A Novel Diversity-based Evolutionary Algorithm for the Traveling Salesman Problem

Carlos Segura
Center for Research in Mathematics A.C.
Guanajuato, Mexico
carlos.segura@cimat.mx

Salvador Botello Rionda
Center for Research in Mathematics A.C.
Guanajuato, Mexico
botello@cimat.mx

Arturo Hernández Aguirre
Center for Research in Mathematics A.C.
Guanajuato, Mexico
artha@cimat.mx

S. Ivvan Valdez Peña
Center for Research in Mathematics A.C.
Guanajuato, Mexico
ivvan@cimat.mx

ABSTRACT

The Traveling Salesman Problem (TSP) is one of the most well-known NP-hard combinatorial optimization problems. In order to deal with large TSP instances, several heuristics and metaheuristics have been devised. In this paper, a novel memetic scheme that incorporates a new diversity-based replacement strategy is proposed and applied to the largest instances of the TSPLIB benchmark. The novelty of our method is that it combines the idea of transforming a single-objective problem into a multi-objective one, by considering diversity as an explicit objective, with the idea of adapting the balance induced between exploration and exploitation to the various optimization stages. In addition, the intensification capabilities of the individual learning method incorporated in the memetic scheme are also adapted by taking into account the stopping criterion. Computational results show the clear superiority of our scheme when compared against state-of-the-art schemes. To our knowledge, our proposal is the first evolutionary scheme that readily solves an instance with more than 30,000 cities to optimality.

Categories and Subject Descriptors

I.2.8 [**Computing Methodologies**]: Artificial Intelligence—*Problem Solving, Control Methods and Search* Heuristic Methods

Keywords

Diversity Preservation; Traveling Salesman Problem; Evolutionary Algorithms; Memetic Algorithms

GECCO '15, July 11 - 15, 2015, Madrid, Spain
© 2015 ACM. ISBN 978-1-4503-3472-3/15/07…$15.00
DOI: http://dx.doi.org/10.1145/2739480.2754802

1. INTRODUCTION

The Traveling Salesman Problem (TSP) is one of the most popular NP-hard combinatorial optimization problems [7]. Given a complete weighted graph, the goal in the TSP is to find a Hamiltonian cycle with the least weight, i.e. a cycle of minimum cost that visits every city. The TSP received its name from the field of logistics. Specifically, it refers to a situation where a traveling salesman must make a round of all the cities in a set and return to the initial position. However, the TSP is probably so popular because it does not only appear in logistics, but also in several different practical applications [7], such as in genome sequencing, chip design and telescope slewing.

Given the complexity and interest of the TSP, several different methods have been devised to deal with it [7]. Some exact approaches [2], such as those based on branch and cut, have been used to solve some large TSP instances. The concorde tool [3] is probably one of the most popular exact tools for the TSP. The largest non-trivial instance solved by concorde consists of 85,900 cities. However, this tool requires a huge amount of computational time and it has not been able to solve other smaller instances [3]. Thus, several approximation techniques have also been devised. These include various heuristics and metaheuristics [2]. In the 80s and 90s, a large number of constructive heuristics were defined [7]. In addition, several improvement heuristics have been devised [7]. In the 2-opt local search, neighbor solutions are created by eliminating two edges and reconnecting the two resulting paths in a different way. This can be generalized to k-opt moves, where k edges are interchanged. Finally, the Lin-Kernighan family of heuristics (LK) —which is the one incorporated into our proposal— is based on conducting a restricted search for larger values of k. These heuristics yield high-quality approximations for large instances in a relatively small amount of time. However, when tighter approximations are required, metaheuristics are used.

Practically every popular metaheuristic has been adopted to tackle the TSP. For instance, Iterated Local Search (ILS) [4], and Evolutionary Algorithms (EAs) [30] have been used. Trajectory-based metaheuristics attain high-quality solutions on reasonable timescales. However, when longer times are admissible, population-based metaheuristics are usually preferred. In fact, some of the best-known solutions for several instances of the TSP have been obtained with EAs [12]. Some of the first successful applications of EAs to the TSP were steady-state schemes that relied in the replace-worst (RW) strategy [27]. In these cases, diversity can be prematurely lost. This drawback can be avoided with the application of diversity-preservation schemes and larger instances can be considered [9, 23]. In fact, the LKH tool [11], which is one of the most efficient tools for dealing with large TSP instances, incorporates an EA that explicitly manages diversity. The LKH tool has reported the best-known solutions for several instances [1].

In recent years, several new methods for dealing with diversity have appeared [31], with those that take diversity into account as an alternative objective and use multi-objective schemes gaining considerable popularity [25]. In this paper, we propose an extension of these techniques and apply them to the TSP. Specifically, we propose a new replacement strategy that combines the idea of transforming a single-objective problem into a multi-objective one, by considering diversity as an explicit objective, with the idea of adapting the balance induced between exploration and exploitation to the various optimization stages. The aim of the paper is two-fold. First, to study the generality and suitability of this kind of diversity management scheme. Second, to improve on the best results reported by state-of-the-art EAs for the TSP. Our experimental validation with some of the largest instances of the TSPLIB benchmark [22] shows the clear benefits of the new model. In fact, to the best of our knowledge, this is the first EA that reports the optimal solution for *pla33810*, an instance that consists of $33,810$ cities.

