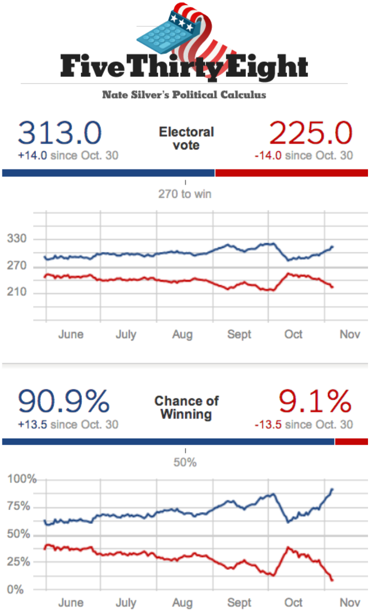
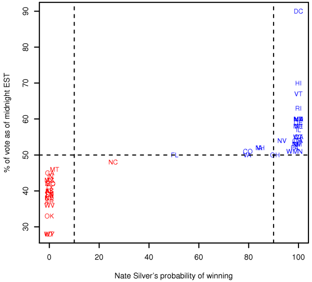
Regression (I)

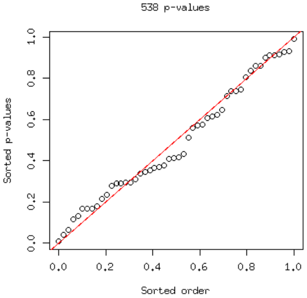
1. A simple linear model
2. Ordinary Least Squares (OLS)
3. Regression output
4. Draft No. 2



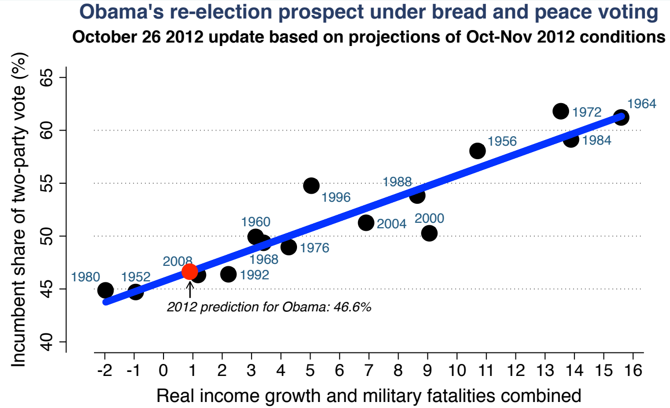
FiveThirtyEight
Nate Silver's Political Calculus
313.0
+14.0 since Oct. 30
Electoral vote
225.0
-14.0 since Oct. 30
270 to win
330
270
210
June
July
Aug
Sept
Oct
Nov
90.9%
+13.5 since Oct. 30
Chance of Winning
9.1%
-13.5 since Oct. 30
50%
100%
75%
50%
25%
0%
June
July
Aug
Sept
Oct
Nov



% of vote as of midnight EST
Nate Silver's probability of winning
DC
HI
VT
RI
NV
NH
CO
VA
FL
OH
NC
MT
OK
0
20
40
60
80
100
30
40
50
60
70
80
90



538 p-values
Sorted p-values
Sorted order
0.0
0.2
0.4
0.6
0.8
1.0



Combination of real growth and fatalities weights each variable by its estimated coefficient.
Estimated effects of fatalities on vote shares: -0.7% in 2008 (Iraq), -7.4% in 1968 (Vietnam), -9.7% in 1952 (Korea); negligible in 1964,1976, 2004, 2012, and null in other years.
Source: www.douglas-hibbs.com October 26 2012

To what extent can trust in government be predicted from variations in economic growth?

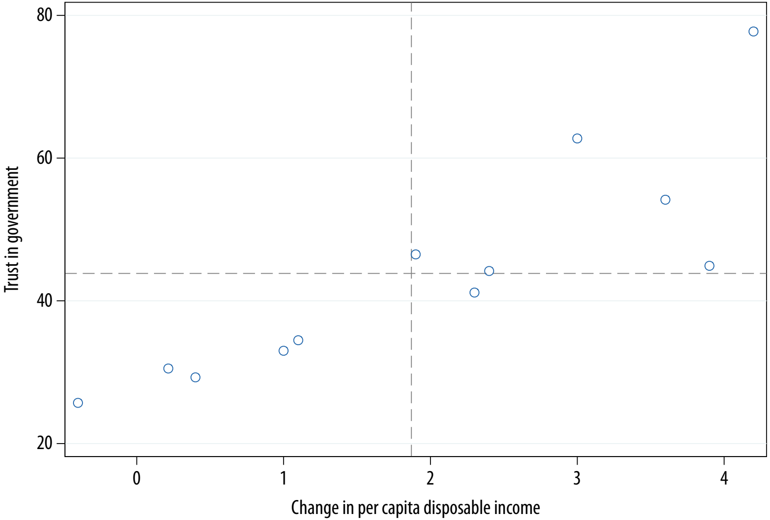
DV: Trust in government

"Just about always/Most of the time" (American National Election Studies)

