LECTURE 5

MARKET DEMAND
EXCHANGE ECONOMY

Where are we?

- Consumer theory
  - Optimal choice
  - Individual demand
  - Consumer welfare
  - Market demand
- Exchange economy
  - Edgeworth box
    - How to represent the economy graphically?
  - Pareto efficiency
    - What is the “best” allocation?
  - Competitive equilibrium

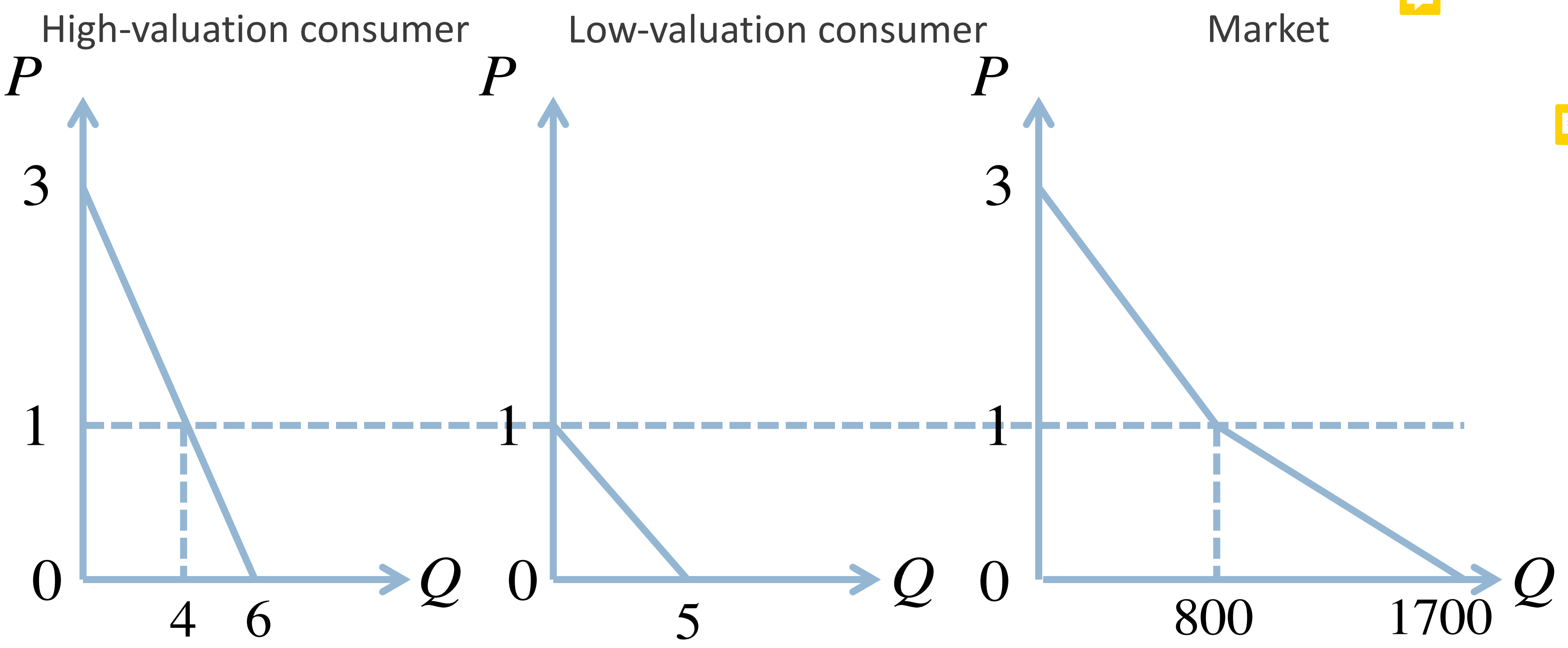
Part 1

Market Demand

Individual Demand Curve and Market Demand Curve

- Market/Aggregate demand curve is the horizontal summation of all individual demand curves
- Suppose there are 200 high-valuation consumers
  - Each high-valuation consumer's demand curve is $Q=6-2P$
- Suppose there are 100 low-valuation consumers
  - Each low-valuation consumer's demand curve is $Q=5-5P$
- What is the market demand curve?

Market Demand Curve in Graph



Equation of Market Demand Curve

- When $P>1$
  - Only high-valuation consumers will buy
  - Market demand curve: $Q=200(6-2P)$
- When $P<=1$
  - Both types of consumers will buy
  - Market demand curve: $Q=200(6-2P)+100(5-5P)$
- Market demand curve is

$$Q = \begin{cases} 1700 - 900P & if \quad P \leq 1 \\ 1200 - 400P & if \quad P > 1 \end{cases}$$

Summary: Consumer Choice

Preference
- What I like

Budget Constraint
- What I can afford

Optimal Choice
- What I buy

Summary: Individual Demand Curve

Optimal Choice
- What I buy at a particular price

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Price Changes

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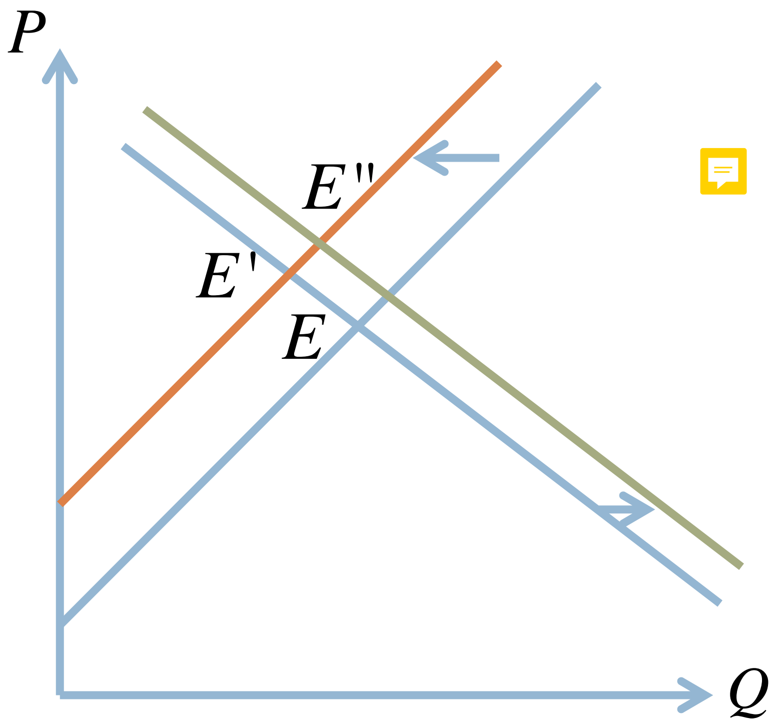
Individual Demand
- What I buy as a function of price

