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A transformation for the mixed general routing problem with turn penalties

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In this paper, we study a generalization of the Mixed General Routing Problem (MGRP) with turn penalties and forbidden turns. Thus, we present a unified model of this kind of extended versions for both node- and arc-routing problems with a single vehicle. We provide a polynomial transformation of this generalization into an asymmetric travelling salesman problem, which can be considered a particular case of the MGRP. We show computational results on the exact resolution on a set of 128 instances of the new problem using a recently developed code for the MGRP.

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Introduction

The overwhelming majority of academic arc-routing models assume that all turns are allowed and that they are not time(cost)-consuming. Nevertheless, in some real-world problems, especially for routes inside cities, in particular by truck, some turns may be considered more time-consuming and/or dangerous than others. Furthermore, at least in big cities, many U-turns and some left turns are forbidden. Therefore, for these kind of problems the solution given by an arc-routing problem can be non-feasible if the traffic signs must be respected.

Early papers that considered turn penalties in closed walks on graphs took into account only some kinds of turn penalties in order to solve heuristically certain real problems, see for example Bodin *et al* (1989) and Roy and Rousseau (1989). But in the last years, several well-known arc-routing problems have been generalized to include turn penalties and forbidden turns in the solution cost. For example, Benavent and Soler (1999) and Corberán *et al* (2002) generalized the Directed Rural Postman Problem and the Mixed Rural Postman Problem (MRPP), respectively (to be brief, the definitions of the single-vehicle routing problems cited here are given in Figure 1), providing theoretical results about complexity and resolution as well as computational results. Clossey *et al* (2001) presented good behaviour heuristics to solve extended versions of the Chinese Postman Problem and the Directed Chinese Postman Problem. Arkin *et al* (2006) presented an algorithmic study of some covering tour problems minimizing the number or cost of turns. With respect to mutivehicle problems, extended versions of the Capacitated Arc-Routing Problem (CARP) with turn penalties have been studied; see for example Bautista and Pereira (2004) and Lacomme *et al* (2004, 2005). The CARP is an arc-routing problem, in which, basically, a fleet of vehicles based on a specified vertex (the depot) and with a known capacity must service a subset of the links of a graph, with minimum total cost and such that the load assigned to each vehicle does not exceed its capacity and each link is serviced by exactly one vehicle.

But as far as we know, no node-routing problem has ever considered turn penalties, although traffic restrictions on U-turns or left turns inside a big city, or the fact that some turns can be more time-consuming than others do not depend on demand locations (arcs or nodes) inside that city.

On the other hand, the Mixed General Routing Problem (MGRP) contains a large number of important arc- and node-routing problems, like, for example the routing problems with a single vehicle mentioned above (see Figure 1). Its particular case (to our aim) in which G is a directed graph, the subset of required arcs is empty and all the vertices are required, corresponds to the Graphical Asymmetric Travelling Sales- man Problem (GATSP) (Chopra and Rinaldi, 1996), a generalization of the classical Asymmetric Travelling Salesman Problem (ATSP) in which graph G does not need to be complete (few variables are needed), and then the solution does not need to be a Hamiltonian cycle but a closed walk passing through each vertex at least once. Note that an ATSP instance can be transformed into a GATSP instance by simply adding a large positive number to each arc cost in order to assure the occurrence of a Hamiltonian cycle in the optimal solution, see Corberán *et al* (2003, 2005) for recent papers on the MGRP.

In this paper, we present a generalization of the MGRP that considers turn penalties and forbidden turns: a tour solution

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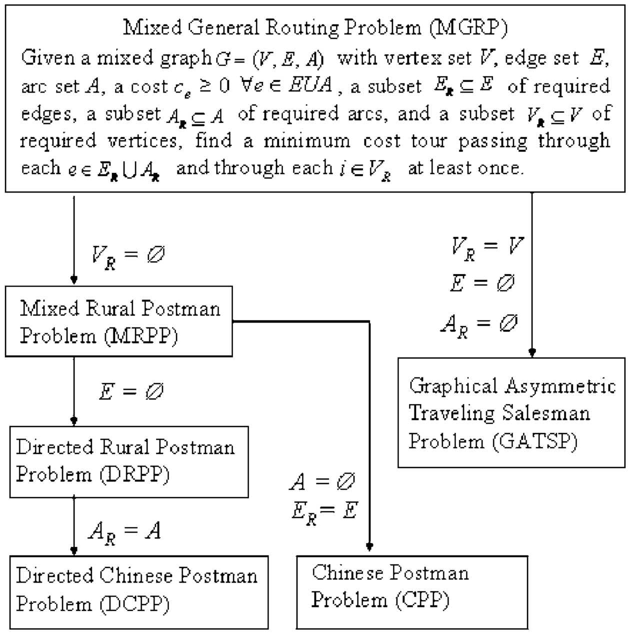

Figure 1 The MGRP and some particular cases.

must traverse all the required service vertices and links of the graph while not making forbidden turns. Its total cost will be the sum of the costs of the traversed arcs and edges together with the penalties associated with the turns made. Following precedent works, the problem consisting of finding such a tour with a total minimum cost will be called the Mixed General Routing Problem with Turn Penalties (MGRPTP).