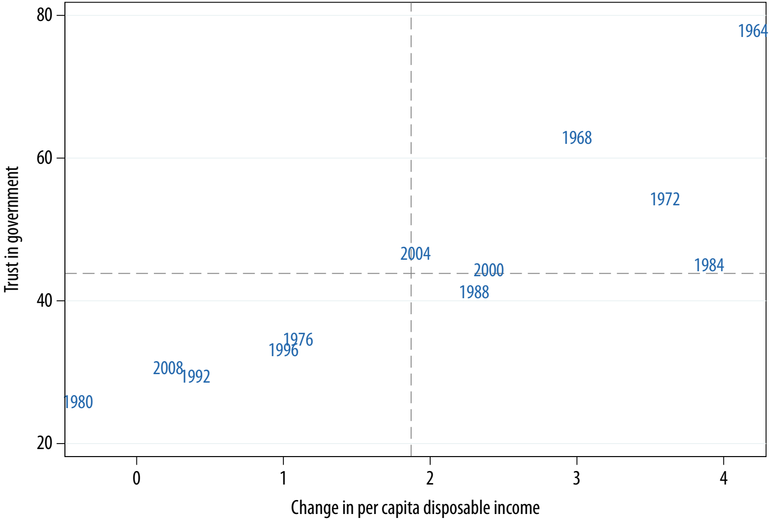
IV: Economic performance

Change in disposable income per capita (Bureau of Economic Analysis)

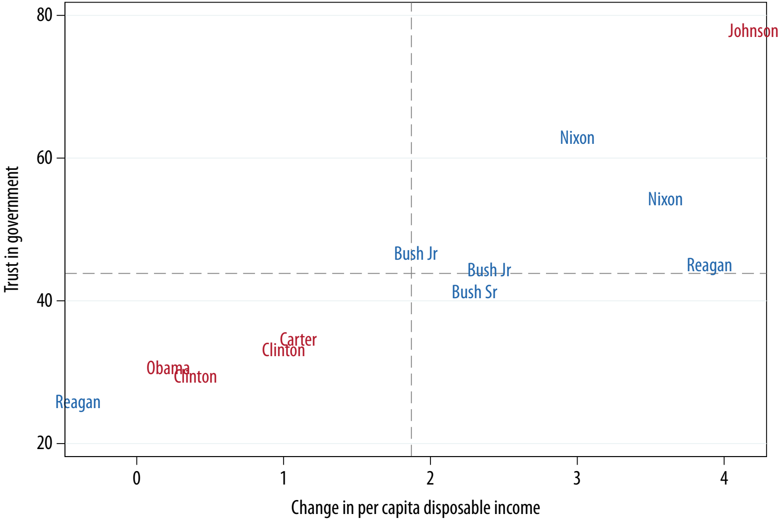
Example kindly provided by John Sides.



Dashed lines at averages. Pearson correlation $\rho = .86$ significant at $p < .01$.



Dashed lines at averages. Pearson correlation $\rho = .86$ significant at $p < .01$.



Dashed lines at averages. Pearson correlation $\rho = .86$ significant at $p < .01$.

