Figure 23: Ackley Heat Map.

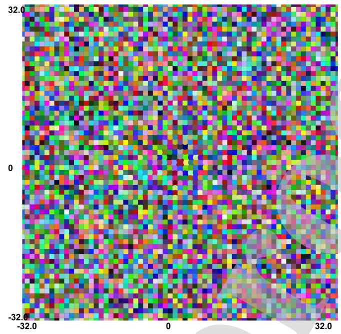


Figure 24: Attraction basins for the Ackley function.

Algorithm	Fitness	ExpBas	ExpDist	Diversity
RWSi	2.4846 ±0.9354	1.0000 ±0.0000	1.0000 ±0.0000	32.0738 ±0.1688
Hill Climbing	11.1492 ±4.1012	0.0064 ±0.0056	0.0031 ±0.0012	30.4865 ±2.6611
JADE	0.0000 ±0.0000	0.0344 ±0.0041	0.0371 ±0.0043	0.4972 ±0.0893
DE-best-1-bin	0.0516 ±0.3649	0.0145 ±0.0013	0.0157 ±0.0013	0.2252 ±0.0340
jDElscop	0.0000 ±0.0000	0.0338 ±0.0042	0.0360 ±0.0046	0.5945 ±0.1353
TLBO	0.0000 ±0.0000	0.0319 ±0.0037	0.0326 ±0.0042	0.6018 ±0.1179

Table 10: The results of the Ackley function.

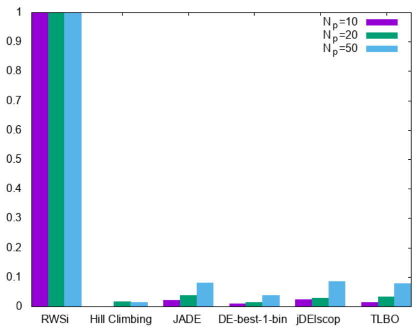


Figure 25: ExpBas for different N_p.

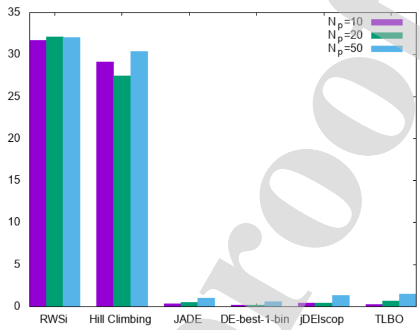


Figure 26: Diversity for different N_p.

Algorithm	ExpBas vs. ExpDist	ExpBas vs. Diversity	ExpDist vs. Diversity
Hill Climbing	-0.2042	-0.2244	-0.2881
JADE	-0.9863	0.7908	0.7938
DE-best-1-bin	-0.9105	0.4296	0.3904
jDElscop	-0.9860	0.8099	0.7829
TLBO	-0.9169	0.7156	0.5632

Table 11: Pearson correlation coefficients for the Ackley function.

4.1.6. Griewank function

The attraction basins of the Griewank 2D function (Eq. 12, Figure 27), $F_{min}(0,0) = 0.$, which were computed from a Heat Map (Figure 28), are presented in Figure 29. From the Heat Map of the Griewank function (left part of Figure 28), which is similar to the Heat Map of the Sphere function (Figure 20), one might be surprised that the number of attraction basins for the Griewank function (Figure 29) is much larger than in most of the cases presented in this Section. However, if we zoom in on the Griewank function (right part of Figure 27) and its Heat Map (right part of Figure 28) we can notice that there are many local optima. Note that each local optimum has its own attraction basin (see Section 3). This explains that there are many attraction basins in Figure 29. Our previous approach to measure exploration and exploitation, *ExpDist*, gave comparable, but not accurate, results to our newly proposed measure *ExpBas* (Table 12). For example, while *ExpBas* and *ExpDist* are similar for Hill Climbing, DE-best-1-bin, and JDElscop, this is not the case for JADE and TLBO, where *ExpDist* is two times bigger than *ExpBas*. This is also shown in Table 13, where it can be noticed that correlation between *ExpBas* and *ExpDist* is strong (except for Hill Climbing), while correlation between *ExpBas* and *Diversity* is only medium (Table 13). Yet another example that *Diversity* is not a good measure of exploration is shown in Table 12, where exploration in JADE ($ExpBas = 0.1426$) is bigger than in jDElscop ($ExpBas = 0.1348$), yet diversity in JADE is smaller ($Diversity = 12.4242$) than in jDElscop ($Diversity = 12.5181$).

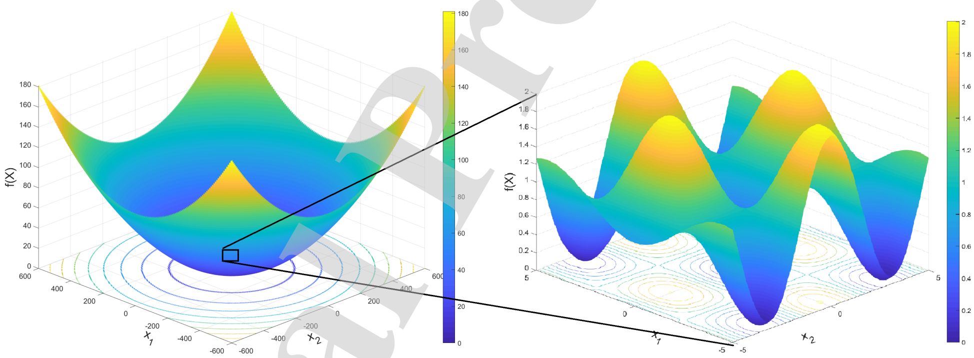


Figure 27: The Griewank function.

$$f(x) = \sum_{i=1}^{d} \frac{x_i^2}{400} - \prod_{i=1}^{d} \cos\left(\frac{x_i}{\sqrt{i}}\right) + 1 \qquad (12)$$

$$-600 \leq x_i \leq 600$$

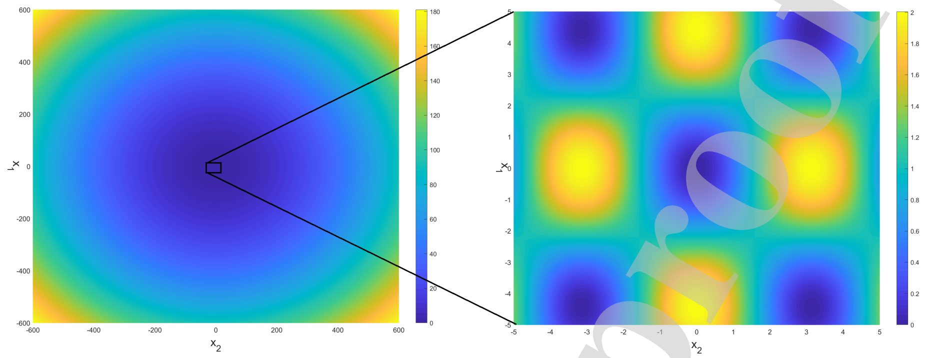


Figure 28: The Heat Map of the Griewank function.

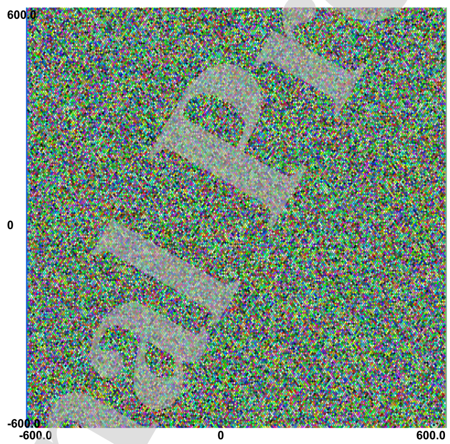


Figure 29: Attraction basins for the Griewank function.

