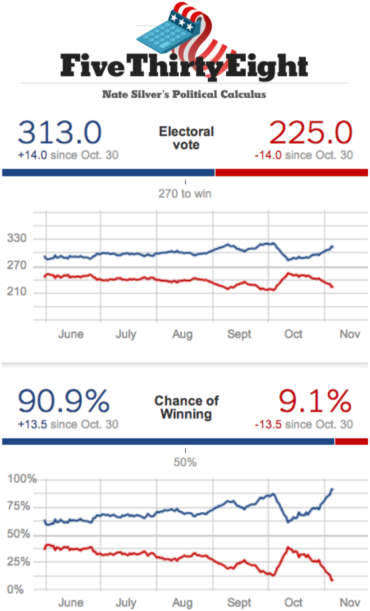
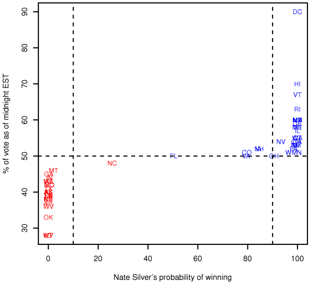
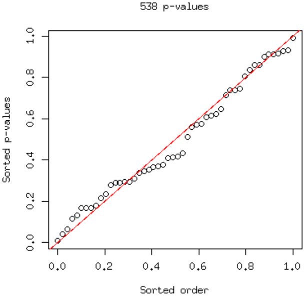
REGRESSION

1. A simple linear model
2. Ordinary Least Squares (OLS)
3. Regression output
4. Practice

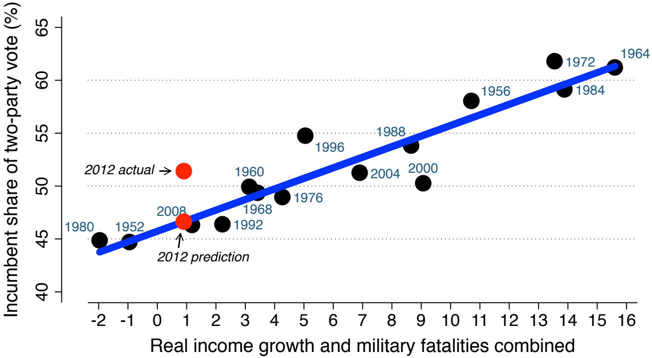

FiveThirtyEight
Nate Silver's Political Calculus
313.0
+14.0 since Oct. 30
Electoral vote
225.0
-14.0 since Oct. 30
270 to win
330
270
210
June
July
Aug
Sept
Oct
Nov
90.9%
+13.5 since Oct. 30
Chance of Winning
9.1%
-13.5 since Oct. 30
50%
100%
75%
50%
25%
0%
June
July
Aug
Sept
Oct
Nov


% of vote as of midnight EST
Nate Silver's probability of winning
DC
HI
VT
RI
NV
NH
CO
VA
FL
OH
NC
MT
OK
WV
WY
0
20
40
60
80
100
30
40
50
60
70
80
90


538 p-values
Sorted p-values
Sorted order
0.0
0.2
0.4
0.6
0.8
1.0

Bread and Peace Voting in 2012

Actual and predicted values in postwar perspective



Combination of real growth and fatalities weights each variable by its estimated coefficient.
Estimated effects of fatalities on vote shares: -0.7% in 2008 (Iraq), -7.4% in 1968 (Vietnam),
-9.7% in 1952 (Korea); negligible in 1964,1976, 2004, 2012, and null in other years.
Source: www.douglas-hibbs.com November 11 2012

To what extent can trust in government be predicted from variations in economic growth?

