Dependence

1. Comparison
2. Chi-squared test
3. *t*-test
4. Correlation

http://f.briatte.org/teaching/quanti/

Statistical comparison

Substantive hypotheses

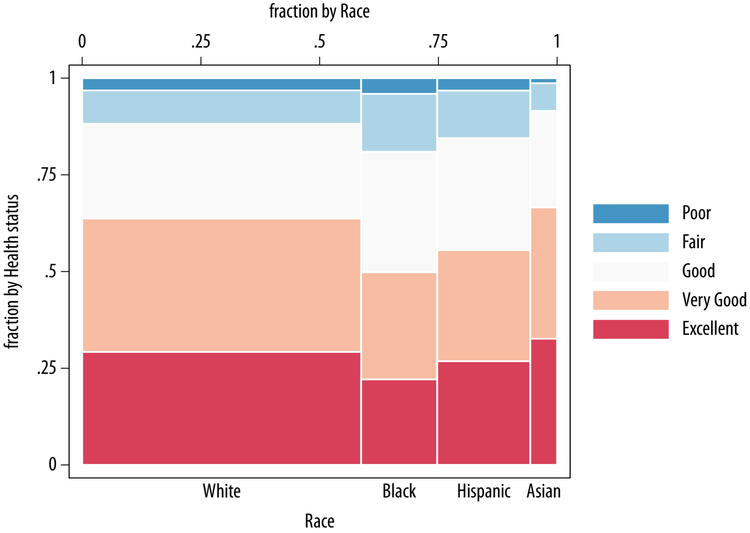
There is an association between X and Y, ...
There is a difference of X between groups of Y, ...

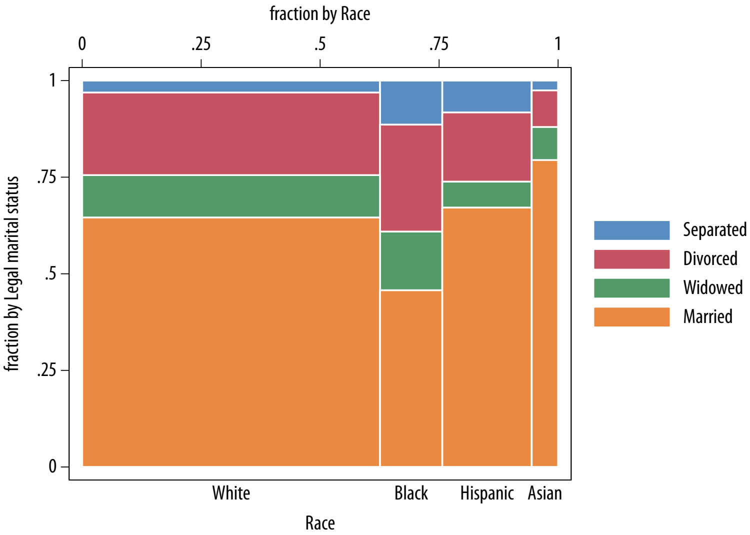
Null hypothesis tests

H_0: the association of X by Y is likely to be random.
H_0: the difference in X between groups of Y is likely to be random.

Rejecting the null

H_0 estimates the likelihood of an association or difference being attributable to sampling error under a certain level of confidence.


fraction by Race
0
.25
.5
.75
1
fraction by Health status
1
.75
.5
.25
0
Poor
Fair
Good
Very Good
Excellent
White
Black
Hispanic
Asian
Race


fraction by Race
0
.25
.5
.75
1
fraction by Legal marital status
1
.75
.5
.25
0
Separated
Divorced
Widowed
Married
White
Black
Hispanic
Asian
Race

Chi-squared test

The Chi-squared test is a nonparametric test of association that measures the deviation in orthogonality between groups:

- Null hypothesis H_0: $\chi^2 = 0$
- Test statistic: $\chi^2 = \sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i}$ (deviation between observed frequencies O_i and expected frequencies E_i for each table cell i)

```
tab v1 v2, exp chi2 V
```