Algorithm	Fitness	ExpBas	ExpDist	Diversity
RWSi	0.2739 ±0.1570	1.0000 ±0.0000	1.0000 ±0.0000	600.9729 ±3.1457
Hill Climbing	5.7238 ±4.7902	0.0181 ±0.0105	0.0185 ±0.0037	580.3882 ±47.2148
JADE	0.0000 ±0.0000	0.1426 ±0.0185	0.2814 ±0.0497	12.4242 ±1.2126
DE-best-1-bin	0.0130 ±0.0225	0.0334 ±0.0043	0.0572 ±0.0089	5.1168 ±0.9111
jDElscop	0.0001 ±0.0010	0.1348 ±0.0252	0.1988 ±0.0324	12.5181 ±1.6760
TLBO	0.0009 ±0.0024	0.2343 ±0.0632	0.6346 ±0.1474	16.5450 ±2.4448

Table 12: The results of the Griewank function.

Algorithm	ExpBas vs. ExpDist	ExpBas vs. Diversity	ExpDist vs. Diversity
Hill Climbing	-0.0333	0.0434	0.2329
JADE	-0.6087	0.4545	0.3432
DE-best-1-bin	-0.8083	0.5596	0.4029
jDElscop	-0.9354	0.5906	0.5383
TLBO	-0.8522	0.5839	0.5451

Table 13: Pearson correlation coefficients for the Griewank function.

4.1.7. Holder Table function

The attraction basins of the Holder Table 2D function (Eq. 13, Figure 30), $F_{min}(x^*) = -19.2085$ at $x^* = (8.05502, 9.66459)$, $(8.05502, -9.66459)$, $(-8.05502, 9.66459)$ and $(-8.05502, -9.66459)$, which were computed from a Heat Map (Figure 31), are presented in Figure 32. This function has four equal optima in the corners of the search region. Due to the regularity of the attraction basins (Figure 32), our previous approach, (*ExpDist*), would work quite well if threshold value X would be increased (since attraction basins for the Holder Table function are larger than for the Rastrigin function). This is also exhibited from Pearson correlation coefficient ρ (Table 15), where the correlation between *ExpBas* and *ExpDist* is strong (except for Hill Climbing). By comparing Figures 33 and 34 it can be also noticed that *Diversity* is not a good measure for exploration. Figure 33 also shows that exploration of TLBO on this problem is much higher than that of JADE, DE-best-1-bin, and jDElscop.

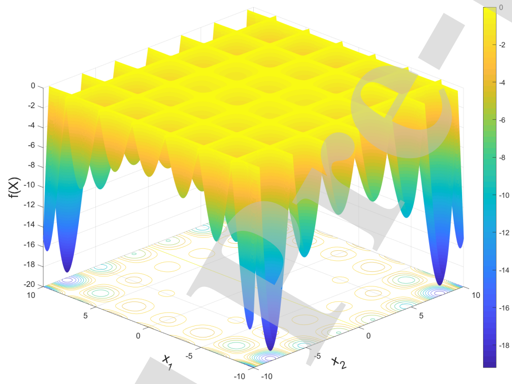


Figure 30: Holder Table function.

$$f(x) = -\left|\sin(x_1)\cos(x_2)\exp\left(\left|1 - \frac{\sqrt{x_1^2 + x_2^2}}{\pi}\right|\right)\right| \quad (13)$$

$$-10 \leq x_i \leq 10$$

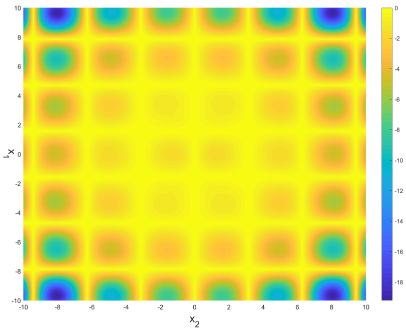


Figure 31: The Heat Map of the Holder Table function.

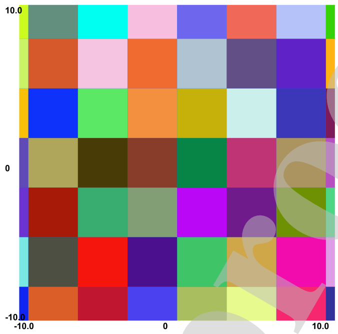


Figure 32: Attraction basins for the Holder Table function.

Algorithm	Fitness	ExpBas	ExpDist	Diversity
RWSi	-19.1436 ±0.0821	1.0000 ±0.0000	1.0000 ±0.0000	9.9929 ±0.0612
Hill Climbing	-13.8389 ±5.0789	0.0030 ±0.0042	0.0295 ±0.0065	9.7369 ±0.8615
JADE	-19.1497 ±0.4159	0.0229 ±0.0499	0.0331 ±0.0551	0.5173 ±0.9463
DE-best-1-bin	-18.7510 ±0.9144	0.0054 ±0.0009	0.0086 ±0.0016	0.1133 ±0.0204
jDElscop	-19.2084 ±0.0004	0.0815 ±0.0214	0.0897 ±0.0232	2.3183 ±1.1078
TLBO	-19.2085 ±0.0000	0.4356 ±0.1123	0.5659 ±0.1177	7.3214 ±2.8166

Table 14: The results of the Holder Table function.

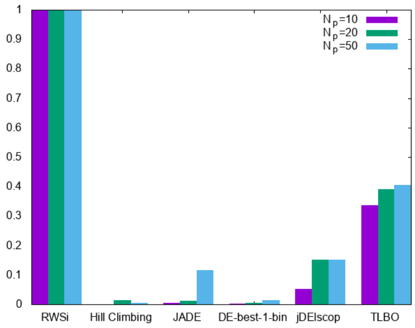


Figure 33: ExpBas for different N_p.

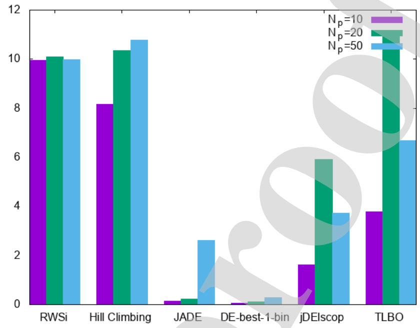


Figure 34: Diversity for different N_p.

Algorithm	ExpBas vs. ExpDist	ExpBas vs. Diversity	ExpDist vs. Diversity
Hill Climbing	0.1770	0.0053	-0.0940
JADE	-0.9994	0.9953	0.9936
DE-best-1-bin	-0.7580	0.7146	0.4201
jDElscop	-0.9980	0.9166	0.9181
TLBO	-0.9186	0.7649	0.8821

Table 15: Pearson correlation coefficients for the Holder Table function.

4.1.8. Shifted Rotated Rastrigin function

Attraction basins of the Shifted Rotated Rastrigin 2D function (Eq. 14, Figure 35), $F_{min}(x^*) = -330.0$ at $x^* = (1.9005, -1.5644)$, which are computed from a Heat Map (Figure 36), are presented in Figure 37. This function is a shifted and rotated version of the Rastrigin function presented in Section 4.1.1. By comparing attraction basins between these two functions (Figures 4 and 37), the effect of rotation can be noticed. From Tables 2 and 16 it can be observed that shifting and rotation have some effect on exploration measures (*ExpBas*, *ExpDist*, *Diversity*) for the metahuristics under discussion. The amount of TLBO exploration (*ExpBas*) for the shifted and rotated Rastrigin function is more than five times bigger than that for a non-shifted and non-rotated Rastrigin function, while the amount of exploration (*ExpBas*) of JADE is about two times bigger for population size $N_p = 50$ (see Figures 7 and 38). This finding requests further investigation on shifted and rotated functions. On the other hand, Pearson correlation coefficient ρ comparing different exploration measures, are similar (see Tables 3 and 17), indicating that the computed measures $ExpBas$, $ExpDist$, and $Diversity$ correlate regardless of wheather the function is rotated and shifted for this problem.