The rest of the paper is organized as follows. A discussion of some of the most important methods for maintaining proper diversity in EAs is given in Section 2. Section 3 offers a review of the literature on the application of EAs to the TSP. In Section 4 the new diversity-based replacement scheme, as well as the rest of the details of our memetic algorithm, are described. Section 5 is devoted to presenting our experimental validation. Some EAs not-based on diversity, as well as the EA of the LKH tool, are used to validate our scheme. Finally, our conclusions and some lines of future work are given in Section 6.

2. DIVERSITY PRESERVATION IN EVOLUTIONARY ALGORITHMS

Premature convergence is one of the most important drawbacks of many population-based metaheuristics [28]. Most of the techniques for dealing with premature convergence are based on directly or indirectly managing the diversity of the population [31]. These methodologies range from general techniques to ad-hoc heuristics. In this section, we briefly review some of the most popular general techniques, placing special emphasis on those techniques that, as our proposal, modify the replacement strategy.

During the 90s, considerable efforts were devoted to devising new parent selection schemes where the selection pressure might be controlled. However, some studies found that parent selectors were not able to maintain proper diversity *per se* [5] even if relatively large populations were used. The population model has also been studied with the aim of improving diversity preservation in EAs. In these schemes, some recombination restrictions are imposed by taking into account the positions of the individuals in the population. These modifications have important effects on diversity [1]. Other popular schemes [31] include those based on imposing mating restrictions and on adapting the variation stage. In the aforementioned schemes, diversity is controlled indirectly, thus complicating the task of adapting the schemes to the requirements of different problems and stopping criteria. In fact, such an indirect management approach is known to cause performance drawbacks in some cases [26].

Some techniques that rely on modifying the replacement phase have also been devised. The basic principle behind these schemes is that by diversifying the survivors, more exploration can be induced. The reason for this is two-fold. First, if the diversity of a given population is large, it means that several regions of the search space are maintained. Second, most crossovers tend to be explorative operators when distant individuals are involved. In our opinion, introducing the diversity preservation mechanism into the survivor selection stage is highly promising. We base this on the fact that while the variation and parent selection stages make decisions that affect the current generation — creation of the offspring —, the survivor selection mechanism makes decisions that might have a more drastic effect on the whole optimization process.

One of the first studies into this topic resulted in Cavicchio's *preselection scheme*, which was extended to create the *crowding* methods [31]. Several other replacement strategies that promote diversity have been proposed since then. One of the most popular is the *clearing* [21] strategy, which can be regarded as an extension of fitness sharing. Another state-of-the-art alternative is the method devised in [14] (CD/RW — Contribution to Diversity / Replace Worst), where a new individual enters the population by replacing another that is worse both in quality and diversity contribution. If such an individual is not found, a *replace-worst* strategy (RW) is applied, i.e., the worst individual in the population is deleted if it is worse than the newly generated individual.

Finally, another quite popular alternative is to explicitly consider diversity as an objective and apply a multi-objective optimization scheme [6]. In these methods, the auxiliary objective is a measure of the diversity introduced by each individual considered in the population. Several different ways of calculating the auxiliary objective have been proposed [25]. One of the most popular is probably the *distance to the closest neighbor* (DCN) metric [6]. In DCN, the auxiliary objective of a given individual is calculated as its distance to the closest member of the population. In addition, it was shown that a replacement phase where the DCN is calculated by taking into account only the members that have already been selected to survive is preferred [24]. In this paper, we propose a new replacement phase that is an extension of the method devised in [24].

3. EVOLUTIONARY ALGORITHMS FOR THE TSP

The application of EAs to the TSP has a long history [32]. In this section, we briefly review three of the areas that have seen the most intense research efforts: the development of

[1] http://www.akira.ruc.dk/~keld/research/LKH/

crossover schemes, the hybridization of improvement heuristics and metaheuristics, and diversity control.

The 80s and 90s saw the appearance of a large number of papers on recombination operators [27]. Among others, basic variants of the well-known maximal preservative crossover (MPX) [18], edge assembly crossover (EAX) [19] and edge recombination crossover (ERX) [32] were devised. While many other recombinations appeared, these three are some of the most widely used nowadays because they are adjacency-based crossovers, i.e., they emphasize preserving the edges of their parents, which has been shown to be a truly beneficial feature [30].

In the TSP, the crossover operators can be divided into two classes: those that use local problem-dependent information and those that do not. The former are greedy schemes that take into account the cost of the different edges to create better tours. The latter are simpler operators that, due to the absence of greedy operations, induce greater levels of diversity. Note that several extensions of the above operators have also been devised. For instance, a greedy variant of the ERX (Edge-T) was developed [29]. Edge-T not only includes a greedy strategy, but also encourages the alternation of parents in edge inheritance with the aim of increasing diversity. Nowadays, it is not clear whether using greedy or non-greedy strategies is preferred. In fact, both kinds of crossovers have attained the best-known solutions for different instances [20, 12]. In the case of greedy crossovers, hybridization is not usually required [12]. However, considering that some of the most powerful greedy variants [12] incorporate local searches to connect certain intermediate subtours, the conceptual difference between this scheme and hybrid methods is not so significant. Alternatively, when non-greedy crossovers are taken into account, hybrid methods are required to deal with large instances [20]. The LKH tool features the ERXT operator. This operator is a variant of the Edge-T that does not include the greedy operations.

