No.: Preference, Budget Constraint, Consumer Choice

Date:

- strictly prefers A to B: $A \succ B$

  indifferent between A & B: $A \approx B$

  Preference ≠ Choice ⇒ Preference doesn't change with prices / income
- Assumptions on preference: { completeness, transitivity
- Indifference curves
  - do not cross
  - downward sloping if "more is better" is satisfied for both goods.

- Marginal rate of substitution of x for y
  - rate at which the consumer is willing to give up y to get more of x, maintaining the same level of satisfaction.

$$MRS_{x,y} = -\frac{dy}{dx}\bigg|_{\text{same } U} = -\frac{\Delta y}{\Delta x}\bigg|_{\text{same } U}$$

  - negative of the slope of indifference curve
  - Diminishing MRS: $MRS_{x,y}$ ↓ as consumer gets more x & less y along same indiff curve.

  → if diminishing MRS + 3 assumptions hold: indiff curve convex to origin

- Utility function
  - assigns a level of utility to each consumption basket so that if $A \succ B$, $U(A) > U(B)$. (represents preference)

- Marginal utility
  - rate at which utility changes as the level of consumption of a good changes.

$$MU_x = \frac{dU}{dx} = \frac{\Delta U}{\Delta x}$$

  - slope of utility function
  - sign of MU
    - +ve: more is better is satisfied
    - −ve: more is not better.
  - Principle of diminishing MU: MU ↓ as consumption level rises.

    Utility ↑ slower as consumption level rises

    Utility function flatter as consumption level rises.
  - $U(x,y)$: $MU_x = \frac{\partial U}{\partial x}$, $MU_y = \frac{\partial U}{\partial y}$

    Diminishing MU
    - MU_x ↓ with x, holding y constant
    - MU_y ↓ with y, holding x constant.

- Utility function → indifference curves

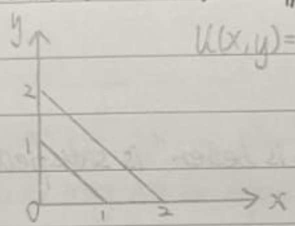
No.: Date:

VII $MRS_{x,y} = \frac{MU_x}{MU_y}$

VIII Perfect substitutes

- MRS constant, indiff curves linear, utility functions linear

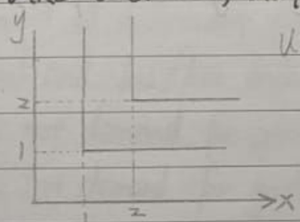
$U(x,y) = x + y$



IX Perfect complements

- MRS 0 or ∞, indiff curves L-shaped, utility functions are "min" functions.

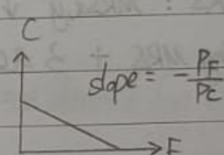
$U(x,y) = \min(x,y)$



X Budget constraint (set) : $P_F F + P_C C \le I$

Budget line : $P_F F + P_C C = I$

$\rightarrow C = \frac{I}{P_C} - \frac{P_F}{P_C} F$



Slope of budget line : rate at which 2 goods can be substituted in the market, that is, based on the prices.

XI Optimal basket

- Consumer choose the basket that gives him/her the highest utility given the budget constraint.
- At optimal basket
  - consumer spends all money $P_F F + P_C C = I$
  - tangency condition : $MRS_{F,C} = \frac{P_F}{P_C}$
- Equal marginal principle

$MRS_{F,C} = \frac{MU_F}{MU_C} = \frac{P_F}{P_C} \Rightarrow \frac{MU_F}{P_F} = \frac{MU_C}{P_C}$ To max utility, ~~extra~~ extra U per dollar spent on food = extra U per dollar spent on clothing.

$\Rightarrow$ If per dollar MU not the same, the consumer can reallocate how to spend the money to get higher utility.

