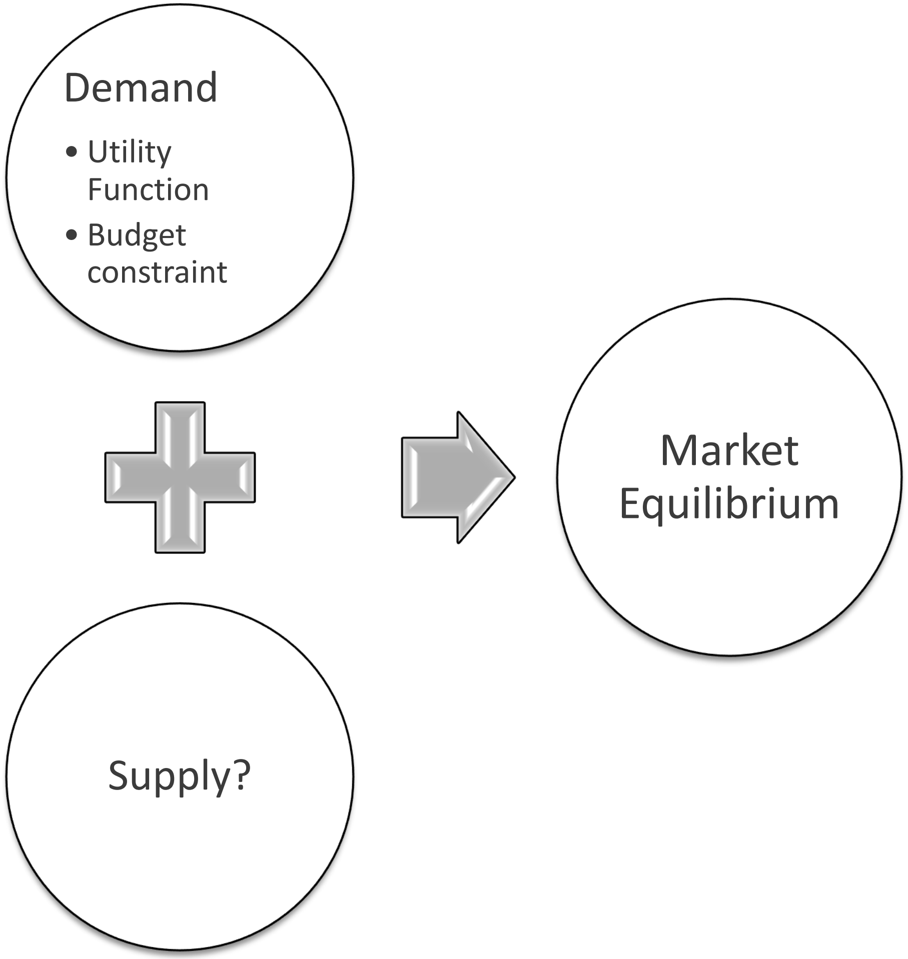
LECTURE 7
PRODUCTION

The Big Picture



Where are we?

- Production function with one variable
  - Marginal and average products
- Production function with two variables
  - Isoquants – representing the production function graphically
  - Marginal rate of technical substitution
  - Uneconomic region of production
- Returns to scale
  - Three types of returns to scale
- Technological progress
  - Three types of technological progress

Part 1

Production Function with One Input

What is production?

- Firms turn inputs to outputs
- *Factors of production* (inputs)
  - Labor
  - Equipment
  - Raw material
  - Land
- Production technology tells us how firms turn inputs into outputs

Production Function

- Suppose the firm needs two inputs, labor (*L*) and capital (*K*), to produce outputs
- Definition 7.1 *Production function* tells us the *maximum* quantity (*Q*) of output the firm can produce given the amount of *L* and *K*

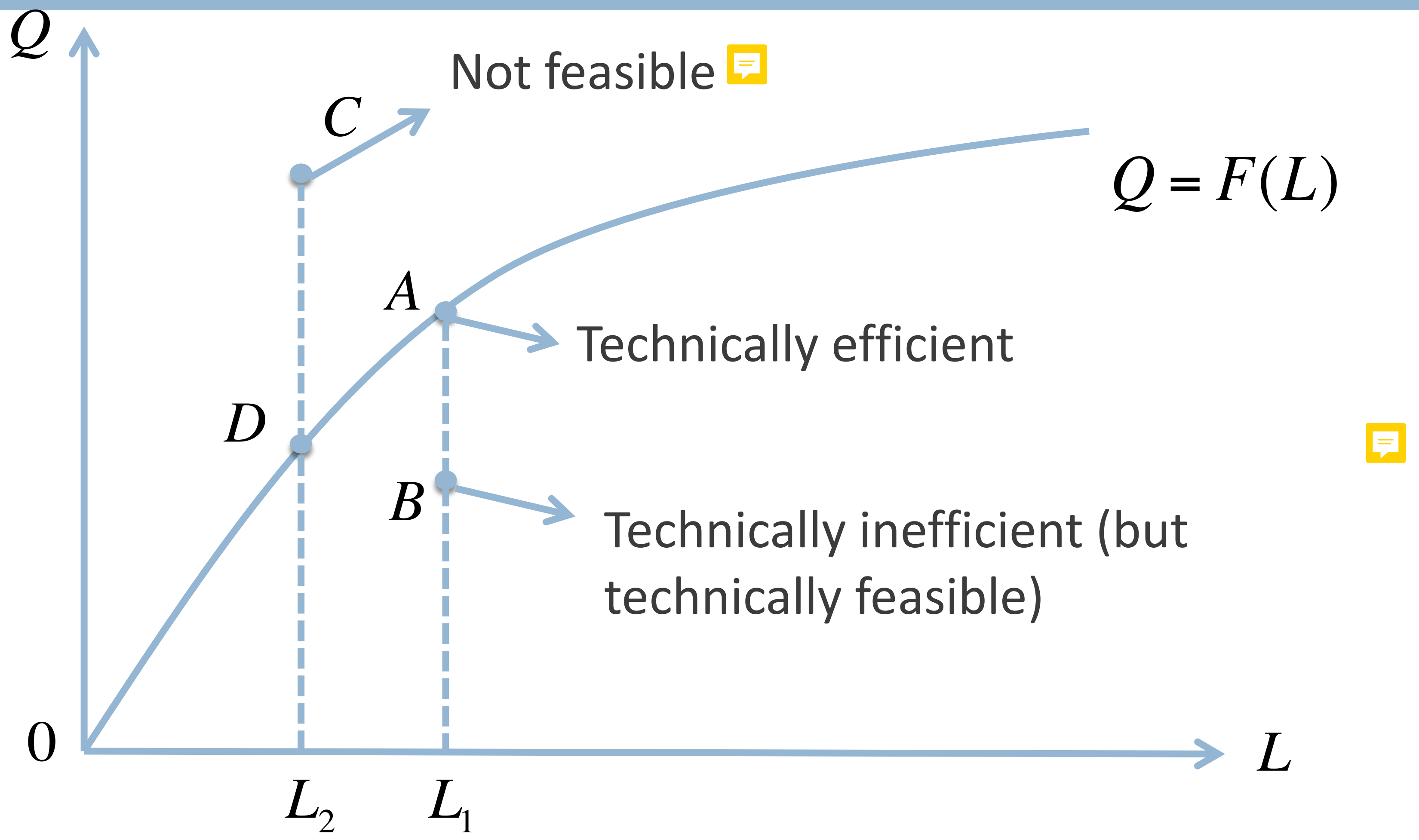
$$Q = F(L, K)$$

Production Function with One Input

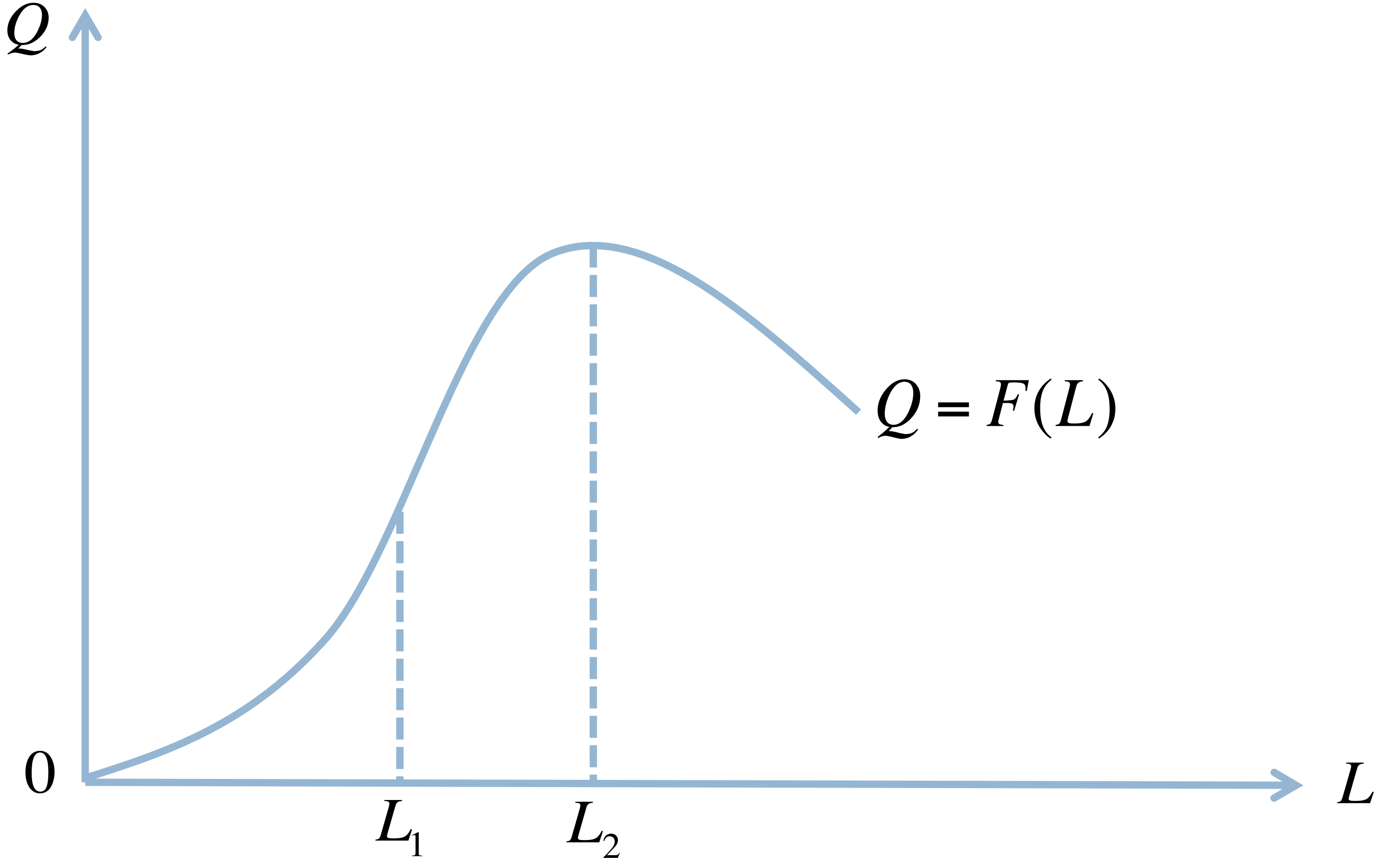
- Short run in production
  - At least one input is fixed
- Long run in production
  - All inputs are variable
- Suppose capital is fixed in the short run
- Firm can only adjust labor
- The production function is