A closely related approach to crossover is the *iterative partial transcription* (IPT) [17]. Similarly to crossover operators, IPT combines two tours with the aim of generating a new, shorter tour. Particularly, IPT identifies parts of the tours that can be exchanged without compromising the feasibility of the newly generated tour. The main drawback is that depending on the tours under consideration, no suitable parts might be identified. In these cases, no changes are made. As a result, IPT should not be used as a crossover. However, in some other cases, impressive improvements can be obtained. This is why in some EAs, as in the one incorporated in the LKH tool, in addition to a traditional crossover scheme, IPT is also incorporated. Specifically, new offspring created with the ERXT are then combined with all the members of the population using IPT. Since this method yields significant improvements, our scheme also incorporates it.

The study and implementation of hybrid schemes for the TSP is another very active area of research. Some of the first memetic algorithms incorporated simple improvement heuristics such as the 2-opt or 3-opt [16]. Subsequently, more complex schemes, especially those based on the LK heuristic, were adopted. Nowadays, the iterated LK (ILK) — which is a variant of ILS — has shown to be one of the best trajectory-based metaheuristics for the TSP. In several cases [20], ILK has been used as the individual-learning scheme in memetic algorithms. It is this scheme, specifically the variant incorporated in LKH, that we have adopted in our paper. Note that this variant is a special case of ILS. Usually, in ILS once a local optimum is found, a perturbation is done. However, in the LKH implementation, a new random tour is created. Then, the local search applied to this tour is biased by the best solution found in the current iteration of ILS. Specifically, k-opt moves in which the first edge to be broken belongs to the best solution found in the current iteration of ILS are not investigated. This attracts the solution towards the current zone being explored. In addition, the search restrictions produce a faster local search. Finally, note that since ILK incorporates mechanisms to create new edges, EAs without the presence of mutation can be used [30].

Some of the first successful EAs applied to the TSP were based on the GENITOR scheme [32], meaning that high levels of intensification were induced. However, some proposals where diversity was taken into account were also devised, showing the advantages of diversifying the search. The first methods applied were very basic. For instance, in ASPARAGOS [9] structured populations were used, while in [23] a method based on hashes to avoid clones was presented. In addition, some other classical diversity management strategies, such as fitness sharing, restarting, multi-chromosomal cramping, large populations and crowding, have been considered [23]. Note that some of these methods rely on functions capable of determining a real-valued measure of the distance between two given tours. Thus, several metrics to measure these distances have been devised [15]. These research efforts have revealed the benefits of taking diversity into account when designing EAs for the TSP. In fact, ASPARAGOS was the best evolutionary strategy for many years [16].

Most current EAs for the TSP include certain actions to control diversity [16, 11]. In fact, nowadays most of the general techniques devised for dealing with the balance between exploration and exploitation have been tested with the TSP. In addition, some techniques specifically designed for the TSP have also appeared. In several cases, structured populations are taken into account, though they are usually combined with other approaches. For instance, in [20] it is used with crowding, while in [13] diversity is taken into account when accepting immigrants. Mating restrictions are also quite popular. For instance, in [30], heterogeneous paring selections are promoted. Combining diversity and the length of the tour in the fitness function has also yielded quite promising results for large instances [12]. In these types of proposals, different entropy-based metrics have been used to measure the diversity. The LKH tool incorporates the CD/RW method and clone avoidance. In this case, the distance between two given individuals is calculated using the k-OPT metric [15]. In this metric, the number of different edges in both individuals is counted. Note that this operation can be completed in linear time with respect to the number of cities in the instance. Finally, we would like to note that while controlling diversity by considering it as an additional optimization objective has gained popularity in recent years [25], we are not aware of the application of this kind of method to the TSP.

4. OUR PROPOSAL

In our new proposal we incorporate a novel replacement strategy (Algorithm 1) in a memetic scheme (Algorithm 2) that applies readily available operators. This section describes both schemes.

Algorithm 1 MULTI_DYN survivor selection scheme

```
CurrentMembers = Population ∪ Offspring
Best = Individual with best f(x) in CurrentMembers
NewPop = { Best }
CurrentMembers = CurrentMembers - { Best }
while (|NewPop| < N) do
    Calculate DCN of CurrentMembers, considering as reference NewPop
    D = D_I - D_I * T_Elapsed / T_End
    Penalize(CurrentMembers, D)
    ND = Non-dominated individuals in CurrentMembers (without repetitions) according to the two defined objectives
    Selected = Randomly select an individual from ND
    NewPop = NewPop ∪ Selected
    CurrentMembers = CurrentMembers - {Selected}
end while
Population = NewPop
```

Algorithm 2 Pseudocode of the Memetic Scheme

```
Initialization: Generate an initial population with N individuals with the initialization incorporated in the LKH tool
Local Search: Apply ILS to every individual in the population
Survivor selection initialization: Initialize D_I
while (the stopping criterion is not fulfilled) do
    Mating selection: select 2 × N parents with binary tournaments
    Crossover: Apply the ERXT to generate N new individuals
    Local Search: Adapt the local-search strength and apply ILS to the offspring population
    Combination: Try to combine each new individual with every member of the current population using the IPT method
    Survivor selection: Apply the MULTI_DYN scheme to select N survivors
end while
```

