# LINEAR REGRESSION

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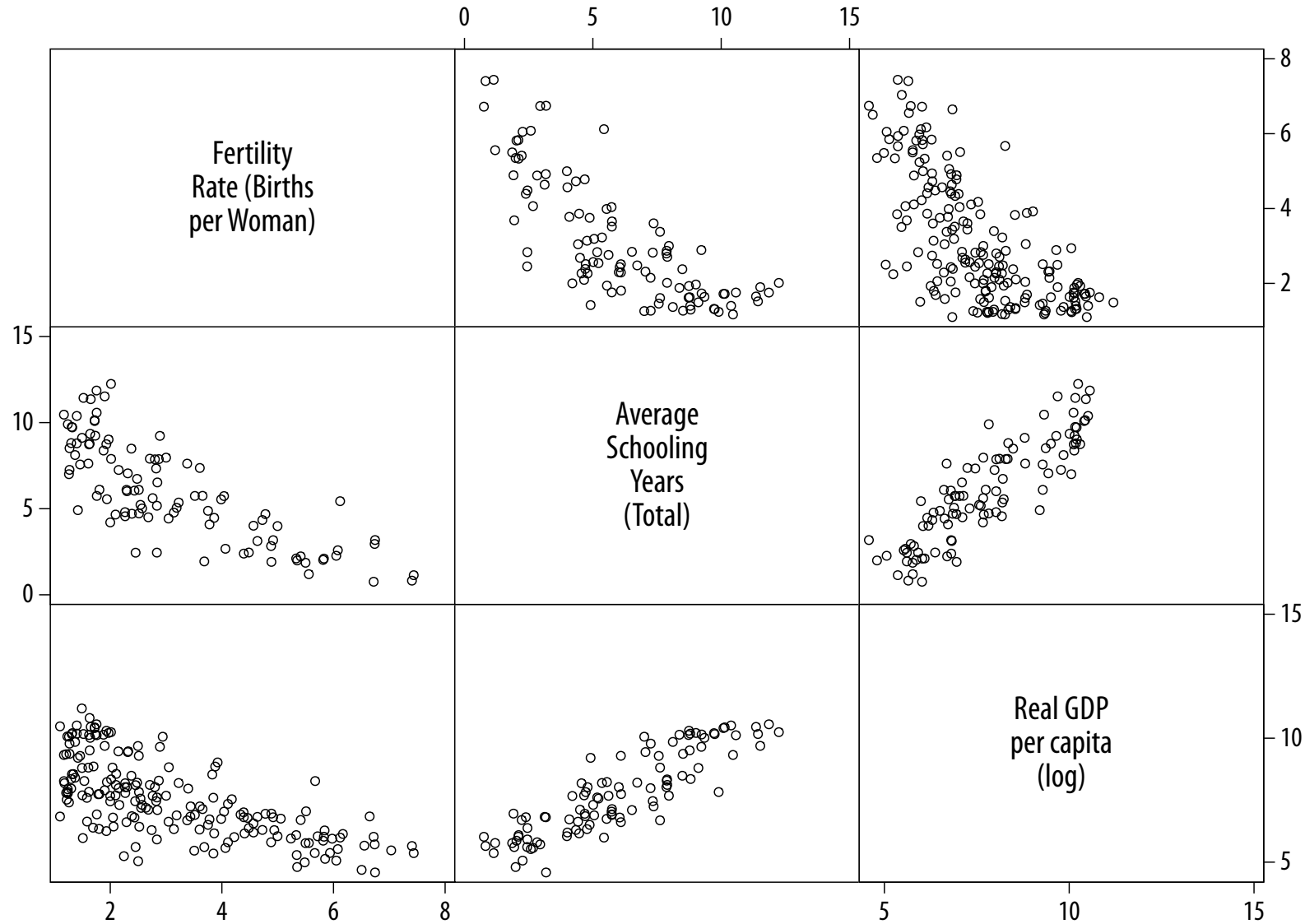
Fertility Rate (Births per Woman)
Average Schooling Years (Total)
Real GDP per capita (log)
0
5
10
15
2
4
6
8
5
10
15

# Multiple linear regression

```
. reg births schooling log_gdpc
```

| Source | SS | df | MS |
|---|---|---|---|
| Model | 150.301883 | 2 | 75.1509417 |
| Residual | 70.475313 | 83 | .849100157 |
| Total | 220.777196 | 85 | 2.59737878 |

| | |
|---|---|
| Number of obs | 86 |
| F( 2, 83) | 88.51 |
| Prob > F | 0.0000 |
| R-squared | 0.6808 |
| Adj R-squared | 0.6731 |
| Root MSE | .92147 |

| births | Coef. | Std. Err. | t | P>\|t\| | [95% Conf. | Interval] |
|---|---|---|---|---|---|---|
| schooling | -.1976117 | .0724595 | -2.73 | 0.008 | -.3417306 | -.0534927 |
| log_gdpc | -.4703416 | .1324501 | -3.55 | 0.001 | -.7337796 | -.2069036 |
| _cons | 7.950304 | .6861182 | 11.59 | 0.000 | 6.585642 | 9.314965 |

# Multiple linear regression

$$Y = \alpha + \beta_1 X_1 + \beta_2 X_2 +, \ldots, + \beta_k X_k + \epsilon$$

## Partial derivatives

Each coefficient is calculated by holding all others constant.