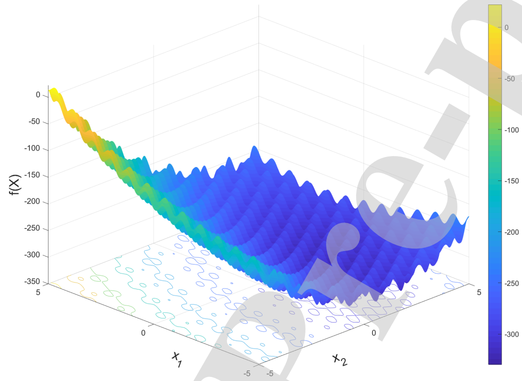


Figure 35: Shifted Rotated Rastrigin function.

$$
\begin{gathered}
f(x) = 10d + \textstyle\sum_{i=1}^{d}(z_i^2 - 10\cos(2\pi z_i)) + f\_bias_{10} \\
z = (x - o) * M \\
M : \text{linear transformation matrix} \\
o : \text{shifted global optimum} \\
-5 \leq x_i \leq 5
\end{gathered}
\tag{14}
$$

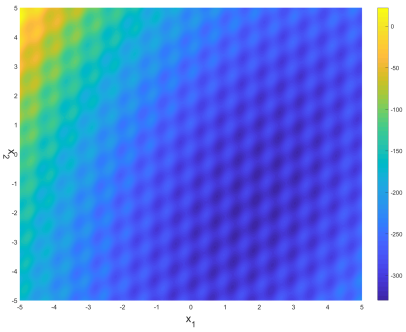


Figure 36: The Heat Map of the Shifted Rotated Rastrigin function.

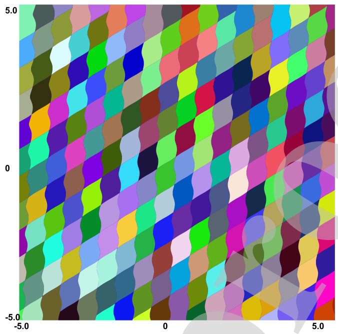


Figure 37: Attraction basins for the Shifted Rotated Rastrigin function.

Algorithm	Fitness	ExpBas	ExpDist	Diversity
RWSi	-328.9586 ±0.5669	1.0000 ±0.0000	1.0000 ±0.0000	4.9969 ±0.0296
Hill Climbing	-323.2935 ±5.2013	0.0082 ±0.0111	0.0020 ±0.0000	4.8330 ±0.5045
JADE	-330.0000 ±0.0000	0.0837 ±0.0132	0.0685 ±0.0115	0.2726 ±0.0445
DE-best-1-bin	-329.1412 ±1.5819	0.0193 ±0.0055	0.0160 ±0.0047	0.0695 ±0.0289
jDElscop	-330.0000 ±0.0000	0.1150 ±0.0137	0.1040 ±0.0122	0.3469 ±0.0513
TLBO	-329.8806 ±0.3266	0.3540 ±0.0202	0.3248 ±0.0204	0.8481 ±0.0720

Table 16: The results of the Shifted Rotated Rastrigin function.

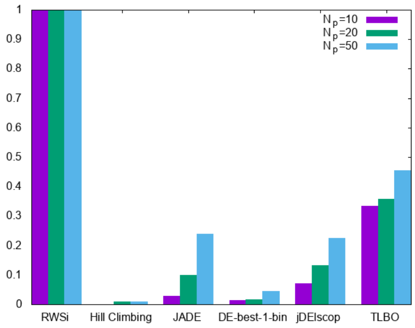


Figure 38: ExpBas for different N_p.

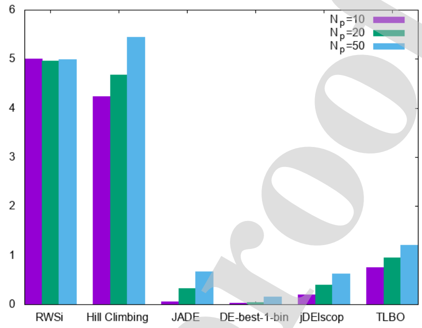


Figure 39: Diversity for different N_p.

Algorithm	ExpBas vs. ExpDist	ExpBas vs. Diversity	ExpDist vs. Diversity
HillClimbing	-1.0000	-0.2748	0.0000
JADE	-0.9363	0.7540	0.6982
DE-best-1-bin	-0.9754	0.7740	0.7570
jDElscop	-0.9776	0.8337	0.8536
TLBO	-0.9239	0.7284	0.6785

Table 17: Pearson correlation coefficients for the Shifted Rotated Rastrigin function.

4.1.9. Split Drop Wave function

The attraction basins of the Split Drop Wave 2D function (Eq. 15, Figure 40), with several optima $F_{min}(x) = -1$, which were computed from a Heat Map (Figure 41), are presented in Figure 42. This function has been included, since its attraction basins are of different, not squared shape. Due to the irregular shapes of attraction basins, our previous measure *ExpDist* is inaccurate, and up to three times bigger than *ExpBas* (Table 18) since the neighborhood has been modeled as a hypercube with the same length in all dimensions. This is not true for this problem. By comparing Figures 43 and 44 it can be noticed that *Diversity* is also not an accurate measure for exploration. Yet another example that *Diversity* is not a good measure of exploration is shown in Table 18, where exploration in JADE ($ExpBas = 0.0661$) is smaller than in jDElscop

($ExpBas = 0.0756$), yet diversity in JADE is bigger ($Diversity = 1.0649$) than in jDElscop ($Diversity = 0.7732$). This is further supported by Pearson correlation coefficient ρ (Table 19), where the correlation between $ExpBas$ and $Diversity$ is strong (except for Hill Climbing). But, the accuracy of $Diversity$ as shown before is low. Pearson correlation coefficient ρ for Hill Climbing exhibits weak correlation between $ExpBas$ and $Diversity$, and even uncorrelation between $ExpDist$ and $Diversity$.

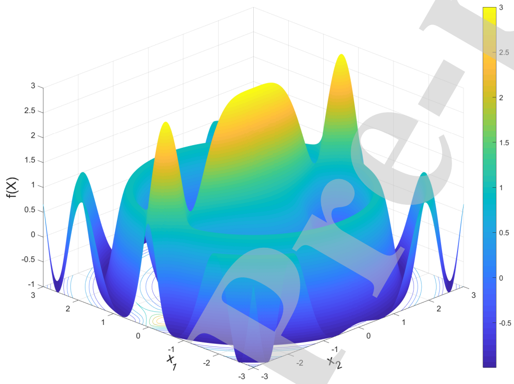


Figure 40: Split Drop Wave function.

$$f(x) = cos(x^2 + y^2) + 2 * e^{-10*y^2} \tag{15}$$
$$-3 \leq x_i \leq 3$$

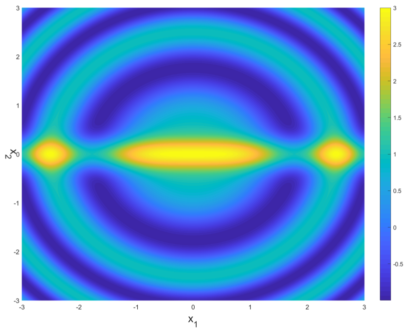


Figure 41: The Heat Map of the Split Drop Wave function.

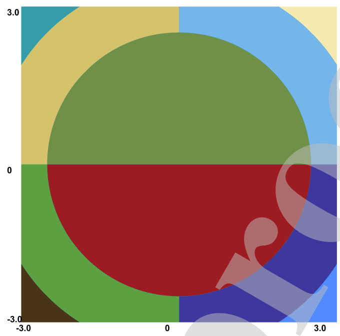


Figure 42: Attraction basins for the Split Drop Wave function.

