Final Practice Problems

Question 1 Consider question 1 in homework 2. Set y as a numeraire. Is the allocation (x^A=10, y^A=0, x^B=6, y^B=12) an equilibrium allocation? Briefly explain.

Question 2 There are two consumers, A and B, in an exchange economy. There are three goods in the economy, x_1, x_2, and x_3.

a) Prove the Walras' law in this two-consumers, three-goods economy.

b) When market for good 1 is in equilibrium, market for good 2 and market for good 3 must also be in equilibrium. Is the above statement true or false? Briefly explain.

c) If there is excess demand for good 1, there must be excess supply for some other good. Is the above statement true or false? Briefly explain.

Question 3 A firm has production function $Q=KL+L+K$. The price of labor is w=1 and the price of capital is r=1.

a) Suppose in the short run, capital is fixed at K=3. To produce an output of Q=8, the short-run cost-minimizing labor choice is $L =$ (1).

b) In the long run, what is the cost-minimizing input combination when the firm produces Q=8?

c) Suppose now w=1 and r=10. What is the long-run cost-minimizing input combination when the firm produces Q=8?

Question 4 A firm's production function is given by $Q = L(K-3)$ where $K \geq 3$. The price of labor is $w > 0$ and the price of capital is $r > 0$.

a) Suppose in the short run, capital is fixed at $K = 6$. The short-run cost-minimizing labor choice is $L =$ (1). The equation of the firm's short-run total cost function is $STC(Q,w,r) =$ (2).

b) Suppose the firm can choose labor and capital freely. That is, consider the scenario in the long run. To minimize cost, the firm will choose labor and capital such that $\frac{w}{r} =$ (3). The demand function for labor is $L =$ (4), and the demand function for capital is $K =$ (5).

c) Consider the firm's expenditure on labor and capital in the long run. The firm's expenditure on (6) is always higher than the expenditure on (7) by the amount of (8).

Question 5 A firm's production function is $Q = \sqrt{L} + 2\sqrt{K}$. The price of labor is $w > 0$ and the price of capital is $r > 0$.

a) Derive the demand functions for labor and capital respectively.

b) What is the long-run total cost function?

c) Suppose in the short run, capital is fixed at $K = 9$. To find the short-run total cost function, is the following solution correct? If not, what is the mistake?

"Solution: Since capital is fixed at 9, we have $Q = \sqrt{L} + 6$, thus the firm needs $L = (Q-6)^2$ units of labor and $STC(Q, w, r) = w(Q-6)^2 + 9r$."

Question 6 A market contains a group of identical price taking firms. Each firm has a short-run marginal cost curve $SMC(Q) = 2Q$, where Q is the annual output of each firm. A study reveals that the minimum level of the average non-sunk cost for each firm is \$20. The market demand curve for the industry is $D(P) = 240 - P/2$, where P is the market price. At the short-run equilibrium market price, each firm produces 20 units. What is the short-run equilibrium market price, and how many firms are in this industry?

Question 7 In a perfectly competitive industry with 80 identical firms, every firm faces the short-run total variable cost curve characterized by $VC(Q) = Q^2 + 8Q$. Each firm's fixed cost is 100, out of which 25 is non-sunk. The market demand curve is $D(P) = 4000 - 40P$.

a) What is the short-run supply curve for each firm? What is the shut down price (the price below which the firm will not produce and above which the firm will produce) for each firm?

b) What is the short-run equilibrium price in this market? What is the producer surplus for each firm?

c) Suppose the fixed cost for each firm is still 100, but we do not know how much of it is non-sunk. What is the possible range of the shut down price?

Question 8 Suppose the market for a type of mineral is perfectly competitive, characterized by price-taking firms and free entry and exit. All producers are identical and currently they have the following short-run average total cost curve, $SAC(Q)=2Q+648/Q$. Assume all fixed cost is sunk in the short run. Moreover, the short-run average total cost curve given above is also the one that touches the long-run average total cost curve of a typical firm at its minimum. Any potential entrant into the market would also have the same cost curves. The market demand curve for the mineral is $D(P)=2340-20P$. Currently, there are 24 firms in the market.

a) The equation of the average non-sunk cost curve is $ANSC(Q) =$ (1). An individual firm's supply curve is $Q =$(2). The short-run equilibrium price of the mineral is (3). In the short-run equilibrium, each firm's total cost is (4) and each firm's profit for is (5).

b) Suppose the market has reached the short-run equilibrium in part a). In the long run, the market price will (6) (answer "increase", "decrease", or "stay the same"). This is because (7). The long-run equilibrium price in this market is (8). Without the long-run cost curves, we can still determine the long-run equilibrium price because (9). In the long run equilibrium, each firm produces (10). There will be (11) firms in the market in the long-run equilibrium.