In this way we present a unified model of turn penalty-based extended version for both node- and arc-routing problems with a single vehicle. Through several steps, we polynomially transform the MGRPTP into an ATSP, for which several heuristic and exact procedures have been successfully tested, see for example Carpaneto *et al* (1995), Fischetti and Toth (1997), Blais and Laporte (2003) and Kwon *et al* (2005). As the ATSP can be considered a particular case of the MGRP, we also show computational results on the exact resolution of the new problem using a recently developed code for the MGRP (Corberán *et al*, 2005), which seems to perform well even for large-size ATSP instances with several thousands of vertices (Albiach *et al*, 2006). Thus, we solve the MGRPTP, a generalization of the MGRP, by transforming it into a particular case of the MGRP.

The rest of the paper is organized as follows. In the next section, we introduce some definitions, notations and auxiliary problems in order to formally define and solve the MGRPTP. In the section MGRPTP transformation, through two intermediate transformations, we prove that the MGRPTP can be transformed in polynomial time into an ATSP on an auxiliary digraph. In the penultimate section, we present computational results on the exact resolution on a set of 128 MGRPTP instances obtained from MRPP with turn penalties instances given in Corberán *et al* (2002) by converting 'isolated' vertices into required ones. The instances have up to 200 vertices, 440 arcs, 176 required arcs, 26 required vertices and 40 (required) edges. In the last section, we present some conclusions about this work.

Definitions, notations and auxiliary problems

To our aim, let us remember the definition of an existing node-routing problem called the Asymmetric Generalized Travelling Salesman Problem (AGTSP):

> *Given a directed graph $G = (V, A)$ with non-negative costs associated with its arcs, such that V is partitioned into k non-empty subsets $\{S_i\}_{i=1}^{k}$, find a minimum cost circuit passing through exactly one vertex of each subset S_i $\forall i \in \{1, \ldots, k\}$.*

Note that the particular case of the AGTSP in which each subset contains a single vertex is the ATSP.

To solve the AGTSP, we have in the literature several polynomial time transformations of this problem into an ATSP; the most efficient seems to be the transformation given in Noon and Bean (1993), which we will use in our process.

On the other hand, given a mixed graph $G = (V, E, A)$, each pair of links $a=(u, v), b=(v, w) \in E \cup A$ has associated a turn at v, which is the turn made going from a to b, denoted as $[ab]$. Moreover, if $a, b \in E$, this pair has another associated turn at v, the one made going from b to a, denoted as $[ba]$. Each edge e incident with v has an associated U-turn at v that, if necessary, will be denoted by $[eve]$. Each link $a \in E \cup A$ has an associated cost $c_a \geqslant 0$, and each turn $[ab]$ in G has an associated penalty $p_{[ab]} \geqslant 0$ ($p_{[ab]} = +\infty$ iff $[ab]$ is a forbidden turn).

Given $a = (u, v), b = (s, t) \in E \cup A$, a $v - s$ *feasible chain from* a to b is an alternating sequence of links and turns $C = \{a_1, [a_1a_2], a_2, \ldots, [a_{r-1}a_r], a_r, [a_rb]\}$, where $a_1 = a$, satisfying that $\{a_1, \ldots, a_r, b\}$ is a chain in G, turn $[a_rb]$ is allowed in G and, if $r > 1$, $[a_ia_{i+1}]$ $i = 1, \ldots, r-1$ are allowed turns in G. For convenience, a feasible chain begins at a link and ends at a turn and if all links involving C are edges, this feasible chain must be traversed only in the specified direction. The cost of a feasible chain C is defined as $c(C) = \sum_{i=1}^{r} (c_{a_i} + p_{[a_ia_{i+1}]})$, where $a_{r+1} = b$.

Given $a=(u, v), b=(s, t) \in E \cup A$, a *shortest $v-s$ feasible chain from a to b* is a $v - s$ feasible chain C from a to b of minimum cost $c(C)$. This feasible chain will be denoted by $s.f.c.(v^a, s^b)$. A $v - s$ feasible chain from a to b is *closed* if $a = b$ and $s \neq v$ $(b = (s, v))$.

The *Mixed Rural Postman Problem with Turn Penalties* (MRPPTP) was defined in Corberán *et al* (2002) as follows:

> Let $G = (V, E, A)$ *be a mixed graph with associated link costs and turn penalties as defined above. Given a non-empty subset $R \subseteq E \cup A$, find a minimum cost feasible closed chain in G containing each link of R at least once.*

To our aim, from the MRPPTP we define an auxiliary problem that we call the *Generalized Mixed Rural Postman Problem with Turn Penalties* (GMRPPTP) as follows:

Let $G=(V, E, A)$ be a mixed graph with associated link costs and turn penalties as defined above. Given subsets $E_R \subseteq E$ and $A_R \subseteq A$, where A_R is partitioned into k non-empty subsets S_i, $i=1, \ldots, k$. Find *a minimum cost feasible closed chain in G containing each link of E_R* and an arc of each *S_i $i=1, \ldots, k$ at least once.*

This problem has been called GMRPPTP due to its similarity with the AGTSP with respect to the ATSP, because in the particular case of the GMRPPTP in which each S_i has a single element we have an MRPPTP.

The formal definition of the problem object of this paper, that is, the *Mixed General Routing Problem with Turn Penalties* (MGRPTP) is

Let $G=(V, E, A)$ be a mixed graph with associated link costs and turn penalties as defined above. Given subsets $E_R \subseteq E$, $A_R \subseteq A$, and $V_R \subseteq V$ such that $E_R \neq \emptyset$ or $A_R \neq \emptyset$ or $V_R \neq \emptyset$, *find a minimum cost feasible closed chain in G containing each link of $E_R \cup A_R$ and each vertex of V_R at least once.*

The links in $E_R \cup A_R$ are called *required links*, the vertices in V_R are called *required vertices* and a feasible closed chain containing each required link and each required vertex at least once is called an MGRPTP *tour*.