Algorithm	Fitness	ExpBas	ExpDist	Diversity
RWSi	-1.0000 ±0.0000	1.0000 ±0.0000	1.0000 ±0.0000	2.9986 ±0.0171
Hill Climbing	-1.0000 ±0.0002	0.0042 ±0.0062	0.0020 ±0.0000	2.8797 ±0.2853
JADE	-1.0000 ±0.0000	0.0661 ±0.0519	0.2180 ±0.1296	1.0649 ±0.6027
DE-best-1-bin	-0.9997 ±0.0022	0.0060 ±0.0019	0.0129 ±0.0049	0.0507 ±0.0224
jDElscop	-1.0000 ±0.0000	0.0756 ±0.0317	0.1381 ±0.0394	0.7732 ±0.3385
TLBO	-1.0000 ±0.0000	0.2359 ±0.0921	0.5099 ±0.1089	2.3012 ±0.4499

Table 18: The results of the Split Drop Wave function.

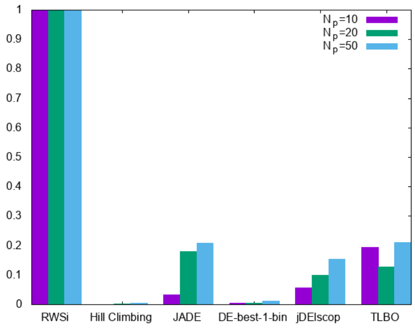


Figure 43: ExpBas for different N_p.

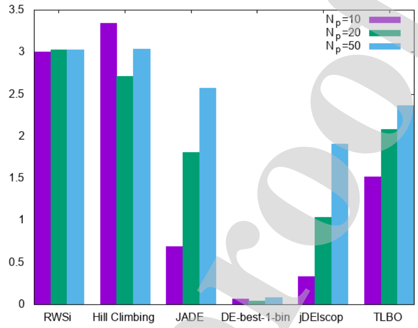


Figure 44: Diversity for different N_p.

Algorithm	ExpBas vs. ExpDist	ExpBas vs. Diversity	ExpDist vs. Diversity
Hill Climbing	-1.0000	0.3301	-0.0000
JADE	-0.8897	0.9308	0.9061
DE-best-1-bin	-0.8335	0.8361	0.6983
jDElscop	-0.5835	0.8785	0.7155
TLBO	-0.8140	0.7447	0.6028

Table 19: Pearson correlation coefficients for the Split Drop Wave function.

4.1.10. Rotated Hybrid Composition function 1

The attraction basins of the Rotated Hybrid Composition Function 1 (Eq. 16, Figure 45), with several optima $F_{min}(x) = 120$, which were computed from a Heat Map (Figure 46), are presented in Figure 47. The Rotated Hybrid Composition Function 1 is composed of several functions: Rastrigin, Sphere, Ackley, Griewank, and Weierstrass. This composition is also reflected in the attraction basins (compare Fig. 47 with Figs. 4, 24, and 29). The shapes and sizes of attraction basins are varied, and our previous exploration measure *ExpDist* can-not cope which such variety. From Table 20 it can be noticed that *ExpDist* is up to four times bigger than *ExpBas*, which is adaptive to the shape and size of attraction basins. Again, *Diversity* is not a good measure of exploration. For example, *Diversity* of Hill Climbing is almost the same as for RWSI, while exploration powers of both algorithms are not comparable. This is also shown by Pearson correlation coefficient ρ (Table 21), which exhibits weak correlation between *ExpBas* and *ExpDist* on jDElscop, and strong correlation for the other algorithms. Similarly, the correlation between *ExpBas* and *Diversity* is weak on Hill Climbing and jDElscop. The latter finding is also shown in Figures 48 and 49.

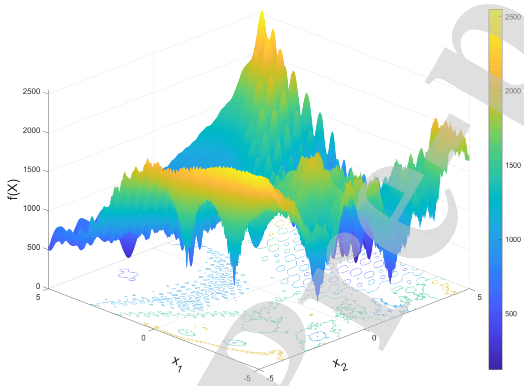


Figure 45: Rotated Hybrid Composition Function 1.

$$
\begin{gathered}
F(x) = \sum_{i=1}^{n} (w_i * (f_i((x - o_i)/\lambda_i + M_i) + bias_i)) + f_{bias} \\
F(x) : \text{new composition function} \\
f_i(x) : i^{th} \text{ basic function} \\
n : \text{number of basic functions} \\
M_i : \text{linear transformation matrix} \\
o_i : \text{shifted global optimum} \\
w_i : \text{weight value for each } f_i(x) \\
\lambda_i : \text{ used to stretch compress the function} \\
-5 \le x_i \le 5
\end{gathered}
\tag{16}
$$

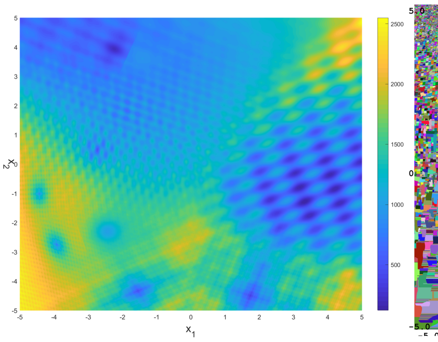


Figure 46: The Heat Map of the Rotated Hybrid Composition function 1.

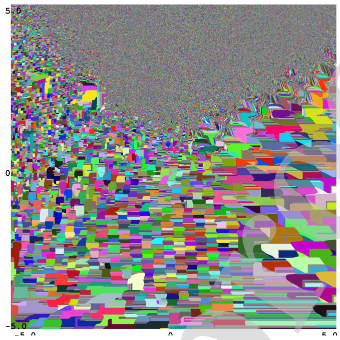


Figure 47: Attraction basins for the Rotated Hybrid Composition function 1.

Algorithm	Fitness	ExpBas	ExpDist	Diversity
RWSi	150.0959 ±20.7715	1.0000 ±0.0000	1.0000 ±0.0000	4.9993 ±0.0259
Hill Climbing	348.6120 ±143.3567	0.0132 ±0.0125	0.0020 ±0.0000	4.8197 ±0.4878
JADE	143.5226 ±62.8912	0.0217 ±0.0183	0.0738 ±0.0390	0.5335 ±0.5191
DE-best-1-bin	274.2004 ±144.4036	0.0041 ±0.0030	0.0100 ±0.0042	0.0595 ±0.0164
jDElscop	165.0834 ±55.3211	0.0195 ±0.0158	0.0854 ±0.0160	0.6343 ±0.2443
TLBO	130.7487 ±27.1311	0.1163 ±0.1259	0.4073 ±0.1072	2.0356 ±1.2025

Table 20: The results of the Rotated Hybrid Composition function 1.

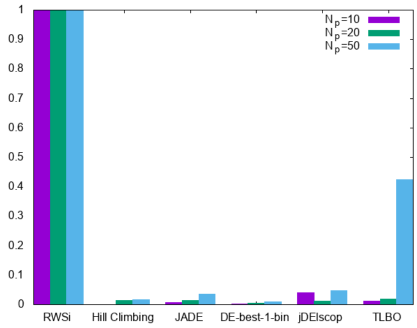


Figure 48: ExpBas for different N_p.

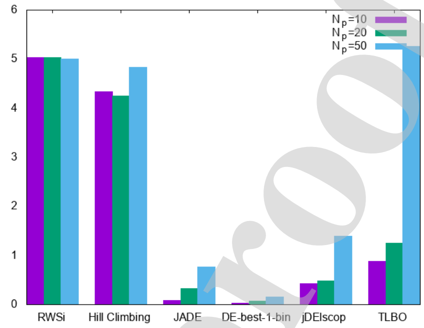


Figure 49: Diversity for different N_p.