- add `V` to measure the association with Cramér's V ($0 < V < 1$)
- use `tabchi` to inspect residuals and `tabodds` for odds ratios

use datasets/nhis2009, clear

- Variables: `d raceb marstat`
- Inspect frequencies (row and column, expected and observed)
- Run a Chi-squared test and analyze the residuals

```
. tab marstat raceb if marstat < 8, chi2 V

                      |                     Race
Legal marital status  |     White      Black   Hispanic      Asian |     Total
──────────────────────┼────────────────────────────────────────────┼───────────
             Married  |     7,151      1,059      2,231        780 |    11,221
             Widowed  |     1,215        352        223         84 |     1,874
            Divorced  |     2,367        641        595         93 |     3,696
           Separated  |       343        264        274         25 |       906
──────────────────────┼────────────────────────────────────────────┼───────────
               Total  |    11,076      2,316      3,323        982 |    17,697

          Pearson chi2(9) = 733.4437   Pr = 0.000
               Cramér's V =   0.1175
```

t-test

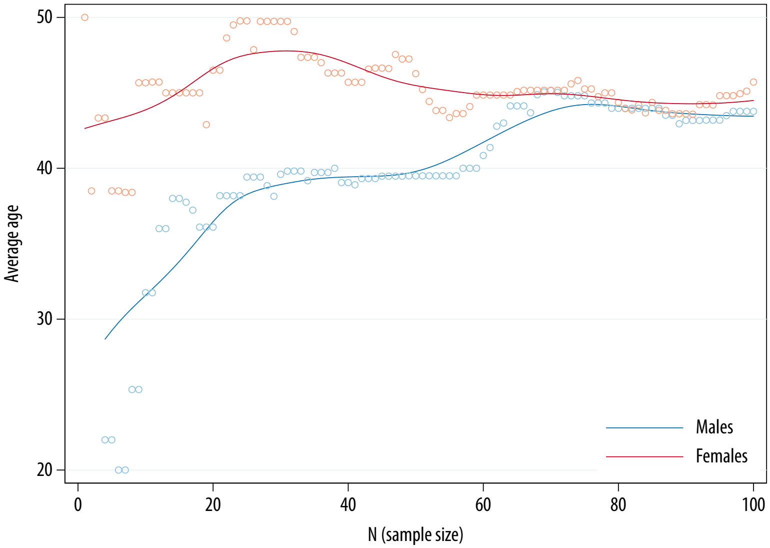
Measuring association as the difference in means between two groups:

- Population notation: $\delta = \mu_1 - \mu_2$
- Sample notation: $D = \bar{X}_1 - \bar{X}_2$

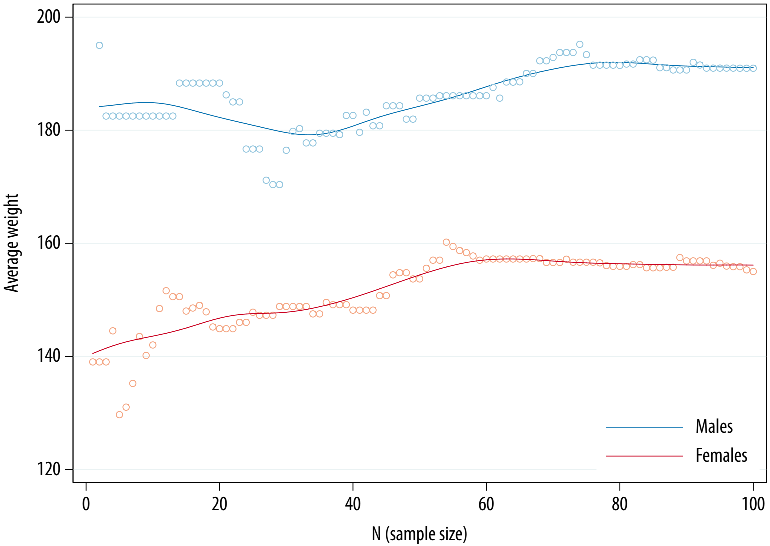
The *t*-test computes a 95% CI around the difference of their means and returns its *p*-value against the *t*-distribution.