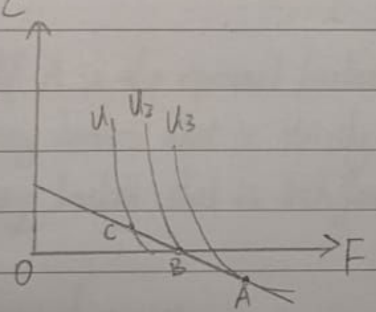
- constrained maximization problem

$$\max_{F,C} FC$$
$$\text{s.t. } 20F + 40C = 600$$

If optimal choice is the point of tangency:

- tangency condition + budget line
- Lagrangian method.

Optimal basket is not always a point of tangency

- Corner solution: an optimal basket at which the consumption of at least 1 good is 0.
  → optimal basket either on horizontal / vertical axis.
  → at corner solutions, indiff curve may not be tangent to budget line.
- Interior solution: an optimal basket in which both goods are consumed.
- 

Cobb-Douglas utility function

- $U(x,y) = Ax^{\alpha}y^{\beta}$, $A>0$, $\alpha>0$, $\beta>0$
- $MU_x = A\alpha x^{\alpha-1}y^{\beta}$, $MU_y = A\beta x^{\alpha}y^{\beta-1}$
  +ve MU → more is better for both goods → indiff curve downward sloping
- $MRS_{x,y} = \frac{\alpha y}{\beta x}$
  diminishing $MRS_{x,y}$ → indiff curve convex
- MU for Cobb-Douglas utility functions may / may not be diminishing
  $\frac{\partial MU_x}{\partial x} = A\alpha(\alpha-1)x^{\alpha-2}y^{\beta}$ → diminishing MU_x when $\alpha<1$
  → diminishing MU_y when $\beta<1$
- Indifference curves do not intersect the axes!

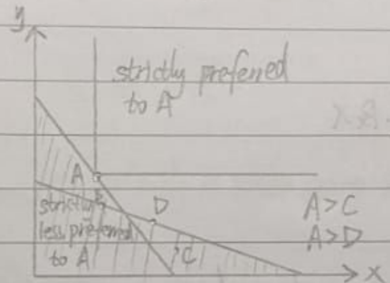
No.: Revealed Preference, Individual Demand

Date:

▥ A is strictly preferred to B : $A \succ B$

A is weakly preferred to B : $A \succsim B$ (either $A \succ B$ or $A \approx B$)

▥ Revealed preference

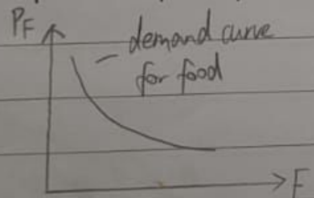
- we know budget constraint + optimal basket chosen → infer preference.
- 

A is weakly preferred to any other basket on budget line.
A is strictly preferred to any other basket in the budget set, but not on the budget line.

- If A is the optimal basket given budget constraint, any basket that is strictly preferred to A cannot be affordable, any basket that is indifferent to A cannot cost less than A.

▥ Demand curve

- A consumer's demand curve for a good is the optimal consumption of the good as a function of its price. (holding income & price of the other good fixed) eg. $F = \frac{5}{P_F}$



- Law of demand: demand curve is downward sloping
  higher price, lower quantity demanded.

eg. $u(F, C) = FC$, $P_C = 2$, $I = 10$

$$\max_{F,C} FC$$

s.t. $P_F F + 2C = 10$

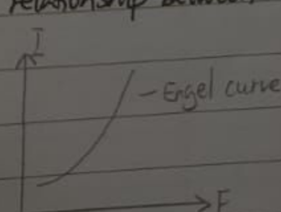
Tangency condition $\frac{P_F}{2} = \frac{C}{F} \rightarrow P_F F = 2C$

Budget line $P_F F + 2C = 10$

∴ demand curve for food is $F = \frac{5}{P_F}$

▥ Engel curve

- A consumer's Engel curve of a good is the curve that shows relationship between income and optimal consumption. (holding other factors fixed)



- normal good : upward sloping Engel curve
  inferior good : downward sloping Engel curve.

