PRE-LECTURE VIDEO

COBB-DOUGLAS UTILITY FUNCTION

Definition

- A utility function of the following form is called a Cobb-Douglas utility function

$$U(x,y) = Ax^{\alpha}y^{\beta}, A > 0, \alpha > 0, \beta > 0$$

- Examples of Cobb-Douglas utility function

$$U(x,y) = xy$$

$$U(x,y) = \frac{1}{3}x^2y^3$$

$$U(x,y) = \sqrt{xy}$$

$$U(x,y) = 4x^{\frac{1}{3}}y^5$$

Marginal Utilities of Cobb-Douglas Utility Functions

- Partially differentiating the utility function

$$MU_x = A\alpha x^{\alpha-1}y^{\beta}$$

$$MU_y = A\beta x^{\alpha}y^{\beta-1}$$

- Both marginal utilities are always positive
- "More is better" satisfied for both goods
- Indifference curves are downward sloping

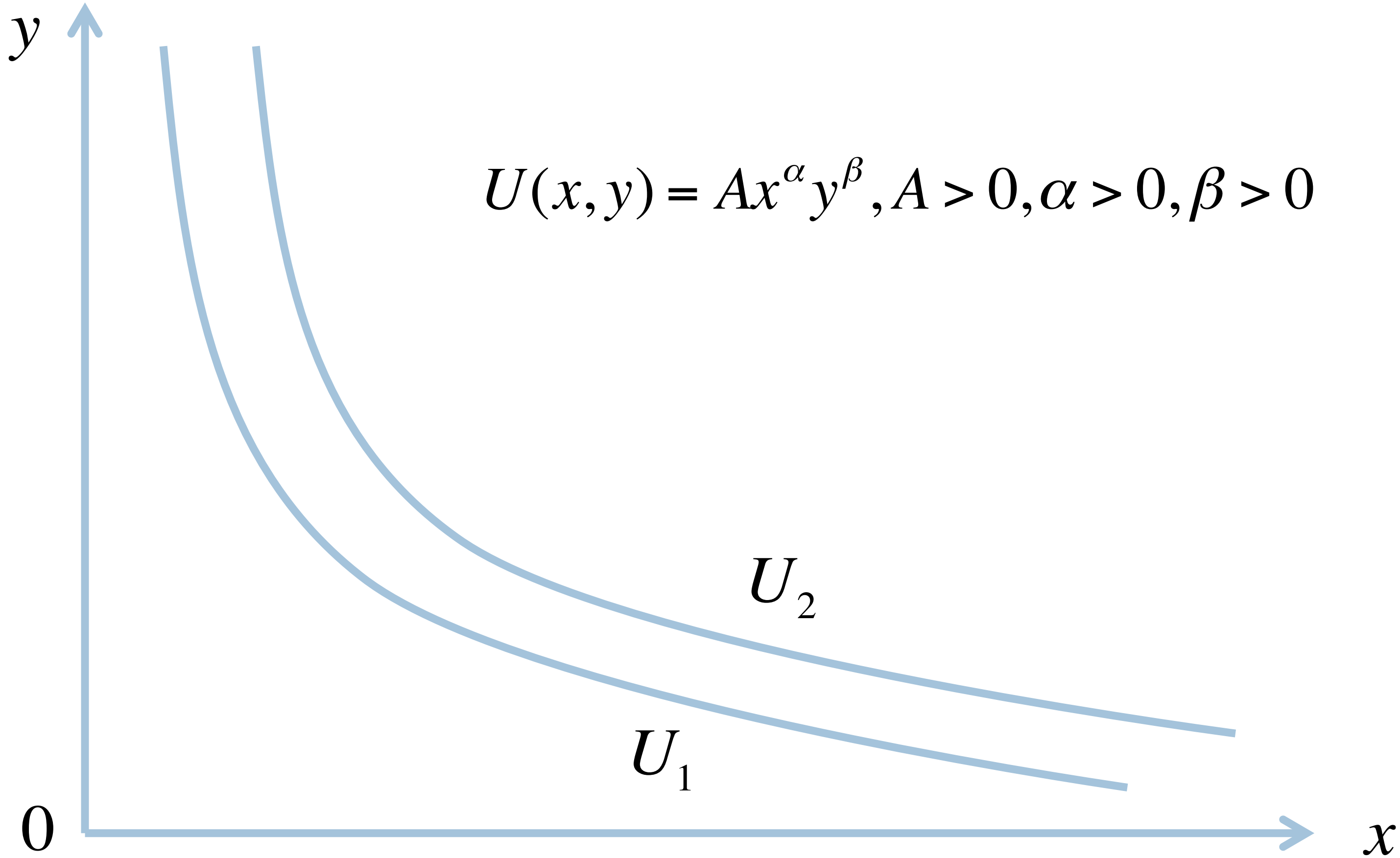
Marginal Rate of Substitution of Cobb-Douglas Utility Functions

- The marginal rate of substitution is

$$MRS_{x,y} = \frac{MU_x}{MU_y} = \frac{A\alpha x^{\alpha-1} y^{\beta}}{A\beta x^{\alpha} y^{\beta-1}} = \frac{\alpha y}{\beta x}$$

- As the consumer gets more *x* and less *y* along the same indifference curve
  - $MRS_{x,y}$ diminishes
- Indifference curves are convex

Typical Indifference Curves for Cobb-Douglas Utility Functions



$$U(x,y) = Ax^{\alpha}y^{\beta}, A > 0, \alpha > 0, \beta > 0$$

Is the principle of diminishing marginal utility satisfied?

- Recall the marginal utilities are

$$MU_x = A\alpha x^{\alpha-1}y^{\beta}$$

$$MU_y = A\beta x^{\alpha}y^{\beta-1}$$

- Differentiating MU_x with respect to x, we get

$$\frac{\partial MU_x}{\partial x} = A\alpha(\alpha-1)x^{\alpha-2}y^{\beta}$$

- The derivative is negative when $\alpha < 1$
- Marginal utility for Cobb-Douglas utility functions may or may not be diminishing

Examples

- Consider the utility function

$$U(x, y) = x^2 y^2$$

- The marginal utilities and the marginal rate of substitution are

$$MU_x = 2xy^2$$

$$MU_y = 2x^2 y$$

$$MRS_{x,y} = \frac{2xy^2}{2x^2 y} = \frac{y}{x}$$

- Marginal utilities are increasing, not diminishing

Examples Cont'

- Consider the utility function

$$U(x,y) = \sqrt{xy}$$

- The marginal utilities and the marginal rate of substitution are

$$MU_x = \frac{1}{2}\sqrt{\frac{y}{x}}$$

$$MU_y = \frac{1}{2}\sqrt{\frac{x}{y}}$$

$$MRS_{x,y} = \frac{y}{x}$$

- Marginal utilities are diminishing

Why do we study Cobb-Douglas Utility Functions?

- Convenient mathematical/economic properties
  - Simple functional form
  - "More is better" satisfied
  - Diminishing marginal rate of substitution
  - Indifference curves do not intersect the axes
- What kind of preferences can be represented by a Cobb-Douglas utility function?