$$Q = F(L)$$

Technically Efficient and Technically Feasible



A Typical Production Function



Marginal Product

- Definition 7.2 *Marginal product of labor* measures the rate at which output level changes as quantity of labor changes

$$MP_L = \frac{dQ}{dL} = \frac{\Delta Q}{\Delta L}$$

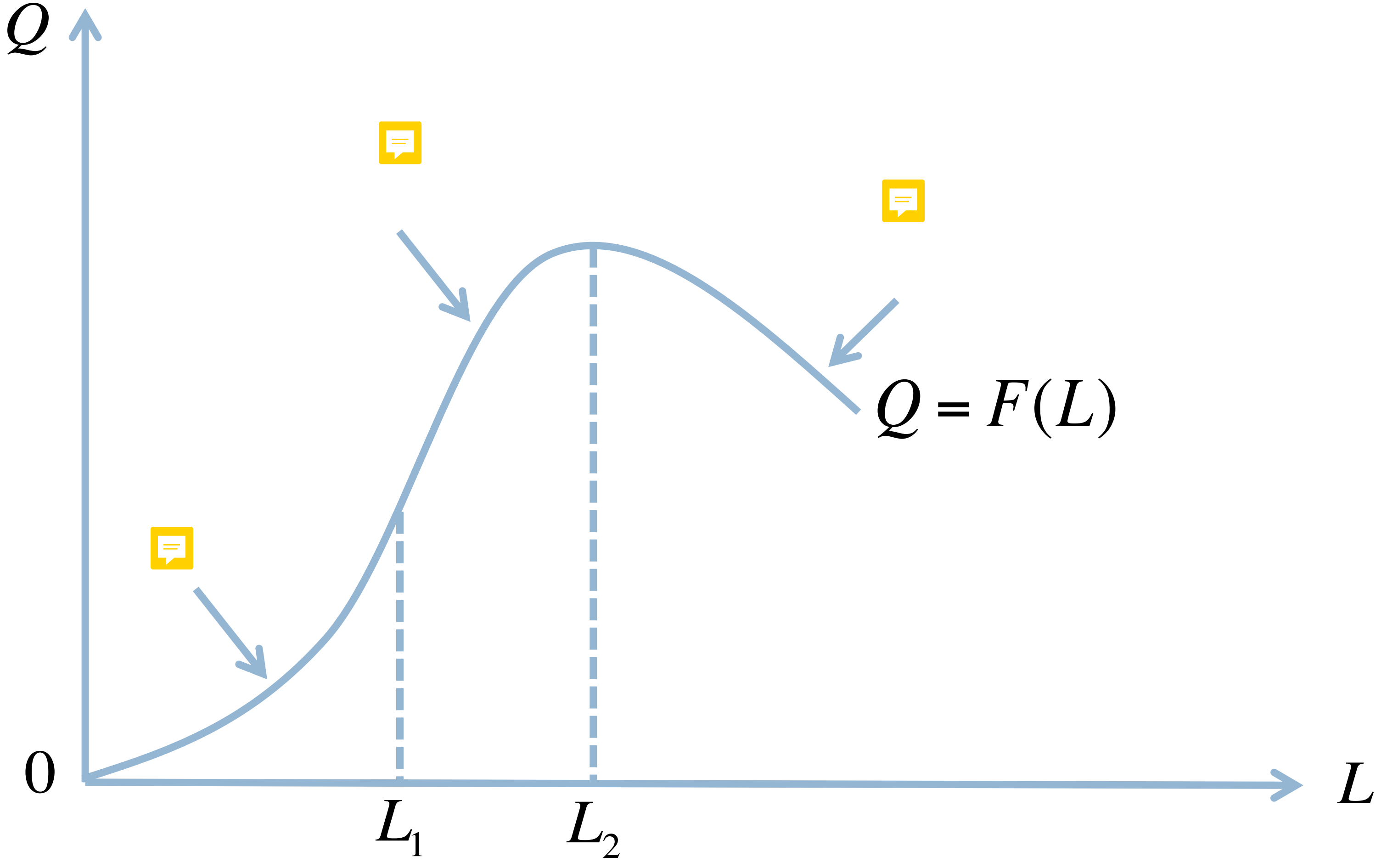
  where ΔL is extremely small

- In graph, it is the slope of the production function

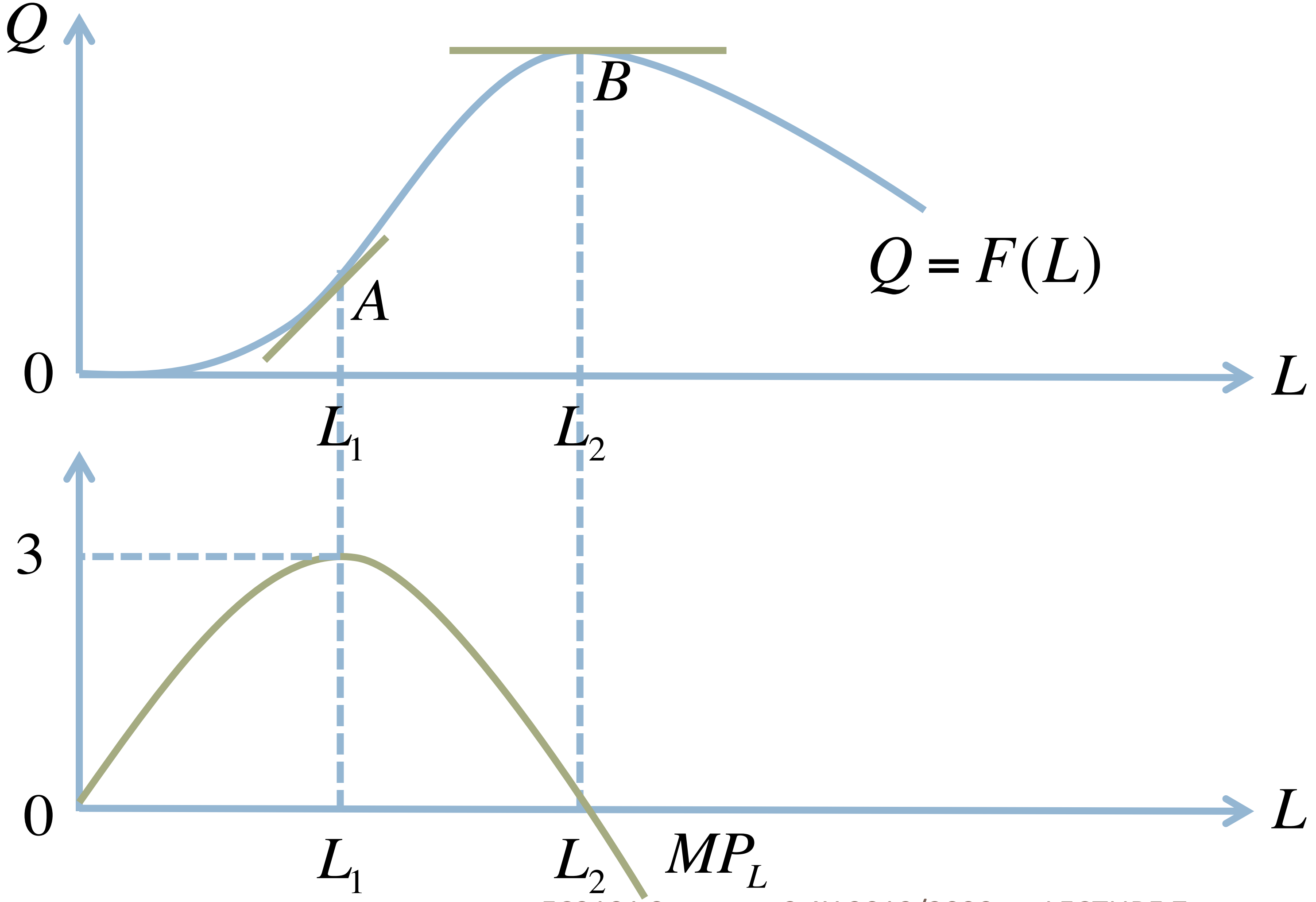
Law of Diminishing Marginal Returns

- Definition 7.3 *Increasing marginal returns*
  - MP_L increases as L increases
- Definition 7.4 *Diminishing marginal returns*
  - MP_L decreases as L increases
- *Law of diminishing marginal returns*
  - Suppose capital is fixed, marginal product of labor will eventually decline as the quantity of labor increases
- Definition 7.5 *Diminishing total returns*
  - Q decreases as L increases
  - MP_L is negative