## Least squares

The model is still optimized by minimizing the squared error terms.

## Sanity check

The model is still assuming *linear*, *additive* relationships.

# Logarithmic coefficients: see UCLA mini-guide

## Linear-linear relationships: $Y = \alpha + \beta_1 X$

An increase of one unit of $X$ is associated with an increase of $\beta_1$ units of $Y$.

## Log-linear relationships: $\ln Y = \alpha + \beta_1 X$

An increase of one unit of $X$ is associated with a $100 \times \beta_1$% increase in $Y$ (true effect: $Y \times \exp(\beta_1)$).

## Linear-log relationships: $Y = \alpha + \beta_1 \ln X$

A 1% increase in $X$ is associated with a $0.01 \times \beta_1$ unit increase in $Y$ (e.g. $\beta_1 \times \log(1.15)$ for +15% in $X$).

## Log-log relationships: $\ln Y = \alpha + \beta_1 \ln X$

A 1% increase in $X$ is associated with a $\beta_1$% increase in $Y$.

# `reg births schooling log_gdpc, beta`

Each variable can be normalized to fit $\mathcal{D} \sim \mathcal{N}(0, 1)$, so that their standardized coefficients have comparable standard deviation units:

| births | Coef. | Std. Err. | t | P>\|t\| | Beta |
|---|---|---|---|---|---|
| schooling | -.1976117 | .0724595 | -2.73 | 0.008 | -.3686479 |
| log_gdpc | -.4703416 | .1324501 | -3.55 | 0.001 | -.4800156 |
| _cons | 7.950304 | .6861182 | 11.59 | 0.000 | . |

*(identical output for overall model fit omitted)*

## Sanity check

Interpret unstandardized coefficients; use standardization only for model comparisons.

# Regression dummies and categorical predictors

## Single coefficient of dummy $X_3$

$$Y = \alpha + \beta_1 X_1 + \beta_2 X_2 + \beta_3(0) + \epsilon$$
$$Y = \alpha + \beta_1 X_1 + \beta_2 X_2 + \beta_3(1) + \epsilon$$

The omitted category $X_3 = 0$ is called the reference category and is part of the baseline model $Y = \alpha$, for which all coefficients are null.

## Example

$$\textit{Income} = \alpha + \beta_1 \cdot \text{age} + \beta_2 \cdot \text{education} + 0 \cdot \text{male} + \epsilon$$
$$\textit{Income} = \alpha + \beta_1 \cdot \text{age} + \beta_2 \cdot \text{education} + 1 \cdot \text{female} + \epsilon$$

# `reg births schooling log_gdpc i.region`

Categorical variables can be used as dummies, i.e. binary recodes of each category that are tested against a reference category to provide regression coefficients for the net effect of each category:

| births | Coef. | Std. Err. | t | P>\|t\| | [95% Conf. | Interval] |
|---|---|---|---|---|---|---|
| schooling | -.0415563 | .0639718 | -0.65 | 0.518 | -.1688888 | .0857763 |
| log_gdpc | -.742187 | .1380037 | -5.38 | 0.000 | -1.016876 | -.4674975 |
| region | | | | | | |
| 2 | -.6523485 | .5803126 | -1.12 | 0.264 | -1.807432 | .5027349 |
| 3 | .3682404 | .254364 | 1.45 | 0.152 | -.1380585 | .8745393 |
| 4 | 1.411177 | .2486027 | 5.68 | 0.000 | .9163457 | 1.906008 |
| 5 | 1.167491 | .337383 | 3.46 | 0.001 | .4959471 | 1.839035 |
| _cons | 8.315004 | .8006456 | 10.39 | 0.000 | 6.721359 | 9.908649 |

*(identical output for overall model fit omitted)*

# Regression diagnostics

## Residuals

- `predict yhat`: fitted values
- `predict r, resid`: residuals
- `predict r, rsta`: standardized residuals

Use `rvfplot` for residuals-versus-fitted values plots.

## Heteroskedasticity

When the residuals are not normally distributed, the model expresses heterogeneous variance (unreliable standard errors).

## Examples: UCLA *Regression with Stata*, ch. 2

There are many more diagnostics on display there.

# Regression diagnostics

## Interaction terms

- Use `vif` to detect variables with $VIF > 10$
- Use # and ## to capture interactions
- Use `c.` to interact continuous variables

(See also `avplot` and partial regression.)

## Variance inflation

Variables that strongly interact induce multicollinearity 'inside' your model, making standard errors unreliable.

## Examples: UCLA Stata FAQ

Search for "comparing coefficients across groups".