No.: Date:

Ⅲ Demand function

- A consumer's demand function for a good is quantity demanded as a function of income and all prices.
- demand function for Cobb-Douglas Utility function

$$\max_{x,y} Ax^{\alpha}y^{\beta}$$
$$\text{s.t. } P_x x + P_y y = I$$

Tangency condition: $\frac{\alpha y}{\beta x} = \frac{P_x}{P_y} \Rightarrow P_y y = \frac{\beta}{\alpha} P_x x$

Budget line $\Rightarrow P_x x + \frac{\beta}{\alpha} P_x x = I$

Demand function for x is $x = \frac{\alpha}{\alpha+\beta} \times \frac{I}{P_x}$ > normal good

Demand function for y is $y = \frac{\beta}{\alpha+\beta} \times \frac{I}{P_y}$

For Cobb-Douglas,

⇒ Demand for one good doesn't depend on the price of the other good.

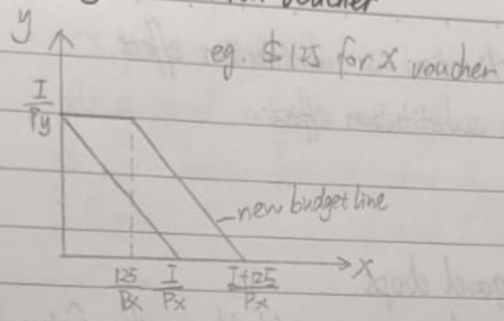
⇒ Consumers always spends a fixed proportion of income on each good.

eg. total expenditure on x is $P_x x = P_x \times \frac{\alpha}{\alpha+\beta} \times \frac{I}{P_x} = \frac{\alpha I}{\alpha+\beta}$

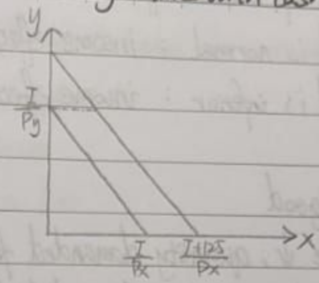
No.: Voucher vs. Cash, Income & Substitution Effects

Date:

▣ Budget line with voucher

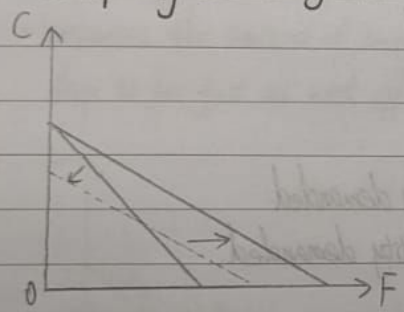


▣ Budget line with cash



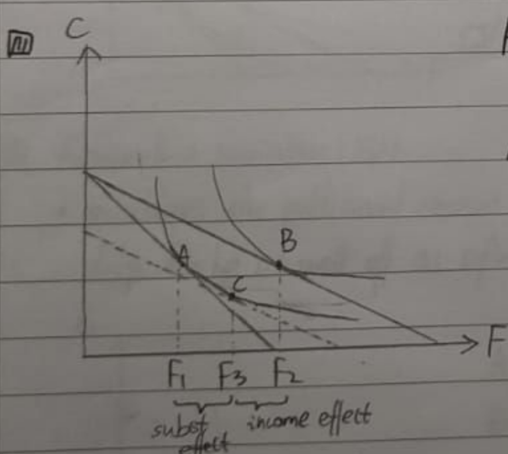
- Cash subsidy is never going to be worse than voucher.

▣ Decomposing the change in budget line



① Lower the price of food but hold purchasing power constant
② Increase purchasing power without changing the price of food.