Algorithm	ExpBas vs. ExpDist	ExpBas vs. Diversity	ExpDist vs. Diversity
HillClimbing	1.0000	0.0216	-0.0000
JADE	-0.9280	0.9569	0.9461
DE-best-1-bin	-0.6043	0.5782	0.5919
jDElscop	0.1870	-0.1582	0.1925
TLBO	-0.8568	0.8743	0.9821

Table 21: Pearson correlation coefficients for the Rotated Hybrid Composition function 1.

4.2. High-dimensional functions

Our newly proposed exploration measure (*ExpBas*) is scalable to high-dimensional problems, but computing attraction basins for high-dimensional problems becomes computationally expensive, and some approximate computation of attraction basins can be used, as suggested in Caamaño et al. (2010); Hernando et al. (2016). However, the computation of attraction basins is not needed to compute the newly proposed measure *ExpBas*. The rationale is as follows. Two search points belong to the same attraction basin if a local search starting in these two search points leads to the same attractor - local optimum. This is the case of exploitation. Two search points belong to different attraction basins if the local search leads to different attractors - local optima. This is the case of exploration. As a local search "Any Ascent" version is used in our experiment, since it is less computationally expensive than "Steepest Ascent" (Garnier & Kallel, 2002; Jones, 1995). In this Section an application of the newly proposed measure *ExpBas* is shown on the following high-dimensional ($D = 10$) functions: Rastrigin, Ackley, and Griewank. The experiment was conducted for $N_p = 20$ and $MaxFEs = 10,000$. It is interesting to investigate how increasing dimension influences exploration measures *ExpBas*, *ExpDist*, and *Diversity*. As expected, with an increase of dimensions the exploration power of algorithms was increased as well (Figures 50, 51, and 52).

For the Rastrigin function we can notice an increase of *ExpBas* for the used algorithms JADE, jDElscop, and TLBO on D=2 from 6%, 6%, and 7%, respectively (Table 2), to 38%, 52% and 61% on D=10, respectively (Table 22). An increase of exploration was the highest for TLBO (Figure 50). Since attraction basins for the Rastrigin functions are regular, our previous measure *ExpDist* is similar to the newly proposed exploration measure *ExpBas* (Table 22). However, an increase of *Diversity* from D=2 to D=10 is not the same as for exploration measure *ExpBas*. For example, the increase of *ExpBas* for JADE and TLBO was six and nine times, respectively. While the increase of *Diversity* for JADE and TLBO was three times and two times, respectively (compare Tables 2 and 22).

Algorithm	Fitness	ExpBas	ExpDist	Diversity
JADE	0.0000	0.3753	0.3591	0.8577
	±0.0000	±0.0714	±0.0662	±0.1247
jDElscop	0.0995	0.5243	0.4782	1.1300
	±0.3146	±0.0378	±0.0410	±0.2392
TLBO	2.7461	0.6114	0.4089	0.9187
	±5.5120	±0.1396	±0.1394	±0.5157

Table 22: The results of compared exploration measures on D=10 for the Rastrigin function.

For the Ackley function we can notice an increase of *ExpBas* for the used algorithms JADE, jDElscop, and TLBO on D=2 from 3% (Table 10) to 15%, 30% and 12% on D=10, respectively (Table 23). The increase of exploration was the highest for jDElscop (Figure 51). There was also an increase of our previous measure of exploration *ExpDist* when D=10 (compare Tables 10 and 23). However, the results are not accurate, since the attraction basins size and shape were anticipated wrongly as the same across the search space using *ExpDist*. *Diversity* was also increased when D=10. However, the increase was not the same as for exploration measure *ExpBas*. For example, an increase of *ExpBas* from D=2 to D=10 for jDElscop and TLBO was ten and four times, respectively, while the increase of *Diversity* for jDElscop and TLBO was four and less than two times, respectively (compare Tables 10 and 23). It is yet more evidence that *Diversity* is not an accurate measure for exploration.

Algorithm	Fitness	ExpBas	ExpDist	Diversity
JADE	0.0000	0.1519	0.1292	1.1473
	±0.0000	±0.0117	±0.0107	±0.1551
jDElscop	0.0000	0.2963	0.2459	2.3764
	±0.0000	±0.0209	±0.0171	±0.4291
TLBO	0.0000	0.1198	0.0709	0.9499
	±0.0000	±0.0067	±0.0059	±0.2104

Table 23: The results of compared exploration measures on D=10 for the Ackley function.

For the Griewank function we can notice an increase of *ExpBas* for the used algorithms JADE, jDElscop, and TLBO on D=2 from 14%, 14%, and 23%, respectively (Table 12), to 53%, 58% and 38% on D=10, respectively (Table 24). The increase of exploration was again the highest for jDElscop (Figure 52). The increase of *ExpDist* was even higher (Table 24), due to the inability to identify attraction basins correctly. Again, the increase in *Diversity* was not the same as for exploration measure *ExpBas*. For example, the increase of *ExpBas* from D=2 to D=10 for JADE and jDElscop was four times, while the increase of *Diversity* for JADE and jDElscop was less than two times and three times, respectively (compare Tables 12 and 24).

Algorithm	Fitness	ExpBas	ExpDist	Diversity
JADE	0.0161	0.5260	0.7030	22.4487
	±0.0138	±0.1094	±0.1224	±2.4474
jDElscop	0.0214	0.5772	0.5905	37.7403
	±0.0193	±0.1556	±0.1548	±5.5594
TLBO	0.0371	0.3840	0.4380	14.3854
	±0.0669	±0.1370	±0.1572	±1.9724

Table 24: The results of compared exploration measures on D=10 for the Griewank function.

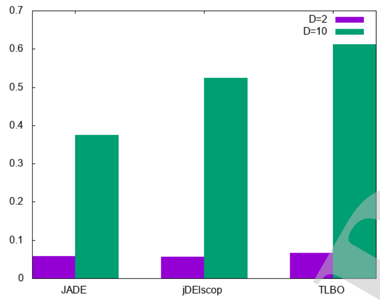


Figure 50: Graph comparing ExpBas on Rastrigin function for D=2 and D=10.

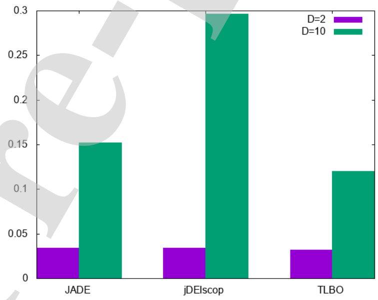


Figure 51: Graph comparing ExpBas on Ackley function for D=2 and D=10.

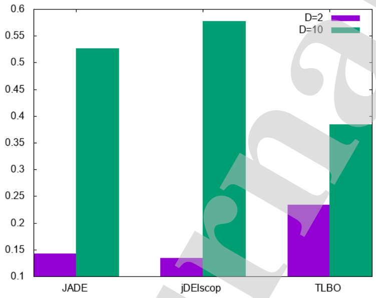


Figure 52: Graph comparing ExpBas on Griewank function for D=2 and D=10.

5. Discussion

With more accurate measures of exploration and exploitation we hope that further progress in the research field of Metaheuristics would be possible. We anticipate the following directions of research:

- It would be possible to distinguish metaheuristics based on how they perform exploration and exploitation. With more precise and direct measures of exploration and exploitation, this would finally be possible. The novelty of the proposed metaheuristics would be easier to identify.
- The shortcomings of particular metaheuristics with regard to exploration and exploitation would be easier to identify and to remedy.
- Given a specific problem by computing attraction basins it would be easier to identify which metaheuristics will work better on this problem.
- The effect of control parameters (including population size) on exploration and exploitation can be quantified better. Furthermore, various approaches were used for better balancing exploration and exploitation (Wu et al., 2016; Paldrak et al., 2016; Cui et al., 2016; Omidvar et al., 2016; Li et al., 2017; Du & Chen, 2019; Zhang et al., 2021). If they are indeed effective, they can be shown with our newly proposed direct measure.
- Investigating how specific approaches, such as long-term memory assistance (Črepinšek et al., 2019a) and neighborhood-based approaches (Weber et al., 2009; Mendes et al., 2004; Epitropakis et al., 2011; Gong & Cai, 2013), can be benefited by attraction basins.