- Null hypothesis H_0: $\mu_1 - \mu_2 = 0$
- Test statistic: $t = \frac{D}{SE_D}$

Type I errors



Type II errors



Stata implementation

```
ttest v1, by(v2)
```

- v1 is continuous, v2 is a dummy; for two dummies, use `prtest`
- use `tab, gen()` to create dummies from categorical variables

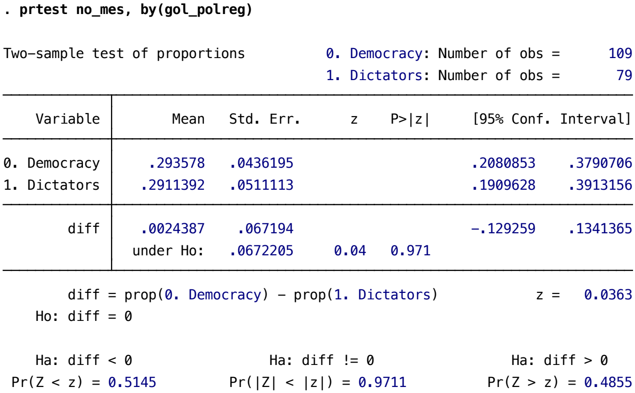
```
use datasets/qog2011, clear
```

- Variables: `d gol_enep gol_est2`
- Create dummies and compare parties across electoral systems.

Stata implementation

```
use datasets/qog2011, clear
```

Explore the variables and interpret the output below.

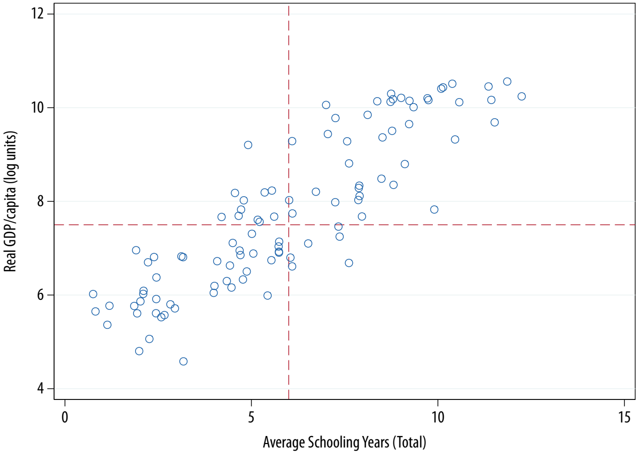


```
. prtest no_mes, by(gol_polreg)

Two-sample test of proportions          0. Democracy: Number of obs =      109
                                        1. Dictators: Number of obs =       79
------------------------------------------------------------------------------
    Variable |       Mean   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
0. Democracy |    .293578   .0436195                      .2080853    .3790706
1. Dictators |   .2911392   .0511113                      .1909628    .3913156
-------------+----------------------------------------------------------------
        diff |   .0024387    .067194                      -.129259    .1341365
             |  under Ho:   .0672205     0.04   0.971
------------------------------------------------------------------------------
        diff = prop(0. Democracy) - prop(1. Dictators)            z =   0.0363
    Ho: diff = 0

    Ha: diff < 0                 Ha: diff != 0                 Ha: diff > 0
 Pr(Z < z) = 0.5145         Pr(|Z| < |z|) = 0.9711          Pr(Z > z) = 0.4855
```

Correlation



Pearson correlation coefficient

Measuring association as the linear dependence of two variables:

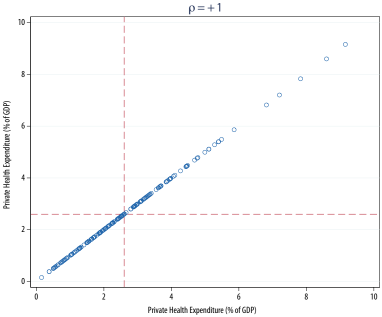
Population notation $\rho = \frac{\mathrm{Cov}(X, Y)}{\mathrm{Var}_X \mathrm{Var}_Y}, \quad -1 \leq \rho \leq 1$