▣



Holding purchasing power constant = holding utility constant

A to C: price of food is lower, income is lower, but utility remains the same

- Substitution effect: change in consumption of 1 good associated with a change in its price, holding the level of utility and other prices constant.

$\Rightarrow F_3 - F_1$

- Income effect: change in consumption of a good associated with a change in purchasing power, holding all prices constant.

$\Rightarrow F_2 - F_3$

- Effect of price change = substitution effect + income effect.

▣ Direction of substitution effect

- If price of food ↓, substitution effect is always non-negative.

No.: Date:

⑪ Direction of income effect

- If food is normal : income effect same direction as substitution effect.
  If food is inferior : income effect opposes substitution effect.

⑫ Giffen good

- as price ↓, quantity demanded for the good drops.
  as price ↑, quantity demanded for the good ~~ge~~ goes up. (holding other factors fixed)
- inferior (if normal good, subst & income effect will be both +ve)
  magnitude of income effect is bigger than subst effect.
- Law of demand
  Demand curve is upward sloping for Giffen good.

⑬ Giffen vs. inferior goods

- Giffen : +ve correlation bet. price and quantity demanded
- Inferior : −ve correlation bet. income and quantity demanded.

⇒ All Giffen goods are inferior.
  All inferior goods are Giffen ? No.

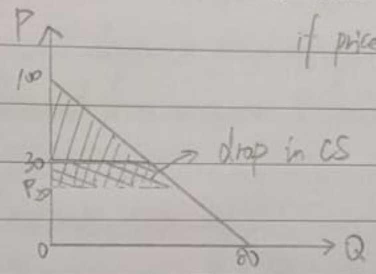
on

No.: Consumer Welfare

Date:

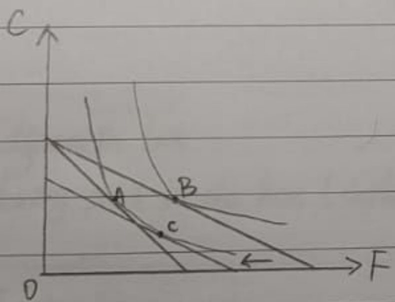
I Consumer surplus (CS)

- CS for an individual consumer is the difference between the consumer's willingness to pay for a good and the cost of purchasing the good.
  ⇒ area below demand curve & above price



II Compensating variation (CV)

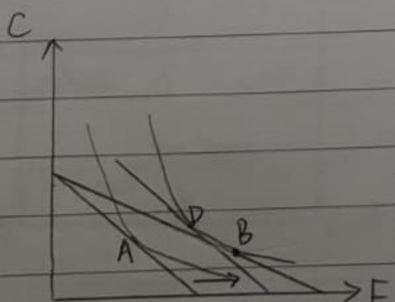
- measures the amount of money (income) the consumer is willing to give up after the price drop to be just as well off as before the price drop.



A: initial optimal basket
B: new optimal basket
C: consumer gets the old utility with the new price
CV = income at A − income at C

III Equivalent variation (EV)

- measures the additional amount of money (income) the consumer needs before the price drop to be as well off as after the price drop.



A: initial optimal basket
B: new optimal basket
D: consumer gets the new utility with the old price
EV = income at D − income at A.

(From A to B, no change in income, only change in price)

III The utility gain from the price decrease is equivalent to $... (CV or EV)

No.: **Market Demand, The Edgeworth Box, Pareto Efficiency** Date:

Market / Aggregate demand curve

- horizontal summation of all individual demand curves

Summary

- preference + budget constraint = optimal choice
- optimal choice + price changes = individual demand
- demand from consumer 1 + demand from consumer 2 = market demand

Exchange economy

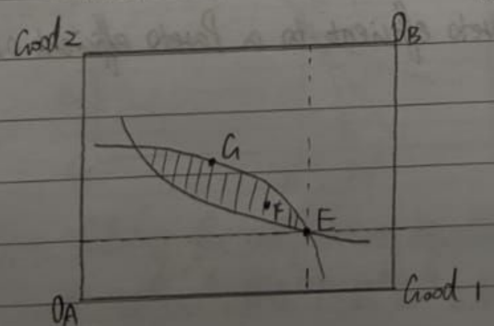
- 2 consumers : A & B , 2 goods : 1 & 2
  - ⇒ allocation : $(x_1^A, x_2^A, x_1^B, x_2^B)$
  - ⇒ endowment allocation : $(w_1^A, w_2^A, w_1^B, w_2^B)$
- An allocation is feasible if $\begin{cases} x_1^A + x_1^B = w_1^A + w_1^B \\ x_2^A + x_2^B = w_2^A + w_2^B \end{cases}$

Edgeworth box

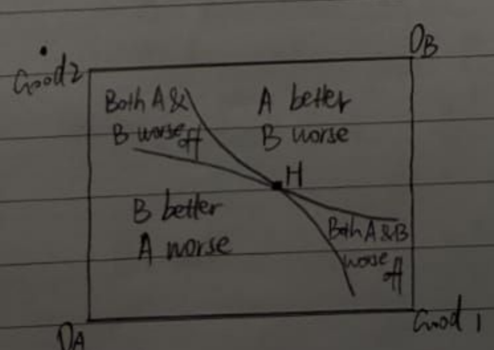
- Every point in the box (including those on boundaries) represents a feasible allocation.

Pareto improvement

- From allocation X to allocation Y is a Pareto improvement if from X to Y, at least 1 consumer is better off and no one else is worse off.



Any allocation in the shaded region is a Pareto improvement of E. (eg. E to F, E to G)

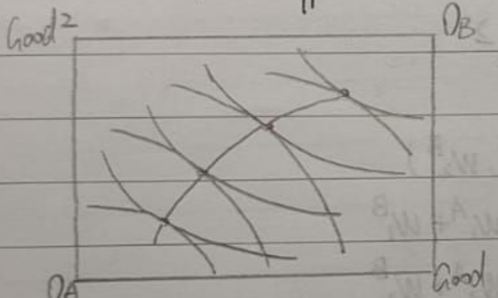


At H, cannot make 1 consumer better off without making the other consumer worse off.

No.: Date:

VII Pareto efficiency

- An allocation is Pareto efficient if there is no way to make 1 consumer better off without making someone else worse off.
  → no room for Pareto improvement
- When each consumer's indiff curves are ~~stoo~~ smooth with diminishing MRS and when we have interior solutions, Pareto efficient allocations will be the tangency points between the 2 indiff curves.
- More than 1 Pareto efficient allocation



→ contract curve : set of all Pareto efficient allocations.
→ if an allocation is Pareto efficient, that allocation will lie on the contract curve.

- Deriving contract curve → only if we have tangency points
  ① Tangency condition : $MRS^A_{1,2} = MRS^B_{1,2}$ (1)
  ② feasible allocation : $x_1^A + x_1^B = w_1^A + w_1^B$ (2)
  $x_2^A + x_2^B = w_2^A + w_2^B$ (3)
  express contract curve in terms of x_1^A & x_2^A or x_1^B & x_2^B.

☆ When moving from an allocation that is not Pareto efficient to a Pareto efficient allocation, it's not necessarily a Pareto improvement!

No.: General Competitive Equilibrium

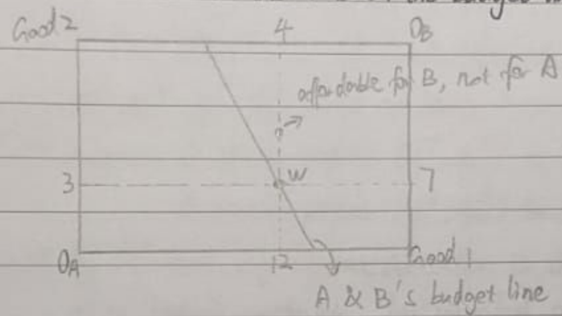
Date:

III Budget set (constraint) in exchange economy.

- $P_1 x_1^A + P_2 x_2^A \le P_1 w_1^A + P_2 w_2^A$

  $P_1 x_1^B + P_2 x_2^B \le P_1 w_1^B + P_2 w_2^B$

- Endowment allocation is on the budget line!



$P_1 x_1^A + P_2 x_2^A = P_1 w_1^A + P_2 w_2^A$

$P_1 x_1^B + P_2 x_2^B = P_1 w_1^B + P_2 w_2^B$

IV General competitive equilibrium

- A pair of prices (P_1, P_2) constitutes a (general) competitive equilibrium if at these prices,

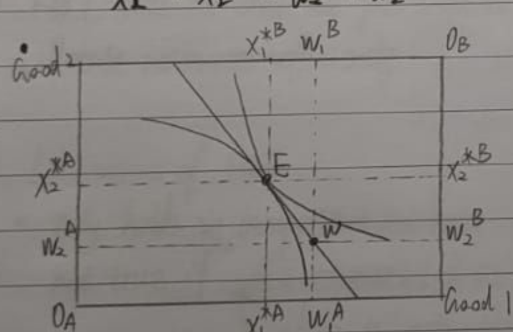
  ① each consumer maximize utility given budget constraint

  $x_1^{*A}, x_2^{*A}, x_1^{*B}, x_2^{*B}$ optimal consumption for each consumer given equilibrium prices.

  ② markets for both goods clear

  $x_1^{*A} + x_1^{*B} = w_1^A + w_1^B$

  $x_2^{*A} + x_2^{*B} = w_2^A + w_2^B$



E is equilibrium.

When will economy reach an equilibrium?

- when optimal basket of the 2 consumers are the same pt. (markets clear)

- Solving for competitive equilibrium

eg. $U^A(x_1^A, x_2^A) = x_1^A x_2^A$, $U^B(x_1^B, x_2^B) = x_1^B x_2^B$,

A's endowment $(10, 6)$, B's endowment is $(10, 4)$

Find equilibrium prices P_1, P_2.

$\Rightarrow$ A's optimal choice: $\frac{x_2^A}{x_1^A} = \frac{P_1}{P_2}$ — ①

$P_1 x_1^A + P_2 x_2^A = 10P_1 + 6P_2$ — ②

B's optimal choice: $\frac{x_2^B}{x_1^B} = \frac{P_1}{P_2}$ — ③

$P_1 x_1^B + P_2 x_2^B = 10P_1 + 4P_2$ — ④

market clearing: $x_1^A + x_1^B = 20$ — ⑤

$x_2^A + x_2^B = 10.$ — ⑥

No.: First Welfare Theorem, Walras' Law

Date:

First welfare theorem

- states that a competitive equilibrium allocation is Pareto efficient.
- proof by contradiction:

Suppose at equilibrium prices P_1 and P_2, equilibrium allocation is $x_1^{*A}, x_2^{*A}, x_1^{*B}, x_2^{*B}$.

→ If this allocation is not Pareto efficient, there must exist another feasible allocation $y_1^A, y_2^A, y_1^B, y_2^B$ where at least 1 consumer is better off & no one is worse off.

→ Suppose A strictly prefers (y_1^A, y_2^A) to (x_1^{*A}, x_2^{*A}),
B weakly prefers (y_1^B, y_2^B) to (x_1^{*B}, x_2^{*B}).