Summary: Market Demand Curve

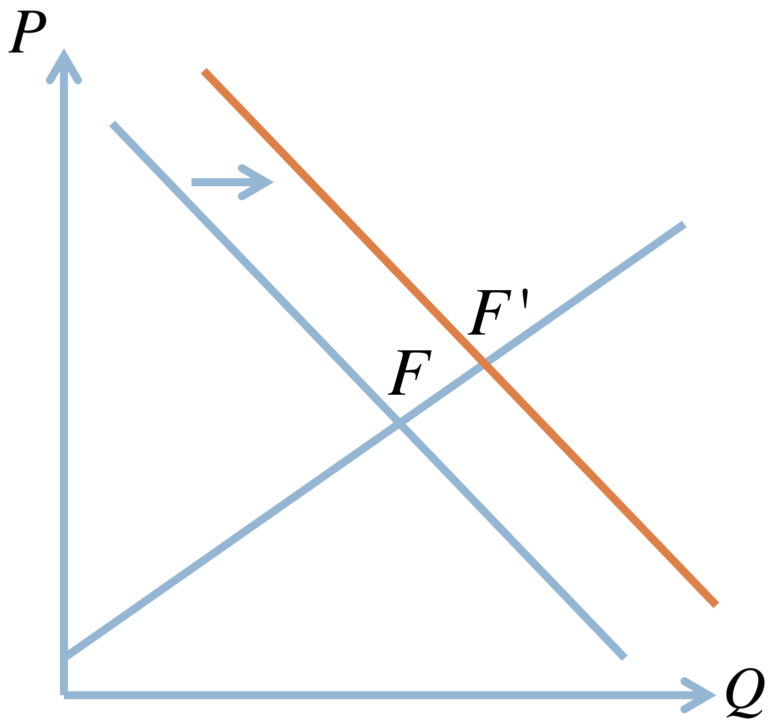
Part 2

The Edgeworth Box

Example: Market for Coffee and Market for Tea



Market for Coffee



Market for Tea

Partial vs. General Equilibrium

- Partial Equilibrium Analysis
  - Finding the equilibrium price and quantity in a single market
  - Holding prices in all other markets fixed
- General Equilibrium Analysis
  - Finding the equilibrium prices and quantities in more than one markets simultaneously

An Exchange Economy

- There are two consumers in the economy, A and B,
- There are two goods in the economy, 1 and 2
- Consumer A's consumption basket is denoted by

$$(x_1^A, x_2^A)$$

- Consumer B's consumption basket is denoted by

$$(x_1^B, x_2^B)$$

- An *allocation* is a pair of consumption baskets

$$(x_1^A, x_2^A, x_1^B, x_2^B)$$

Endowment Allocation

- Each consumer has some amount of each good to start with
  - There is no money/income
  - They can trade with each other
- The allocation the consumers start with is the *endowment allocation* denoted by

$$(\omega_1^A, \omega_2^A, \omega_1^B, \omega_2^B)$$

- E.g. consumer A's endowment is (6, 4) and consumer B's endowment is (2, 2)
  - The total amount of good 1 is 8 and the total amount of good 2 is 6

Feasible Allocation

- Definition 5.1 An allocation is *feasible* if

$$x_1^A + x_1^B = \omega_1^A + \omega_1^B$$

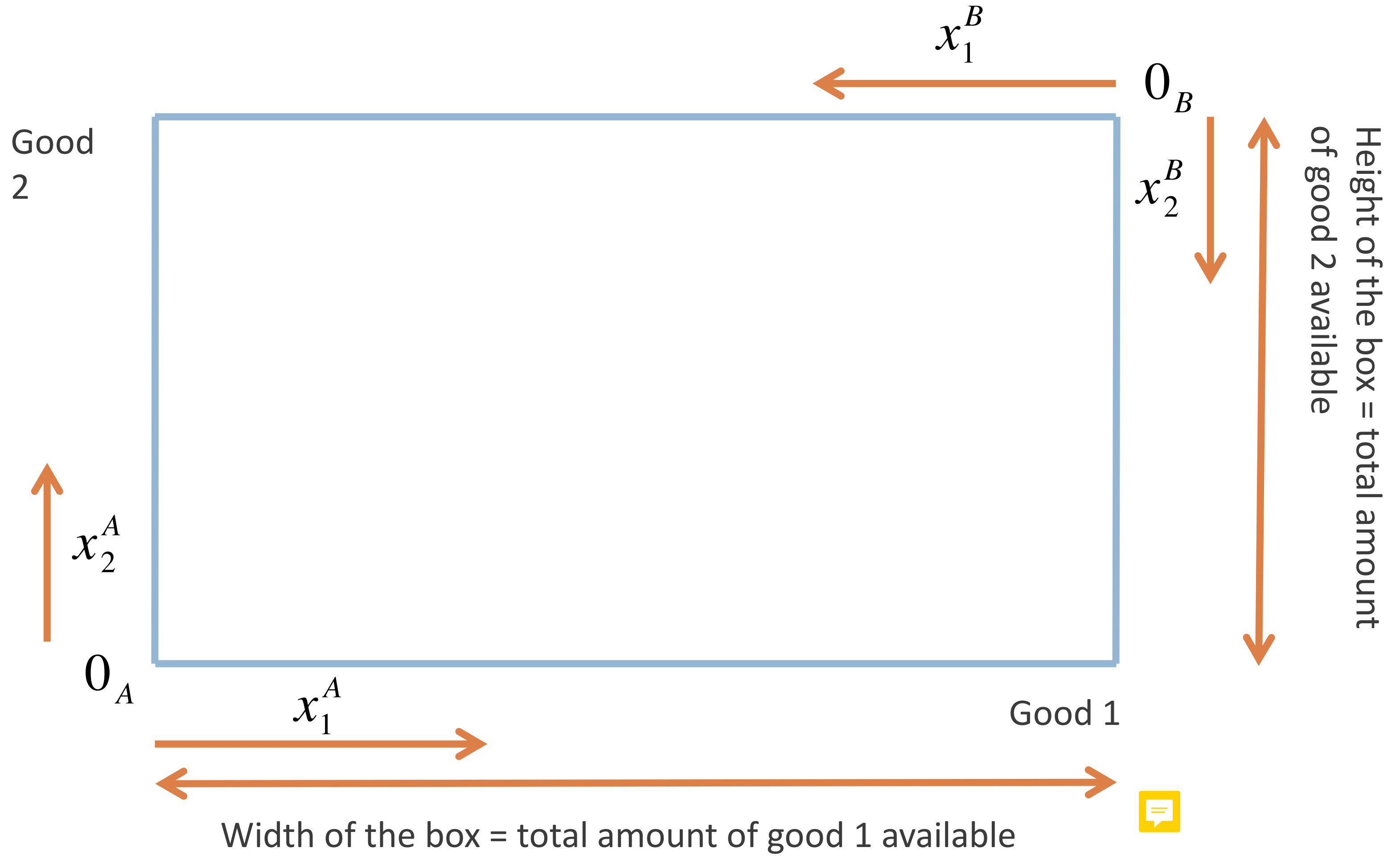
$$x_2^A + x_2^B = \omega_2^A + \omega_2^B$$