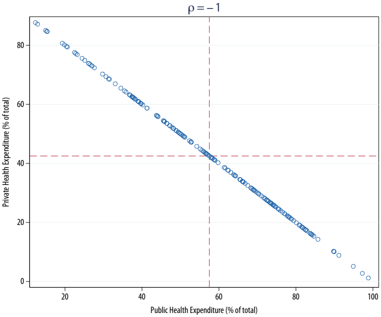
Sample notation $r = \frac{1}{n-1} \sum_{i=1}^{n} (\frac{X_i - \bar{X}}{s_X})(\frac{Y_i - \bar{Y}}{s_Y})$

Detects linear correlation

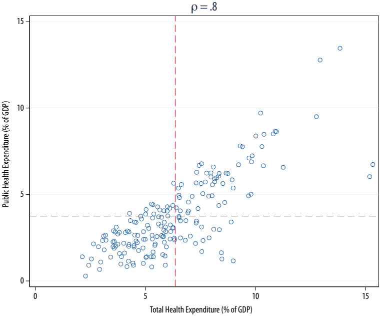
- Uncorrelated $\neq$ unrelated
- Correlated $\neq$ unconfounded

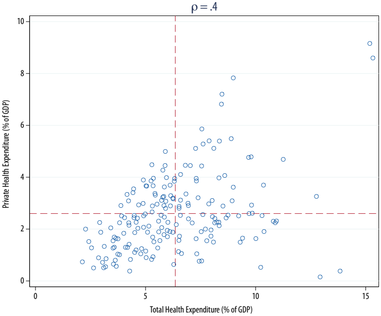
Perfect positive/negative correlation



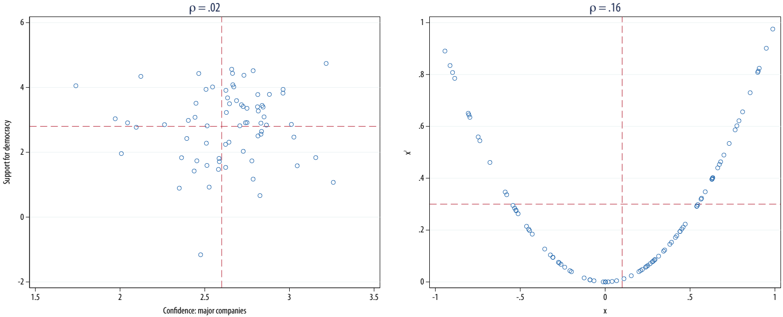


Significant (moderate–strong) correlation





Insignificant (weak, non-linear) correlation



Pearson correlation coefficient

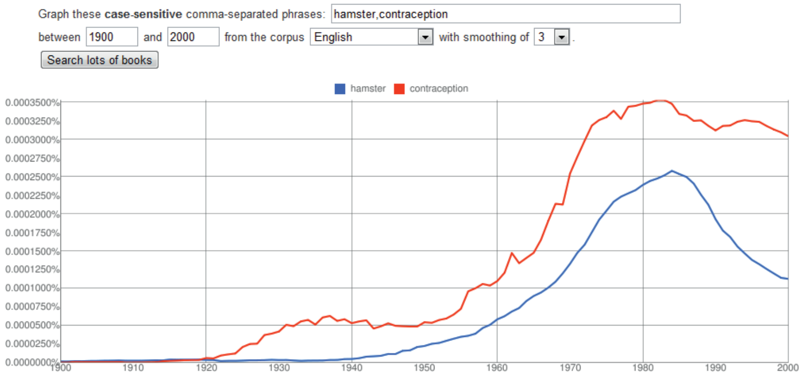
Significance test:

$$\text{Null hypothesis } H_0 \quad r = 0$$

$$\text{Test statistic} \quad T = r\sqrt{\frac{n-2}{1-r^2}}$$

Sanity check

- Uncorrelated $\neq$ independent
- Correlated $\neq$ causally related



Graph these **case-sensitive** comma-separated phrases: hamster,contraception

between 1900 and 2000 from the corpus English with smoothing of 3 .

Figure 1: Frequencies of the words “hamster” and “contraception” in Google Books, 1900–2000

Source: Harkness, “Seduced by Stats?”, *Significance*, 2012.