From now on and as in similar papers, each non-required edge will be replaced by two arcs of the same cost and opposite direction; then we assume $E_R=E$. We also assume that a required vertex is not incident with a required link, otherwise, traversing the required link involves crossing the required vertex, and then the restriction 'required' is redundant for that vertex. For simplicity, we will not write the middle turns of a feasible chain.

Note that when $V_R=\emptyset$ the MGRPTP becomes an MRPPTP, which is an NP-hard problem (Corberán *et al*, 2002), then the MGRPTP is also NP-hard.

MGRPTP transformation

In this section, we transform the MGRPTP into an AGTSP and then into an ATSP or into a particular case of the MGRP, so that it can be solved with several codes, some of which have been already mentioned in the first section.

Let G be a mixed graph where an MGRPTP is defined, $E \cup A_R$ being the required link set, and V_R the required vertex set. From G we construct an auxiliary graph G' as follows:

- Initially $G'=G$
- Divide subset V_R into two subsets, V_{R_1} and V_{R_2}, where V_{R_1} comprises only the vertices containing all allowed zero-penalty turns, and V_{R_2} comprises the vertices containing forbidden or positive-penalty turns.

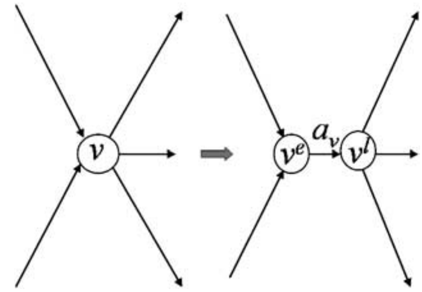

Figure 2 Transformation of a vertex $v \in V_{R_1}$ in G'.

(i) If $v \in V_{R_1}$, replace vertex v in G' by two vertices v^{e} and v^l, so that v^{e} has only entering arcs (the arcs entering at v) and v^l has only leaving arcs (the arcs leaving from v). Add a required arc $a_v=(v^{\mathrm{e}}, v^l)$ to G' such that all turns at v^{e} and v^l are allowed with penalty zero. Traversing arc a_v in G' is then equivalent to passing through vertex v in G (see Figure 2).

(ii) If $v \in V_{R_2}$, follow two steps:

Step 1: Replace vertex v in G' by as many vertices v_{ij} as allowed turns $[a_i b_j]$ are at v, so that each one of these copies has only one entering arc (a_i) and one leaving arc (b_j), with their corresponding allowed turn. Note that if a_i is an entering arc at v, G' will contain at least as many copies of the entering arc a_i as allowed turns involving a_i are found at v, and similarly for a leaving arc b_j from v.

Step 2: Proceed as in case (i) above replacing each one of these vertices v_{ij} by two vertices, v_{ij}^{e} and v_{ij}^l, and a required arc $a_{v_{ij}}=(v_{ij}^{\mathrm{e}}, v_{ij}^l)$ between them with cost zero, such that $p_{[a_i a_{v_{ij}}]}=p_{[a_i b_j]}$ and $p_{[a_{v_{ij}} b_j]}=0$, that is, the penalty that was in the turn at v_{ij} is moved to vertex v_{ij}^{e}. Then, traversing one of these required arcs $a_{v_{ij}}$ in G' involves passing through vertex v in G (see Figure 3).

Let us see an example of the construction of G'. Figure 4 shows a mixed graph G with one required arc $a=(3, 2)$, one required edge $b=(3, 5)$ and two required vertices 4 and 6. The link costs are shown in the figure; all U-turns are forbidden; going straight ahead (in this graph) has penalty 0; any right turn has penalty 1; and any left turn has penalty 3, except at vertex 4 where all turns are allowed with penalty 0. Remember that we do not consider the presence of non-required edges nor of required vertices incident with required links.

Note that we consider $[ab]$ a right (left) turn if, according to the drawing of the graph, passing from a to b implies turn right (left) making an angle $\widehat{ab}$ with $0° < \widehat{ab} < 180°$, that is any kind of right (left) turn, not only those with $\widehat{ab}=90°$. For instance, in Figure 4 turns [(3, 5) (5, 4)] and [(3,5) (5, 6)] are

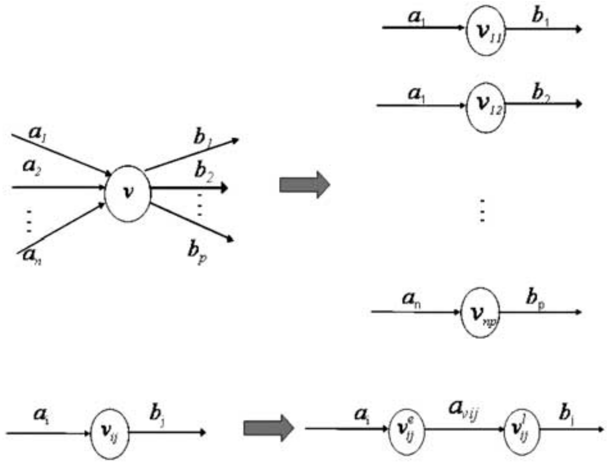

Figure 3 Transformation of a vertex $v \in V_{R_2}$.