A Typical Production Function



From Production Function to *MP*



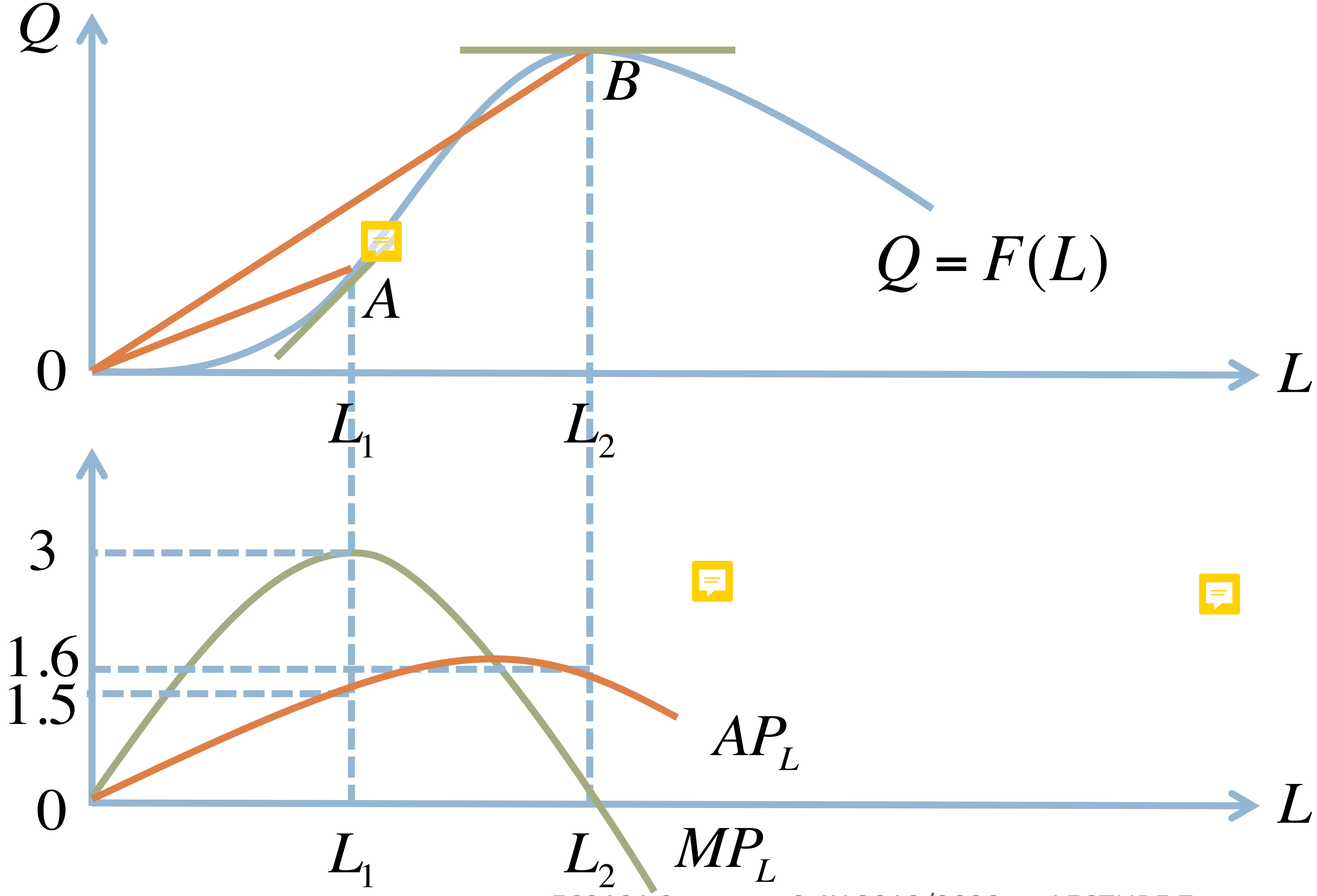
Average Product

- Definition 7.6 *Average product of labor* measures the output per unit of labor

$$AP_L = \frac{Q}{L}$$

- The slope of the ray connecting the origin and the point (*L*,*F*(*L*))

From Production Function to *AP*



Average Value and Marginal Value

- Suppose you bought 5 apples and it cost you \$5 in total
- You paid an average price of \$1 per apple
- Suppose you bought 1 additional apple and the average price you paid became \$0.9 per apple
- Did the 6^{th} apple cost you more than \$1 or less than \$1?

MP crosses *AP* at its highest point

- When AP_L rises as L increases
  - As quantity of labor increases, average product of labor goes up
  - Output generated by an extra unit of labor is pulling up the average
  - $MP_L > AP_L$
- When AP_L falls as L increases
  - As quantity of labor increases, average product of labor goes down
  - Output generated by an extra unit of labor is pulling down the average
  - $MP_L < AP_L$

MP and *AP*: A Mathematical Explanation

- Since

$$AP(L) = \frac{Q(L)}{L}$$

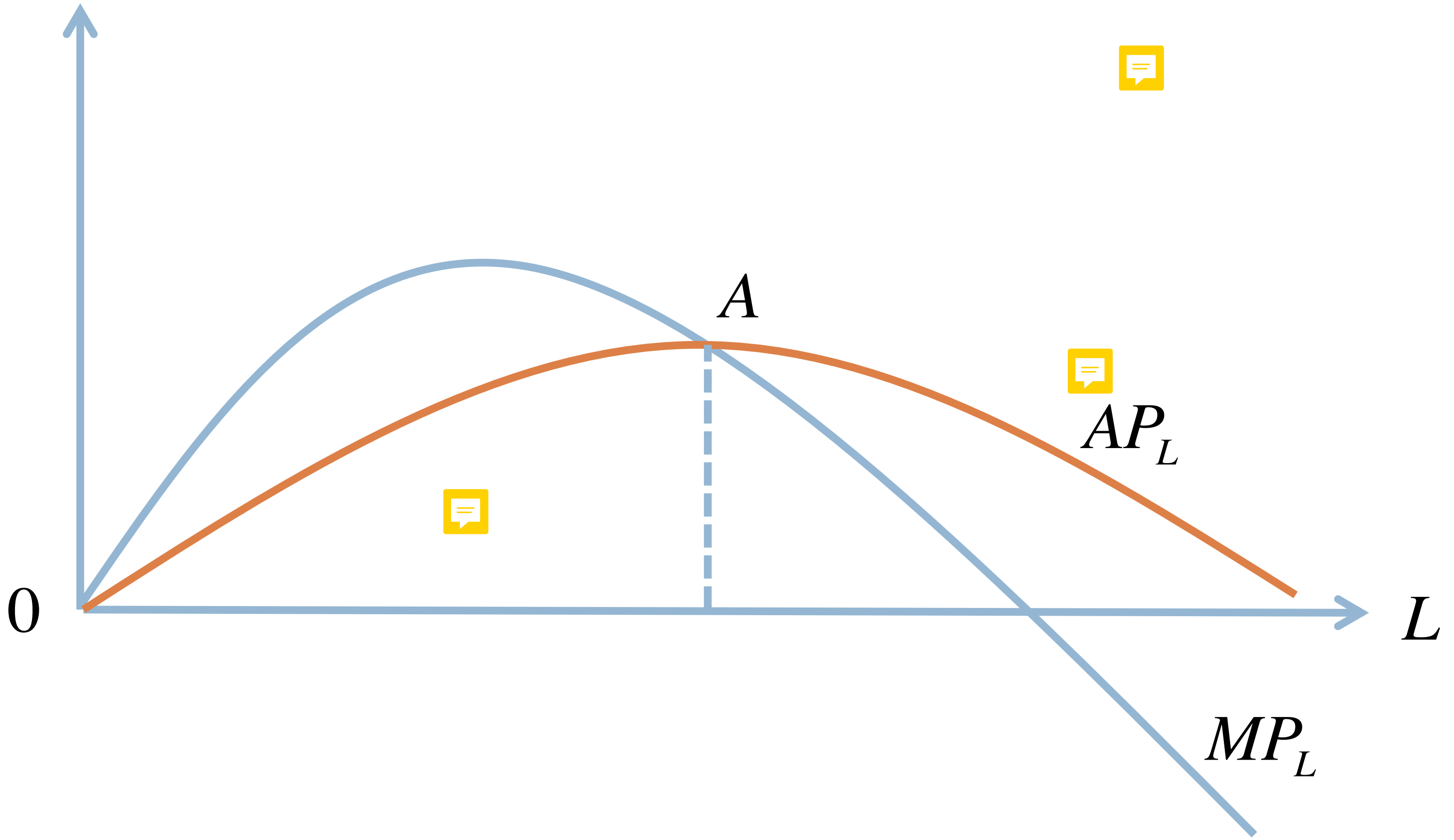
- We have

$$\frac{dAP(L)}{dL} = \frac{d\left(\frac{Q(L)}{L}\right)}{dL} = \frac{MP(L)L - Q(L)}{L^2} = \frac{MP(L) - AP(L)}{L}$$

- If as *L* increases *AP* increases, then

$$\frac{dAP(L)}{dL} > 0 \Rightarrow \frac{MP(L) - AP(L)}{L} > 0 \Rightarrow MP(L) > AP(L)$$

MP and *AP* in Graph



Analogy to Consumer Theory

- Production function
  - Utility function
- Marginal product
  - Marginal utility
- Diminishing marginal returns
  - Diminishing marginal utility

Part 2

Production Function with Two Inputs

Production Function with Two Inputs

- Suppose the firm can adjust both labor and capital
- Production function is

$$Q = F(L, K)$$

- Marginal products

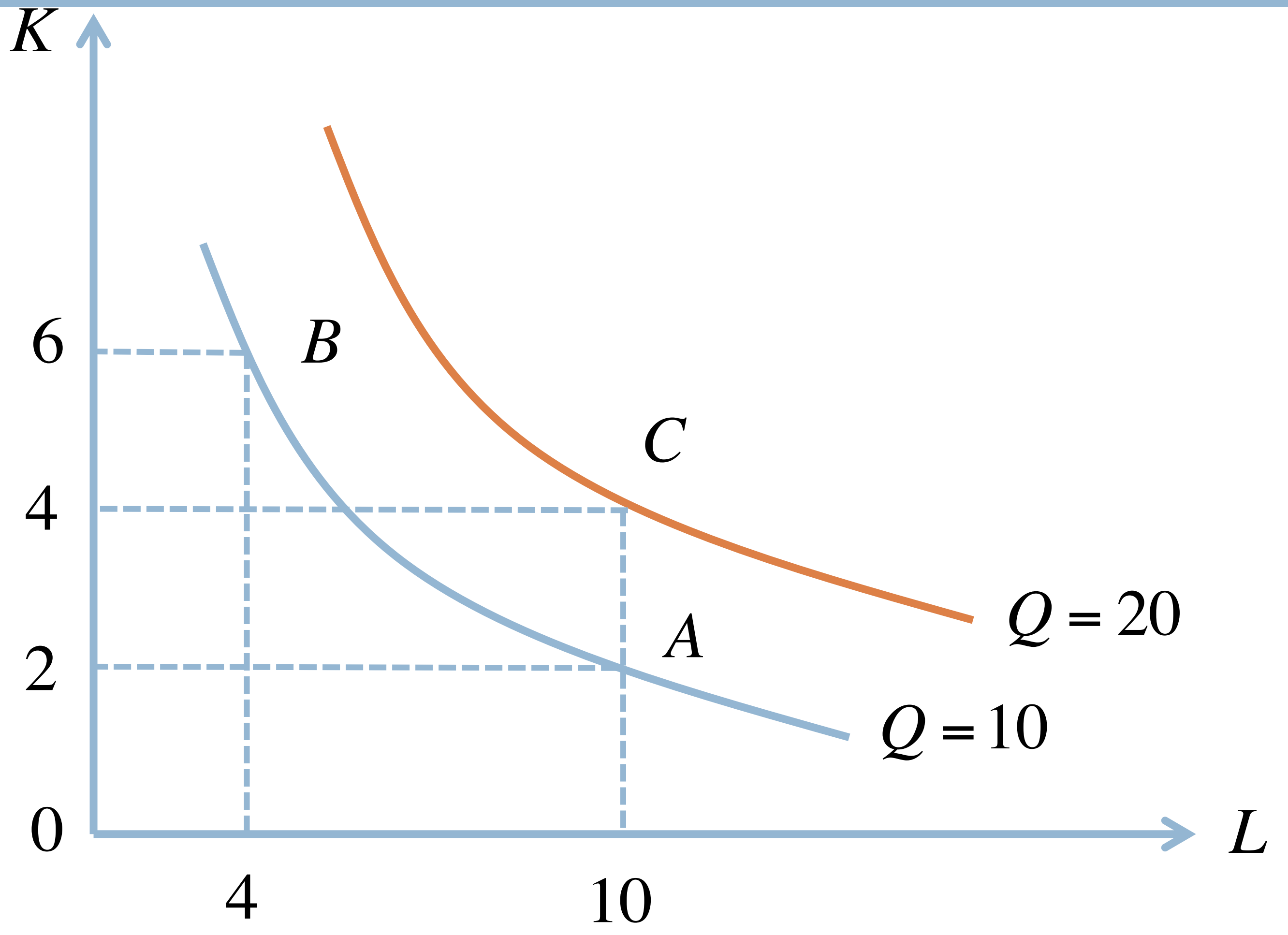
$$MP_L = \frac{\partial Q}{\partial L}$$

$$MP_K = \frac{\partial Q}{\partial K}$$

Isoquants

- We can describe production function using isoquants
- Definition 7.7 An *isoquant* is a curve that connects all combinations of labor and capital that generate the same level of output

Isoquants in Graph



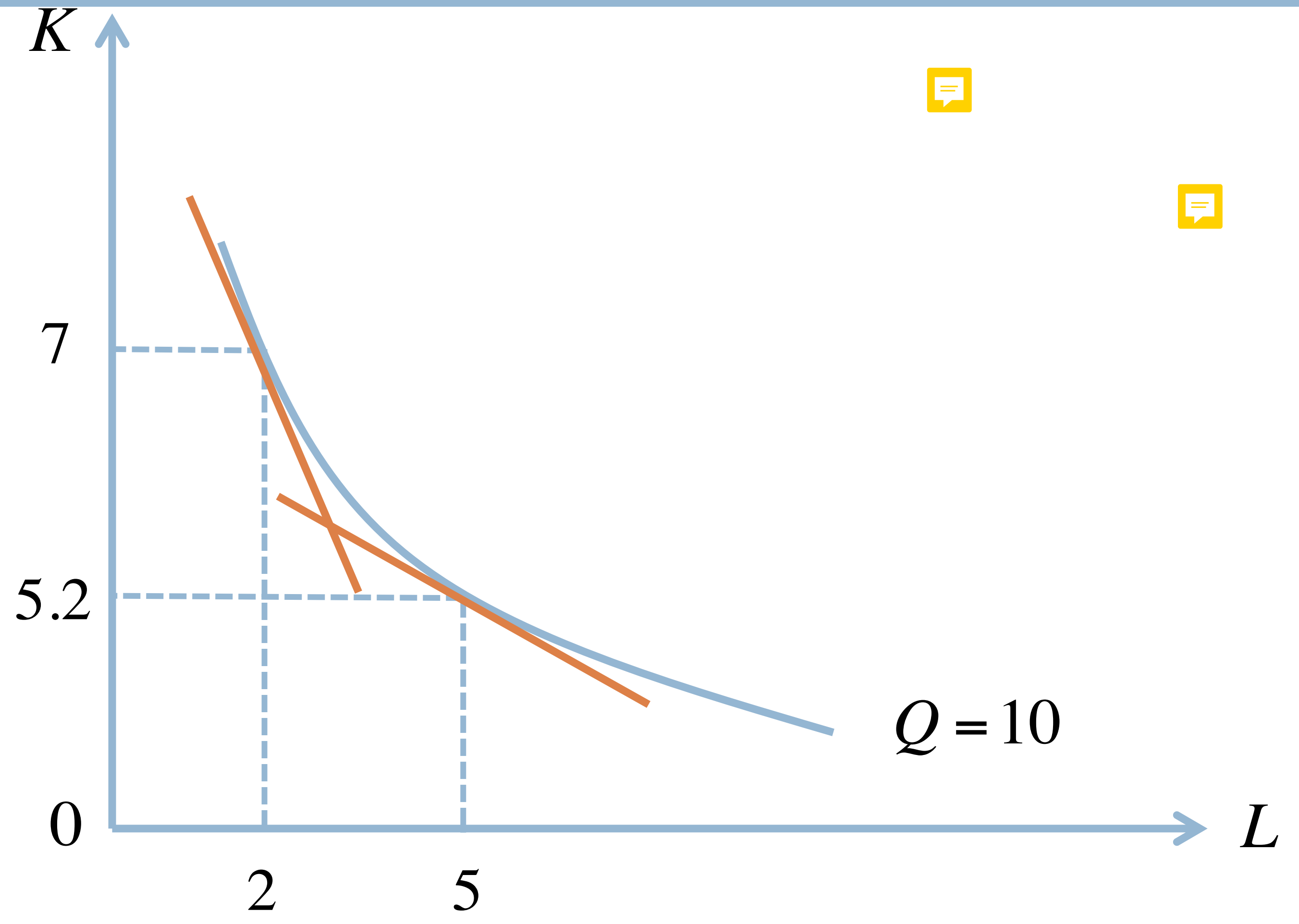
Marginal Rate of Technical Substitution

- Definition 7.8 *Marginal rate of technical substitution* of labor for capital is the rate at which the firm can reduce the quantity of capital for more labor, holding the output level fixed

$$MRTS_{L,K} = -\left.\frac{dK}{dL}\right|_{Same\quad Q} = -\left.\frac{\Delta K}{\Delta L}\right|_{Same\quad Q}$$

  where ΔL is extremely small
- *MRTS* is the negative of the slope of the isoquant

Diminishing Marginal Rate of Technical Substitution



MRTS and *MP*

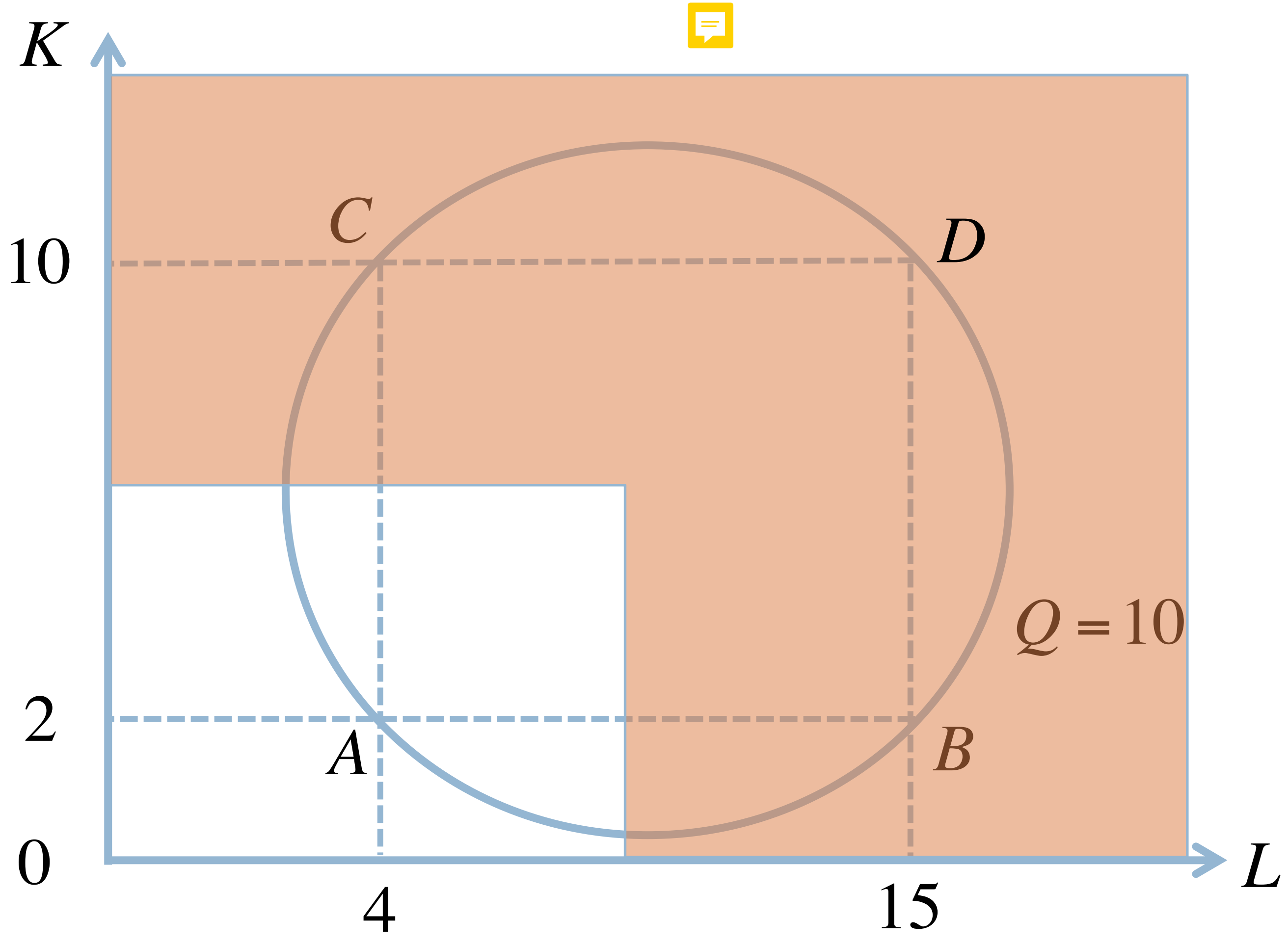
- Suppose the firm changes the quantity of labor and capital, but keeps the output level fixed
- The total change in output is
- The total change in output must be 0
- Thus

$$\frac{MP_L}{MP_K} = -\frac{\Delta K}{\Delta L} = MRTS_{L,K}$$

Analogy to Consumer Theory

- Isoquant
  - Indifference curve
- Marginal rate of technical substitution
  - Marginal rate of substitution
- Diminishing marginal rate of technical substitution
  - Diminishing marginal rate of substitution

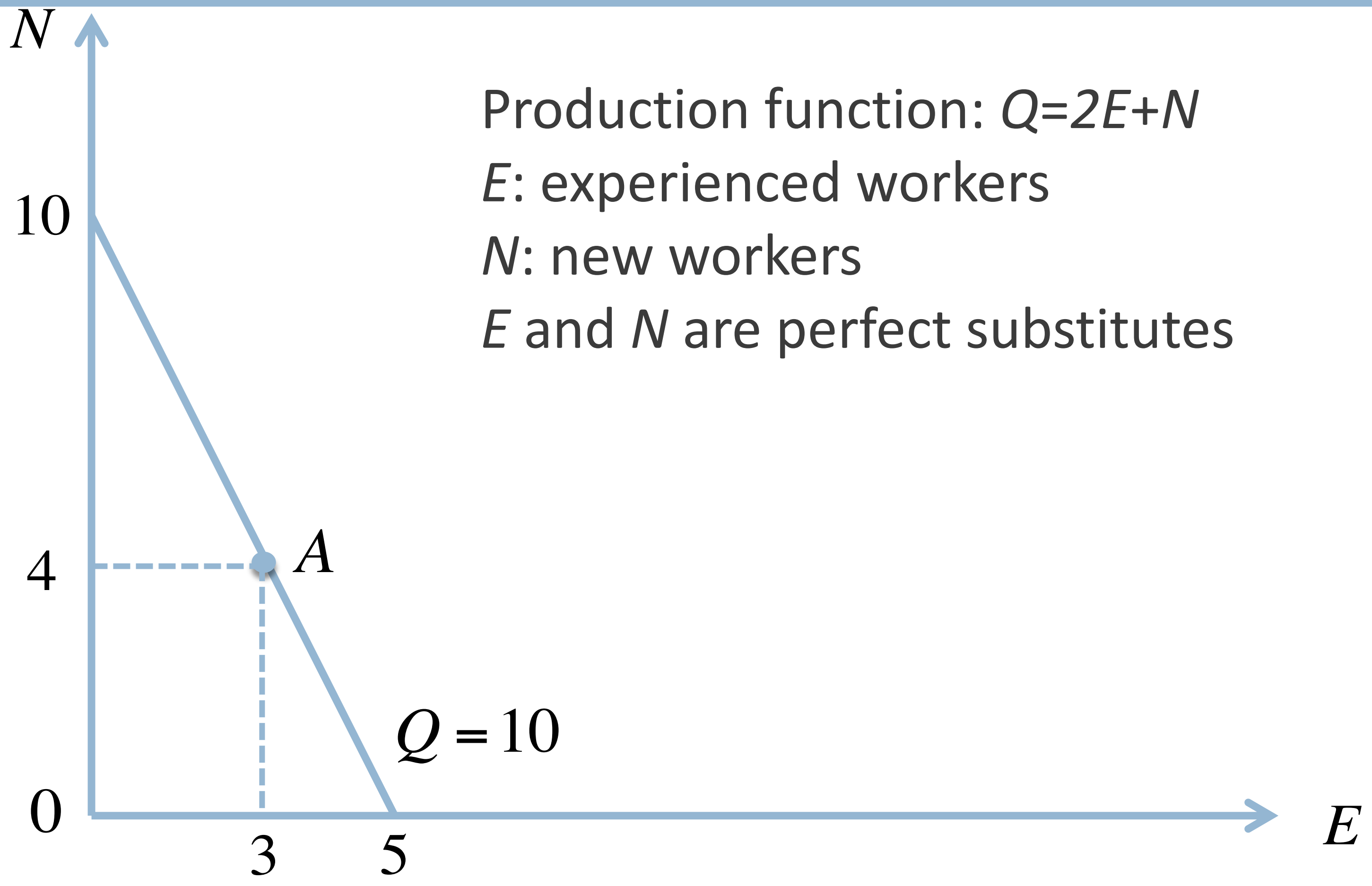
Uneconomic Region of Production



Marginal Product and Uneconomic Region of Production

- Definition 7.9 In the *uneconomic region of production*
  - At least one marginal product is negative
- Cost-minimizing firms never produce in the uneconomic region of production
  - E.g., if the firm produces at point B, it uses 15 labor and 2 capital
  - The firm can produce the same quantity at point A with 4 labor and 2 capital

Linear Production Function



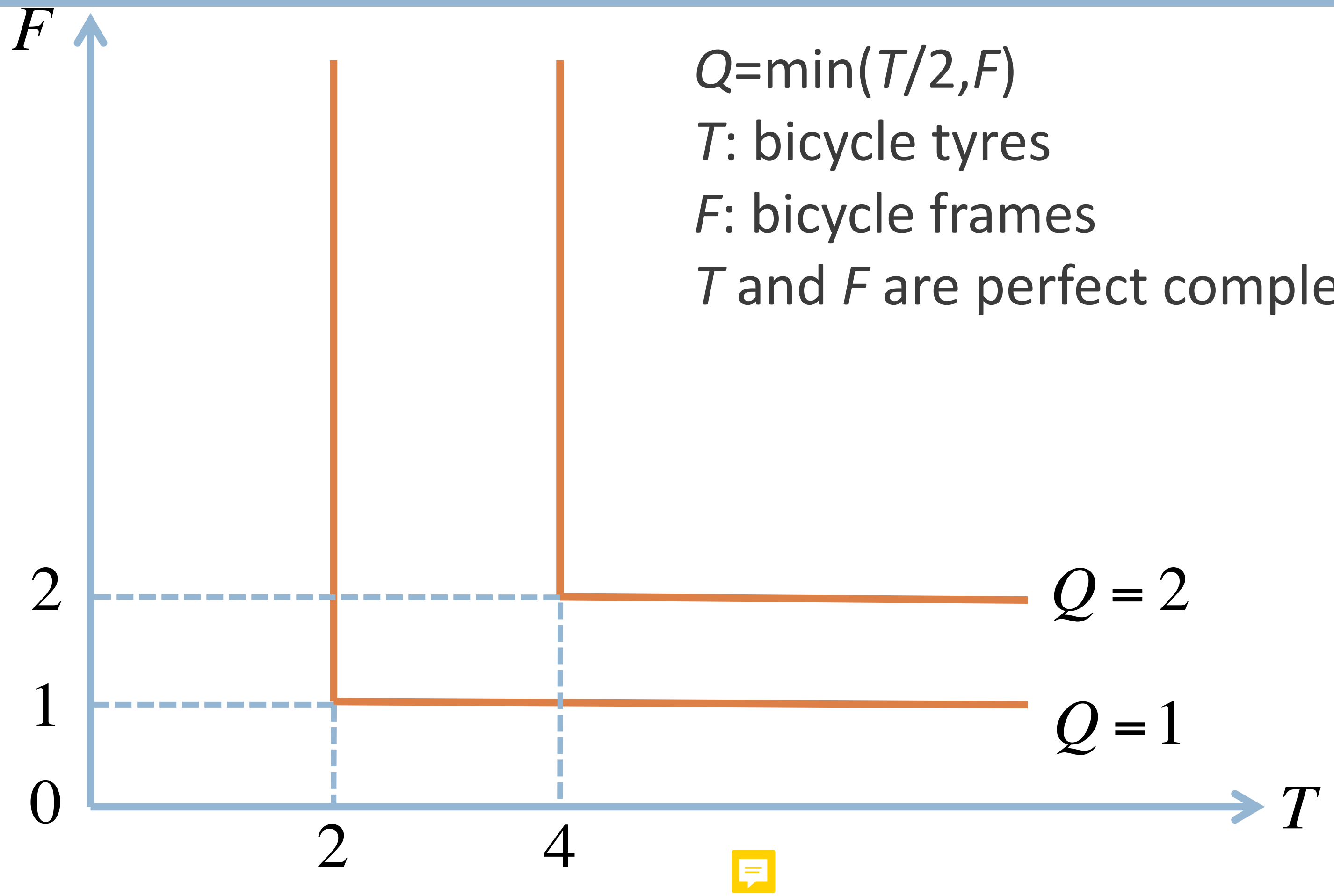
Production function: *Q=2E+N*

E: experienced workers

N: new workers

E and *N* are perfect substitutes

Fixed Proportions Production Function



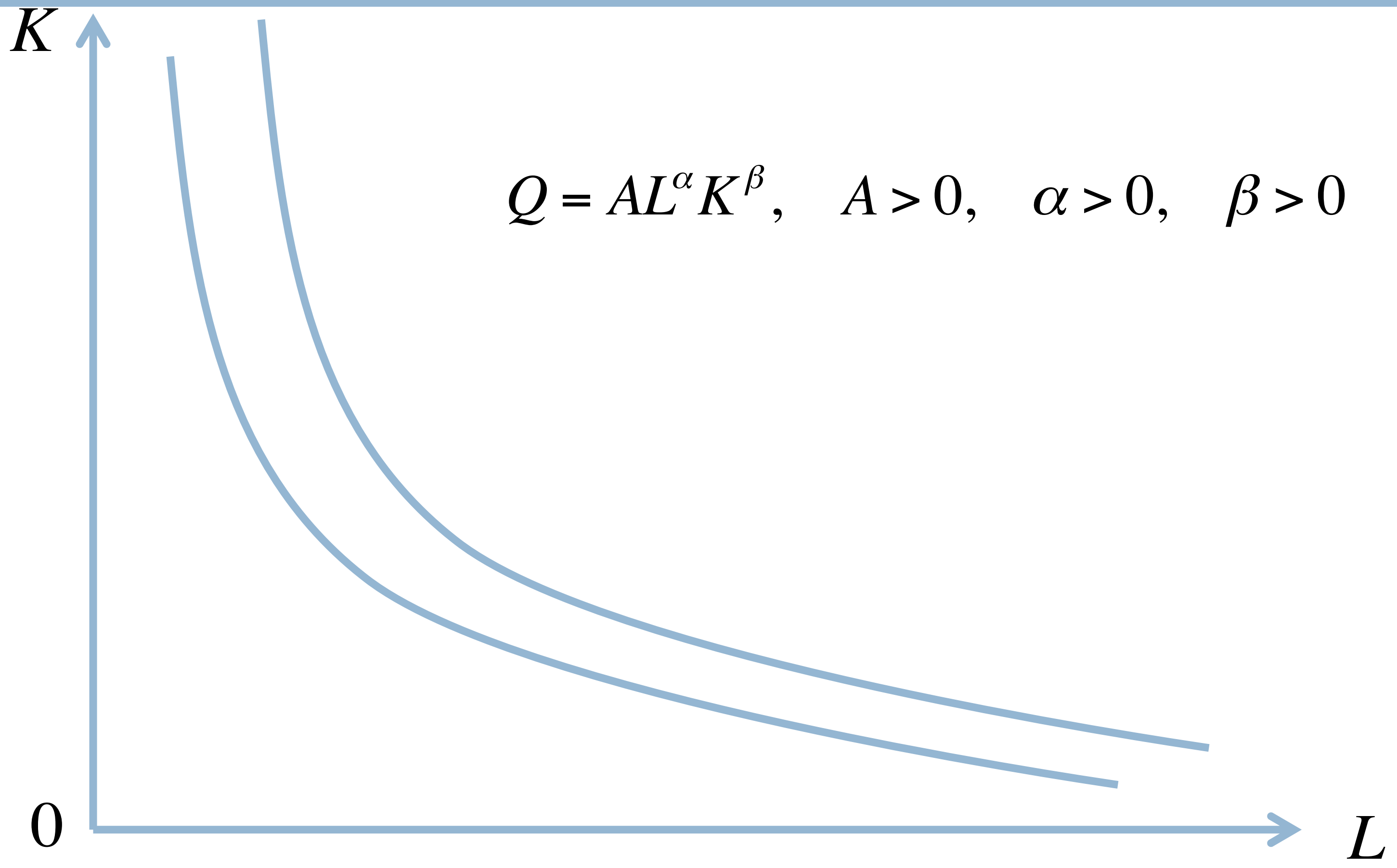
Q=min(T/2,F)

T: bicycle tyres

F: bicycle frames

T and F are perfect complements

Cobb-Douglas Production Function



$$Q = AL^{\alpha}K^{\beta}, \quad A > 0, \quad \alpha > 0, \quad \beta > 0$$

Part 3

Returns to Scale and Technological Progress

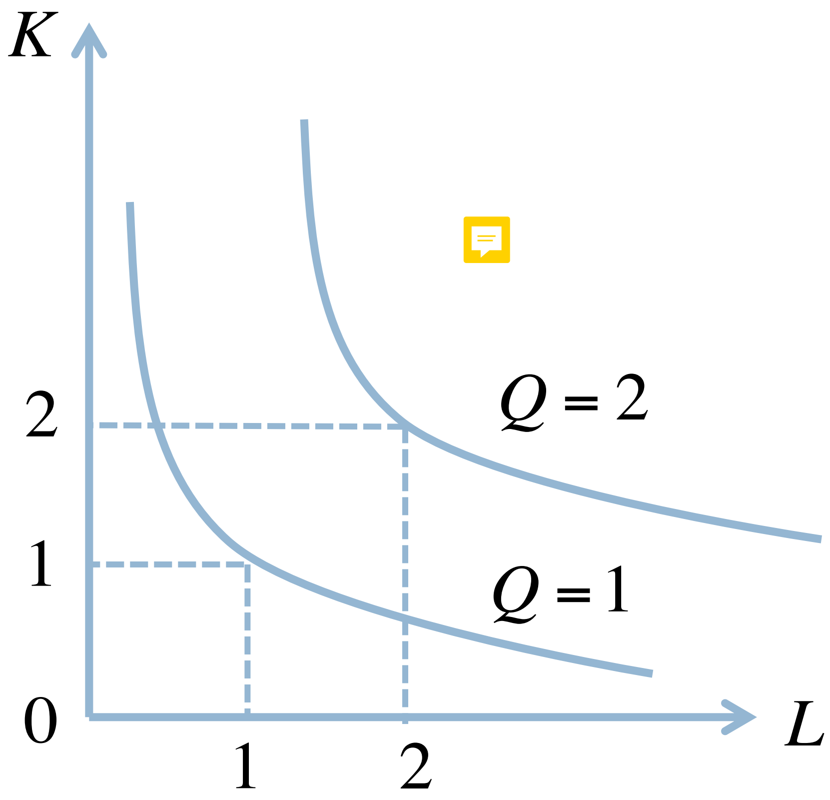
Returns to Scale

- How much more Q can the firm produce when using more L and K?
- *Returns to scale* measures the rate at which output increases when all inputs increase proportionately
  - E.g., how much will output increase if both labor and capital increase by 25%?
  - E.g., how much will output increase if both labor and capital increase by 100%?

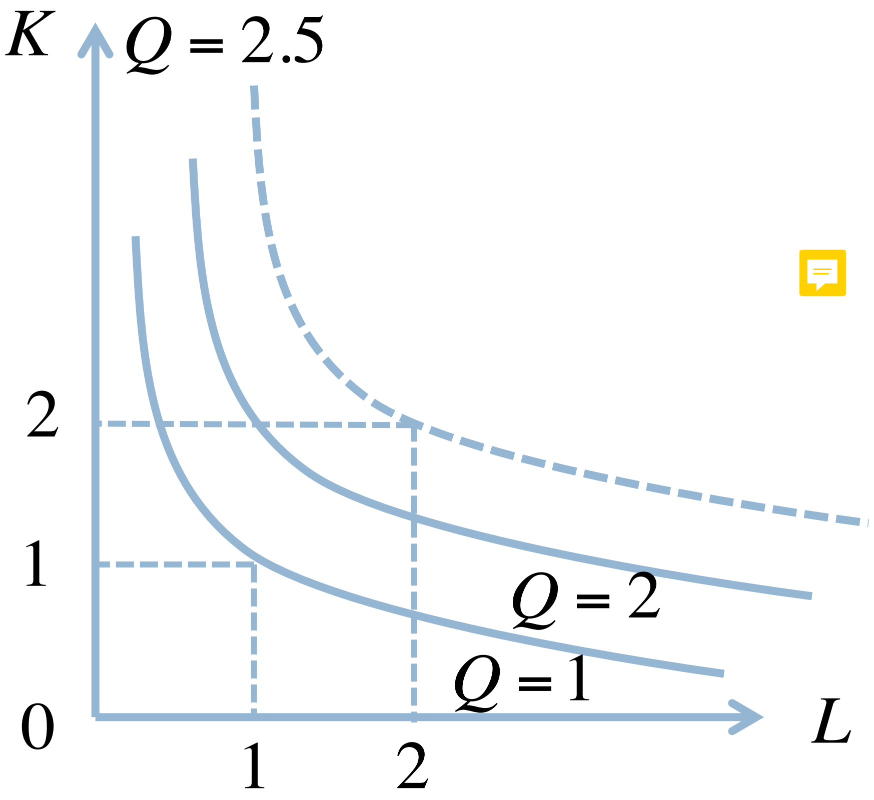
Interpreting Returns to Scale

- Suppose when L increases to aL and K increases to aK ($a>1$)
- Output increases to bQ
- Definition 7.10 *Increasing returns to scale*
  - If $b>a$
- Definition 7.11 *Constant returns to scale*
  - If $b=a$
- Definition 7.12 *Decreasing returns to scale*
  - If $b<a$

Returns to Scale and Isoquants



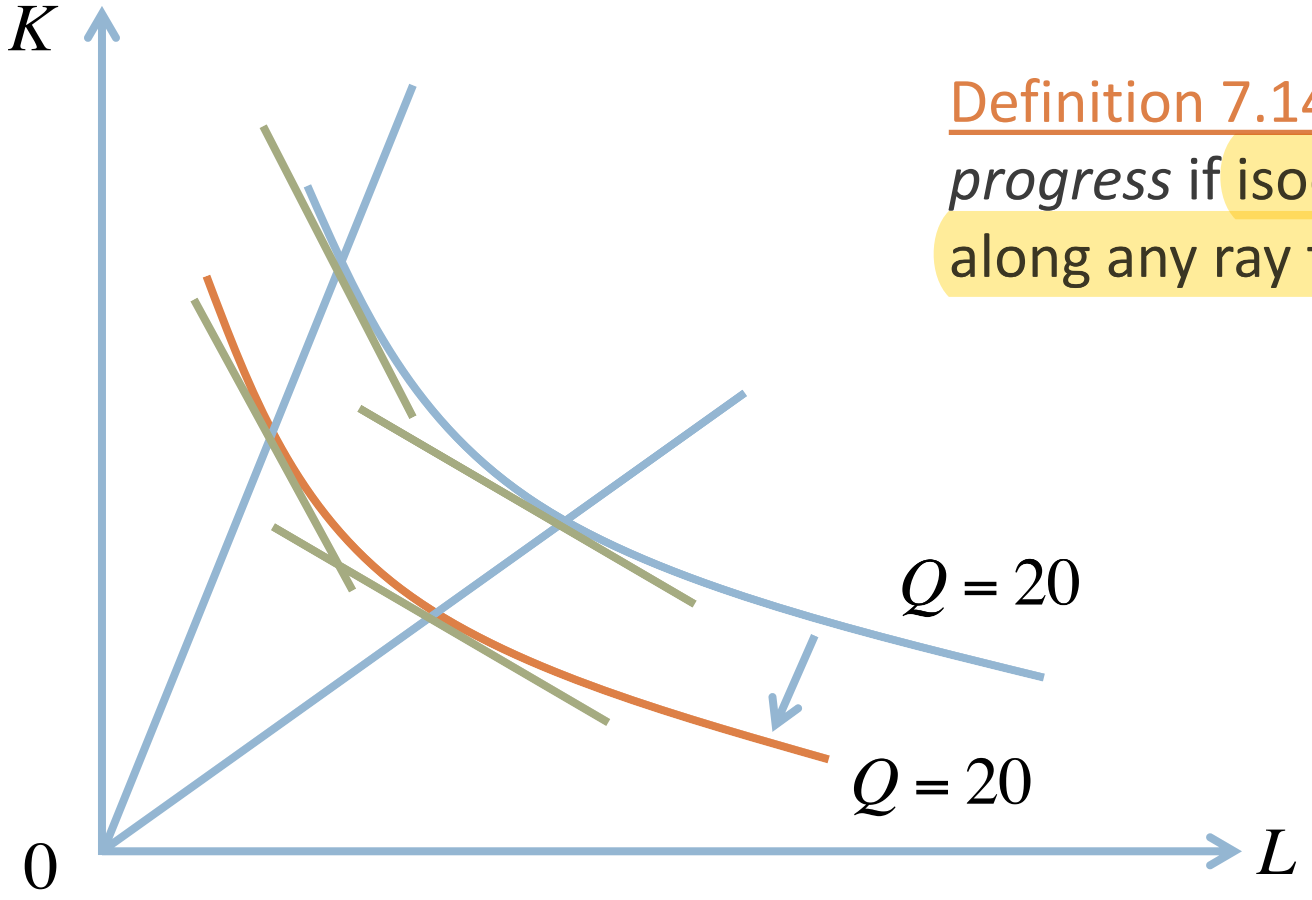
Constant Returns to Scale



Technological Progress

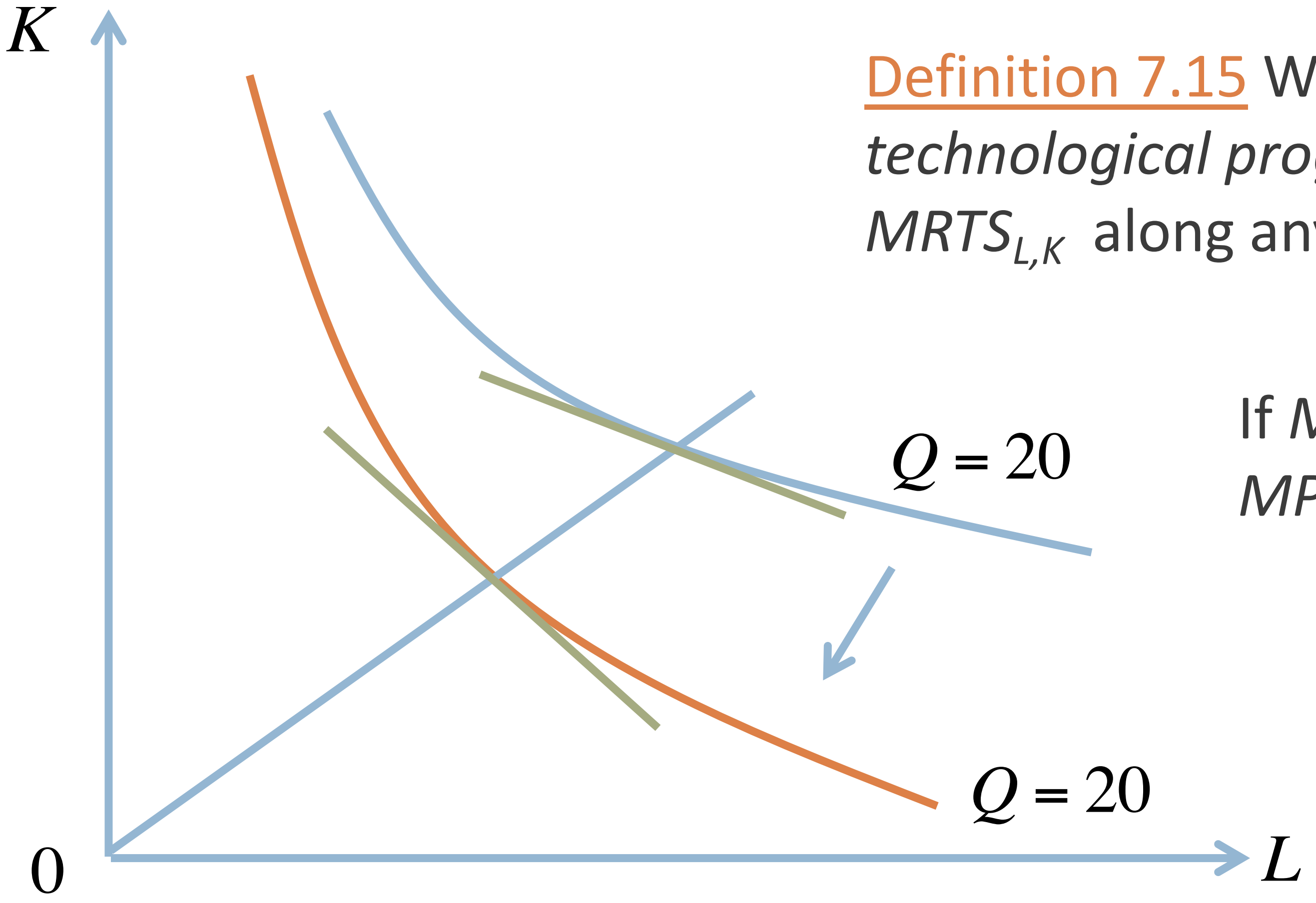
- So far we assumed production technology is fixed
  - Production function is fixed
- What if technology improves?
- Definition 7.13 We have *technological progress* if for any given combination of inputs, the firm produces higher Q
  - Or, to produce any Q, the firm uses less input

Neutral Technological Progress



Definition 7.14 We have a *neutral technological progress* if isoquant shifts inward and $MRTS_{L,K}$ along any ray from the origin remains the same