4.1 Diversity-based Replacement Strategy

Our proposal builds on the multi-objective replacement strategy presented in [24] (MULTI) and modifies it with the aim of avoiding certain drawbacks. The MULTI strategy operates as follows. First, the population of the previous generation and the offspring are combined in a temporary set. Then, the best individual, i.e., the tour with the lowest length, is selected to form part of the new population. Then, until the new population is filled with N individuals, the following steps are executed. First, the distance to closest neighbor (DCN) objective is calculated. The k-OPT metric is used to calculate the distances in our case. The calculation considers the currently selected individuals as the reference, i.e., for each pending individual, the distance to the nearest individual previously selected is taken into account. Then, considering the individuals that have not been selected, the non-dominated front is calculated. The two objectives considered are the tour length and the DCN. The tour length should be minimized whereas the DCN should be maximized. This front is computed as a set with no repetitions. Finally, a non-dominated individual is randomly selected to survive. While survivor selections similar to this one have been successful in other cases [25], initial experimentation with long executions in the TSP revealed that this scheme is not appropriate. The reason is that it can accept very close tours even in the initial phases and tends to create clusters of distant tours that are maintained throughout the execution. Since the number of clusters maintained is neither large nor adaptive, schemes capable of inducing larger levels of diversity are required for long executions.

The main advantage of the newly devised replacement strategy (MULTI_DYN) is that the balance between exploration and exploitation is dynamically adjusted based on the given stopping criterion. Thus, the stopping criterion, as well as the elapsed time, are used as inputs to the replacement strategy. This way, for shorter stopping criteria, MULTI_DYN induces a faster reduction in diversity than for longer stopping criteria. One of the basic principles behind MULTI_DYN (Algorithm 1) is that individuals that contribute too little to diversity —the contribution is measured with the DCN value— should not survive regardless of their tour length. In our approach, individuals whose DCN value is lower than a value D are penalized by setting their tour length to a very low quality value. Specifically, we set it to the maximum value representable in a *long* data type. Then, individuals are ranked taking into account the modified values. While this approach is quite logical, one of the key choices is how to evaluate whether an individual contributes enough or not, i.e., how to set the value of D, which should depend on the optimization stage. Specifically, this value should be reduced as the stopping criterion is approached. In our scheme, the initial D_I value is set by calculating the maximum distance appearing between any two individuals of the initial population and dividing it by two. Note that in most instances, global and local optima share a large number of edges [16], which is why we decided to initialize D_I in this way. This induce a high but not exceedingly large exploration in the initial phases. In our initial experiments we also used some larger values related to the number of cities, though this resulted in higher levels of fruitless exploration. Subsequently, a linear reduction of D is done during the execution. The reduction is calculated in such a way that by the end of the execution, the resulting value is 0, meaning that in the final stages our proposal behaves as MULTI. In this paper, the stopping criterion is set based on time. Thus, if T_{End} is the number of seconds allocated to the run and $T_{Elapsed}$ is the elapsed time, D can be calculated as $D = D_I - D_I * \frac{T_{Elapsed}}{T_{End}}$.

4.2 Memetic Algorithm

The newly proposed EA (MULTI_DYN-MA - Algorithm 2) is quite similar to the one incorporated in the LKH tool. Initially, N individuals are generated. So as to start with solutions that are already fit, completely random solutions are not used. Instead, they are generated with the initialization procedure used in LKH [10] and the ILS variant of the LKH tool is applied to them. Note that the initialization scheme does not produce very high-quality solutions. However, since promising edges are selected, the time invested in the local search is reduced. Afterwards, the maximum distance between individuals is calculated with the aim of initializing the D_I value, as described above. Finally, a set of generations is evolved until the stopping criterion is fulfilled. In each generation the parents are selected through binary

Table 1: Results obtained by the different schemes in the long-term

	MULTI_DYN-MA		RW-MA		ELI_GEN-MA		LKH-MA	
	Mean	SR	Mean	SR	Mean	SR	Mean	SR
RL5915	**565530**	**100%**	565542.8	76.6%	565542.8	76.6%	**565530**	**100%**
RL5934	**556045**	**100%**	**556045**	**100%**	**556045**	**100%**	**556045**	**100%**
PLA7397	**23260728**	**100%**	**23260728**	**100%**	**23260728**	**100%**	**23260728**	**100%**
RL11849	**923288**	**100%**	**923288**	**100%**	**923288**	**100%**	**923288**	100%
USA13509	**19982859**	**100%**	19983011.2	60%	19983053.5	60%	**19982859**	**100%**
BRD14051	**469386.1**	**63.3%**	469391.9	10%	469389.2	10%	469388.3	23.3%
D15112	**1573084**	**100%**	1573120.4	6.6%	1573100.2	6.6%	1573084.2	96.6%
D18512	**645242.2**	0%	645248	3.3%	645242.6	3.3%	645243.4	**6.66%**
PLA33810	**66049417**	**76.6%**	66052783.2	10%	66050892.1	10%	66052732.5	0%
PLA85900	**142399695.8**	0%	142402714.9	0%	142400081.4	0%	142400364.8	0%

tournaments, the ERXT operator is applied to create N new individuals, and ILS is used to improve each generated individual. Since ILS already perturbs solutions, mutation is not used. Note that ILS accepts a parameter (*MoveType* in LKH) to define the strength of the local search. The default value of this parameter in the LKH tool is 5. In order to increase the number of generations evolved in our EA, we adapt this value. Specifically, in the first third of the execution it is set to 3; in the second third it is set to 4; and finally in the last third it is set to 5. In addition, the IPT scheme is used to combine each new individual with every member of the population. If a better tour is generated, this new tour replaces the offspring. Finally, the survivors are selected with the MULTI_DYN scheme.

5. EXPERIMENTAL EVALUATION

In this section, the experiments conducted with our new scheme (MULTI_DYN-MA) are described. The analyses were performed with the 10 largest instances of the TSPLIB benchmark [22]. Their sizes range from $5,915$ to $85,900$ cities. Tests were run in the cluster "El Insurgente" at CIMAT. In consists of bi-processor nodes with 32Gb RAM. Each processor is an Intel(R) Xeon(TM) CPU E5-2620 at 2.10GHz. Since stochastic algorithms were considered, each execution was repeated 30 times and comparisons were carried out by applying a set of statistical tests, following some of the recommendations given in [8]. Specifically, the following tests were applied, assuming a significance level of 5%. First, for each instance, a multiple-comparison among all methods was done with the Friedman's test. In the cases where the p-value was lower than the established significance level, the following pair-wise comparisons were carried out. First, a *Shapiro-Wilk test* was performed to check whether or not the values of the results followed a Gaussian distribution. If so, the *Levene test* was used to check the variances for homogeneity. If samples had equal variances, an ANOVA *test* was done; if not, a *Welch test* was performed. For non-Gaussian distributions, the *Kruskal-Wallis* test was used to test whether samples were drawn from the same distribution. In this work, the sentence "algorithm A is better than algorithm B" means that the differences between them are statistically significant, and that the mean and median obtained by A are lower than the mean and median achieved by B.

In order to show the benefits of the new model, three additional algorithms were executed. In two of them, the only difference with respect to MULTI_DYN-MA is the replacement strategy. In the RW-MA scheme, the RW strategy is used, while in the ELI_GEN-MA method a generational replacement with elitism is considered, i.e. in each generation $N-1$ new individuals are created. These new individuals and the best from the previous generation are then selected to survive. The aim of including these two schemes is to show the advantages of the MULTI_DYN replacement strategy. Finally, the state-of-the-art EA included in the LKH tool (LKH-MA) was also taken into account. The main difference with respect to MULTI_DYN-MA is the way in which diversity is managed in the replacement strategy. Note that LKH-MA applies the CD/RW strategy, which resorts to the use of the RW strategy in some cases. This means that in the long term, diversity can be greatly reduced. In addition, this scheme does not adapt the strength of the local search.

We divided our experiments into two sections. Given that maintaining higher levels of diversity is usually helpful, especially for long-term executions, in our first analyses we focus on the long long-term behavior of these algorithms. In any case, analyzing the convergence properties and the behavior in other phases is also helpful in providing a better insight into the behavior of the different schemes. Our second analyses are based on this principle.

5.1 Analysis of the long-term behavior

Our first experiments were focused on analyzing the long-term behavior of the various schemes. Specifically, in the 5 smallest instances, the stopping criterion was set to 4 days, while in the largest instances it was set to 10 days. While we do not report comparisons in base of the generations, it is interesting to remark that for the smallest instances more that $5,000$ generations could be evolved, while in the largest ones about 300 were evolved. In the schemes considered in this paper, two parameters have to be set: the population size and the number of perturbations done in each iteration of the ILS. These were set experimentally to 50 and 25, respectively. Note that the behavior of the ILS in the LKH tool can be slightly modified with a large number of parameters. In our experiments we used the default values. Similarly, other ways of initializing D_I might be tested. We are aware that even better results might be obtained by properly tuning these parameters. However, given the computational cost associated with these tasks, such a study is beyond the scope of this paper.

Table 1 shows, for each scheme, the mean obtained for every TSP instance. Other important information, such as the median of the results, is not shown due to space limitations, but similar conclusions can be drawn for the median. Since the optimal tours of these instances are known, we also report the success rate (SR), i.e. the probability of

Table 2: **Statistical comparison of the different methods**

	MULTI-DYN MA		RW MA		ELI_GEN MA		LKH MA	
	↑	↓	↑	↓	↑	↓	↑	↓
RL5915	2	0	0	2	0	2	2	0
USA13509	2	0	0	2	0	2	2	0
BRD14051	3	0	0	3	1	2	2	1
D15112	2	0	0	3	1	2	2	0
D18512	1	0	0	3	1	0	1	0
PLA33810	3	0	0	2	2	1	0	2
PLA85900	1	0	0	2	1	0	0	0
Total	14	0	0	17	6	9	9	3