Simple linear regression

Equations

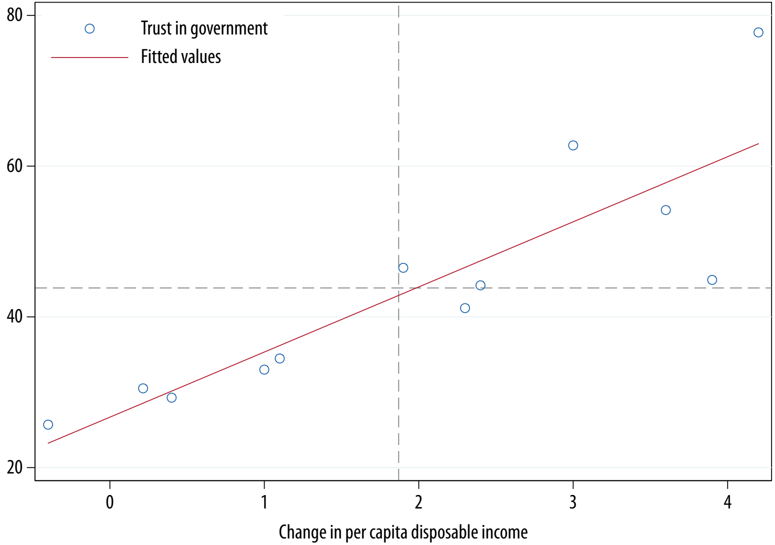
$Y = \alpha + \beta X + \epsilon \quad \hat{Y} = \alpha + \beta X \quad \epsilon = Y - \hat{Y}$

Parameters

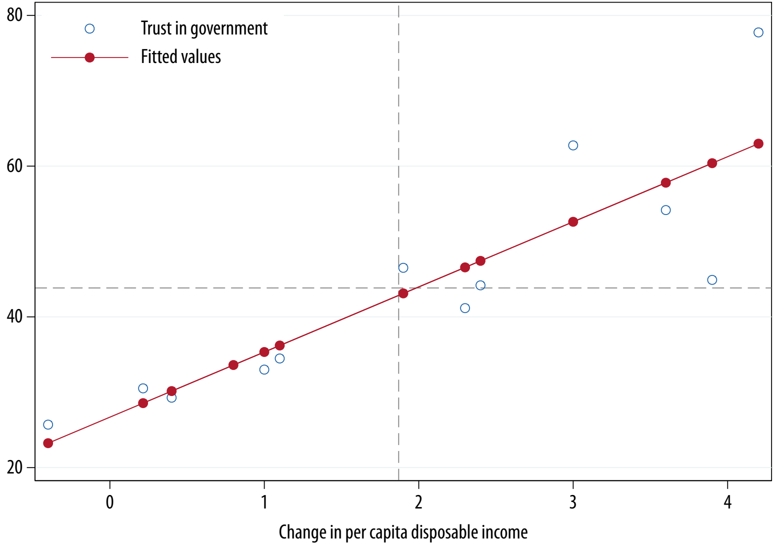
- Y is the dependent variable and $\hat{Y}$ its predicted value
- X is the independent variable used as a predictor of Y
- α is the constant (intercept)
- β is the regression coefficient (slope)
- ϵ is the error term (residuals)

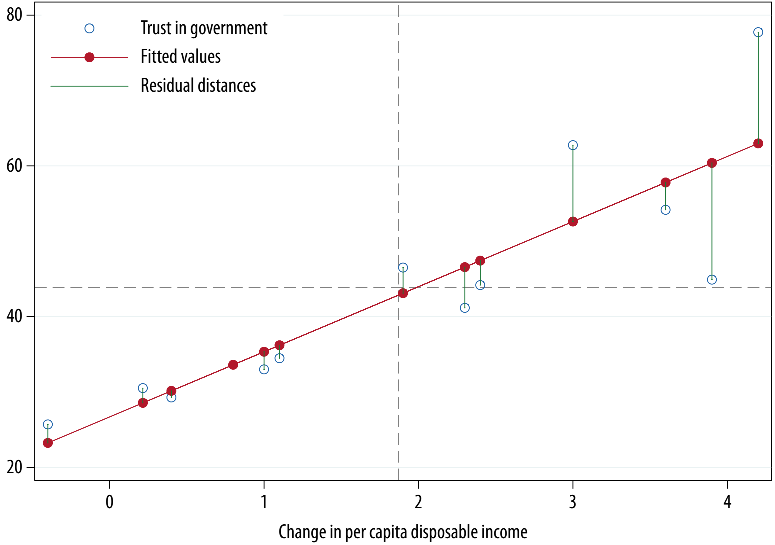
Warning

The model assumes a *linear*, *additive* relationship.



