Regression diagnostics

Statistical Reasoning
and Quantitative Methods

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Session 11

Outline

Diagnostics

Diagnostics

Regression models produce **fitted** (predicted) values and residuals that hold the unexplained variance for each data point.
Issues that arise in that context are:

- **unreliable coefficients** due to multicollinearity, i.e. interactions between independent variables
- **unreliable significance tests** due to heteroskedasticity, i.e. heterogeneous variance in the residuals
- **unreliable predictions** due to outliers and influential points in the data that either do not fit or 'overfit' the model

Note: the model still assumes a **linear, additive** relationship between Y and $X_1, X_2, \ldots X_k$. That assumption can also be violated among other matters.

Fitting a **multiple** linear regression model

The model also fits a **linear function** to the data, of the form:

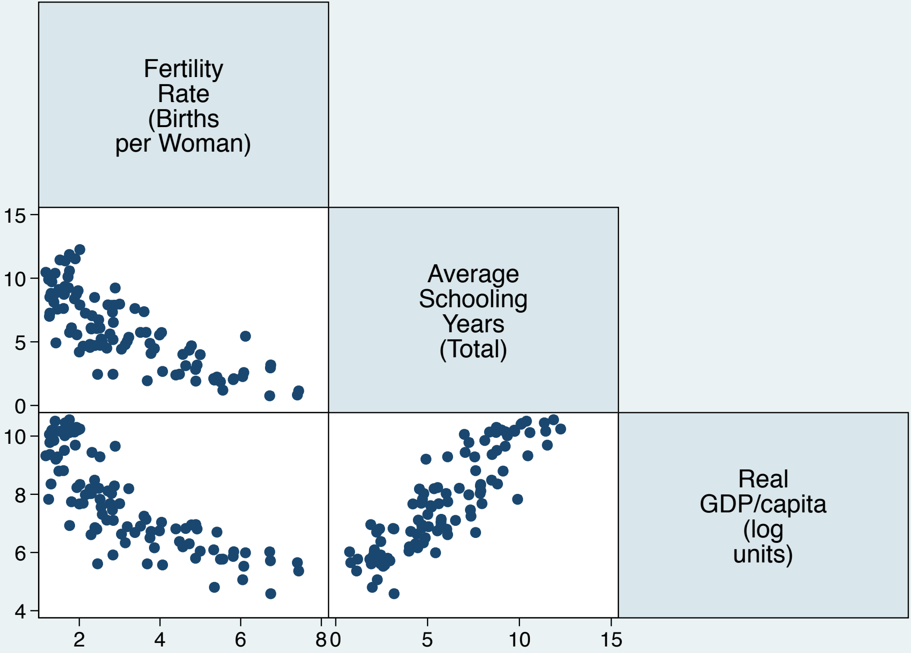
$$Y = \alpha + \beta_1 X_1 + \beta_2 X_2 + \ldots + \beta_k X_k + \epsilon$$

where:

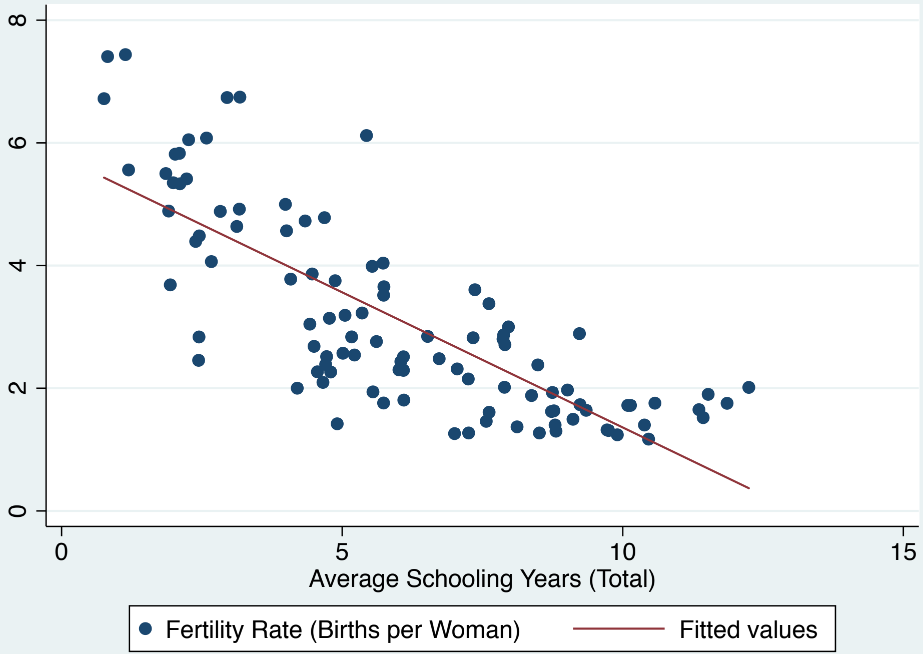
- Y is the **dependent variable** (response)
- X is a **vector** of **independent variables** (predictors)
- α is the **constant**
- $\beta_1 X_1 + \beta_2 X_2 + \ldots + \beta_k X_k$ is a **vector** of **regression coefficients**
- ϵ is the **error term** (residuals)