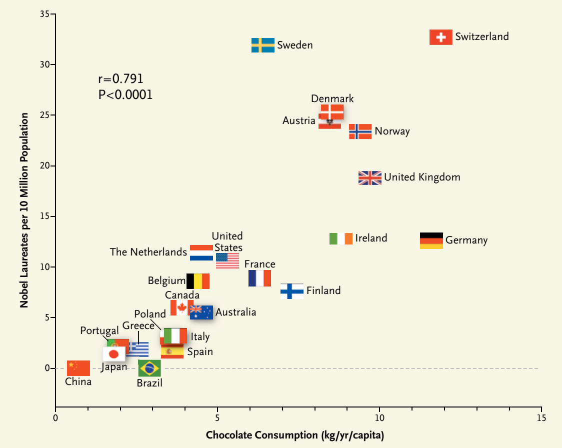


Figure 1. **Correlation between Countries' Annual Per Capita Chocolate Consumption and the Number of Nobel Laureates per 10 Million Population.**

Source: Messerli, "Chocolate Consumption, Cognitive Function, and Nobel Laureates", *New England Journal of Medicine*, 2012.

Stata implementation

```
pwcorr [varlist], [obs sig]
```

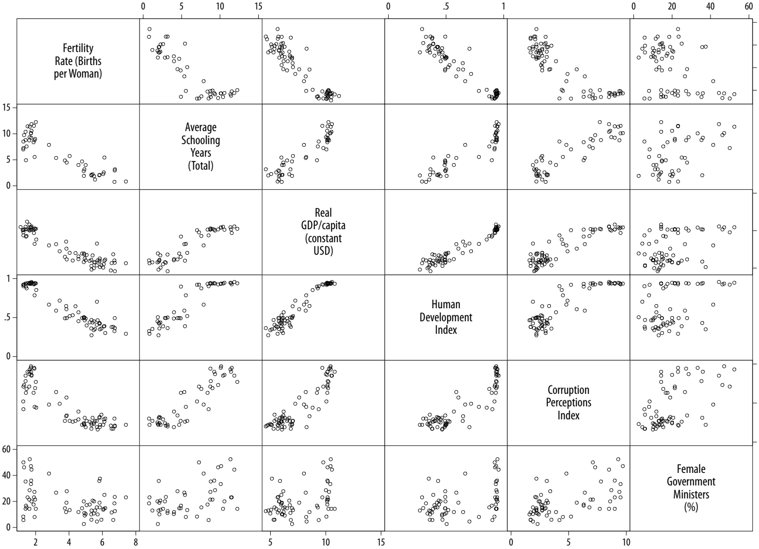
- `obs` shows the number of observations
- `sig` shows the coefficient's *p*-value

```
gr mat [varlist], [half etc.]
```

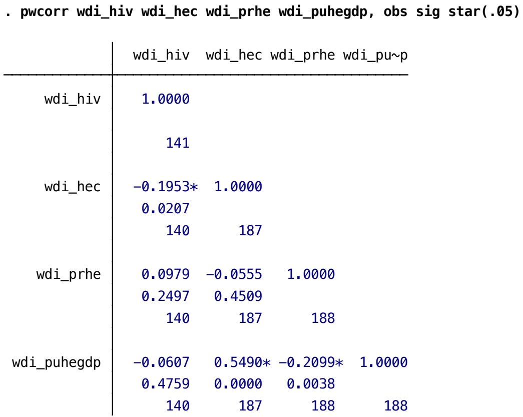
- `half` plots only half of all graphs (quicker)
- accepts scatterplot options (`jitter`, `mlab`, etc.)

```
use datasets/qog2011, clear
```

- Variables: `d wdi_brd wdi_mege wdi_pb2 wdi_the`
- Inspect and plot the correlation matrix.