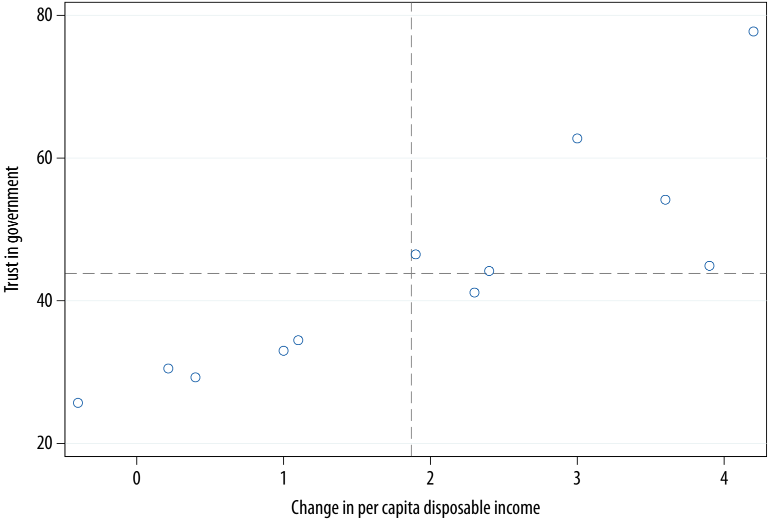
DV: Trust in government

"Just about always/Most of the time"
(American National Election Studies)

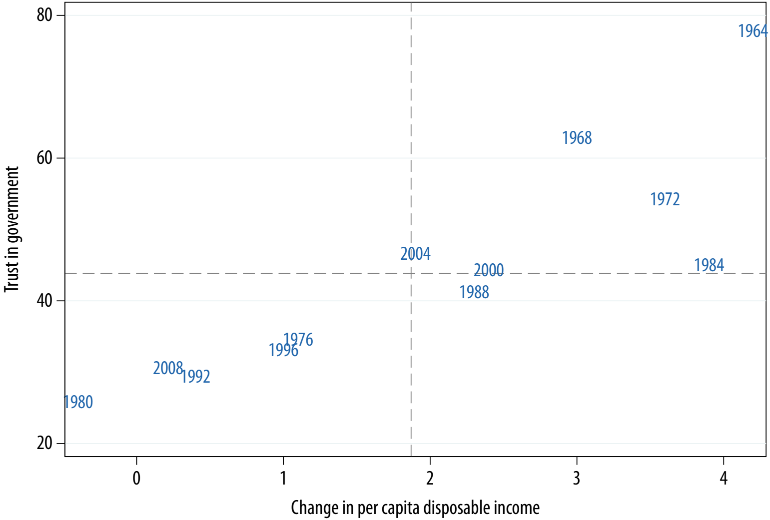
IV: Economic performance

Change in per capita disposable income
(Bureau of Economic Analysis)

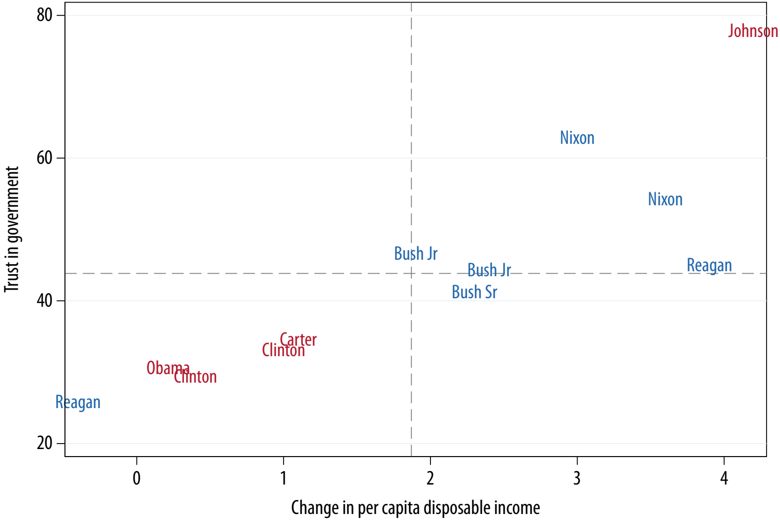
Example and data provided by John Sides.