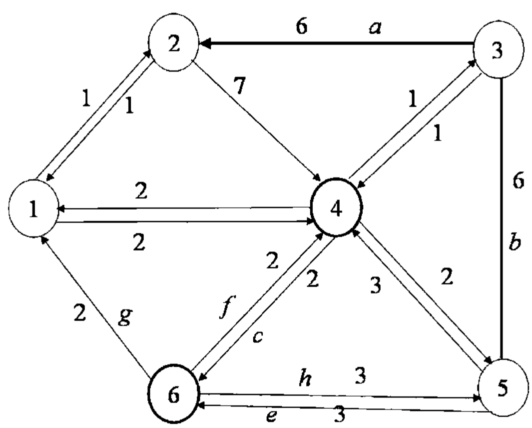

Figure 4 Graph G with $A_R = \{a\}$, $E_R = \{b\}$ and $V_R = \{4, 6\}$.

both right turns and then, for simplicity in this example, both are with penalty 1, although in a real-word problem it should be more appropriate to assign different penalties to right turns with different angles (remember that in the definition of the MGRPTP each turn $[ab]$ has its own penalty $p_{[ab]}$).

First, we consider vertex 4 which has no forbidden turns and all turn penalties are zero. Figure 5 represents the graph G' obtained at this point of the process.

Second, we consider the transformation of vertex 6, which belongs to V_{R_2}, into four vertices, that is as many as allowed turns in it. Figure 6 shows G' after Step 1 at vertex 6, and Figure 7 shows G' after Step 2, and then, in its final form.

Given an MGRPTP in G, we define a GMRPPTP in G' with the same required edge set as in G ($E'_R = E$), and for the required arcs we have A'_R which is partitioned into the following subsets: $S_a = \{a\}\ \forall a \in A_R$, $S_v = \{a_v\}\ \forall v \in V_{R_1}$ and $S_v = \{a_{v_{ij}} : [a_i b_j]$ is an allowed turn at $v\}\ \forall v \in V_{R_2}$. Thus, in the graph G' shown in Figure 7, $E'_R = \{b\}$ and $A'_R = S_a \cup S_4 \cup S_6$ with $S_a = \{a\}$, $S_4 = \{a_4\}$ and $S_6 = \{a_{6cg}, a_{6ch}, a_{6ef}, a_{6eg}\}$.

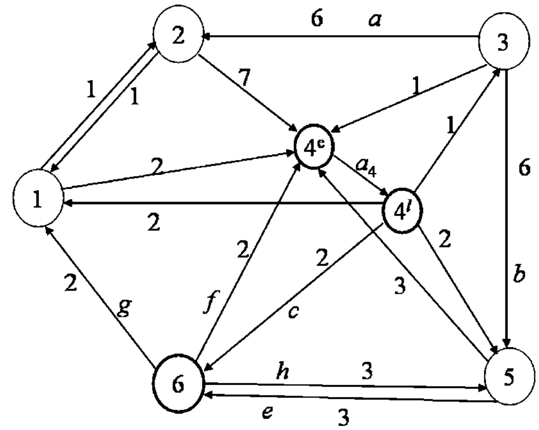

Figure 5 Transformation of required vertex 4.

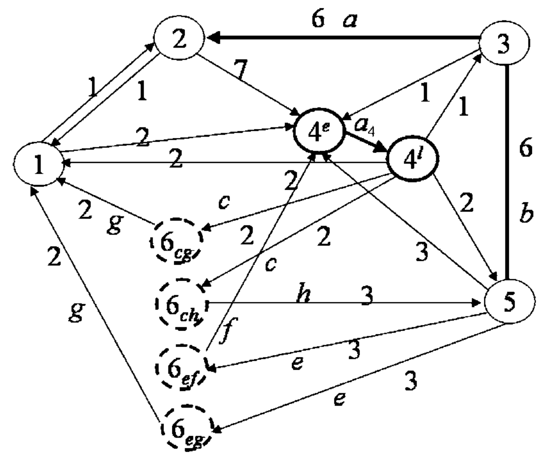

Figure 6 Step 1 of the transformation of required vertex 6.

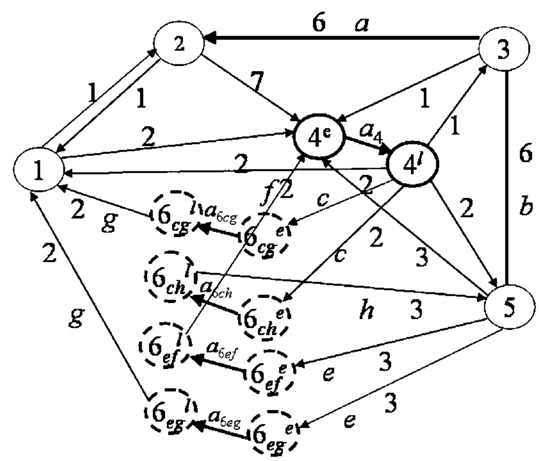

Figure 7 Transformed graph G'.

