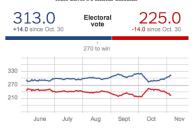
# Simple regression

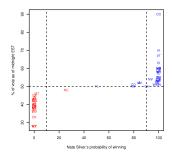
- 1 A simple linear model
- 2 Ordinary Least Squares (OLS)
- 3 Regression output
- 4 Draft No. 2

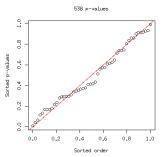


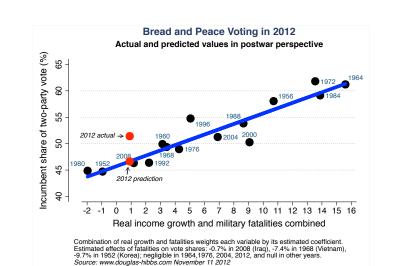
Nate Silver's Political Calculus











To what extent can trust in government be predicted from variations in economic growth?

DV: Trust in government

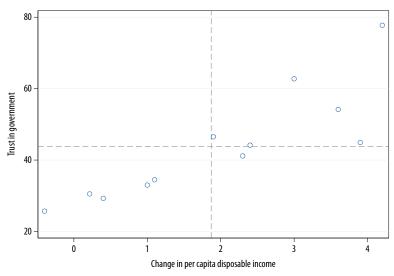
"Just about always/Most of the time" (American National Election Studies)

IV: Economic performance

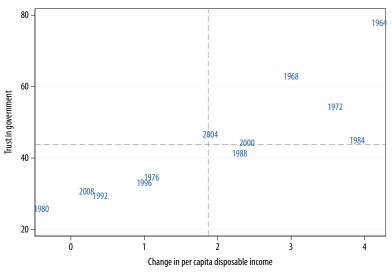
Change in per capita disposable income (Bureau of Economic Analysis)

Example and data provided by John Sides.

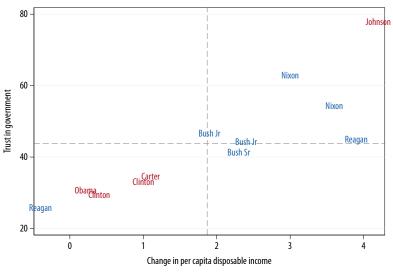




Dashed lines at averages. Pearson correlation  $\rho = .86$  significant at p < .01.



Dashed lines at averages. Pearson correlation  $\rho = .86$  significant at p < .01.



Dashed lines at averages. Pearson correlation  $\rho = .86$  significant at p < .01.

# Simple linear regression

## Equations

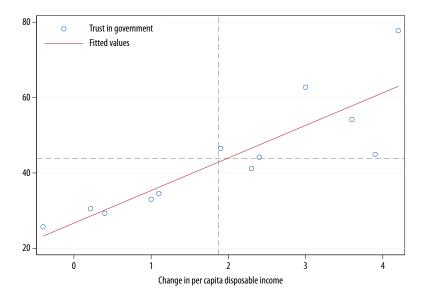
$$Y = \alpha + \beta X + \epsilon$$
  $\hat{Y} = \hat{\alpha} + \hat{\beta} X + \hat{\epsilon}$   $\epsilon = Y - \hat{Y}$ 

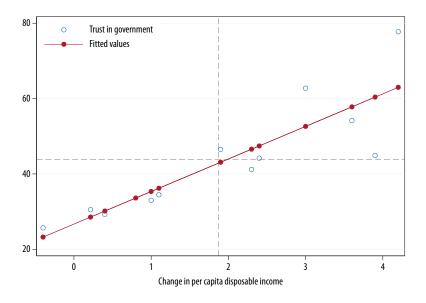
#### **Parameters**

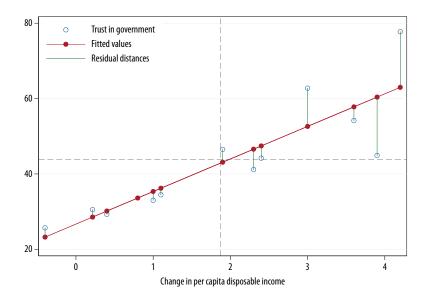
- lacksquare Y is the dependent variable and  $\hat{Y}$  its predicted value
- $\blacksquare$  X is the independent variable used as a predictor of Y
- $\blacksquare$   $\alpha$  is the constant (intercept)
- $\blacksquare$   $\beta$  is the regression coefficient (slope)
- $\bullet$  is the error term (residuals)

# Warning

The model assumes a linear, additive relationship.







# Ordinary Least Squares (OLS)

#### Error term

In a simple linear model  $Y=\alpha+\beta X+\epsilon$ , the regression coefficient  $\beta$  is calculated as to minimize the residual sum of squares

$$RSS = \sum_{i=1}^{n} (Y_i - \hat{Y}_i)^2 = \sum_{i=1}^{n} \epsilon^2$$

where  $Y_i - \hat{Y}_i$  is the residual (or error term) of each observation.

#### Parameter estimation

$$\beta = \frac{\mathsf{Cov}(X,Y)}{\mathsf{Var}_X} = \frac{\sum_{i=1}^n (X_i - \bar{X})(Y_i - \bar{Y})}{\sum_{i=1}^n (X_i - \bar{X})^2} \quad \alpha = \bar{Y} - \beta \bar{X}$$

#### reg y x

#### . regress trust income

Source	ource SS		MS	
Model Residual	1908.80221 643.906248	1 10	1908.80221 64.3906248	
Total	2552.70846	11	232.064405	

Number of obs = 12 F( 1, 10) = 29.64 Prob > F = 0.0003 R-squared = 0.7478 Adj R-squared = 0.7225 Root MSE = 8.0244

trust	Coef.	Std. Err.	t	P> t	[95% Conf.	Interval]
income	i	1.586767	5.44	0.000	5.103836	12.17491
_cons		3.888016	6.87	0.000	18.03197	35.35805

Top left: ANOVA table. Top right: model fit.

Bottom: regression coefficients.

# Interpretation of fit

Number of observations N, significance test  $H_0: \beta = 0$ , coefficient of determination  $R^2$ , root mean square error (RMSE).

Saurce	33	41	765.		Busher of one	
Model.	1988,88221	1.1	988,86223		F( 1, 10)	· 29.7
Residual	641,10020	30.0	4,3101201		R-squared	+ 6.76
Total	2112.79888	33 2	12.00000		Adj R-squares Next MSE	- 8.72 - 8.63
tows	Coef.	114, 14	v. 1	Pr151	late Cost.	

#### Goodness of fit

$$R^2=1-rac{\sum_{i=1}^n(\mathbf{Y}_i-\hat{Y}_i)^2}{\sum_{i=1}^n(\mathbf{Y}_i-ar{Y}_i)^2}=rac{ ext{residual sum of squares}}{ ext{total sum of squares}}$$

As the fit improves,  $RSS \rightarrow 0$  and  $R^2 \rightarrow 1$ .

Number of	obs	=	12
F( 1,	10)	=	29.64
Prob > F		=	0.0003
R-squared		=	0.7478
4 11' D			0 7005

Adj R-squared = 0.7225Root MSE = 8.0244

# Sanity check

Focus on getting  ${\it N}$  and the RMSE right.

# Interpretation of regression coefficients

A regression coefficient estimates the variation in Y predicted by a change in one unit of X (recall that  $Y = \alpha + \beta X + \epsilon$ )



trust	Coef.	Std. Err.	t	P> t	[95% Conf.	Interval]
income	8.639373	1.586767	5.44	0.000	5.103836	12.17491
_cons	26.69501	3.888016	6.87	0.000	18.03197	35.35805

- The coefficient is the slope  $\beta$  of the regression line and the constant is its intercept, the coordinate of origin  $\alpha = \hat{Y}_{X=0}$ .
- The standard error, *t*-value and *p*-value test whether the coefficient is significantly different from 0.

# Logarithmic coefficients: see UCLA mini-guide

# Linear-linear relationships: $Y = \alpha + \beta X$

An increase of one unit of X is associated with an increase of  $\beta_1$  units of Y.