Dashed lines at averages. Pearson correlation $\rho = .86$ significant at $p < .01$.



Dashed lines at averages. Pearson correlation $\rho = .86$ significant at $p < .01$.



Dashed lines at averages. Pearson correlation $\rho = .86$ significant at $p < .01$.

Simple linear regression

Equations

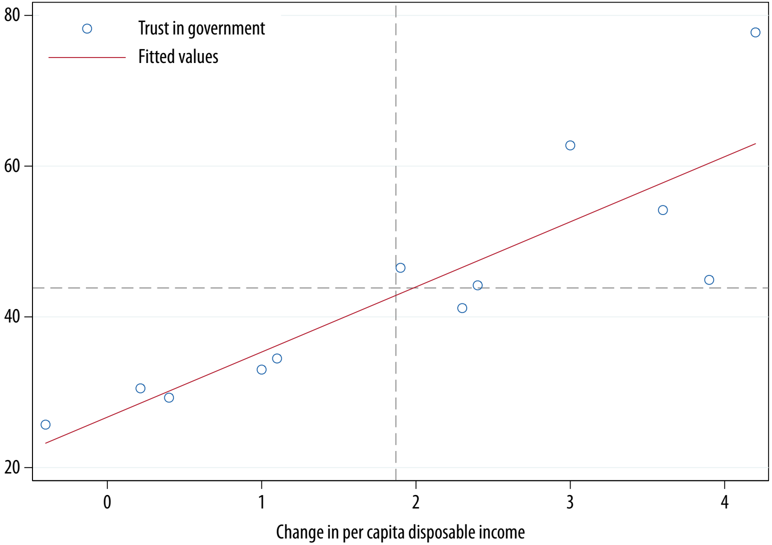
$Y = \alpha + \beta X + \epsilon \quad \hat{Y} = \hat{\alpha} + \hat{\beta} X + \hat{\epsilon} \quad \epsilon = Y - \hat{Y}$

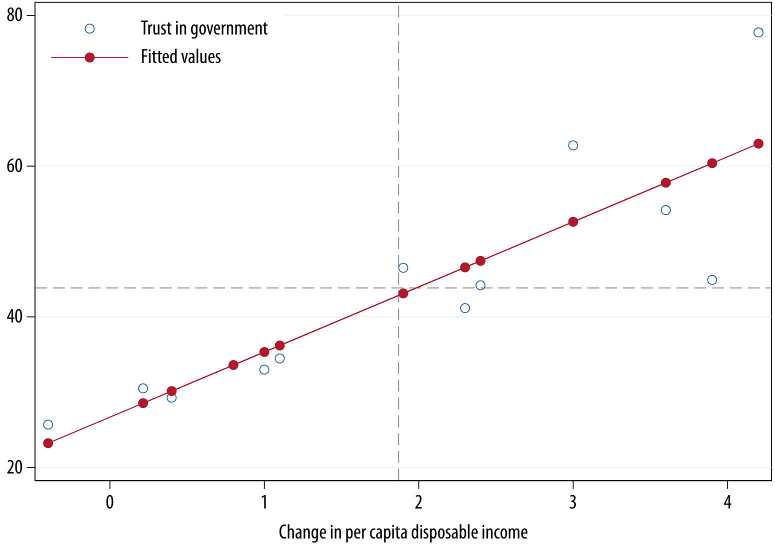
Parameters

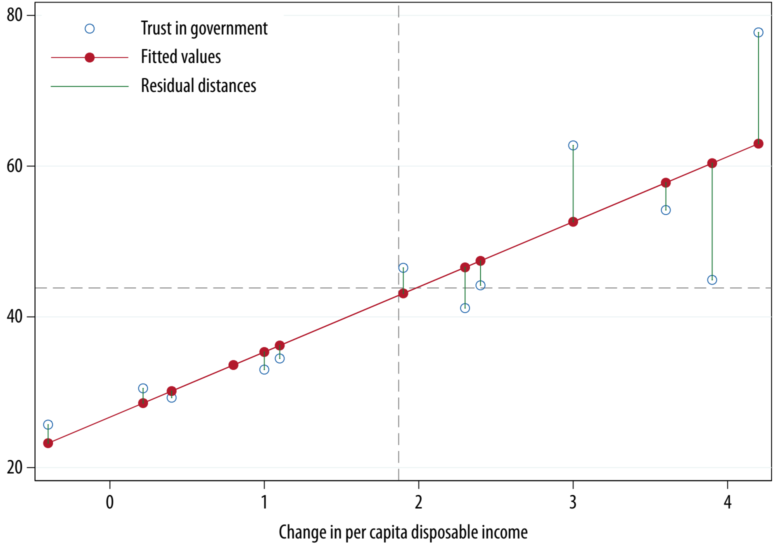
- Y is the dependent variable and $\hat{Y}$ its predicted value
- X is the independent variable used as a predictor of Y
- α is the constant (intercept)
- β is the regression coefficient (slope)
- ϵ is the error term (residuals)

Warning

The model assumes a *linear*, *additive* relationship.


Trust in government
Fitted values
80
60
40
20
0
1
2
3
4
Change in per capita disposable income


Trust in government
Fitted values
80
60
40
20
0
1
2
3
4
Change in per capita disposable income



Trust in government
Fitted values
Residual distances
80
60
40
20
0
1
2
3
4
Change in per capita disposable income

Ordinary Least Squares (OLS)

Error term

In a simple linear model $Y = \alpha + \beta X + \epsilon$, the regression coefficient β is calculated as to minimize the residual sum of squares

$$RSS = \sum_{i=1}^{n} (Y_i - \hat{Y}_i)^2 = \sum_{i=1}^{n} \epsilon^2$$

where $Y_i - \hat{Y}_i$ is the residual (or error term) of each observation.

Parameter estimation

$$\beta = \frac{\mathrm{Cov}(X, Y)}{\mathrm{Var}_X} = \frac{\sum_{i=1}^{n} (X_i - \bar{X})(Y_i - \bar{Y})}{\sum_{i=1}^{n} (X_i - \bar{X})^2} \quad \alpha = \bar{Y} - \beta \bar{X}$$

reg y x

```
. regress trust income

      Source |       SS           df       MS      Number of obs   =        12
-------------+----------------------------------   F(1, 10)        =     29.64
       Model |  1908.80221         1  1908.80221   Prob > F        =    0.0003
    Residual |  643.906248        10  64.3906248   R-squared       =    0.7478
-------------+----------------------------------   Adj R-squared   =    0.7225
       Total |  2552.70846        11  232.064405   Root MSE        =    8.0244

------------------------------------------------------------------------------
       trust |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      income |   8.639373   1.586767     5.44   0.000     5.103836    12.17491
       _cons |   26.69501   3.888016     6.87   0.000     18.03197    35.35805
------------------------------------------------------------------------------
```

Top left: ANOVA table. Top right: model fit.
Bottom: regression coefficients.

Interpretation of fit

Number of observations N, significance test $H_0 : \beta = 0$, coefficient of determination R^2, root mean square error (RMSE).

```
. regress trust income

      Source |       SS       df       MS              Number of obs =      12
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------------------------------------------------------------------------------
```

Goodness of fit

$$R^2 = 1 - \frac{\sum_{i=1}^{n}(Y_i - \hat{Y}_i)^2}{\sum_{i=1}^{n}(Y_i - \bar{Y}_i)^2} = \frac{\text{residual sum of squares}}{\text{total sum of squares}}$$

As the fit improves, $RSS \to 0$ and $R^2 \to 1$.

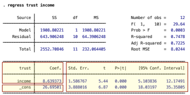
```
Number of obs =      12
F(  1,    10) =   29.64
Prob > F      =  0.0003
R-squared     =  0.7478
Adj R-squared =  0.7225
Root MSE      =  8.0244
```

Sanity check

Focus on getting N and the RMSE right.

Interpretation of regression coefficients

A regression coefficient estimates the variation in Y predicted by a change in one unit of X (recall that $Y = \alpha + \beta X + \epsilon$)



```
. regress trust income

      Source |       SS       df       MS              Number of obs =      12
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------------------------------------------------------------------------------
       trust |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      income |   8.639373   1.586767     5.44   0.000     5.103836    12.17491
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------------------------------------------------------------------------------
```

trust	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
income	8.639373	1.586767	5.44	0.000	5.103836	12.17491
_cons	26.69501	3.888016	6.87	0.000	18.03197	35.35805

- The coefficient is the slope β of the regression line and the constant is its intercept, the coordinate of origin $\alpha = \hat{Y}_{X=0}$.
- The standard error, t-value and p-value test whether the coefficient is significantly different from 0.

Logarithmic coefficients: see UCLA mini-guide

Linear-linear relationships: $Y = \alpha + \beta X$

An increase of one unit of X is associated with an increase of β_1 units of Y.

Log-linear relationships: $\ln Y = \alpha + \beta X$

An increase of one unit of X is associated with a $100 \times \beta_1$% increase in Y (true effect: $Y \times$ `exp`(β_1)).

Linear-log relationships: $Y = \alpha + \beta \ln X$

A 1% increase in X is associated with a $0.01 \times \beta_1$ unit increase in Y (e.g. $\beta_1 \times$ `log(1.15)` for +15% in X).

Log-log relationships: $\ln Y = \alpha + \beta \ln X$

A 1% increase in X is associated with a β_1% increase in Y.

Factor coefficients

```
reg trust income i.republican
```

- For Democrat presidents, `republican == 0`

$$Y = \alpha + \beta X_0 = \alpha$$

- For Republican presidents, `republican == 1`

$$Y = \alpha + \beta X_1$$

Dummies in regression equations

- The first category (0) is used as the 'baseline' category, which is omitted.
- The same logic applies to any categorical variable passed to `reg` with `i.`

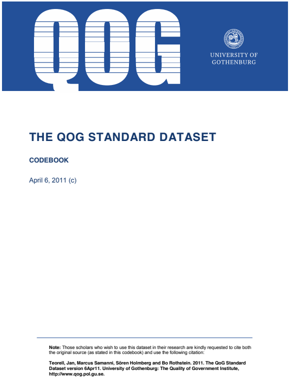
Practice: QOG dataset

Data:

- Quality of Government (QOG)
- Sample: countries, c. 2002

Variables:

- Fertility rate
- Education years
- Corruption Perceptions Index
- Human Development Index
- Female ministers


QOG

UNIVERSITY OF GOTHENBURG

THE QOG STANDARD DATASET

CODEBOOK

April 6, 2011 (c)

Note: Those scholars who wish to use this dataset in their research are kindly requested to cite both the original source (as stated in this codebook) and use the following citation:

Teorell, Jan, Marcus Samanni, Sören Holmberg and Bo Rothstein. 2011. The QoG Standard Dataset version 6Apr11. University of Gothenburg: The Quality of Government Institute, http://www.qog.pol.gu.se.

Practice session

Class

```
* Get the do-file for this week.
srqm fetch week8.do
* Open to read and replicate.
doedit code/week8
```

Coursework

- Finish the do-file and read all comments at home.
- Correct your do-file and add significance tests.
- Correct your paper and substantiate its hypotheses.

Exercise

Ex 8.1. Quality of Government 2011

- Variables: `d wdi_gdpc wdi_mege wdi_pb2 wdi_the`
- Plot correlations and estimate simple linear regressions.

Ex 8.2. Quality of Government 2011

- Variables: `d wdi_pb2 gol_polreg`
- Plot correlations and estimate simple linear regressions, using `wdi_pb2`