Theorem 1 *An MGRPTP defined in G can be transformed in polynomial time into the corresponding GMRPPTP in G'.*

Proof If T is an MGRPTP tour in G, we construct a GMRPPTP tour T' in G' with the same cost as follows:

Let $(u, v)[(u, v)(v, w)]$ be a link-turn sequence in T:

- if $v \notin V_R$, the same sequence will be defined in T'.
- if $v \in V_{R_1}$, the sequence $(u, v)[(u, v)(v, w)]$ in G will be replaced by $(u, v^e)(v^e, v^l)[(v^e, v^l)(v^l, w)]$ in G' with the same cost by the way G' is constructed.
- if $v \in V_{R_2}$, the sequence $(u, v)[(u, v)(v, w)]$ indicates that $\exists i, j$ such that v_{ij} is a copy of vertex v with $a_i = (u, v)$ and $b_j = (v, w)$; this situation gives rise to an arc $a_{v_{ij}}$ between two vertices v^e_{ij} and v^l_{ij} in S_v in G'. Then the sequence $(u, v)[(u, v)(v, w)]$ will be replaced in G' by $(u, v^e_{ij})(v^e_{ij}, v^l_{ij})[(v^e_{ij}, v^l_{ij})(v^l_{ij}, w)]$ with the same cost by the way G' is constructed.

It is evident that T' is a GMRPPTP tour in G' of the same cost as T.

Reciprocally, a GMRPPTP tour T' in G' generates an MGRPTP tour T in G of the same cost, simply reversing the process; that is, by replacing each sequence in T' of the form $(u, v^e)(v^e, v^l)[(v^e, v^l)(v^l, w)]$ with $a_v = (v^e, v^l) \in S_v$ if $v \in V_{R_1}$, and each sequence of the form $(u, v^e_{ij})(v^e_{ij}, v^l_{ij})[(v^e_{ij}, v^l_{ij})(v^l_{ij}, w)]$ with $a_{v_{ij}} = (v^e_{ij}, v^l_{ij}) \in S_v$ if $v \in V_{R_2}$, by $(u, v)[(u, v)(v, w)]$. Any other link-turn sequence in T' is replaced by itself in T. □

Now we have transformed the MGRPTP defined in G into a GMRPPTP defined in G', but this problem has not been studied in the OR literature, so we do not know how to solve our problem yet. Let us see how the GMRPPTP can be transformed into an AGTSP, and then into an ATSP; in this way, we will have a manner to solve the MGRPTP. To do this, associated with G', we construct a directed auxiliary graph G'' as follows:

- For each required arc $a \in A_R$ associate a vertex x_a in G''.
- For each subset S_v with $v \in V_{R_1}$ associate a vertex x_{a_v} in G''.
- For each subset S_v with $v \in V_{R_2}$ associate a vertex set S_{x_v} in G'' with a vertex $x_{v_{ij}}$ for each $a_{v_{ij}} \in S_v$.
- For each edge $e = (u, v) \in E_R$, G'' has two vertices, u_e and v_e, and two arcs, (u_e, v_e), (v_e, u_e), both with cost $-M$, M being a very large positive number.
- For each pair of different edges $a = (u, v)$, $b = (s, t) \in E_R$, add the following arcs to G'' (if the corresponding s.f.c. exists in G'):
  - ◦ (u_a, s_b) and (s_b, u_a) whose cost is that of *s.f.c.*(u^a, s^b) and *s.f.c.*(s^b, u^a) in G', respectively.
  - ◦ (u_a, t_b) and (t_b, u_a) whose cost is that of *s.f.c.*(u^a, t^b) and *s.f.c.*(t^b, u^a) in G', respectively.
  - ◦ (v_a, s_b) and (s_b, v_a) whose cost is that of *s.f.c.* (v^a, s^b) and *s.f.c.*(s^b, v^a) in G', respectively.
  - ◦ (v_a, t_b) and (t_b, v_a) whose cost is that of *s.f.c.*(v^a, t^b) and *s.f.c.*(t^b, v^a) in G', respectively.
- For each pair of different arcs $a = (u, v)$, $b = (s, t) \in A'_R$, add arcs (x_a, x_b) and (x_b, x_a) to G'', whose cost is that of *s.f.c.*(v^a, s^b) and *s.f.c.*(t^b, u^a) in G' (if they exist), respectively, except if a and b belong to the same S_v with $v \in V_{R_2}$.
- For each pair consisting of an arc $a = (u, v) \in A'_R$ and an edge $b = (s, t) \in E_R$, add the following arcs to G'' (if the corresponding s.f.c. exists in G'):
  - ◦ (x_a, s_b) and (s_b, x_a) whose cost is that of *s.f.c.*(v^a, s^b) and *s.f.c.*(s^b, u^a) in G', respectively.
  - ◦ (x_a, t_b) and (t_b, x_a) whose cost is that of *s.f.c.*(v^a, t^b) and *s.f.c.*(t^b, u^a) in G', respectively.

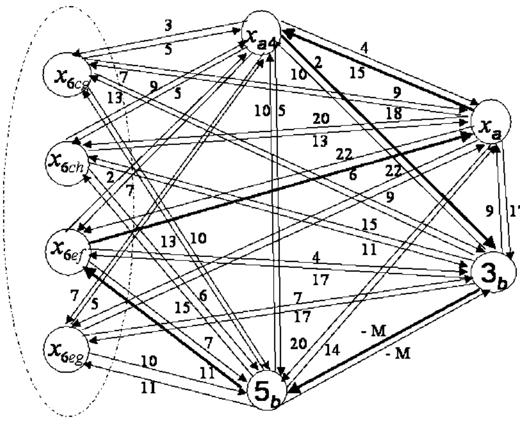

Figure 8 Directed graph G'' associated with G'.