With the proposed new direct measure of exploration and exploitation ($ExpBas$) we anticipate that the novelty of the newly proposed metaheuristics can be easier to identify, solving the problem of different metaphors as described vividly in Tilahun & Tawhid (2019), who wrote: "*However, there is a heavy criticism that some of these algorithms lack novelty. In fact, some of these algorithms are the same in terms of the updating operators, but with different mimicking scenarios and names. The performance of a metaheuristic algorithm depends on how it balances the degree of the two basic search mechanisms, namely, intensification* *and diversification. Hence, introducing novel algorithms that contribute to a new way of search mechanism is welcome, but not for a mere repetition of the same algorithm with the same or perturbed operators but different metaphor.*"

Many authors tried to propose new genetic operators that influence exploration and exploitation. For example, the authors in Ji et al. (2017) identified an imbalance between exploitation and exploration in the standard Teaching-Learning-Based Optimization (TLBO) algorithm (Venkata Rao et al., 2012). To improve its balance they proposed a new variant called I-TLBO: "*The design of a self-feedback learning phase seeks the maintaining of good exploitation ability, while the introduction of the mutation and crossover phase aims at the improvement of exploration ability in the original TLBO. The I-TLBO algorithm has significant advantages due to its ability to balance between exploitation and exploration.*" With the proposed new direct exploration and exploitation measure, the authors can verify more easily if their newly proposed genetic operators indeed improve the balance between exploration and exploitation.

Based on the shape and number of attraction basins, we anticipate it would be possible to identify a family of metaheuristics that perform well on such shape of attraction basins. To achieve this, the state-of-the-art metaheuristics must be studied deeply with respect to the proposed exploration and exploitation measures on a diverse set of benchmarks (artificially constructed (Das & Suganthan, 2010; Liang et al., 2013; Wu et al., 2017), as well as real world problems (Jesenik et al., 2018, 2020; Črepinšek et al., 2019b; Rathee & Chhabra, 2019; Kovačević et al., 2020; Panić et al., 2020)).

Until now, achieving a balance between exploration and exploitation has been controlled implicitly by proper parameter settings, off-the-fly or on-the-fly. With the newly proposed direct measure of exploration and exploitation (*ExpBas*), the effect of different settings of parameters can be quantified better. Similarly, approaches for balancing exploration and exploitation, such as learning strategies (Cui et al., 2016; Li et al., 2017; Du & Chen, 2019), ensemble mechanisms (Paldrak et al., 2016; Zhang et al., 2021), and cooperative methods (Wu et al., 2016; Omidvar et al., 2016) can be verified.

Many neighborhood-based variants of EAs have been proposed in the past where population topology was constructed in different manners. For example, it can be based on the order of indices in the population (Mendes et al., 2004), based on the Euclidean distance between solutions (Epitropakis et al., 2011), based on fitness values (Gong & Cai, 2013), and based on the explorative/exploitative roles (Weber et al., 2009), to name a few. We anticipate that attraction basins might be even better criteria to form a population topology.

6. Conclusion

Although exploration and exploitation have been well recognised by their influence on search processes in the Metaheuristic research community, surprisingly there have not been direct measures commonly accepted among peer scholars. This paper, as a sequel of our previous work, introduces a direct measure of exploration and exploitation that is based on attraction basins, where exploitation is identified when an offspring remains in the same basin as its parents, while exploration means an offspring and its parent fall into two different basins. By applying the attraction basin concepts, this paper introduces a dynamic boundary of exploration and exploitation based on problem-dependent landscapes. Several benchmark functions, both unimodal and multimodal, were selected to measure exploration and exploration using diversity ($Diversity$), our previous measure ($ExpDist$), and the newly introduced one ($ExpBas$). From the experimental results the advantages of $ExpBas$ become obvious: (1) Diversity has been proved as an unreliable measure for exploration. Specifically, in most of the experiments, at least some algorithm has lower exploration power, but higher diversity than another algorithm. For example, Hill Climbing is known for its premature convergence to a local optimum. Whilst the $ExpBas$ values are small, the diversity values of *all* the selected benchmark functions using Hill Climbing algorithm are almost as large as the diversity values of those functions using the RWSi algorithm. By observing the Pearson correlation coefficient results for *all* the selected algorithms, we proved that *Diversity* and *ExpBas* have a diverse correlation range, from uncorrelated (e.g., the Easom and Sphere functions) to strongly correlated (e.g., for the Split Drop Wave function). A less than strong correlation between *Diversity* and *ExpDist* can also be found in some experiments (e.g., DE-best-1-bin for the Ackley, Griewank, and Holder Table functions, as well as JADE for the Griewank function). Hence, we should not use diversity as an accurate measure for exploration. (2) *ExpDist* used a fixed threshold to delimit exploration from exploitation, which is considered as a hyper-cube with a fixed length in all dimensions. It is rigid, problem dependent, and, hence, less reliable, which can be proved by observing the experimental results of the Schwefel, Easom, and Sphere functions: *ExpDist* exaggerates exploration based on the predetermined threshold and its problem landscape. The "exaggeration rate" could be three times (e.g., Easom) or up to a hundred times more (e.g., Sphere). For the Easom function, correlations between *ExpDist* and *ExpBas* also range from uncorrelated to strongly correlated for different algorithms. All of these prove *ExpDist* less convincing. (3) Most importantly, by defining the boundary of each attraction basin adaptively based on a problem's landscape, *ExpBas* provides a more accurate exploration measure. This can be proved by observing the experimental results of benchmark functions with irregular basins. For example, for the Schwefel function, *ExpDist* value is 0.1881 for Hill Climbing and is 0.0965 for JADE. This can be interpreted as Hill Climbing has more exploration than JADE. However, it does not reflect the real behaviors of Hill Climbing and JADE well-known by peer scholars. Conversely, the *ExpBas* value is 0.0028 for Hill Climbing and is 0.0255 for JADE. Apparently, *ExpBas*'s values reflect the common beliefs. Even for the functions with regular basins, the same observations can be found. For the Sphere function, *ExpBas* shows the exact same value among *all* the benchmark functions, to represent that the exploration only occurred during initialization. Unfortunately, *ExpDist* values are varied, and do not reflect to the definition of exploration. In summary, our intensive experiments have proved that *ExpBas* is a more accurate exploration measure that could address the problems found in *Diversity* and *ExpDist* measures.

In our future work we will address the research problems discussed in Section 5, as well as we intend to measure "*failed exploration*", "*deceptive exploration*", "*successful exploration*", and "*successful rejection*" (Chen et al., 2019), due to their importance to understand fully how different metaheuristics perform exploration and exploitation of the search space.

Acknowledgements

The authors acknowledge the financial support from the Slovenian Research Agency (Research Core Funding No. P2-0041).

References

Albunian, N., Fraser, G., & Sudholt, D. (2020). Measuring and maintaining population diversity in search-based unit test generation. In A. Aleti, & A. Panichella (Eds.), *Search-Based Software Engineering* (pp. 153–168). Cham: Springer International Publishing.

Allawi, Z. T., Ibraheem, I. K., & Humaidi, A. J. (2019). Fine-tuning metaheuristic algorithm for global optimization. *Processes*, *7*.

Angus, D., & Fister, I. (2020). Tracking the exploration and exploitation in stochastic population-based nature-inspired algorithms using recurrence plots. In A. Zamuda, S. Das, P. N. Suganthan, & B. K. Panigrahi (Eds.), *Swarm, Evolutionary, and Memetic Computing and Fuzzy and Neural Computing* (pp. 166–176). Cham: Springer International Publishing.