Question 9 Consider a perfectly competitive market. Every firm is identical and each faces the long-run total cost curve $LTC(Q) = \begin{cases} F + 2Q^2, & Q > 0 \\ 0, & Q = 0 \end{cases}$, where F is the licensing fee each firm pays to the government. The market demand curve is $Q = \dfrac{10,000}{P}$.

a) What is the long-run equilibrium price? What is the equilibrium number of firms?

b) In the long run, who bear the cost when the licensing fee increases, the producers or the consumers? (Hint: how does the increase in licensing fee affect the profit of each firm? How does it affect the consumer surplus in the market? You do not need to calculate the consumer surplus.)

c) Suppose the number of firms in this market is fixed at 100 even in the long run. In other words, entry or exit is not possible in the long run. For simplicity, assume F is low enough that it is always profitable for every firm to produce. What is the new long-run equilibrium price? Reconsider part b), what is your answer now?

Solutions

Question 1

Suppose this allocation is an equilibrium allocation. For this allocation to lie on the budget line, the price of x must be 1 because the budget line has to pass through this allocation and the endowment allocation. The budget line for consumer B is thus $x^B+y^B=18$. The tangency point between the budget line and B's indifference curve is where $\frac{y^B}{x^B}=1$. Thus given this budget line, consumer B's optimal basket is $x^B=9$, $y^B=9$. Therefore the allocation ($x^A=10$, $y^A=0$, $x^B=6$, $y^B=12$) is not an equilibrium allocation because ($x^B=6$, $y^B=12$) is not the utility maximizing basket for consumer B.

Question 2

a) To maximize utility, consumer A must consume a basket on the budget line, thus $P_1x_1^A+P_2x_2^A+P_3x_3^A=P_1\omega_1^A+P_2\omega_2^A+P_3\omega_3^A$.
Rearranging, we have $P_1(x_1^A-\omega_1^A)+P_2(x_2^A-\omega_2^A)+P_3(x_3^A-\omega_3^A)=0$.

Similarly, consumer B must consume a basket on the budget line, thus $P_1x_1^B+P_2x_2^B+P_3x_3^B=P_1\omega_1^B+P_2\omega_2^B+P_3\omega_3^B$.

Rearranging, we have $P_1(x_1^B-\omega_1^B)+P_2(x_2^B-\omega_2^B)+P_3(x_3^B-\omega_3^B)=0$.

Adding up the two equations, we have the Walras' law:
$P_1(x_1^A+x_1^B-\omega_1^A-\omega_1^B)+P_2(x_2^A+x_2^B-\omega_2^A-\omega_2^B)+P_3(x_3^A+x_3^B-\omega_3^A-\omega_3^B)=0$.

b) False. When market 1 is in equilibrium, $x_1^A+x_1^B-\omega_1^A-\omega_1^B=0$. By the Walras' law, we just need $P_2(x_2^A+x_2^B-\omega_2^A-\omega_2^B)+P_3(x_3^A+x_3^B-\omega_3^A-\omega_3^B)=0$, which does not require $x_2^A+x_2^B-\omega_2^A-\omega_2^B=0$ or $x_3^A+x_3^B-\omega_3^A-\omega_3^B=0$. For example, it is possible that there is excess demand for good 2 and excess supply for good 3. Thus market 2 and market 3 do not have to be in equilibrium.

c) True. If there is excess demand for good 1, we have $x_1^A+x_1^B-\omega_1^A-\omega_1^B>0$. Based on the Walras' law, we must have $x_2^A+x_2^B-\omega_2^A-\omega_2^B<0$, or $x_3^A+x_3^B-\omega_3^A-\omega_3^B<0$, or both. Thus there must be excess supply for some good.

Question 3

a) (1) 1.25.

b) $MP_L=K+1$ and $MP_K=L+1$. To minimize cost, the firm needs $(K+1)/(L+1)=w/r=1$, which gives us $K=L$. To produce 8 units, it must be that $L^2+2L=8$, therefore $L=2$ and $K=2$.

c) At interior solution, the cost-minimizing input must satisfy $(K+1)/(L+1)=w/r=1/10$. To produce 8 units, we need $KL+K+L=8$. The solution to the two equations is non-positive. Since K cannot be negative, in the optimal combination, it must the case that $K=0$, i.e., we have a corner solution. Thus to produce 8 units, we need $L=8$.

Question 4

a) (1) $Q/3$.

(2) $\frac{wQ}{3}+6r$.

b) (3) $\frac{K-3}{L}$.

(4) $\sqrt{\frac{rQ}{w}}$.

(5) $\sqrt{\frac{wQ}{r}}+3$.

c) (6) capital.
(7) labor.
(8) $3r$.