Figure 8 shows the graph G'' with its arc costs, corresponding to graph G' of Figure 7.

Theorem 2 *The GMRPPTP defined in G' can be transformed in polynomial time into an AGTSP defined in G''.*

Proof Let us suppose that V_R is not empty. Otherwise we have an MRPPTP, which can be transformed in polynomial time into an ATSP (Corberán *et al*, 2002).

In graph $G'' = (V'', A'')$ obtained from G' we define an AGTSP in which V'' is partitioned into the following subsets: $\{x_a\}$ for each $a \in A_R$, $\{x_{a_v}\}$ for each $v \in V_{R_1}$, $\{u_e\}$ and $\{v_e\}$ for each $e = (u, v) \in E$, and $\{x_{v_{ij}} : a_{v_{ij}} \in S_v\}$ for each $v \in V_{R_2}$.

From an AGTSP circuit solution T'' in G'', we obtain a GMRPPTP tour T' in G' as follows:

Each time an arc different from (u_e, v_e) with $e \in E$ is traversed in G'' by T'', T' traverses the shortest feasible chain in G' that has given rise to the cost of this arc in G''. It is easy to see that T' is a GMRPPTP tour in G' containing at least each edge of E'_R and an arc of each of the subsets into which A'_R has been partitioned, due to the relationship between the sets that define the partition into A'_R and the sets that define the partition into V''. Moreover, if T'' is an optimal AGTSP solution in G'', the cost of T' is the cost of T'' plus kM with $k=|E'_R|$.

On the other hand, if T' is a GMRPPTP tour in G', we construct an AGTSP circuit solution T'' in G'' that takes into account the orientation and the order in which a required edge appears for the first time in T', and the order in T' of the first arc that appears in T' of each subset into which A'_R has been partitioned. Starting T' at a fixed link, each pair of these required links, consecutive with this ordering (the last pair contains the last of these links in T' and the first of these links in T') generates a section in T'' that depends on the type of these two links:

Given $a=(u,v), b=(w,t)\in A'_R, a\neq b, c=(s,r), d=(p,q)\in E_R, c\neq d$:

- $(u,v),(w,t)$ generates the arc in T'' (x_a,x_b).
- $(u,v),(s,r)$ generates the arc in T'' (x_a,s_c).
- $(s,r),(u,v)$ generates the section in T'' $(s_c,r_c),(r_c,x_a)$.
- $(s,r),(p,q)$ generates the section in T'' $(s_c,r_c),(r_c,p_d)$.

Following the order in which the above mentioned required links appear first time in T' and replacing each pair of these consecutive elements by its corresponding arc or section in T'', it is evident that T'' is an AGTSP circuit solution in G''.

Moreover, $c'(T')\geqslant c''(T'')+kM$ because the section cost in T' between two of these consecutive links is greater than or equal to the cost of the shortest feasible chain between them.

Let T''^* be an optimal AGTSP solution in G'' and let T'^* be the GMRPPTP tour in G' obtained from T''^*. For each GMRPPTP tour T' in G' we have:

$$c'(T')\geqslant c''(T'')+kM\geqslant c''(T''^*)+kM=c'(T'^*) \qquad (1)$$

Therefore, T^* is an optimal GMRPPTP tour in G'. $\square$

Following with our example, in Figure 8 arcs with a thicker line in G'' represent an optimal AGTSP solution, that is, the circuit $(x_a,x_{a_4}),(x_{a_4},3_b),(3_b,5_b),(5_b,x_{6ef}),(x_{6ef},x_a)$. From this circuit we obtain the corresponding optimal GMRPPTP tour in G' (remember that the middle turns in a chain have been removed for simplicity):

- (x_a,x_{a_4}) corresponds in G' to the *s.f.c.*$(2^a,4^{ea_4})$: $(3,2)(2,1)(1,4^{\mathrm{e}})$ $[(1,4^{\mathrm{e}})(4^{\mathrm{e}},4^l)]$ with cost $6+\mathbf{3}+1+\mathbf{3}+2+\mathbf{0}=15$ (bold numbers correspond to turn penalties).
- $(x_{a_4},3_b)$ corresponds to the *s.f.c.*$(4^{la_4},3^{\mathrm{b}})$: $(4^{\mathrm{e}},4^l)(4^l,3)$ $[(4^l,3)(3,5)]$ with cost $0+\mathbf{0}+1+\mathbf{1}=2$.
- $(3_b,5_b)(5_b,x_{6ef})$ corresponds to the *s.f.c.*$(5^b,6_{ef}^{\mathrm{e}(a_{6ef})})$: $(3,5)(5,6^{\mathrm{e}}_{ef})[(5,6^{\mathrm{e}}_{ef}),(6^{\mathrm{e}}_{ef},6^l_{ef})]$ with cost $6+\mathbf{1}+3+\mathbf{1}=11$.
- (x_{6ef},x_a) corresponds to the *s.f.c.*$(6_{ef}^{l(a_{6ef})},3^a)$: $(6^{\mathrm{e}}_{ef},6^l_{ef})(6^l_{ef},4^{\mathrm{e}})(4^{\mathrm{e}},4^l)(4^l,3)[(4^l,3)(3,2)]$ with cost $0+\mathbf{0}+2+\mathbf{0}+0+\mathbf{0}+1+\mathbf{3}=6$.

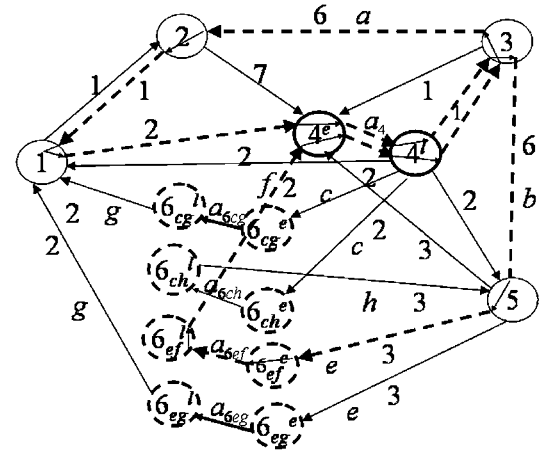

Figure 9 Optimal solution to the GMRPPTP defined in G' (broken line).

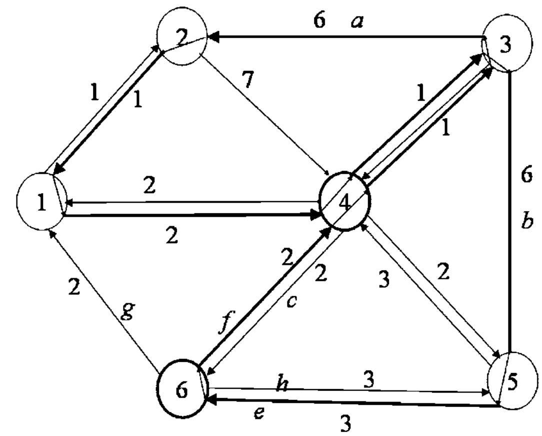

Figure 10 Optimal solution to the MGRPTP in G.

The broken line in Figure 9 represents this optimal GMRPPTP solution in G': $\{(3,2)(2,1)(1,4^{\mathrm{e}})(4^{\mathrm{e}},4^l)(4^l,3)(3,5)\ (5,6^{\mathrm{e}}_{ef})(6^{\mathrm{e}}_{ef},6^l_{ef})(6^l_{ef},4^l)(4^l,3)[(4^l,3)(3,2)]\}$ which generates the optimal MGRPTP solution in G $\{(3,2)(2,1)(1,4)(4,3)(3,5)(5,6)(6,4)(4,3)[(4,3)(3,2)]\}$ with cost $6+\mathbf{3}+1+\mathbf{3}+2+\mathbf{0}+1+\mathbf{1}+6+\mathbf{1}+3+\mathbf{1}+2+\mathbf{0}+1+\mathbf{3}=34$ (see Figure 10).

Computational experiments

To check the efficiency of our transformation, from the MRPPTP instances given in Corberán *et al* (2002) we have generated a set of 128 MGRPTP instances with both required links and required vertices, by randomly selecting required vertices among the 'isolated' vertices (those vertices non-incident with any required link), according to different percentages (from 40 to 100%) that depend on the number of the isolated vertices. We have generated MGRPTP instances with up to 200 vertices, 440 arcs, 176 required arcs, 26

Table 1 Computational results

Ins	*ARA*	$\|E\|$	*ARV*	AV_{ATSP}	*Opt*	*AT*
6	51	0	6	82	6	14.48
4	84	0	8	127	4	65.04
6	108	0	8	149	6	392.81
5	125	0	12	195	5	348.34
7	145	0	14	212	7	932.43
6	164	0	16	248	6	1269.71
5	54	10	3	86	5	42.65
4	84	10	8	137	4	478.80
7	107	10	9	175	7	490.38
6	126	10	11	201	6	819.55
5	141	10	13	231	5	1273.89
6	156	10	14	245	6	1037.18
6	172	10	15	265	6	3211.59
5	61	20	3	119	5	139.92
6	105	20	5	167	6	1464.62
4	126	20	9	208	4	516.69
6	142	20	11	240	6	3095.38
4	156	20	9	236	4	3539.15
3	168	20	12	236	3	3646.25
4	58	30	3	129	4	846.88
6	100	30	5	187	6	1218.35
6	65	40	3	156	5	2437.78
6	113	40	6	218	4	2931.00
5	140	40	6	191	0	—

required vertices and 40 required edges that give rise to GATSP instances with up to 315 vertices.

Encouraged by the results obtained in Albiach *et al* (2006) in the resolution of ATSP instances using the exact algorithm for the MGRP given in Corberán *et al* (2005), we decided to solve the MGRPTP with this exact algorithm, which is a cutting-plane procedure based on the polyhedral study of the MGRP. The algorithm is coded in *C* and run on a PC with a 1.8 GHz Pentium IV processor, using CPLEX 8.0 (CPLEX, 2002) as an LP solver.

We have grouped the 128 instances into sets of about five similar instances, according to the number of required arcs and the number of required edges, as in Corberán *et al* (2002). In Table 1 each row corresponds to one of these sets, with the following notation: Ins = number of instances in the set; ARA = average number of required vertices in the set; $|E|$= number of (required) edges for all the instances in the set; *ARV*= average number of required vertices in the set; AV_{ATSP}= average number of vertices in the ATSP instances obtained by the transformation; *Opt*= number of instances optimally solved in less than 2 h; and AT= average time in seconds to obtain the optimal solution in the solved (in less than two hours) GATSP instances obtained from G''.