  - The total amount of each good consumed equals to the total amount available
- Using the example from the previous slide
  - The total amount of good 1 is 8 and the total amount of good 2 is 6
  - (3, 1) and (5, 5) is feasible
  - (4, 4) and (2, 6) is not feasible

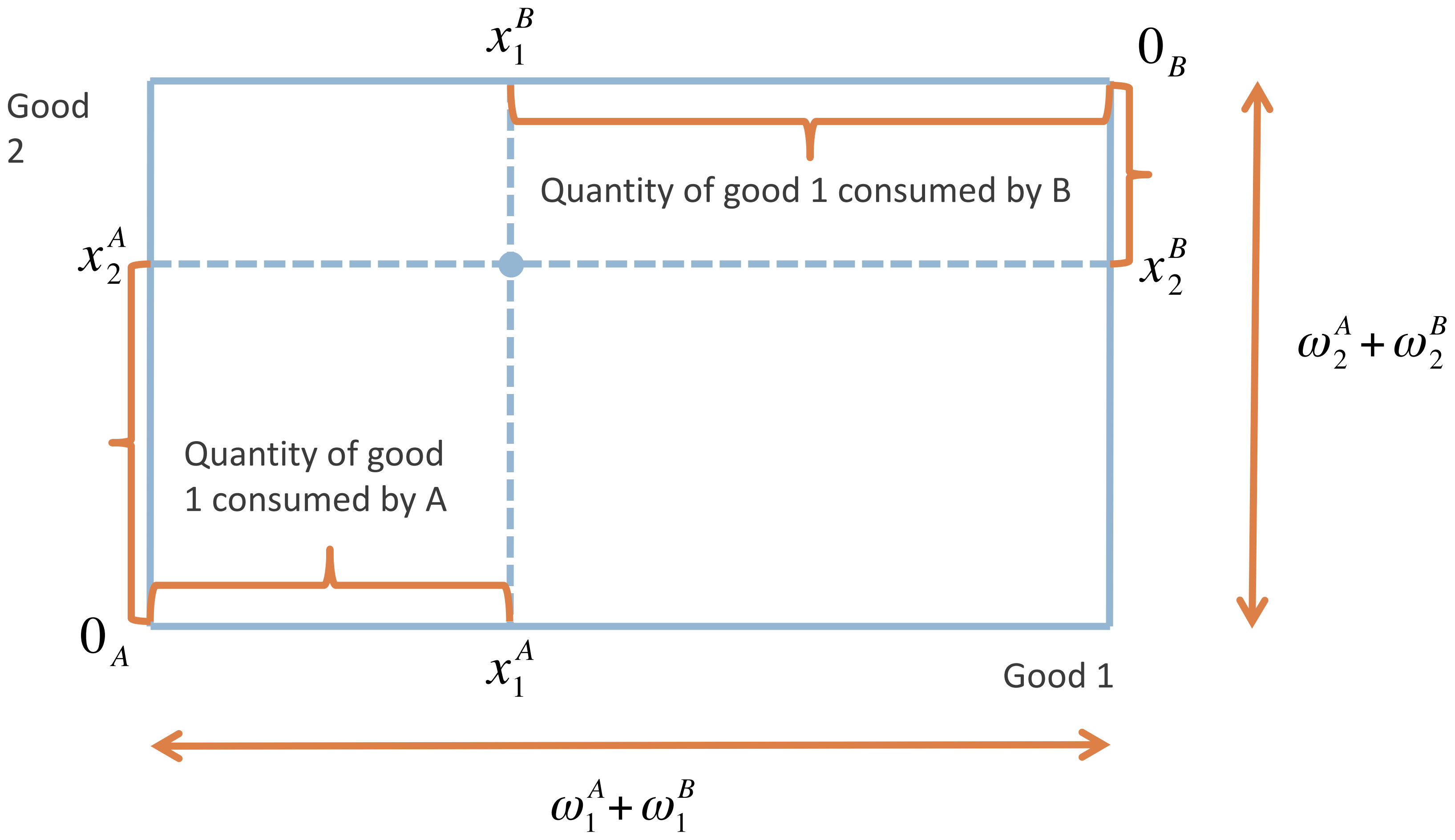
Edgeworth Box

- An *Edgeworth box* is used to graphically show all feasible allocations of the two goods between the two consumers
- Every point in the box, including those on the boundaries, represents a feasible allocation

