Linear regression

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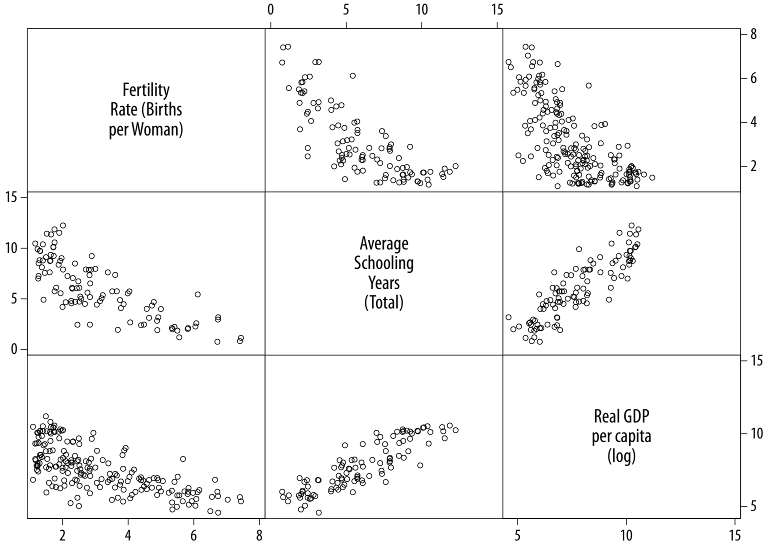

Fertility Rate (Births per Woman)
Average Schooling Years (Total)
Real GDP per capita (log)
0
5
10
15
8
6
4
2
15
10
5
0
15
10
5
2
4
6
8
5
10
15

Multiple linear regression

```
. reg births schooling log_gdpc
```

Source	SS	df	MS
Model	150.301883	2	75.1509417
Residual	70.475313	83	.849100157
Total	220.777196	85	2.59737878

Number of obs =	86
F(2, 83) =	88.51
Prob > F =	0.0000
R-squared =	0.6808
Adj R-squared =	0.6731
Root MSE =	.92147

births	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
schooling	-.1976117	.0724595	-2.73	0.008	-.3417306	-.0534927
log_gdpc	-.4703416	.1324501	-3.55	0.001	-.7337796	-.2069036
_cons	7.950304	.6861182	11.59	0.000	6.585642	9.314965

[-1] or, as my girlfriend calls it, ultimate regression

Multiple linear regression

$$Y = \alpha + \beta_1 X_1 + \beta_2 X_2 +, \ldots, + \beta_k X_k + \epsilon$$

Partial derivatives

Each coefficient is calculated by holding all others constant.

Least squares

The model is still optimized by minimizing the squared error terms.

Sanity check

The model is still assuming *linear*, *additive* relationships.

Logarithmic coefficients: see UCLA mini-guide

Linear-linear relationships: $Y = \alpha + \beta_1 X$

An increase of one unit of X is associated with an increase of β_1 units of Y.

Log-linear relationships: $\ln Y = \alpha + \beta_1 X$

An increase of one unit of X is associated with a $100 \times \beta_1\%$ increase in Y (true effect: $Y \times$ $\exp(\beta_1)$).

Linear-log relationships: $Y = \alpha + \beta_1 \ln X$

A 1% increase in X is associated with a $0.01 \times \beta_1$ unit increase in Y (e.g. $\beta_1 \times$ $\log(1.15)$ for +15% in X).

Log-log relationships: $\ln Y = \alpha + \beta_1 \ln X$

A 1% increase in X is associated with a $\beta_1\%$ increase in Y.

reg births schooling log_gdpc, beta

Each variable can be normalized to fit $\mathcal{D} \sim \mathcal{N}(0, 1)$, so that their standardized coefficients have comparable standard deviation units:

births	Coef.	Std. Err.	t	P>|t|	Beta
schooling	-.1976117	.0724595	-2.73	0.008	-.3686479
log_gdpc	-.4703416	.1324501	-3.55	0.001	-.4800156
_cons	7.950304	.6861182	11.59	0.000	.

(identical output for overall model fit omitted)

Sanity check

Interpret unstandardized coefficients; use standardization only for model comparisons.

Categorical and dummy predictors

Single coefficient of dummy X_3

$$Y = \alpha + \beta_1 X_1 + \beta_2 X_2 + \beta_3(0) + \epsilon$$
$$Y = \alpha + \beta_1 X_1 + \beta_2 X_2 + \beta_3(1) + \epsilon$$

The omitted category $X_3 = 0$ is called the reference category and is part of the baseline model $Y = \alpha$, for which all coefficients are null.

Example

$$Income = \alpha + \beta_1 \cdot \text{age} + \beta_2 \cdot \text{education} + 0 \cdot \text{male} \qquad + \epsilon$$
$$Income = \alpha + \beta_1 \cdot \text{age} + \beta_2 \cdot \text{education} + 1 \cdot \text{female} \qquad + \epsilon$$

reg births schooling log_gdpc i.region

Categorical variables can be used as dummies, i.e. binary recodes of each category that are tested against a reference category to provide regression coefficients for the net effect of each category:

births	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
schooling	-.0415563	.0639718	-0.65	0.518	-.1688888	.0857763
log_gdpc	-.742187	.1380037	-5.38	0.000	-1.016876	-.4674975
region						
2	-.6523485	.5803126	-1.12	0.264	-1.807432	.5027349
3	.3682404	.254364	1.45	0.152	-.1380585	.8745393
4	1.411177	.2486027	5.68	0.000	.9163457	1.906008
5	1.167491	.337383	3.46	0.001	.4959471	1.839035
_cons	8.315004	.8006456	10.39	0.000	6.721359	9.908649

(identical output for overall model fit omitted)

Regression diagnostics

Residuals

- `predict yhat`: fitted values
- `predict r, resid`: residuals
- `predict r, rsta`: standardized residuals

Use `rvfplot` for residuals-versus-fitted values plots.

Interaction terms

- Use `vif` to detect variables with $VIF > 10$
- Use `c.` and # for interactions

Thanks for your attention

Project

- Name your paper and do-file like Briatte_Petev_2
- Make sure to print your paper to a slick PDF

Readings

- *Stata Guide*, Sec. 10, 11, 13 and 15

Practice

- Replicate do-file
- Use its structure for Draft No. 2