Brest, J., & Maučec, M. S. (2011). Self-adaptive differential evolution algorithm using population size reduction and three strategies. *Soft Computing*, *15*, 2157–2174.

Caamaño, P., Prieto, A., Becerra, J. A., Bellas, F., & Duro, R. J. (2010). Real-valued multimodal fitness landscape characterization for evolution. In

K. W. Wong, B. S. U. Mendis, & A. Bouzerdoum (Eds.), *Neural Information Processing. Theory and Algorithms* (pp. 567–574). Berlin, Heidelberg: Springer Berlin Heidelberg.

Caraffini, F., Neri, F., & Epitropakis, M. (2019). Hyperspam: A study on hyperheuristic coordination strategies in the continuous domain. *Information Sciences*, *477*, 186 – 202.

Chen, S., Bolufé-Röhler, A., Montgomery, J., & Hendtlass, T. (2019). An analysis on the effect of selection on exploration in particle swarm optimization and differential evolution. In *2019 IEEE Congress on Evolutionary Computation (CEC)* (pp. 3037–3044).

Chen, Y., & He, J. (2020). Exploitation and exploration analysis of elitist evolutionary algorithms: A case study. `arXiv:2001.10932`.

Cui, L., Li, G., Lin, Q., Chen, J., & Lu, N. (2016). Adaptive differential evolution algorithm with novel mutation strategies in multiple sub-populations. *Computers & Operations Research*, *67*, 155–173.

Das, S., & Suganthan, P. N. (2010). Problem definitions and evaluation criteria for cec 2011 competition on testing evolutionary algorithms on real world optimization problems. *Jadavpur University, Nanyang Technological University, Kolkata*, (pp. 341–359).

De Lorenzo, A., Medvet, E., Tušar, T., & Bartoli, A. (2019). An analysis of dimensionality reduction techniques for visualizing evolution. In *Proceedings of the Genetic and Evolutionary Computation Conference Companion* GECCO '19 (pp. 1864–1872). New York, NY, USA: ACM.

Du, Z., & Chen, K. (2019). Enhanced artificial bee colony with novel search strategy and dynamic parameter. *Computer Science and Information Systems*, *16*, 939 – 957.

Eiben, A. E., & Schippers, C. A. (1998). On evolutionary exploration and exploitation. *Fundam. Inf.*, *35*, 35–50.

Einakian, S., & Newman, T. S. (2019). An examination of color theories in map-based information visualization. *Journal of Computer Languages*, *51*, 143 – 153.

Elreedy, D., F. Atiya, A., & I. Shaheen, S. (2019). A novel active learning regression framework for balancing the exploration-exploitation trade-off. *Entropy*, *21*.

Epitropakis, M. G., Tasoulis, D. K., Pavlidis, N. G., Plagianakos, V. P., & Vrahatis, M. N. (2011). Enhancing differential evolution utilizing proximity-based mutation operators. *IEEE Transactions on Evolutionary Computation*, *15*, 99–119.

Gabor, T., & Belzner, L. (2017). Genealogical distance as a diversity estimate in evolutionary algorithms. In *Proceedings of the Genetic and Evolutionary Computation Conference Companion* GECCO '17 (pp. 1572–1577). New York, NY, USA: ACM.

Garnier, J., & Kallel, L. (2002). Efficiency of local search with multiple local optima. *SIAM J. Discret. Math.*, *15*, 122–141.

Gong, W., & Cai, Z. (2013). Differential evolution with ranking-based mutation operators. *IEEE Transactions on Cybernetics*, *43*, 2066–2081.

Gonzalez-Fernandez, Y., & Chen, S. (2015). Leaders and followers — a new metaheuristic to avoid the bias of accumulated information. In *2015 IEEE Congress on Evolutionary Computation (CEC)* (pp. 776–783).

Gupta, S., Deep, K., Mirjalili, S., & Kim, J. H. (2020). A modified sine cosine algorithm with novel transition parameter and mutation operator for global optimization. *Expert Systems with Applications*, (p. 113395).

Hernando, L., Mendiburu, A., & Lozano, J. A. (2016). Estimating attraction basin sizes. In O. Luaces, J. A. Gámez, E. Barrenechea, A. Troncoso, M. Galar, H. Quintián, & E. Corchado (Eds.), *Advances in Artificial Intelligence* (pp. 458–467). Cham: Springer International Publishing.

Hussain, K., Salleh, M. N. M., Cheng, S., & Shi, Y. (2018). On the exploration and exploitation in popular swarm-based metaheuristic. *Neural Computing and Applications*, *31*, 7665—7683.

Ishibuchi, H., Yoshida, T., & Murata, T. (2003). Balance between genetic search and local search in memetic algorithms for multiobjective permutation flowshop scheduling. *IEEE Transactions on Evolutionary Computation*, *7*, 204–223. Cited By 608.

Jesenik, M., Mernik, M., Črepinšek, M., Ravber, M., & Trlep, M. (2018). Searching for soil models' parameters using metaheuristics. *Applied Soft Computing*, *69*, 131–148.

Jesenik, M., Mernik, M., & Trlep, M. (2020). Determination of a hysteresis model parameters with the use of different evolutionary methods for an innovative hysteresis model. *Mathematics*, *8*.

Ji, X., Ye, H., Zhou, J., Yin, Y., & Shen, X. (2017). An improved teaching-learning-based optimization algorithm and its application to a combinatorial optimization problem in foundry industry. *Applied Soft Computing*, *57*, 504 – 516.

Jones, T. (1995). *Evolutionary Algorithms, Fitness Landscapes and Search*. Doctoral dissertation.

Kovačević, Ž., Mernik, M., Ravber, M., & Črepinšek, M. (2020). From grammar inference to semantic inference—an evolutionary approach. *Mathematics*, *8*.

Li, X., Ma, S., & Hu, J. (2017). Multi-search differential evolution algorithm. *Applied Intelligence*, *47*, 231—-256.

Liang, J. J., Qu, B. Y., & Suganthan, P. N. (2013). Problem definitions and evaluation criteria for the cec 2014 special session and competition on single objective real-parameter numerical optimization. *Computational Intelligence Laboratory, Zhengzhou University, Zhengzhou China and Technical Report, Nanyang Technological University, Singapore*, *635*.

Liu, S.-H., Mernik, M., & Bryant, B. R. (2009). To explore or to exploit: An entropy-driven approach for evolutionary algorithms. *Int. J. Know.-Based Intell. Eng. Syst.*, *13*, 185–206.

Liu, S.-H., Mernik, M., Hrnčič, D., & Črepinšek, M. (2013). A parameter control method of evolutionary algorithms using exploration and exploitation measures with a practical application for fitting sovova's mass transfer model. *Applied Soft Computing*, *13*, 3792 – 3805.

Locatelli, M., & Wood, G. (2005). Objective function features providing barriers to rapid global optimization. *Journal of Global Optimization*, *31*, 549 – 565.

Luan, F., Cai, Z., Wu, S., Liu, S., & He, Y. (2019). Optimizing the low-carbon flexible job shop scheduling problem with discrete whale optimization algorithm. *Mathematics*, *7*.

Mendes, R., Kennedy, J., & Neves, J. (2004). The fully informed particle swarm: simpler, maybe better. *IEEE Transactions on Evolutionary Computation*, *8*, 204–210.

Morales-Castañeda, B., Zaldívar, D., Cuevas, E., Fausto, F., & Rodríguez, A. (2020). A better balance in metaheuristic algorithms: Does it exist? *Swarm and Evolutionary Computation*, *54*, 100671.

Neri, F., Iacca, G., & Mininno, E. (2011). Disturbed exploitation compact differential evolution for limited memory optimization problems. *Information Sciences*, *181*, 2469 – 2487.