Generally and as expected, the running time of this exact and exponential algorithm increases with the total number of vertices in the ATSP instances obtained with the transformation, and increases significantly with the number of required edges, as can be seen by comparing the average time for the five blocks of instances with a similar number of required arcs and vertices but with different number of required edges (0, 10, 20, 30 and 40). The last is due to the fact that for each edge $e = (u, v) \in E_R$, G'' has two vertices, u_e and v_e, and two arcs, (u_e, v_e), (v_e, u_e), both with cost $-M$, M being a very large positive number. Then, the first linear programming problem solved by the exact procedure introduces a 'very attractive' subtour $\{(u_e, v_e), (v_e, u_e)\}$ for each edge, which implies to solve new linear programming problems to break them.

Note that only eight out of the 128 instances were not solved after 2 h of running time, all of them corresponding to the block of 40 edges. Therefore, we believe that, given the complexity of the new problem, the exact procedure used here is a good tool to optimally solve at least medium-size real MGRPTP instances.

Conclusions

Recently, to approach the mathematical models more closely to some real-world problems, different authors have considered extended versions of well-known arc-routing problems, taking into account turn penalties and forbidden turns in the cost, even in the multivehicle case. But to our knowledge, no node-routing problem has been extended to consider turn penalties, although traffic restrictions on U-turns or left turns, or the fact that some turns can be more time-consuming or dangerous than others in big cities do not depend on demand locations (arcs or nodes).

We have presented here a generalization of the MGRP with turn penalties, which basically consist of finding a minimal cost-closed walk on a mixed graph G, which traverses a given subset of required links and vertices, without forbidden turns. Its cost is the sum of the costs of the traversed links together with the penalties associated with the turns made. In this way, we have presented an unified model of turn-penalty extended version extension for both node- and arc-routing problems with a single vehicle. We have seen that this generalization can be transformed into an ATSP and then into a particular case of the MGRP. Using a very recent code for the exact resolution of the MGRP we have presented computational results on the exact resolution of this generalization on a set of instances obtained from others existing in the literature. Almost all instances were optimally solved in less than 2 h, and many of them in at the most a few minutes, so we think that the results obtained are reasonably good if we consider the complexity of the new problem.

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References

Albiach J, Sanchis JM and Soler D (2006). An asymmetric TSP with time windows and with time-dependent travel times and costs: An exact solution through a graph transformation. *Eur J Opl Res*, doi: 10.1016/j.ejor.2006.09.099.

Arkin EM, Bender MA, Demaine ED, Fekete SP, Mitchell JSB and Sethia S (2006). Optimal covering tours with turn costs. *Siam J Comput* **35**: 531–566.

Bautista J and Pereira J (2004). *Ant Algorithms for Urban Waste Collection Routing*. Lecture Notes in Computer Science, Vol. 3172. Springer: Berlin. pp 302–309.

Benavent E and Soler D (1999). The directed rural postman problem with turn penalties. *Transp Sci* **33**: 408–418.

Blais M and Laporte G (2003). Exact solution of the generalized routing problem through graph transformations. *J Opl Res Soc* **54**: 906–910.

Bodin L, Fagin G, Welebny R and Greenberg J (1989). The design of a computerized sanitation vehicle routing and scheduling for the town of Oyster Bay, New York. *Comput Opl Res* **16**: 45–54.

Carpaneto G, Dell'Amico M and Toth P (1995). Exact solution of large-scale, asymmetric traveling salesman problems. *ACM Trans Math Softw* **21**: 394–409.

Chopra S and Rinaldi G (1996). The graphical asymmetric traveling salesman polyhedron: Symmetric inequalities. *SIAM J Discrete Math* **9**: 602–624.

Clossey J, Laporte G and Soriano P (2001). Solving arc routing problems with turn penalties. *J Opl Res Soc* **52**: 433–439.

Corberán A, Martí R, Martínez E and Soler D (2002). The rural postman problem on mixed graphs with turn penalties. *Comput Opl Res* **29**: 887–903.

Corberán A, Romero A and Sanchis JM (2003). The mixed general routing polyhedron. *Math Program Ser A* **96**: 103–137.

Corberán A, Mejía G and Sanchis JM (2005). New results on the mixed general routing problem. *Opns Res* **53**:363–376.

CPLEX 8.0 (2002). http://www.ilog.com/products/cplex. ILOG S.A., France.

Fischetti M and Toth P (1997). A polyhedral approach to the asymmetric traveling salesman problem. *Mngt Sci* **43**: 1520–1536.

Kwon SH, Kim HT and Kang MK (2005). Determination of the candidate arc set for the asymmetric traveling salesman problem. *Comput Opl Res* **32**: 1045–1057.

Lacomme P, Prins C and Ramadane-Cherif W (2004). Competitive memetic algorithms for arc routing problems. *Ann Opl Res* **131**: 159–185.

Lacomme P, Prins C and Ramadane-Cherif W (2005). Evolutionary algorithms for periodic arc routing problems. *Eur J Opl Res* **165**: 535–553.

Noon CE and Bean JC (1993). An efficient transformation of the generalized traveling salesman problem. *INFOR* **31**: 39–44.

Roy S and Rousseau JM (1989). The capacitated Canadian postman problem. *INFOR* **27**: 58–73.

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