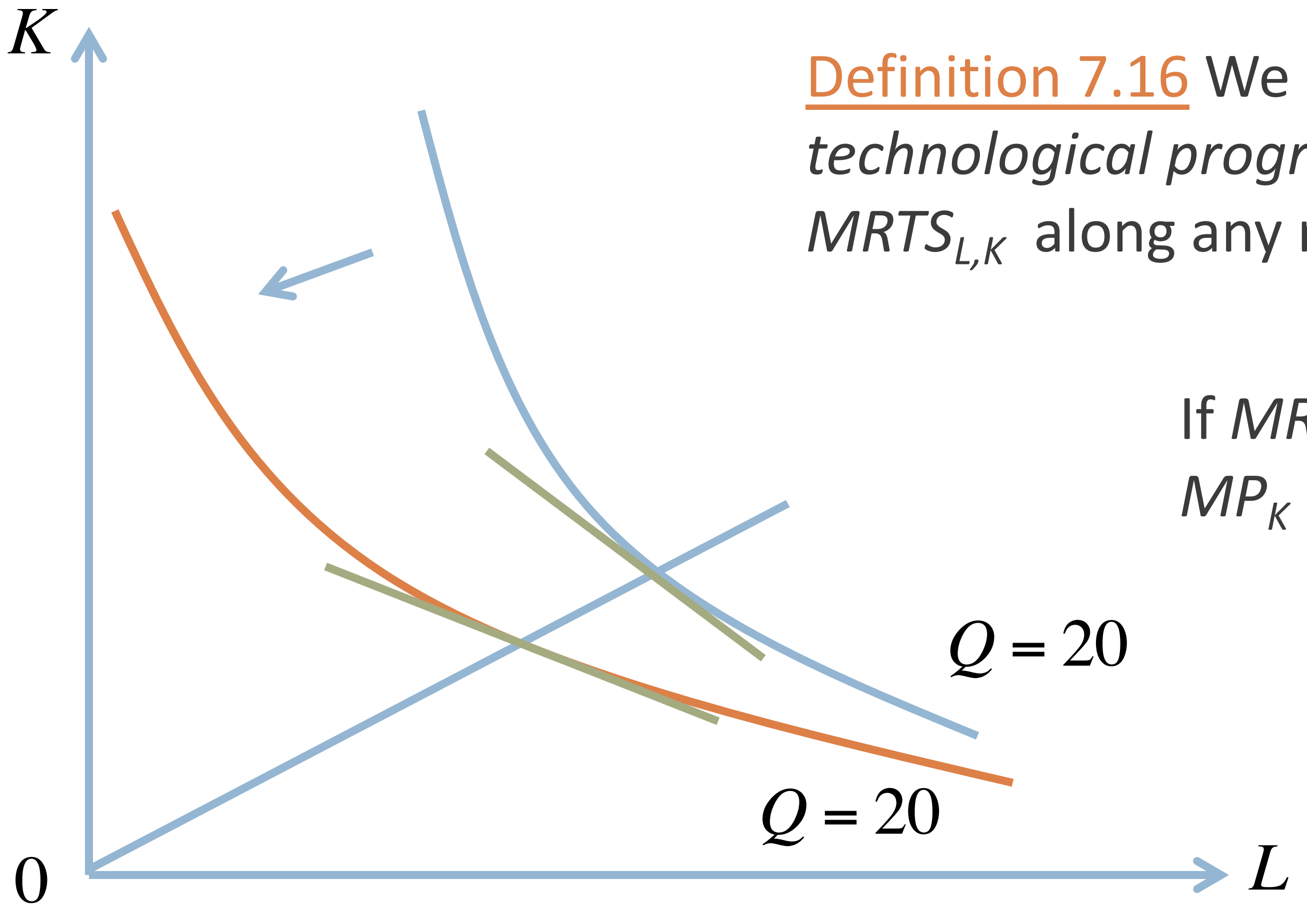
Capital-Saving Technological Progress



Definition 7.15 We have a *capital-saving technological progress* if isoquant shifts inward and $MRTS_{L,K}$ along any ray from the origin increases

If $MRTS_{L,K} = MP_L/MP_K$ increases MP_L increases relative to MP_K

Labor-Saving Technological Progress



Definition 7.16 We have a *labor-saving technological progress* if isoquant shifts inward and $MRTS_{L,K}$ along any ray from the origin decreases

If $MRTS_{L,K} = MP_L/MP_K$ decreases MP_K increases relative to MP_L

Does Neutral Technological Progress Mean *MRTS* does not change?

- Suppose the initial production function is

$$Q^1 = KL + K$$

- The new production function is

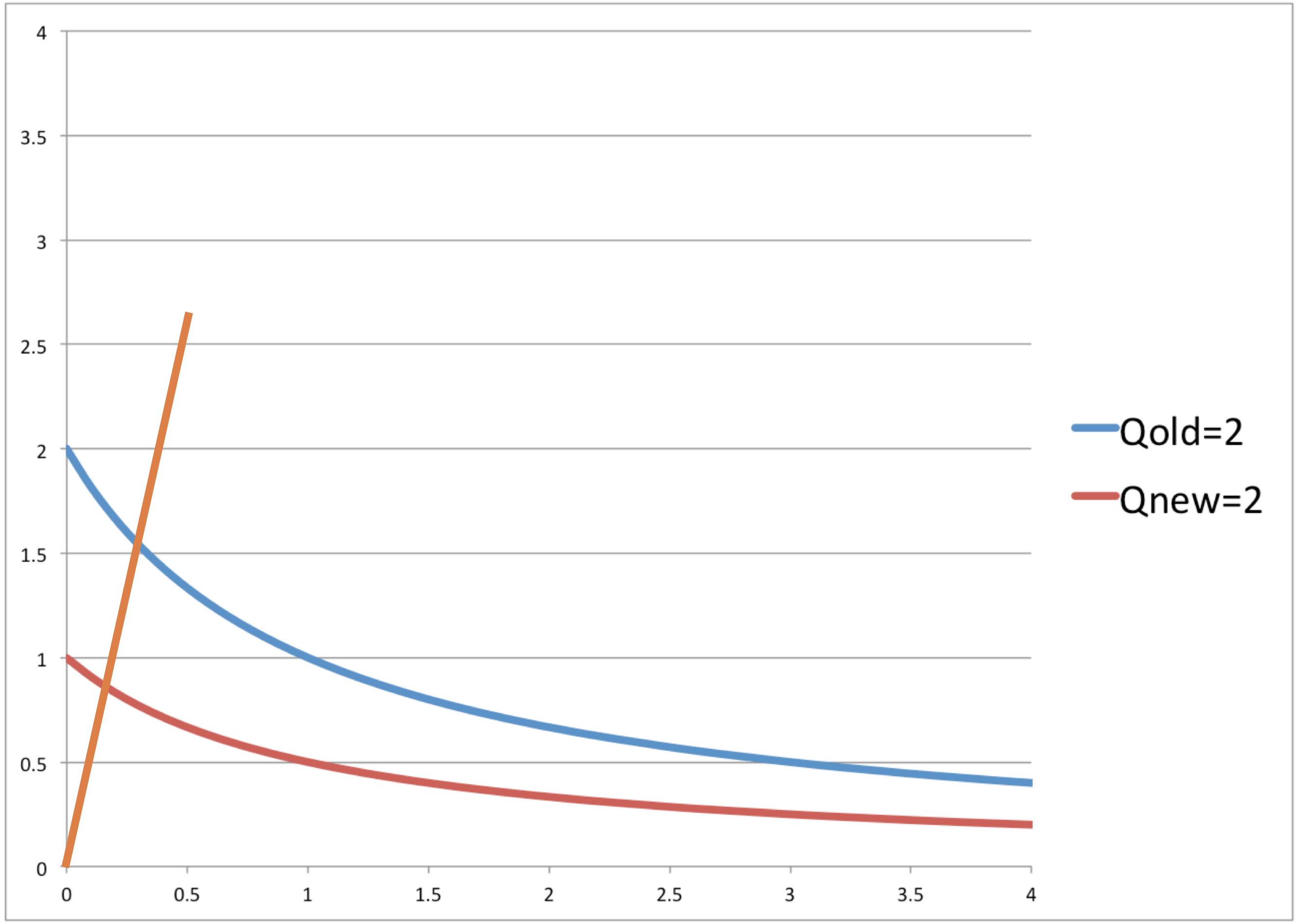
$$Q^2 = 2(KL + K)$$

- $MRTS_{L,K}$ does not change

$$MRTS^1_{L,K} = MRTS^2_{L,K} = \frac{K}{L+1}$$

- Is this neutral technological progress?

Isoquants of Q=2 before and after



MRTS along a ray from the origin not the same

- With the initial production function
  - When $L=K=1$, $Q=2$
  - $(L=1, K=1)$ is on the ray $K=L$
  - At this point $MRTS_{L,K}=1/(1+1)=0.5$
- With the new production function
  - The point on $Q=2$ and $K=L$ is $(L=0.62, K=0.62)$
  - At this point $MRTS_{L,K}=0.62/(0.62+1)=0.38$
- $MRTS_{L,K}$ along the ray $K=L$ not the same!

Technological Progress for Cobb-Douglas Production Functions

- Suppose the initial production function is

$$Q^1 = KL$$

- The new production function is

$$Q^2 = 2KL$$

- $MRTS_{L,K}$ does not change

$$MRTS^1_{L,K} = MRTS^2_{L,K} = \frac{K}{L}$$

- This is indeed a neutral technological progress

We can just Compare *MRTS* for Cobb-Douglas Production Functions

- With the initial production function
  - When L=1, K=2, Q=2
  - (L=1, K=2) is on the ray K=2L
  - At this point, $MRTS_{L,K}$=2
- With the new production function
  - The point on Q=2 and K=2L is (L=0.71, K=1.41)
  - At this point $MRTS_{L,K}$=2
- $MRTS_{L,K}$ along the ray K=2L are the same
- Same applies to any ray K=aL