Ollion, C., & Doncieux, S. (2011). Why and how to measure exploration in behavioral space. In *Proceedings of the 13th Annual Conference on Genetic and Evolutionary Computation* GECCO '11 (pp. 267–274). New York, NY, USA: ACM.

Omidvar, M. N., Kazimipour, B., Li, X., & Yao, X. (2016). Cbcc3 — a contribution-based cooperative co-evolutionary algorithm with improved exploration/exploitation balance. In *2016 IEEE Congress on Evolutionary Computation (CEC)* (pp. 3541–3548).

Paldrak, M., Tasgetiren, M. F., Suganthan, P. N., & Pan, Q. (2016). An ensemble of differential evolution algorithms with variable neighborhood search for constrained function optimization. In *2016 IEEE Congress on Evolutionary Computation (CEC)* (pp. 2610–2617).

Panić, B., Klemenc, J., & Nagode, M. (2020). Optimizing the estimation of a histogram-bin width—application to the multivariate mixture-model estimation. *Mathematics*, *8*.

Piotrowski, A. P. (2017). Review of differential evolution population size. *Swarm and Evolutionary Computation*, *32*, 1 – 24.

Pitzer, E., Affenzeller, M., & Beham, A. (2010). A closer look down the basins of attraction. In *2010 UK Workshop on Computational Intelligence (UKCI)* (pp. 1–6).

Rathee, A., & Chhabra, J. K. (2019). A multi-objective search based approach to identify reusable software components. *Journal of Computer Languages*, *52*, 26 – 43.

Russell, S., & Norvig, P. (2002). Artificial intelligence: a modern approach, .

Salehinejad, H., Rahnamayan, S., Tizhoosh, H. R., & Chen, S. Y. (2014). Micro-differential evolution with vectorized random mutation factor. In *2014 IEEE Congress on Evolutionary Computation (CEC)* (pp. 2055–2062).

Salgotra, R., Singh, U., & Saha, S. (2018). New cuckoo search algorithms with enhanced exploration and exploitation properties. *Expert Systems with Applications*, *95*, 384 – 420.

Spitzer, F. (2013). *Principles of random walk* volume 34. Springer Science & Business Media.

Squillero, G., & Tonda, A. (2016). Divergence of character and premature convergence: A survey of methodologies for promoting diversity in evolutionary optimization. *Information Sciences*, *329*, 782 – 799. Special issue on Discovery Science.

Storn, R., & Price, K. (1997). Differential evolution–a simple and efficient heuristic for global optimization over continuous spaces. *Journal of global optimization*, *11*, 341–359.

Tilahun, S. L. (2017). Prey predator hyperheuristic. *Applied Soft Computing*, *59*, 104 – 114.

Tilahun, S. L., & Tawhid, M. A. (2019). Swarm hyperheuristic framework. *Journal of Heuristics*, *25*, 809–836.

Ursem, R. K. (2002). Diversity-guided evolutionary algorithms. In J. J. M. Guervós, P. Adamidis, H.-G. Beyer, H.-P. Schwefel, & J.-L. Fernández-Villacañas (Eds.), *Parallel Problem Solving from Nature — PPSN VII* (pp. 462–471). Berlin, Heidelberg: Springer Berlin Heidelberg.

Čepinšek, M., Liu, S.-H., & Mernik, L. (2012). A note on teaching-learning-based optimization algorithm. *Inf. Sci.*, *212*, 79–93.

Črepinšek, M., Liu, S.-H., Mernik, L., & Mernik, M. (2016). Is a comparison of results meaningful from the inexact replications of computational experiments? *Soft Computing*, *20*, 223–235.

Črepinšek, M., Liu, S.-H., & Mernik, M. (2013). Exploration and exploitation in evolutionary algorithms: A survey. *ACM Computing Surveys*, *45*, 35:1–35:33.

Črepinšek, M., Liu, S.-H., Mernik, M., & Ravber, M. (2019a). Long term memory assistance for evolutionary algorithms. *Mathematics*, *7*.

Črepinšek, M., Mernik, M., & Shih-Hsi, L. (2011). Analysis of exploration and exploitation in evolutionary algorithms by ancestry trees. *International Journal of Innovative Computing and Applications*, *3*, 11–19.

Črepinšek, M., Ravber, M., Mernik, M., & Kosar, T. (2019b). Tuning multi-objective evolutionary algorithms on different sized problem sets. *Mathematics*, *7*.

Venkata Rao, R., Savsani, V., & P. Vakharia, D. (2012). Teaching–learning-based optimization: An optimization method for continuous non-linear large scale problems. *Information Sciences*, *183*, 1–15.

Wang, H., Jin, Y., & Yao, X. (2017). Diversity assessment in many-objective optimization. *IEEE Transactions on Cybernetics*, *47*, 1510–1522.

Weber, M., Neri, F., & Tirronen, V. (2009). Distributed differential evolution with explorative–exploitative population families. *Genetic Programming and Evolvable Machines volume*, *10*.

Wei, J., Chen, Y., Yu, Y., & Chen, Y. (2019). Optimal randomness in swarm-based search. *Mathematics*, *7*.

Wu, G., Mallipeddi, R., Suganthan, P., Wang, R., & Chen, H. (2016). Differential evolution with multi-population based ensemble of mutation strategies. *Information Sciences*, *329*, 329–345. Special issue on Discovery Science.

Wu, G., Mallipeddi, R., & Suganthan, P. N. (2017). Problem definitions and evaluation criteria for the cec 2017 competition on constrained real-parameter optimization. *National University of Defense Technology, Changsha, Hunan, PR China and Kyungpook National University, Daegu, South Korea and Nanyang Technological University, Singapore, Technical Report*, .

Xu, J., & Zhang, J. (2014). Exploration-exploitation tradeoffs in metaheuristics: Survey and analysis. In *Proceedings of the 33rd Chinese Control Conference* (pp. 8633–8638).

Zhang, H., Sun, J., Liu, T., Zhang, K., & Zhang, Q. (2019). Balancing exploration and exploitation in multiobjective evolutionary optimization. *Information Sciences*, *497*, 129 – 148.

Zhang, H., Wang, Z., Chen, W., Heidari, A. A., Wang, M., Zhao, X., Liang, G., Chen, H., & Zhang, X. (2021). Ensemble mutation-driven salp swarm algorithm with restart mechanism: Framework and fundamental analysis. *Expert Systems with Applications*, *165*, 113897.

Zhang, J., & Sanderson, A. C. (2009). JADE: Adaptive differential evolution with optional external archive. *IEEE Transactions on Evolutionary Computation*, *13*, 945–958.

Highlights

- A novel direct measure of exploration and exploitation based on attraction basins.
- Clear identification of exploration and exploitation of the search process.
- Comparison to the common indirect diversity measure and our previous measure.
- Our proposed method obtained more accurate results than compared measures.

CRediT author statement

Jernej Jerebic: Methodology, Software, Investigation, Writing - Original Draft, Writing - Review & Editing, Visualization. **Marjan Mernik:** Conceptualization, Methodology, Validation, Writing – Original Draft, Writing - Review & Editing, Supervision. **Shih-Hsi Liu:** Methodology, Validation, Writing - Original Draft, Writing - Review & Editing, Supervision. **Miha Ravber:** Methodology, Software, Validation, Investigation, Writing - Original Draft, Writing - Review & Editing. **Mihael Baketarić:** Methodology, Software, Validation, Investigation, Writing - Original Draft, Writing - Review & Editing, Visualization. **Luka Mernik:** Validation, Formal Analysis, Writing - Original Draft, Writing - Review & Editing. **Matej Črepinšek:** Conceptualization, Methodology, Validation, Writing - Original Draft, Writing - Review & Editing, Supervision.

Declaration of interests

☒ The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

☐The authors declare the following financial interests/personal relationships which may be considered as potential competing interests: