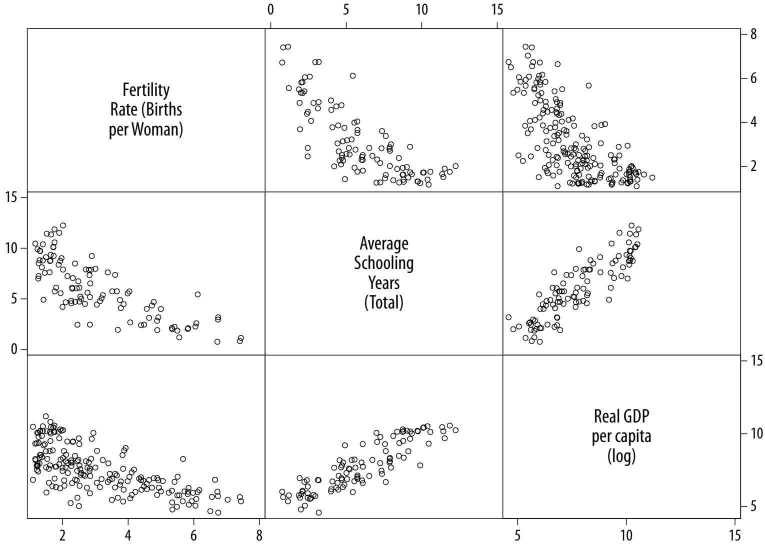
Linear regression

1. Multiple linear regression
2. Standardized coefficients
3. Regression dummies
4. Regression diagnostics
5. Final paper


Fertility Rate (Births per Woman)
Average Schooling Years (Total)
Real GDP per capita (log)
0
5
10
15
8
6
4
2
15
10
5
0
15
10
5
2
4
6
8
5
10
15

Multiple linear regression

```
. reg births schooling log_gdpc

      Source |       SS           df       MS      Number of obs   =        86
-------------+----------------------------------   F(2, 83)        =     88.51
       Model |  150.301883         2  75.1509417   Prob > F        =    0.0000
    Residual |   70.475313        83  .849100157   R-squared       =    0.6808
-------------+----------------------------------   Adj R-squared   =    0.6731
       Total |  220.777196        85  2.59737878   Root MSE        =    .92147

------------------------------------------------------------------------------
      births |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
   schooling |  -.1976117   .0724595    -2.73   0.008    -.3417306   -.0534927
    log_gdpc |  -.4703416   .1324501    -3.55   0.001    -.7337796   -.2069036
       _cons |   7.950304   .6861182    11.59   0.000     6.585642    9.314965
------------------------------------------------------------------------------
```

Multiple linear regression

$$Y = \alpha + \beta_1 X_1 + \beta_2 X_2 +, \ldots, + \beta_k X_k + \epsilon$$

Partial derivatives

Each coefficient is calculated by holding all others constant.

Least squares

The model is still optimized by minimizing the squared error terms.

Sanity check

The model is still assuming *linear*, *additive* relationships.

Logarithmic coefficients: see UCLA mini-guide

Linear-linear relationships: $Y = \alpha + \beta_1 X$

An increase of one unit of X is associated with an increase of β_1 units of Y.

Log-linear relationships: $\ln Y = \alpha + \beta_1 X$

An increase of one unit of X is associated with a $100 \times \beta_1\%$ increase in Y (true effect: $Y \times$ `exp`(β_1)).

Linear-log relationships: $Y = \alpha + \beta_1 \ln X$

A 1% increase in X is associated with a $0.01 \times \beta_1$ unit increase in Y (e.g. $\beta_1 \times$ `log(1.15)` for +15% in X).

Log-log relationships: $\ln Y = \alpha + \beta_1 \ln X$

A 1% increase in X is associated with a $\beta_1\%$ increase in Y.

reg births schooling log_gdpc, beta

Each variable can be normalized to fit $\mathcal{D} \sim \mathcal{N}(0,1)$, so that their standardized coefficients have comparable standard deviation units:

```
─────────────┬──────────────────────────────────────────────────────────────
      births │      Coef.   Std. Err.      t    P>|t|                  Beta
─────────────┼──────────────────────────────────────────────────────────────
   schooling │  -.1976117   .0724595    -2.73   0.008             -.3686479
    log_gdpc │  -.4703416   .1324501    -3.55   0.001             -.4800156
       _cons │   7.950304   .6861182    11.59   0.000                     .
─────────────┴──────────────────────────────────────────────────────────────
```

(identical output for overall model fit omitted)

Sanity check

Interpret unstandardized coefficients; use standardization only for model comparisons.

Regression dummies and categorical predictors

Single coefficient of dummy X_3

$$
\begin{aligned}
Y &= \alpha + \beta_1 X_1 + \beta_2 X_2 + \beta_3(0) + \epsilon \\
Y &= \alpha + \beta_1 X_1 + \beta_2 X_2 + \beta_3(1) + \epsilon
\end{aligned}
$$

The omitted category $X_3 = 0$ is called the reference category and is part of the baseline model $Y = \alpha$, for which all coefficients are null.

Example

$$
\begin{aligned}
\mathit{Income} &= \alpha + \beta_1 \cdot \text{age} + \beta_2 \cdot \text{education} + 0 \cdot \text{male} \qquad + \epsilon \\
\mathit{Income} &= \alpha + \beta_1 \cdot \text{age} + \beta_2 \cdot \text{education} + 1 \cdot \text{female} \qquad + \epsilon
\end{aligned}
$$

reg births schooling log_gdpc i.region

Categorical variables can be used as dummies, i.e. binary recodes of each category that are tested against a reference category to provide regression coefficients for the net effect of each category:

```
------------------------------------------------------------------------------
      births |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
   schooling |  -.0415563   .0639718    -0.65   0.518    -.1688888    .0857763
    log_gdpc |   -.742187   .1380037    -5.38   0.000    -1.016876   -.4674975
             |
      region |
          2  |  -.6523485   .5803126    -1.12   0.264    -1.807432    .5027349
          3  |   .3682404    .254364     1.45   0.152    -.1380585    .8745393
          4  |   1.411177   .2486027     5.68   0.000     .9163457    1.906008
          5  |   1.167491    .337383     3.46   0.001     .4959471    1.839035
             |
       _cons |   8.315004   .8006456    10.39   0.000     6.721359    9.908649
------------------------------------------------------------------------------
```

(identical output for overall model fit omitted)

Regression diagnostics

Residuals

- `predict yhat`: fitted values
- `predict r, resid`: residuals
- `predict r, rsta`: standardized residuals

Use `rvfplot` for residuals-versus-fitted values plots.

Heteroskedasticity

When the residuals are not normally distributed, the model expresses heterogeneous variance (unreliable standard errors).

Examples: UCLA *Regression with Stata*, ch. 2

There are many more diagnostics on display there.

Regression diagnostics

Interaction terms

- Use `vif` to detect variables with $VIF > 10$
- Use # and ## to capture interactions
- Use `c.` to interact continuous variables

(See also `avplot` and partial regression.)

Variance inflation

Variables that strongly interact induce multicollinearity ‘inside’ your model, making standard errors unreliable.

Examples: UCLA Stata FAQ

Search for “comparing coefficients across groups”.

Final paper

Univariate statistics

- Introduction
- Datasets
- Distributions
- Estimation

Assignment No. 1

Bivariate statistics

- Significance
- Crosstabs
- Correlation
- Regression

Assignment No. 2

corrected
revised
appended

Statistical modelling

- Basics
- Extensions
- Diagnostics
- Conclusion

Final paper

Final paper

Essential instructions

- Review paper template: `http://goo.gl/7u8oa`
- Respect paragraph limits mentioned in the template
- Improve formatting (styles, citations, fonts. . .)

3-step guide

1. Rewrite from top to bottom
2. Select what you report
3. Balance evidence and analysis

Systematic interpretation

If you forget to interpret your output, you will be thrown into the gaping mouth of the sarlaac that inhabits the Great Pit of Carkoon on planet Tatooine ($p < 0.01$).

And kittens will get hurt.

Systematic references

If you forget to reference your sources, the hideous terror of Cthulhu will arise from the sunken city of R'lyeh to spread the abject curse of the Great Old Ones onto this world ($p < 0.01$).

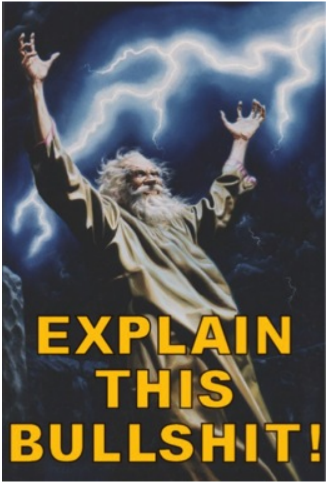
And kittens will get hurt. Again. Different ones.

Systematic proofreading

If you forget to proofread your work, a gigantic hole might open in the earth under your feet, and you might burn in the flames of the monstrous Moloc'h for eternity ($p < 0.01$).

And your graders will get angry at their laptops.

All remaining kittens will be decimated without any sign of human mercifulness.



Thanks for your attention

Final paper

- Name your paper and do-file like Briatte_Petev_final
- Make sure to send your paper as a PDF document
- Email both with subject “SRQM: Final paper, *[your names]*”
- Do not forget to copy your partner on all emails

Readings

- Paper template (download again)
- *Stata Guide*, Sec. 11 and 13–16

THIS
IS
STATA
Warner Bros

Congratulations, and thank you.

```
exit, clear
```