Fertility Rate (Births per Woman)
Average Schooling Years (Total)
Real GDP/capita (constant USD)
Human Development Index
Corruption Perceptions Index
Female Government Ministers (%)
0
5
10
15
0
.5
1
0
20
40
60
8
6
4
2
15
10
5
0
15
10
5
1
.5
0
10
5
0
60
40
20
0
2
4
6
8
5
10
15
0
5
10

In Stata



```
. pwcorr wdi_hiv wdi_hec wdi_prhe wdi_puhegdp, obs sig star(.05)

             |  wdi_hiv  wdi_hec wdi_prhe wdi_pu~p
-------------+------------------------------------
     wdi_hiv |   1.0000 
             |
             |      141
             |
     wdi_hec |  -0.1953*  1.0000 
             |   0.0207
             |      140      187
             |
    wdi_prhe |   0.0979  -0.0555   1.0000 
             |   0.2497   0.4509
             |      140      187      188
             |
 wdi_puhegdp |  -0.0607   0.5490* -0.2099*  1.0000 
             |   0.4759   0.0000   0.0038
             |      140      187      188      188
```

In print

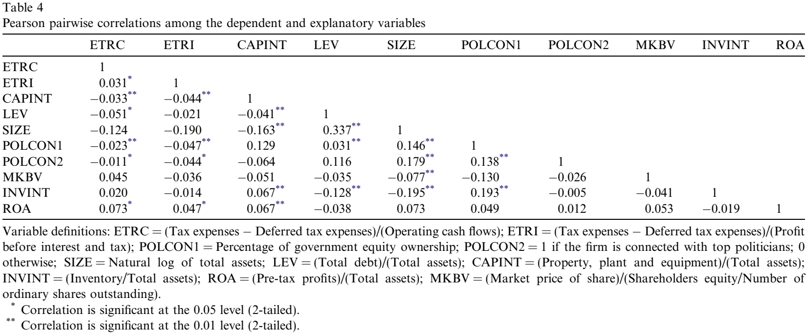


Table 4
Pearson pairwise correlations among the dependent and explanatory variables

	ETRC	ETRI	CAPINT	LEV	SIZE	POLCON1	POLCON2	MKBV	INVINT	ROA
ETRC	1									
ETRI	0.031*	1								
CAPINT	−0.033**	−0.044**	1							
LEV	−0.051*	−0.021	−0.041**	1						
SIZE	−0.124	−0.190	−0.163**	0.337**	1					
POLCON1	−0.023**	−0.047**	0.129	0.031**	0.146**	1				
POLCON2	−0.011*	−0.044*	−0.064	0.116	0.179**	0.138**	1			
MKBV	0.045	−0.036	−0.051	−0.035	−0.077**	−0.130	−0.026	1		
INVINT	0.020	−0.014	0.067**	−0.128**	−0.195**	0.193**	−0.005	−0.041	1	
ROA	0.073*	0.047*	0.067**	−0.038	0.073	0.049	0.012	0.053	−0.019	1

Variable definitions: ETRC = (Tax expenses − Deferred tax expenses)/(Operating cash flows); ETRI = (Tax expenses − Deferred tax expenses)/(Profit before interest and tax); POLCON1 = Percentage of government equity ownership; POLCON2 = 1 if the firm is connected with top politicians; 0 otherwise; SIZE = Natural log of total assets; LEV = (Total debt)/(Total assets); CAPINT = (Property, plant and equipment)/(Total assets); INVINT = (Inventory/Total assets); ROA = (Pre-tax profits)/(Total assets); MKBV = (Market price of share)/(Shareholders equity/Number of ordinary shares outstanding).

* Correlation is significant at the 0.05 level (2-tailed).

** Correlation is significant at the 0.01 level (2-tailed).

Source: Adhikari *et al.*, “Public policy, political connections, and effective tax rates: Longitudinal evidence from Malaysia”, *Journal of Accounting and Public Policy*, 2006.

Correlation matrixes

```
mkcorr [varlist], lab num sig log(corr.txt) replace
```

- `ssc install` the command if needed
- `lab num sig` add labels, numbers and *p*-values

Computer skills

- Import as a table in a spreadsheet editor.
- Convert from text to table in a rich text editor.

```
use datasets/qog2011, clear
```

- Variables: `d wdi_brd wdi_mege wdi_pb2 wdi_the`
- Export and import the correlation matrix.