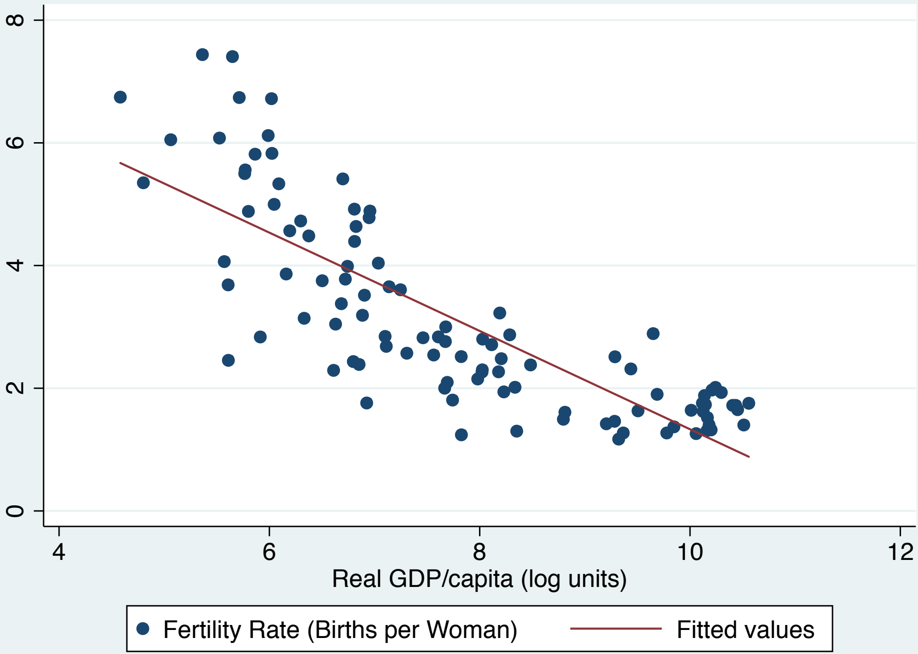
Note: the model assumes that the relationship is **linear** and **additive**.

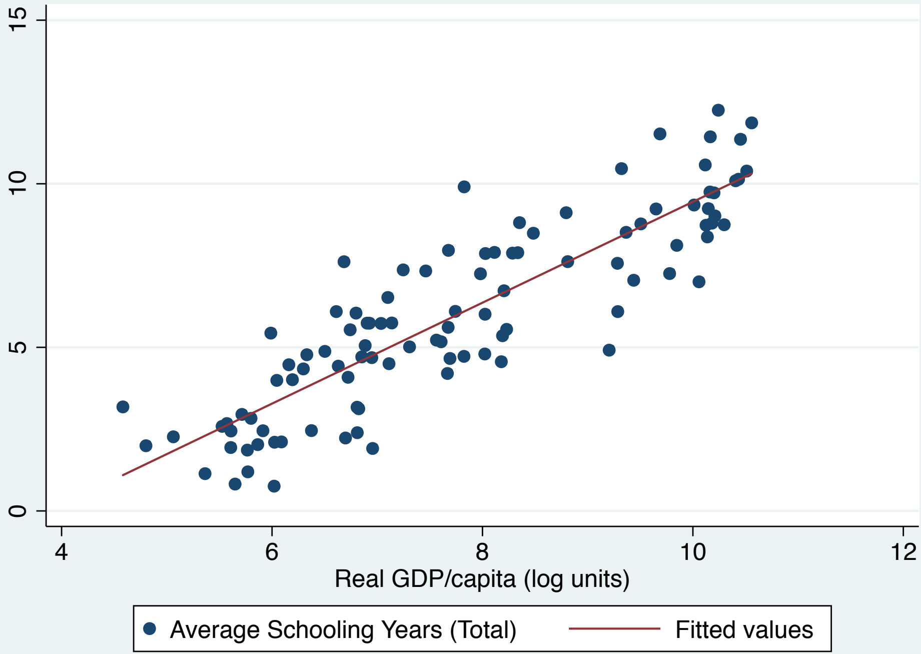
The estimation of regression coefficients in a k-dimensional space is computationally more intensive, but is also based on least squares.