Trust in government
Fitted values
80
60
40
20
0
1
2
3
4
Change in per capita disposable income


80
60
40
20
Trust in government
Fitted values
0
1
2
3
4
Change in per capita disposable income



80
60
40
20
0
1
2
3
4
Trust in government
Fitted values
Residual distances
Change in per capita disposable income

Ordinary Least Squares (OLS)

Error term

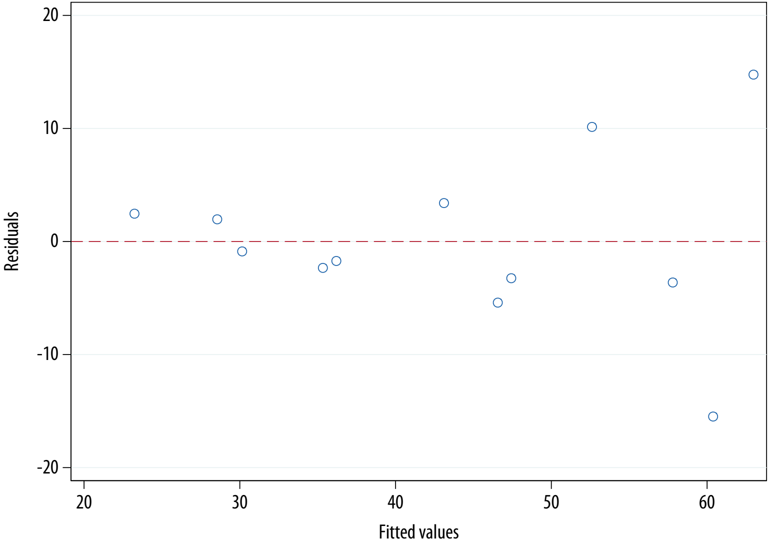
In a simple linear model $Y = \alpha + \beta X + \epsilon$, the regression coefficient β is calculated as to minimize the residual sum of squares

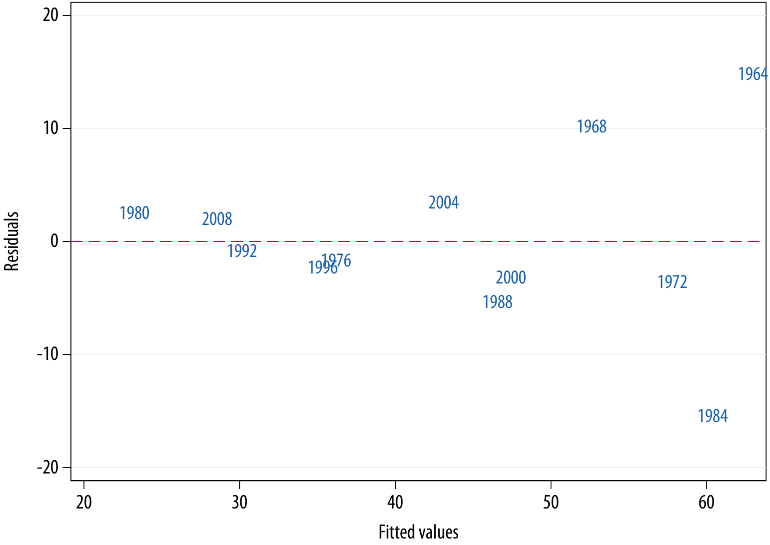
$$RSS = \sum_{i=1}^{n} (Y_i - \hat{Y}_i)^2 = \sum_{i=1}^{n} \epsilon^2$$

where $Y_i - \hat{Y}_i$ is the residual (or error term) of each observation.

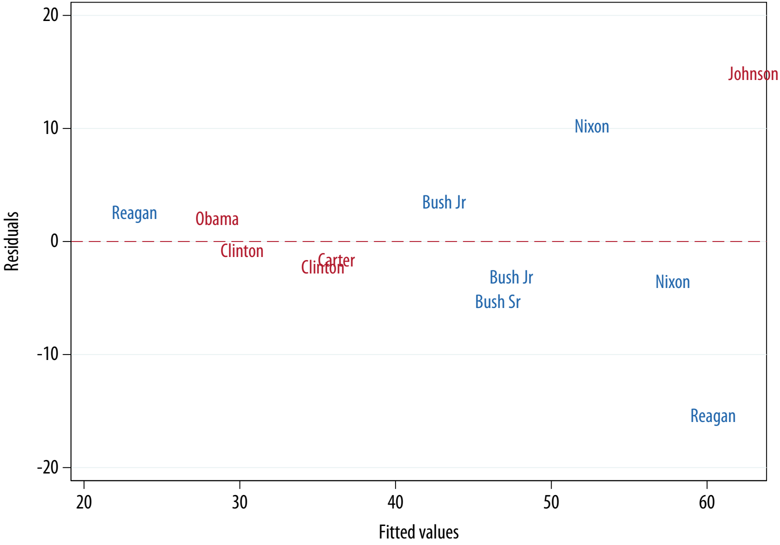
Parameter estimation

$$\beta = \frac{\mathrm{Cov}(X, Y)}{\mathrm{Var}_X} = \frac{\sum_{i=1}^{n}(X_i - \bar{X})(Y_i - \bar{Y})}{\sum_{i=1}^{n}(X_i - \bar{X})^2} \quad \alpha = \bar{Y} - \beta \bar{X}$$