# Log-linear relationships: In $Y = \alpha + \beta X$

An increase of one unit of X is associated with a  $100 \times \beta_1\%$  increase in Y (true effect:  $Y \times \exp(\beta_1)$ ).

# Linear-log relationships: $Y = \alpha + \beta \ln X$

A 1% increase in X is associated with a  $0.01 \times \beta_1$  unit increase in Y (e.g.  $\beta_1 \times \log(1.15)$  for +15% in X).

# Log-log relationships: $\ln Y = \alpha + \beta \ln X$

A 1% increase in X is associated with a  $\beta_1$ % increase in Y.

#### Where we are now

# Univariate statistics

- Introduction
- Dataset
- Variables

Assignment No. 1

corrected revised appended

# Bivariate statistics

- Associations
- Correlations
- Simple OLS

Assignment No. 2



# Statistical modelling

- Regressions
- Diagnostics
- Conclusion

Final paper



#### Essential instructions

#### Revise Draft No. 1

- go through corrections
- remove technical content
- rewrite until concision

Pay attention to paragraph limits and scientific style (esp. sources).

# Explore associations

- between DV and IVs (covariates, controls), or between two IVs
- with graphs and then with significance tests

Write up substantive results as sentences; cite significance tests and other statistics in brackets, e.g. ( $\rho = .7, p < .05$ ).

# Structure and style

#### Paper template



### White (2005)

LYNN WHITE University of Nebrusko-Lincoln

Writes of Passage: Writing an Empirical Journal Article

This article provides adults about preparing research reports for administors by prefusional journals in general and Jeurnal of Marriage and Family in particular. In addition to working through all the major parts of a research paper, particle and present administration of a research paper, and revising. The article is intended to they are prefusionate improve the quality of their journal ambituitions and the liberthood of macrospid publication.

Writing research articles for professional jourrals is an art requiring good research skills, a clear sense of problem, and strong writing and editing skills. Assuming that years of graduate school have provided good research skills, I focus on the other requirements of writing a research article. My advice reflects the issues I most often raise when I review articles and 30 years of experience writing (and revising) research articles. I review ruidelines for the major sections of the typical empirical research report and conclude with some suggestions about writing professionally. The emphasis is on writing for Journal of Marriage and Family (JMF), but the general principles apply across journals and substantive areas

WORKENG THROUGH A RESEARCH PAPER The format for a research paper is not set in stone. Each research problem is different, and

Department of Sociology, University of Nebrasks—Lincoln, Lincoln, NE 68588-0324 (behin-lighed ada).

Ere Words: research, theory, writing.

Journal of Marriage and Family 67 (November 2005): 791-798

the organization of the paper will depend on whether it is explorately research after than theory testing. In addition, authors have some lastale in developing a present inject. Generally, however, each article model an simedestic, deservable, the contract of the product of the contract of the description of method, results, and conclusion. The organization of the piece, the titles of various sections, and the relative weight of these sections vary from paper to paper and from journal to journal, thu some greated guidelines, apply to the product of the proting of the product of the product

#### . .

An abstract should summaries your study, in a few short sentences, it should state for research hypothesis, the sample, sample size, data used, and the findings. A studing sentence such as "Using data form a national somple of a vossess interviewed by platepone is 2026, we canamic interviewed by platepone is 2026, we canamic structured by the summaries of the sample of the year to species a lot of information into a few years to species a lot of information into a few years to species a lot of information into a few years to species a lot of information in the words. In a lamb consent fashion, without hyperlede or exaggnration, state the findings of fewal of the beauty of the sample of the sample of the state of the sample of the sample of the state of the sample of the s

#### . . . . .

The introduction is critical to capturing the reader's attention and setting the tone for the paper. In approximately a single page, it should specify the research question, the data to be used, and the strengths of the design, and it

### Coursework

## Project

Stata Guide, Sec. 13-15

- Correct and improve Draft No. 1
- Finalize association tests and interpretations
- Name your paper (PDF) and do-file like Briatte\_Petev\_2

# Readings

- Making History Count, ch. 4 (simple regression)
- Stata Guide, Sec. 10 (association)
- Stata Guide, Sec. 11 (correlation and simple regression)

Enjoy your day and see you soon!