attaining the optimal solution. The clear superiority of the MULTI_DYN-MA is readily apparent. In fact, the mean success rate for MULTI_DYN-MA is 73.9%, which is larger than that attained by any other scheme. Specifically, LKH-MA, ELI_GEN-MA and RW-MA yielded means of 62.6%, 48.2% and 46.6% respectively. In addition, MULTI_DYN-MA attained the lowest mean in every instance. Notice that the largest differences arise in some of the largest instances, meaning that properly managing diversity is more helpful when dealing with large instances. Also worth noting is the fact that MULTI_DYN was able to achieve the optimal tour in the *pla33810* instance. To our knowledge, this is the first EA to achieve this accomplishment. Note that RW-MA and ELI_GEN-MA also obtained the optimal tours in some executions. However, they obtained lower success rates. Given that LKH-MA was not able to achieve the optimal solution, we hypothesized that adapting the local search strength might yield the most important improvements. As a result, we incorporated the adaptation strength into LKH-MA and executed the new scheme with the *pla33810* instance. While the results indeed improved in this case, the success rate only increased to 30%, meaning that, as expected, additional benefits come from the MULTI_DYN replacement strategy. In addition, it is important to note that in most cases MULTI_DYN-MA provides important benefits in terms of the mean achieved. However, when compared to ELI_GEN-MA in the *d18512* and *pla85900* instances, the differences diminish. In fact, in these two cases, the differences between MULTI_DYN-MA and ELI_GEN-MA are not statistically significant. The reasons have to do with the convergence properties in these problems and are analyzed in the next section.

Finally, the results were also subjected to statistical tests, as shown in Table 2. For each instance where the Friedman's test rejected the null hyphotesis, the column with the ↑ shows the number of cases where the method listed in the corresponding column is better than the other methods in the pairwise comparisons. Similarly, the column with the ↓ shows the number of cases where the model is worse. The only method that was not statistically inferior in any instance was the MULTI_DYN-MA, demonstrating once again its superiority. Due to space constraints, the p-values are not reported but they were quite low. For instance, in the case of the BRD14051, the maximum p-value obtained in the comparisons where MULTI_DYN-MA was taken into account was lower than 10^{-5}.

5.2 Analysis of the convergence properties

One of the advantages of explicitly controlling the diversity in EAs is that premature convergence might be avoided.

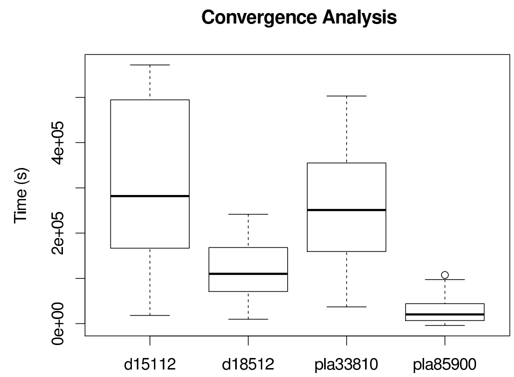

Figure 1: Box-plots of the time elapsed since the last improvement in ELI_GEN-MA

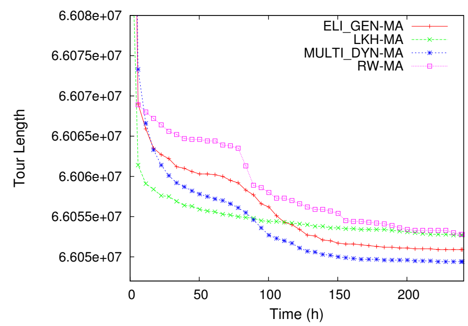

Figure 2: Evolution of tour length in the pla33810 instance

In the previous analyses, we saw how in two of the instances (D18512 and PLA85900) our new proposal did not provide significant benefits when compared to ELI_GEN-MA. In order to shed some light on the reasons behind this behavior, we re-executed ELI_GEN-MA with the four largest instances using the seeds that did not provide the optimal solution. In these executions, we stored the time elapsed between the last improvement made during the execution and the end of the execution. Figure 1 shows the box-plots of these times. We can clearly see that these times are short in D18512 and PLA85900, meaning that in these instances, premature convergence did not appear even after 10 days of execution. The reason is that the results were not very close to the optimal values, even after such a long execution time. Thus, in order to profit from explicitly controlling diversity, longer executions are needed. However, in the two other instances tested (D15112 and PLA33810), it is clear that premature convergence is appearing in ELI_GEN-MA. As a result, in these instances the advantages of using the MULTI_DYN-MA are more significant, both in terms of the success rate and of the mean values attained.

In addition, it is quite interesting to show the evolution of the best tour found for the different schemes. Figures 2 and 3 show how the tour length evolved for the schemes considered in the PLA33810 and PLA85900 instances, respectively. In the case of the PLA33810 intance, the MULTI_DYN-MA suffers from slow convergence in the first phases of the optimization, as expected. However, maintaining such a large diversity allows for better exploration of the different regions,

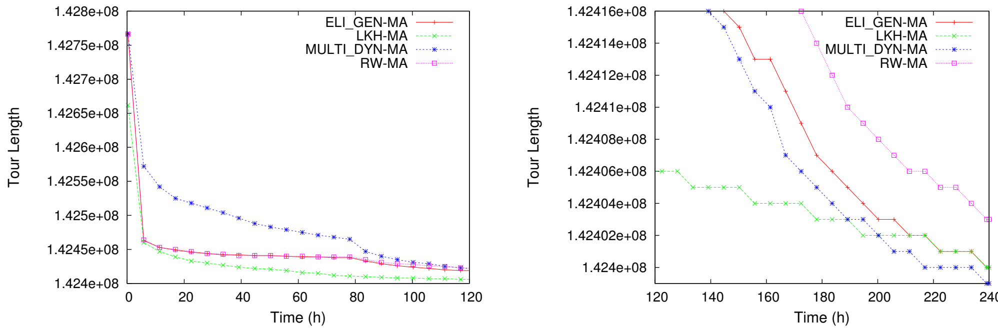

Figure 3: Evolution of tour length in the pla85900 instance

and in the long-term the benefits are clear. Also note that LKH-MA has the faster convergence. The reasons for this are two-fold. First, the strength of the local search is not adapted. Second, in large instances, LKH-MA relies too much on the RW strategy. As a result, it quickly loses diversity. In fact, in the two largest instances, differences between the results obtained by LKH-MA and RW-MA are not statistically significant. The PLA85900 instance exhibits similar effects, some of them more clearly. In the first phases, we see that the convergence in MULTI_DYN-MA is very slow. In fact, after 120 hours, MULTI_DYN-MA reports the worst tours. The time invested in the LK heuristics grows exponentially, meaning that in the longest instances much fewer generations are evolved. In addition, more promising zones can be detected because the search space is larger, meaning that much more time might be required to reach the optimal solution. In fact, in this case we can see that even after 240 hours, no scheme has converged. Also note that in spite of the initial slow convergence, MULTI_DYN-MA attains the lowest mean at the end of the executions. In any case, since the differences between the results of MULTI_DYN-MA and ELI_GEN-MA are not significant, it seems that longer executions are required in order to profit from the application of the MULTI_DYN-MA scheme. However, since these executions are already very long, parallelizing this EA seems quite promising. Finally, we would like to note that, to our knowledge, the mean achieved by MULTI_DYN-MA for the PLA85900 instance is the lowest value reported by an EA.

6. CONCLUSIONS

The TSP is one of the most challenging and studied combinatorial optimization problems. Several different schemes have been devised to address the TSP, including a large number of heuristics and metaheuristics. Among these methods, population-based schemes such as EAs have yielded quite promising results. Several studies have revealed that by properly managing the diversity induced by these methods, larger instances can be handled. In this paper, a new diversity-based memetic scheme for the TSP has been devised. The most important contribution is a novel replacement strategy that accepts the stopping criterion as a parameter, the goal being to properly alter the balance between exploration and exploitation. The realization of this idea is based on the application of certain principles that arise in the multi-objective field. This strategy has been integrated with state-of-the-art crossovers and local search operators. In addition, the strength of the local search is adapted. The new scheme has been compared against state-of-the-art EAs with the largest instances of the TSPLIB benchmark. The results show the clear superiority of our proposal. However, some instances were identified whose results were similar to those obtained by other schemes. The reason is that in these cases, convergence is not obtained even after a 10-day execution runtime. Even in these cases, however, by properly altering the balance between exploration and exploitation, the results reported by our proposal are not inferior. Thus, the newly devised scheme has yielded better or similar results than any other scheme in every instance tested. Two of the most important achievements of the new scheme are the following: the PLA33810 instance has been solved to optimality and, to our knowledge, the mean reported for the PLA85900 is lower than any mean reported by any other state-of-the-art EAs.

Since we have shown that for the largest instances, convergence is not obtained even after 10 days, one of the future lines of research is to develop a parallel scheme to handle even larger instances. We might first develop a straightforward master-slave scheme. Then, in order to be able to profit from a larger number of processors, we would like to parallelize it with an island-based approach. However, prior to implementing it, the implications that integrating structured populations into our proposals have on diversity should be studied. Finally, since there are several other crossovers and local search operators that have yielded high-quality solutions, new designs incorporating some of these schemes might be in order.

Acknowledgments

The first author acknowledges the financial support from CONCYTEG as part of the plan “Investigadores Jóvenes - DPP-2014” (project 14-IJ-DPP-Q182-11).

7. REFERENCES

[1] E. Alba. *Parallel Metaheuristics: A New Class of Algorithms*. Wiley series on parallel and distributed computing. Wiley-Interscience, 2005.

[2] S. Anbuudayasankar, K. Ganesh, and S. Mohapatra. Survey of methodologies for tsp and vrp. In *Models for Practical Routing Problems in Logistics*, pages 11–42. Springer International Publishing, 2014.

[3] D. Applegate, R. Bixby, V. Chvátal, and W. Cook. Available at www.tsp.gatech.edu.

[4] D. Applegate, W. Cook, and A. Rohe. Chained lin-kernighan for large traveling salesman problems. *INFORMS J. on Computing*, 15(1):82–92, Jan. 2003.

[5] T. Blickle and L. Thiele. A Comparison of Selection Schemes used in Evolutionary Algorithms. *Evolutionary Computation*, 4(4):361–394, 1996.

[6] L. T. Bui, H. A. Abbass, and J. Branke. Multiobjective optimization for dynamic environments. In *2005 IEEE Congress on Evolutionary Computation*, volume 3 of *CEC'05*, pages 2349 – 2356 Vol. 3, 2005.

[7] W. Cook. *In Pursuit of the Traveling Salesman: Mathematics at the Limits of Computation*. Princeton University Press. Princeton University Press, 2012.

[8] J. Derrac, S. García, D. Molina, and F. Herrera. A practical tutorial on the use of nonparametric statistical tests as a methodology for comparing evolutionary and swarm intelligence algorithms. *Swarm and Evolutionary Computation*, 1(1):3 – 18, 2011.

[9] M. Gorges-Schleuter. ASPARAGOS an Asynchronous Parallel Genetic Optimization Strategy. In *Proceedings of the Third International Conference on Genetic Algorithms*, pages 422–427, San Francisco, CA, USA, 1989. Morgan Kaufmann Publishers Inc.

[10] K. Helsgaun. An effective implementation of the lin-kernighan traveling salesman heuristic. *European Journal of Operational Research*, 126:106–130, 2000.

[11] K. Helsgaun. General k-opt submoves for the lin-kernighan tsp heuristic. *Mathematical Programming Computation*, 1(2-3):119–163, 2009.

[12] K. Honda, Y. Nagata, and I. Ono. A parallel genetic algorithm with edge assembly crossover for 100,000-city scale tsps. In *2013 IEEE Congress on Evolutionary Computation (CEC)*, pages 1278–1285, June 2013.

[13] C. Li, J. Wang, X. Yan, and G. Hu. Subpopulation diversity based accepting immigrant in distributed evolutionary algorithms. In *Parallel and Distributed Systems (ICPADS), 2013 International Conference on*, pages 422–423, Dec 2013.

[14] M. Lozano, F. Herrera, and J. R. Cano. Replacement strategies to preserve useful diversity in steady-state genetic algorithms. *Information Sciences*, 178(23):4421 – 4433, 2008. Including Special Section: Genetic and Evolutionary Computing.

[15] K.-T. Mak and A. J. Morton. Distances between traveling salesman tours. *Discrete Applied Mathematics*, 58(3):281 – 291, 1995.

[16] P. Merz and B. Freisleben. Memetic algorithms for the traveling salesman problem. *Complex Systems*, 13:297–345, 2001.

[17] A. Möbius, B. Freisleben, P. Merz, and M. Schreiber. Combinatorial optimization by iterative partial transcription. *Phys. Rev. E*, 59:4667–4674, Apr 1999.

[18] H. Mühlenbein. Evolution in Time and Space - The Parallel Genetic Algorithm. In *Foundations of Genetic Algorithms*, pages 316–337, San Francisco, CA, USA, 1991. Morgan Kaufmann.

[19] Y. Nagata and S. Kobayashi. Edge Assembly Crossover: A High-power Genetic Algorithm for the Travelling Salesman Problem. In T. Bäck, editor, *Proceedings of the Seventh International Conference on Genetic Algorithms (ICGA97)*, pages 450–457, San Francisco, CA, 1997. Morgan Kaufmann.

[20] H. D. Nguyen, I. Yoshihara, K. Yamamori, and M. Yasunaga. Implementation of an effective hybrid ga for large-scale traveling salesman problems. *IEEE Transactions on Systems, Man, and Cybernetics, Part B*, 37(1):92–99, 2007.

[21] A. Petrowski. A clearing procedure as a niching method for genetic algorithms. In *Proceedings of IEEE International Conference on Evolutionary Computation 1996 (CEC'96)*, pages 798–803, May 1996.

[22] G. Reinelt. TSPLIB - A Traveling Salesman Problem Library. *ORSA Journal on Computing*, 3(4):376–384, Nov. 1991.

[23] S. Ronald. Preventing Diversity Loss in a Routing Genetic Algorithm with Hash Tagging. *Complexity International*, 2, 1995.

[24] C. Segura, C. Coello, E. Segredo, G. Miranda, and C. León. Improving the diversity preservation of multi-objective approaches used for single-objective optimization. In *2013 IEEE Congress on Evolutionary Computation (CEC)*, pages 3198–3205, June 2013.

[25] C. Segura, C. A. Coello Coello, G. Miranda, and C. León. Using multi-objective evolutionary algorithms for single-objective optimization. *4OR*, 11(3):201–228, 2013.

[26] C. Segura, C. A. Coello Coello, E. Segredo, and C. León. An analysis of the automatic adaptation of the crossover rate in differential evolution. In *2014 IEEE Congress on Evolutionary Computation (CEC'14)*, pages 459–466, July 2014.

[27] T. Starkweather, S. McDaniel, K. Mathias, D. Whitley, and C. Whitley. A comparison of genetic sequencing operators. In R. Belew and L. Booker, editors, *Proceedings of the Fourth International Conference on Genetic Algorithms*, pages 69–76, San Mateo, CA, 1991. Morgan Kaufman.

[28] E.-G. Talbi. *Metaheuristics: From Design to Implementation*. Wiley Publishing, 2009.

[29] C.-K. Ting. Improving edge recombination through alternate inheritance and greedy manner. In J. Gottlieb and G. Raidl, editors, *Evolutionary Computation in Combinatorial Optimization*, volume 3004 of *Lecture Notes in Computer Science*, pages 210–219. Springer Berlin Heidelberg, 2004.

[30] H.-K. Tsai, J.-M. Yang, Y.-F. Tsai, and C.-Y. Kao. An evolutionary algorithm for large traveling salesman problems. *Trans. Sys. Man Cyber. Part B*, 34(4):1718–1729, Aug. 2004.

[31] M. Črepinšek, S.-H. Liu, and M. Mernik. Exploration and exploitation in evolutionary algorithms: A survey. *ACM Computing Surveys*, 45(3):35:1–35:33, July 2013.

[32] L. D. Whitley, T. Starkweather, and D. Fuquay. Scheduling problems and traveling salesmen: The genetic edge recombination operator. In *Proceedings of the 3rd International Conference on Genetic Algorithms*, pages 133–140, San Francisco, CA, USA, 1989. Morgan Kaufmann Publishers Inc.