Fertility Rate (Births per Woman)
Average Schooling Years (Total)
Real GDP/capita (log units)
15
10
5
0
10
8
6
4
2
4
6
8
0
5
10
15


0
2
4
6
8
0
5
10
15
Average Schooling Years (Total)
● Fertility Rate (Births per Woman)
Fitted values

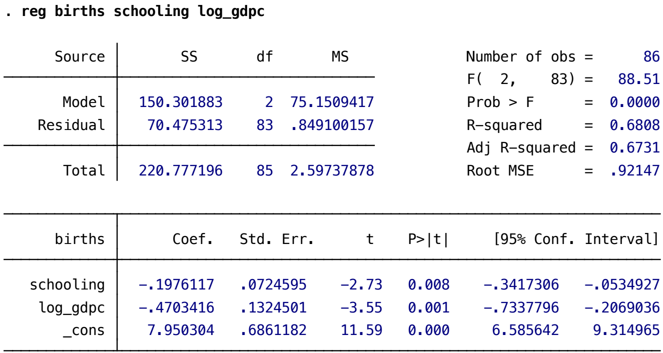

0
2
4
6
8
4
6
8
10
12
Real GDP/capita (log units)
● Fertility Rate (Births per Woman)
Fitted values


0
5
10
15
4
6
8
10
12
Real GDP/capita (log units)
● Average Schooling Years (Total)
Fitted values

Multiple regression output

`reg births schooling log_gdpc`

The `reg` command can take any number of **continuous** variables as arguments, and shows unstandardised coefficients by default, using their original metric and possible transformation:



```
. reg births schooling log_gdpc

      Source |       SS       df       MS              Number of obs =      86
-------------+------------------------------           F(  2,    83) =   88.51
       Model |  150.301883     2  75.1509417           Prob > F      =  0.0000
    Residual |   70.475313    83  .849100157           R-squared     =  0.6808
-------------+------------------------------           Adj R-squared =  0.6731
       Total |  220.777196    85  2.59737878           Root MSE      =  .92147

------------------------------------------------------------------------------
      births |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
   schooling |  -.1976117   .0724595    -2.73   0.008    -.3417306   -.0534927
    log_gdpc |  -.4703416   .1324501    -3.55   0.001    -.7337796   -.2069036
       _cons |   7.950304   .6861182    11.59   0.000     6.585642    9.314965
------------------------------------------------------------------------------
```

Standardised coefficients

```
reg births schooling log_gdpc, beta
```

The `beta` option provides standardised coefficients, which use the **standard deviation of regressors** (or predictor, i.e. the independent variables) in order to provide coefficients with comparable units:

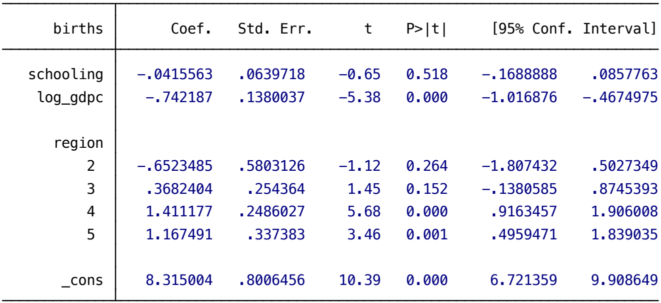
```
------------------------------------------------------------------------------
      births |      Coef.   Std. Err.      t    P>|t|                     Beta
-------------+----------------------------------------------------------------
   schooling |  -.1976117   .0724595    -2.73   0.008                -.3686479
    log_gdpc |  -.4703416   .1324501    -3.55   0.001                -.4800156
       _cons |   7.950304   .6861182    11.59   0.000                        .
------------------------------------------------------------------------------
```

(identical output for overall model fit omitted)

Dummies

```
reg births schooling i.region
```

Categorical variables can be used as dummies, i.e. binary recodes of each category that are tested against a **reference category** to provide regression coefficients for net effect of that category alone:



```
------------------------------------------------------------------------------
      births |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
   schooling |  -.0415563   .0639718    -0.65   0.518    -.1688888    .0857763
    log_gdpc |   -.742187   .1380037    -5.38   0.000    -1.016876   -.4674975
             |
      region |
          2  |  -.6523485   .5803126    -1.12   0.264    -1.807432    .5027349
          3  |   .3682404    .254364     1.45   0.152    -.1380585    .8745393
          4  |   1.411177   .2486027     5.68   0.000     .9163457    1.906008
          5  |   1.167491    .337383     3.46   0.001     .4959471    1.839035
             |
       _cons |   8.315004   .8006456    10.39   0.000     6.721359    9.908649
------------------------------------------------------------------------------
```

(identical output for overall model fit omitted)