Residuals
20
10
0
-10
-20
20
30
40
50
60
Fitted values



Residuals
Fitted values
20
10
0
-10
-20
20
30
40
50
60
1964
1968
2004
1980
2008
1992
1976
1996
2000
1988
1972
1984



Residuals
20
10
0
-10
-20
20
30
40
50
60
Fitted values
Johnson
Nixon
Reagan
Obama
Bush Jr
Clinton
Carter
Clinton
Bush Jr
Nixon
Bush Sr
Reagan

reg y x

```
. regress trust income

      Source |       SS           df       MS      Number of obs   =        12
-------------+----------------------------------   F(1, 10)        =     29.64
       Model |  1908.80221         1  1908.80221   Prob > F        =    0.0003
    Residual |  643.906248        10  64.3906248   R-squared       =    0.7478
-------------+----------------------------------   Adj R-squared   =    0.7225
       Total |  2552.70846        11  232.064405   Root MSE        =    8.0244

------------------------------------------------------------------------------
       trust |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      income |   8.639373   1.586767     5.44   0.000     5.103836    12.17491
       _cons |   26.69501   3.888016     6.87   0.000     18.03197    35.35805
------------------------------------------------------------------------------
```

Top left: ANOVA table. Top right: model fit.
Bottom: regression coefficients.

Interpretation of reg output

Number of observations N, significance test $H_0 : \beta = 0$, coefficient of determination R^2.

```
. regress trust income

      Source |       SS       df       MS              Number of obs =      12
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------------------------------------------------------------------------------
```

Goodness of fit

$$R^2 = 1 - \frac{\sum_{i=1}^{n}(Y_i - \hat{Y}_i)^2}{\sum_{i=1}^{n}(Y_i - \bar{Y}_i)^2} = \frac{\text{residual sum of squares}}{\text{total sum of squares}}$$

As predicted variance increases, $RSS \to 0$ and $R^2 \to 1$, indicating a more efficient fit.

```
Number of obs =      12
F(  1,    10) =   29.64
Prob > F      =  0.0003
R-squared     =  0.7478
Adj R-squared =  0.7225
Root MSE      =  8.0244
```

Sanity check

The most important statistic here is the actual number of observations in the model.

Interpretation of regression coefficients

A regression coefficient estimates the variation in Y predicted by a change in one unit of X (recall that $Y = \alpha + \beta X + \epsilon$)

```
. regress trust income

      Source |       SS       df       MS              Number of obs =      12
-------------+------------------------------           F(  1,    10) =   29.64
       Model |  1908.80221     1  1908.80221           Prob > F      =  0.0003
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------------------------------------------------------------------------------
```

trust	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
income	8.639373	1.586767	5.44	0.000	5.103836	12.17491
_cons	26.69501	3.888016	6.87	0.000	18.03197	35.35805

- The coefficient is the slope β of the regression line and the constant is its intercept, the coordinate of origin $\alpha = \hat{Y}_{X=0}$.
- The standard error, t-value and p-value test whether the coefficient is significantly different from 0.

Draft No. 2

Univariate statistics

- Introduction
- Datasets
- Distributions
- Estimation

Assignment No. 1

corrected
revised
appended

Bivariate statistics

- Significance
- Crosstabs
- Correlation
- Simple OLS

Assignment No. 2

Statistical modelling

- Regression
- Extensions
- Diagnostics
- Conclusion

Final paper

First steps

Revise Draft No. 1

- go through corrections
- remove technical content
- rewrite until concision

Explore associations

- between DV and IVs
- between two IVs

Write up substantive results as sentences; cite significance tests and other statistics in brackets, e.g. $(\rho = .7, p < .05)$.

Thanks for your attention

Project

- Correct and improve first draft
- Finalize association tests and interpretations

Readings

- *Stata Guide*, Sec. 11
- *Making History Count*, ch. 4

Practice

- Replicate do-file
- Include simple OLS results in your second draft