→ By definition, equilibrium allocation is the utility-maximizing basket for each consumer given budget constraint, by revealed preference,

$$P_1 y_1^A + P_2 y_2^A > P_1 W_1^A + P_2 W_2^A \quad - ①$$
$$P_1 y_1^B + P_2 y_2^B \geq P_1 W_1^B + P_2 W_2^B \quad - ②$$

→ ① + ②: $P_1(y_1^A + y_1^B) + P_2(y_2^A + y_2^B) > P_1(W_1^A + W_1^B) + P_2(W_2^A + W_2^B)$

→ allocation $y_1^A, y_2^A, y_1^B, y_2^B$ feasible: $y_1^A + y_1^B = W_1^A + W_1^B$, $y_2^A + y_2^B = W_2^A + W_2^B$ contradiction!

FWT tells us that we just need to create a competitive market and the market will allocate resources efficiently.

Limitations of FWT

- only holds in competitive markets
  not true if
  - consumers / firms have price setting power
  - there is externality
  - there is asymmetric information
- efficiency ≠ equity
  A pareto efficient allocation may or may not be an equitable allocation.

Pareto efficiency vs. Competitive equilibrium

- Pareto efficiency
  - doesn't depend on price
  - doesn't depend on endowment
- Competitive equilibrium
  - depends on prices (a pair of prices such that markets clear and everyone maximizes utility given budget constraint)
  - depends on endowment (endowment & prices determine budget constraints)

→ A pareto efficient allocation may not be an equilibrium allocation.

No.: Date:

Gross demand at any given prices

- P_1, P_2 any pair of prices (may / may not be equilibrium prices)
  Let (x_1^A, x_2^A) be A's gross demand & (x_1^B, x_2^B) be B's gross demand given P_1, P_2.
  → gross demand: optimal basket
- Since P_1, P_2 may not be equilibrium prices, possible that $x_1^A + x_1^B \neq w_1^A + w_1^B$
  $x_2^A + x_2^B \neq w_2^A + w_2^B$

Net demand

- Net demand of a consumer for a good is the diff between gross demand for that good and his/her endowment for that good.
  eg. A's net demand for good 1 : $x_1^A - w_1^A$
  A's net demand for good 2 : $x_2^A - w_2^A$

Aggregate net demand

- Aggregate net demand for a good is the sum of the net demand for that good for the 2 consumers
  $x_1^A + x_1^B - w_1^A - w_1^B$, $x_2^A + x_2^B - w_2^A - w_2^B$
- when aggregate net demand for a good is
  - +ve : excess demand for that good
  - −ve : excess supply for that good
  - 0 : markets clear

Walras' Law

- Total value of the aggregate net demand for the 2 goods is 0.
  $P_1(x_1^A + x_1^B - w_1^A - w_1^B) + P_2(x_2^A + x_2^B - w_2^A - w_2^B) = 0$
- proof: $P_1 x_1^A + P_2 x_2^A = P_1 w_1^A + P_2 w_2^A$ ((x_1^A, x_2^A) lies on A's budget line)
  → $P_1(x_1^A - w_1^A) + P_2(x_2^A - w_2^A) = 0$ — ①
  → total value of A's net demand for 2 goods is 0
  → $P_1(x_1^B - w_1^B) + P_2(x_2^B - w_2^B) = 0$ — ②
  → ① + ② = Walras' Law

Implications of Walras' Law

- In 2-good exchange economy, if 1 market in equilibrium, the other market must also be in equilibrium.
- an excess supply in 1 market implies an excess demand in the other market.

Walras' Law vs. Competitive equilibrium

- Walras' Law holds for any prices! (not just equilibrium prices)
- At equilibrium prices, $P_1\underbrace{(x_1^A + x_1^B - w_1^A - w_1^B)}_{=0} + P_2\underbrace{(x_2^A + x_2^B - w_2^A - w_2^B)}_{=0} = 0$
  aggregate net demand for each good = 0.
- At non-equilibrium prices, $P_1\underbrace{(x_1^A + x_1^B - w_1^A - w_1^B)}_{\neq 0} + P_2\underbrace{(x_2^A + x_2^B - w_2^A - w_2^B)}_{\neq 0} = 0$