Question 5

a) The marginal product of labor is $MP_L = \frac{1}{2\sqrt{L}}$ and the marginal product of capital is $MP_K = \frac{1}{\sqrt{K}}$. The tangency condition is $\frac{\sqrt{K}}{2\sqrt{L}} = \frac{w}{r}$. Substituting into the production function, the demand function for labor is $L(w,r,Q) = \frac{r^2}{(r+4w)^2}Q^2$ and the demand function for capital is $K(w,r,Q) = \frac{4w^2}{(r+4w)^2}Q^2$.

b) Using the demand functions, the long-run total cost function is

$$LTC(w,r,Q) = wL + rK = \frac{rw}{r+4w}Q^2.$$

c) It is not correct. To produce less than or equal to 6 units, the firm does not need any labor, and the total cost is $9r$. Therefore, the short-run total cost function is

$$STC(w,r,Q) = \begin{cases} 9r, & Q \le 6 \\ w(Q-6)^2 + 9r, & Q > 6 \end{cases}$$

Question 6

When P is at least 20, each individual firm chooses Q such that P=SMC. Thus each individual firm's supply curve is $Q = \frac{P}{2}$, when $P \geq 20$, and Q=0 when P<20.

Given the supply curve of each individual firm, when each firm produces 20 tons per year, $Q = \frac{P}{2} = 20 \Rightarrow P = 40$. Thus the current equilibrium price is \$40.

At the equilibrium price of \$40, market demand is $D(P) = 240 - \frac{P}{2} = 220$. Because market is in equilibrium when P=40, the total quantity supplied by all firms is also 220 tons. Since each firm produces 20, the number of firms in the industry is 220/20=11.

Question 7

a) The total non-sunk cost is $TNSC(Q) = Q^2 + 8Q + 25$. The average non-sunk cost is $ANSC(Q) = Q + 8 + \frac{25}{Q}$. The minimum point of the $ANSC$ is when Q=5. And min($ANSC$)=18. The marginal cost is $SMC(Q) = 2Q + 8$. Thus the firm's supply curve is $Q = \begin{cases} \frac{P}{2} - 4, & P \geq 18 \\ 0, & P < 18 \end{cases}$.

The shut down price is 18. When the price is higher than 18, the firm will produce and the when the price is lower than 18, the firm will not produce.

b) Since there are 80 firms, the market supply curve for price 18 or above is $S(P) = 40P - 320$. Equating the supply to the demand, we get the equilibrium price to be P=54, which is indeed greater than 18.

The producer surplus for each firm is its total revenue minus total non-sunk cost. At P=54, each firm produces 23 units. Thus PS=54*23-23*23-8*23-25=504.

c) If all fixed cost is sunk, the total non-sunk cost is $TNSC(Q) = Q^2 + 8Q$. The average non-sunk cost is $ANSC(Q) = Q + 8$ and its minimum level is 8. In this case, the firm will not produce when the price is less than 8.

If all fixed cost is non-sunk, the total non-sunk cost is $TNSC(Q) = Q^2 + 8Q + 100$. The average non-sunk cost is $ANSC(Q) = Q + 8 + \frac{100}{Q}$ and its minimum level is 28. In this case, the firm will not produce when the price is less than 28.

Thus the shut down price, P_{shut}, is between 8 and 28, that is, 8<= P_{shut}<=28.

Question 8

a) (1) $2Q$.
(2) $P/4$.
(3) 90.
(4) 1660.5.
(5) 364.5.

b) (6) decrease.
(7) Firms are making positive profit, there will be entry in the long run and the market price will decrease.
(8) 72.
(9) The *SAC* given is the one that touches the *LAC* at its minimum, thus the long-run equilibrium price is min(*LAC*)=min(*SAC*).
(10) 18.
(11) 50.

Question 9

a) The long-run equilibrium price is the minimum of *LAC*, which is $P = 2\sqrt{2F}$. The total quantity demanded at this price is $Q = \frac{5{,}000}{\sqrt{2F}}$. Since each firm produces $Q = \sqrt{\frac{F}{2}}$, there will be $\frac{5{,}000}{F}$ firms in the long-run equilibrium.

b) Since the long-run equilibrium price is $P = 2\sqrt{2F}$, which is increasing in F, when the licensing fee increases, the long-run equilibrium price will be higher. The total quantity demanded and produced in the long-run equilibrium, $Q = \frac{5{,}000}{\sqrt{2F}}$, will be lower. As the number of firms is $\frac{5{,}000}{F}$, there will be fewer firms in the market when the licensing fee increases. In the new long-run equilibrium, the remaining firms will make zero profit and the firms that have left the market also make zero profit. So the firms do not bear the cost of the licensing fee.

For the consumers, when the licensing fee increases, the market price will be higher and the total quantity consumed will be lower. Thus consumer surplus in the market will reduce. Therefore, the consumers bear the cost in the long run.

c) To maximize profit, each firm produces $Q=P/4$. Since there are always 100 firms in the market, the market supply curve is $S(P)=25P$. In the long-run equilibrium, demand equals supply, thus $25P = \frac{10{,}000}{P}$. The long-run equilibrium price is 20.

The long-run equilibrium price is independent of the licensing fee. When the licensing fee increases, the market price remains the same, hence the total quantity consumed and produced remains the same. Therefore, the consumer surplus does not change.

Hence the consumers do not bear the cost. Each firm produces 5 units and its total profit is $50-F$. For each firm, when the licensing fee increases, the profit is lower. Thus the producers bear the cost.