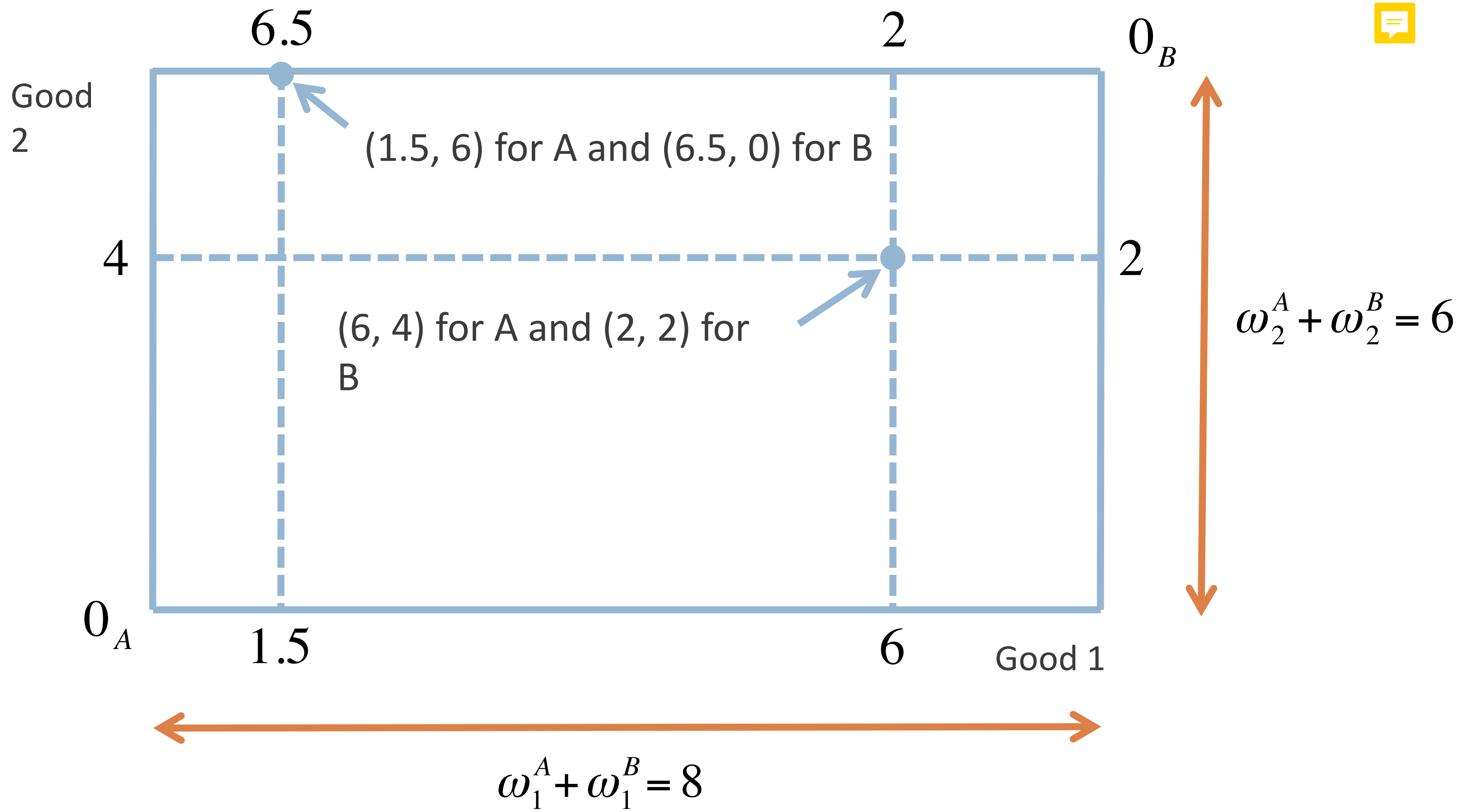
Setting up an Edgeworth Box



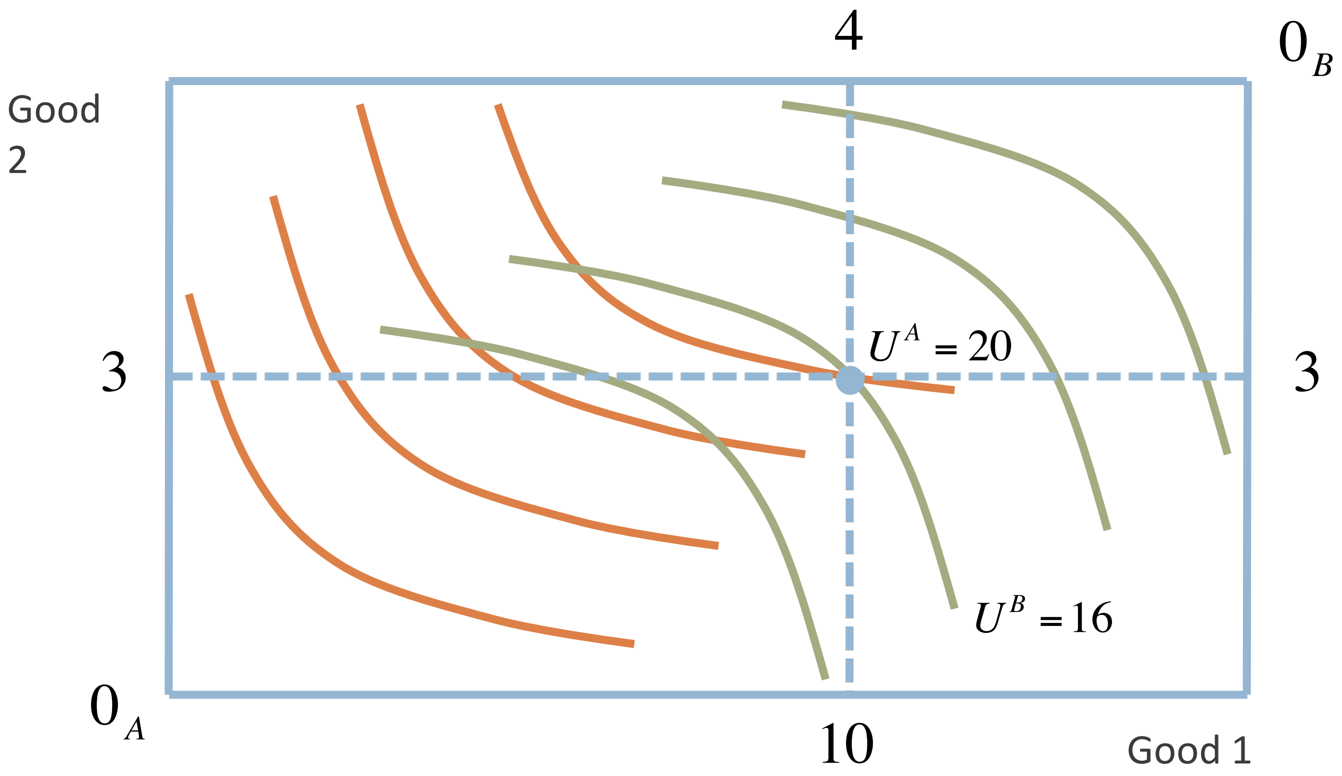
Representing a Feasible Allocation in the Edgeworth Box



Edgeworth Box: An Example



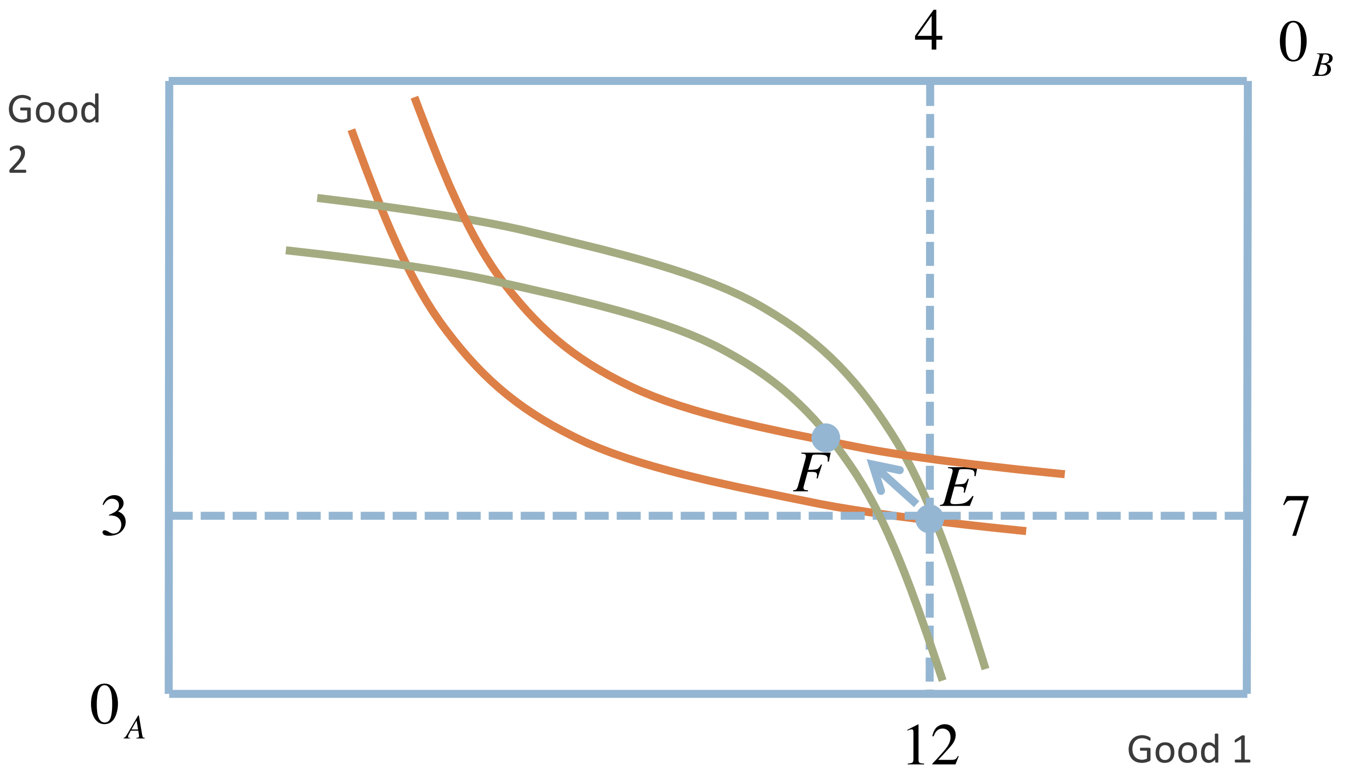
Adding Preferences to the Box



Part 3

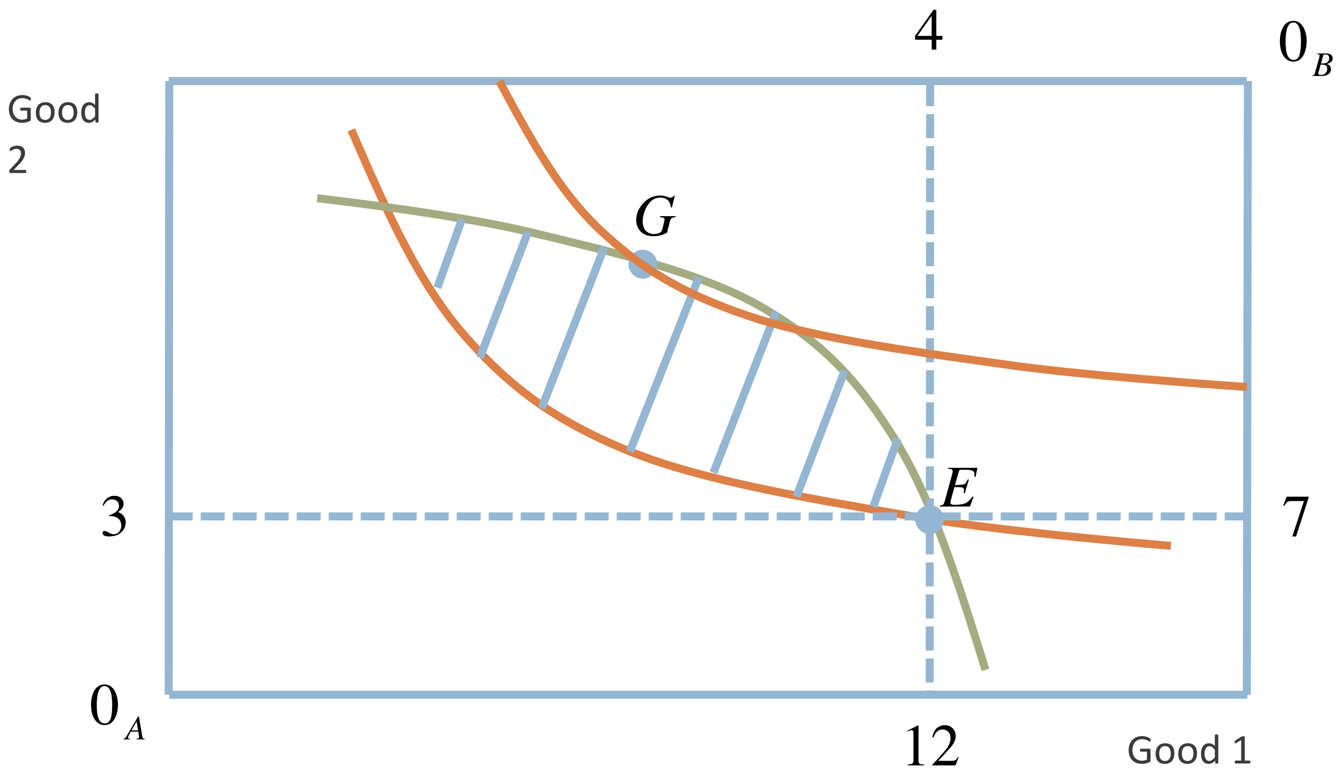
Pareto Efficiency

Is there an allocation where both consumers are better off than at E?



At point F, both consumers get higher utility compared to point E

Both Consumers are at Least as Well off as at E in the Shaded Region

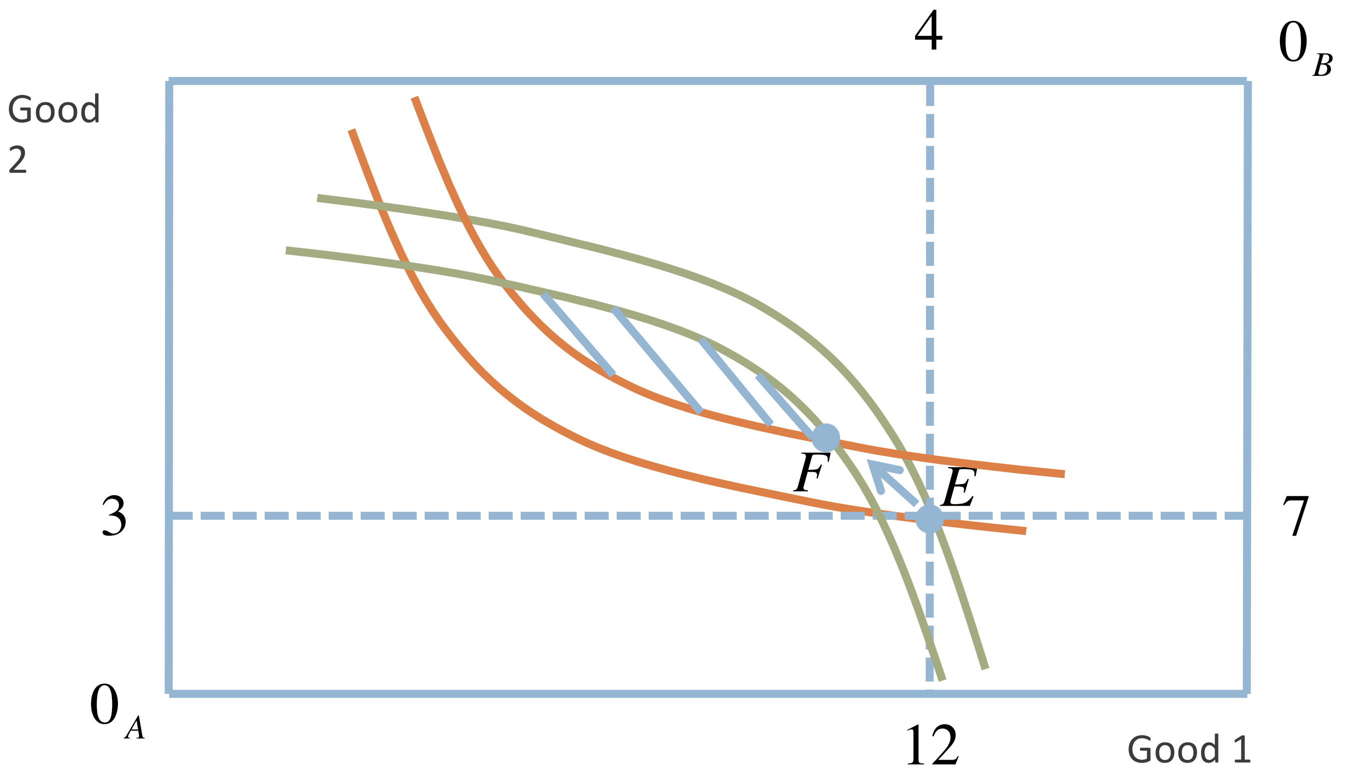


At any allocation in the shaded region, each consumer's utility is either higher than or the same as the utility at E

Pareto Improvement

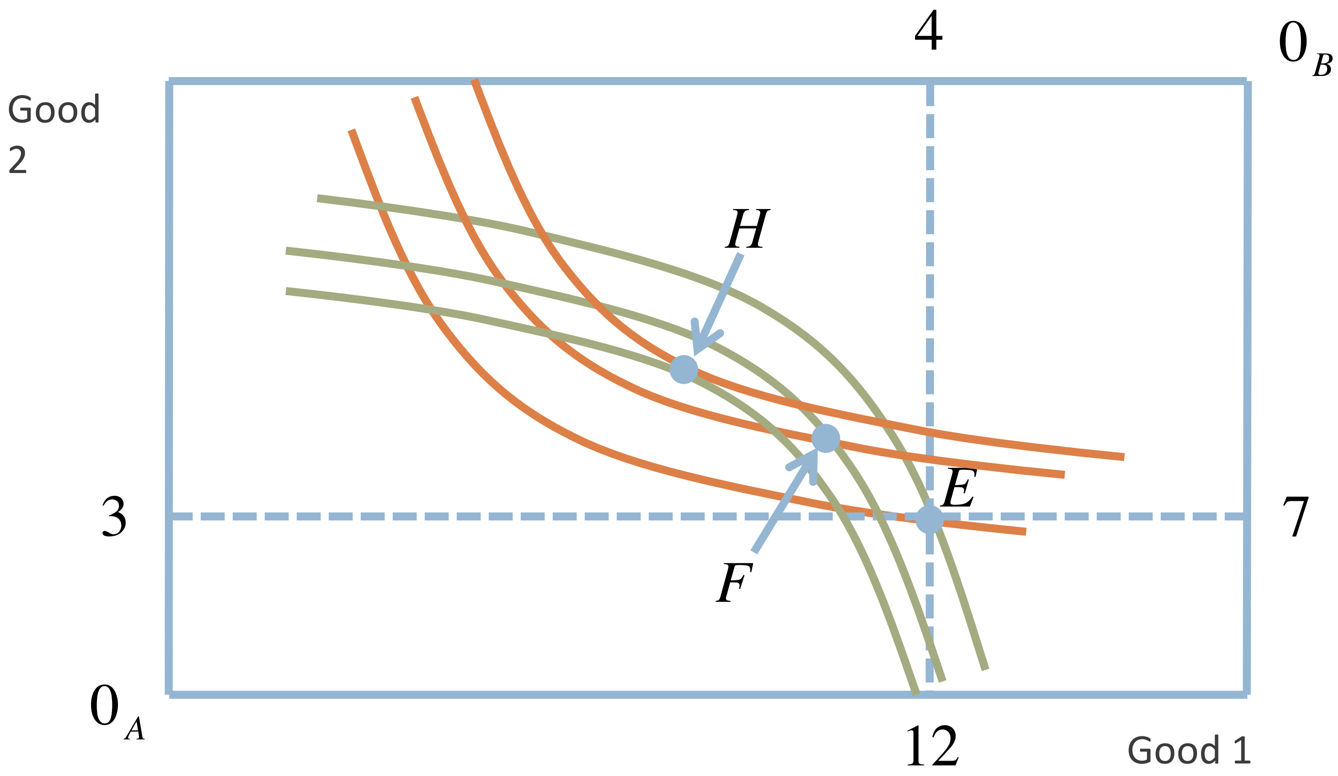
- Definition 5.2 From some allocation X to some other allocation Y is a *Pareto improvement* if from X to Y, at least one consumer is better off and no one else is worse off
- On slide 22
  - E to F is a Pareto improvement
- On slide 23
  - E to G is a Pareto improvement

At F, can we make a Pareto improvement?

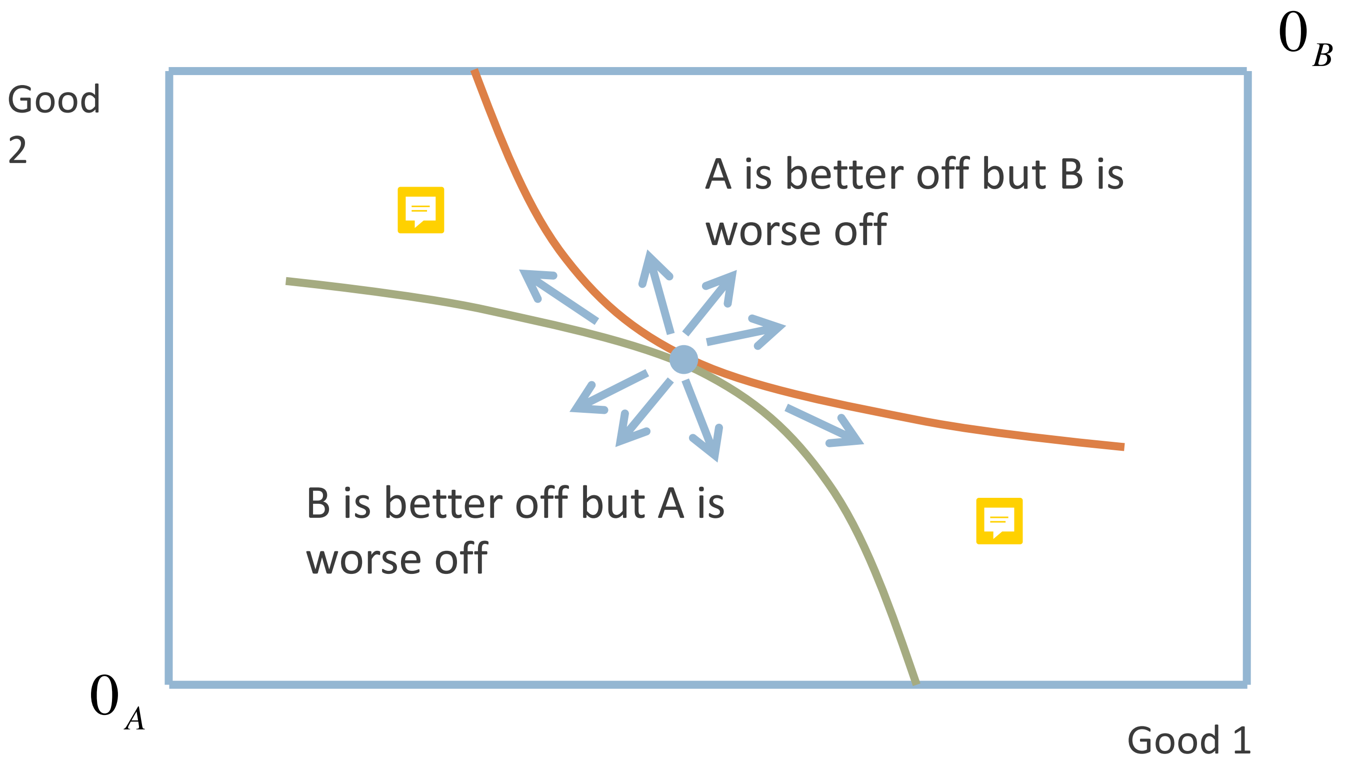


Yes, any allocation in the shaded region is a Pareto improvement of F

At H, can we make a Pareto improvement?



A Closer Look at Point H



At point H, we cannot make one consumer better off without making the other consumer worse off

Pareto Efficiency

- <u>Definition 5.3</u> An allocation is *Pareto efficient* if there is no way to make one consumer better off without making someone else worse off

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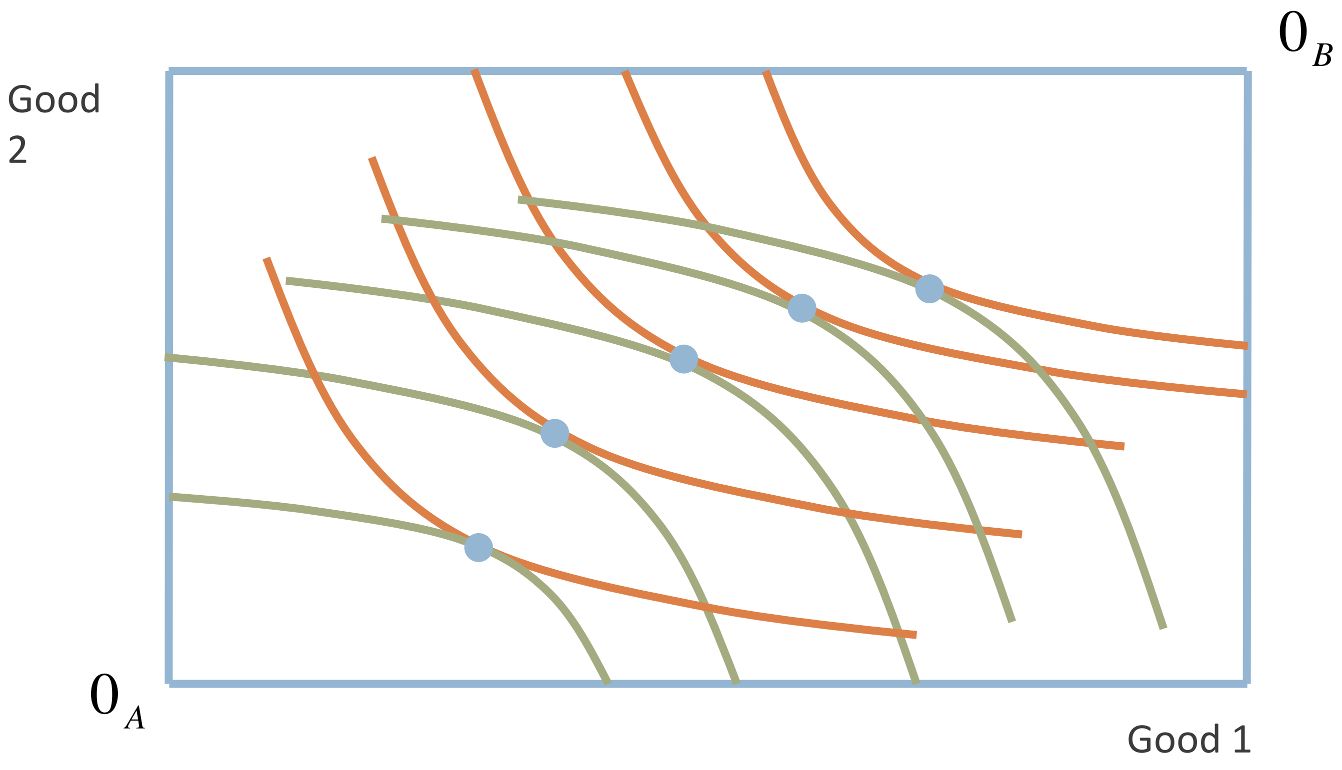
A Pareto efficient allocation can be described as an allocation where:

1. ~~There is no way to make all the people involved better off; or~~
2. there is no way to make some individual better off without making someone else worse off; or
3. ~~all of the gains from trade have been exhausted; or~~
4. ~~there are no mutually advantageous trades to be made, and so on.~~

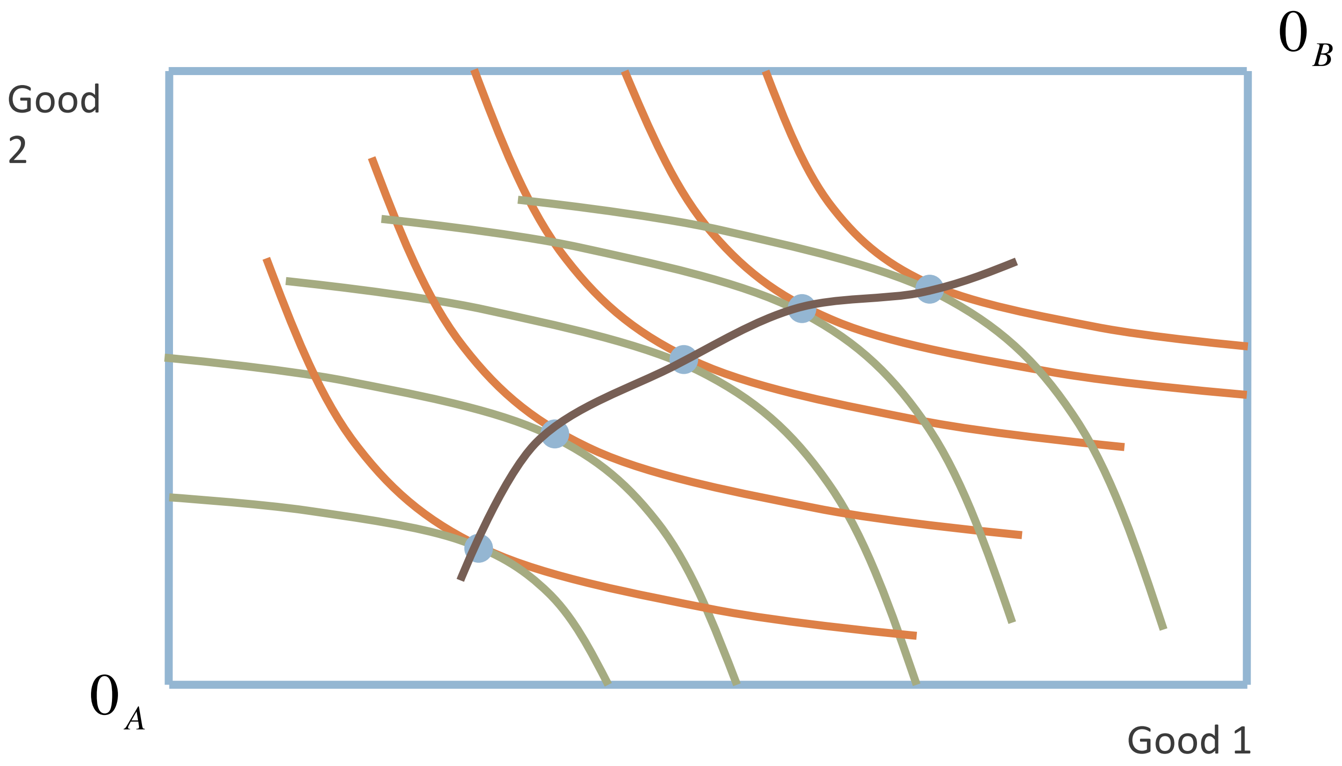
Pareto Efficiency and Pareto Improvement

- If an allocation is Pareto efficient
  - There is no room for Pareto improvement
- When each consumer's indifference curves are smooth with diminishing *MRS* and when we have interior solutions, Pareto efficient allocations will be the tangency points between the two indifference curves
  - When indifference curves are not tangent to each other, it is not Pareto efficient
  - Hence there is room for Pareto improvement

There is More than One Pareto Efficient Allocation



Contract Curve



Definition 5.4 The *contract curve* is the set of all Pareto efficient allocations

Deriving the Contract Curve Mathematically

- Tangency condition

$$MRS^A_{1,2} = MRS^B_{1,2} \quad (1)$$

- The allocation must be feasible

$$x_1^A + x_1^B = \omega_1^A + \omega_1^B \quad (2)$$

$$x_2^A + x_2^B = \omega_2^A + \omega_2^B \quad (3)$$

- Substituting (2) and (3) into (1), we can express the contract curve in terms of $x^A{}_1$ and $x^A{}_2$ or $x^B{}_1$ and $x^B{}_2$

Part 4

General Competitive Equilibrium

Budget Set in the Exchange Economy

- Given an endowment allocation, which allocation will the consumers end up consuming?
  - Each consumer will choose the utility-maximizing basket given the budget constraint
  - Budget constraint determined by prices and endowments
- Suppose the market for each good is perfectly competitive
  - That is, consumers are price takers
- Let P_1 be the price of good 1 and P_2 be the price of good 2, the two consumers' budget constraints are

$$P_1 x_1^A + P_2 x_2^A \le P_1 \omega_1^A + P_2 \omega_2^A \quad (A)$$

$$P_1 x_1^B + P_2 x_2^B \le P_1 \omega_1^B + P_2 \omega_2^B \quad (B)$$

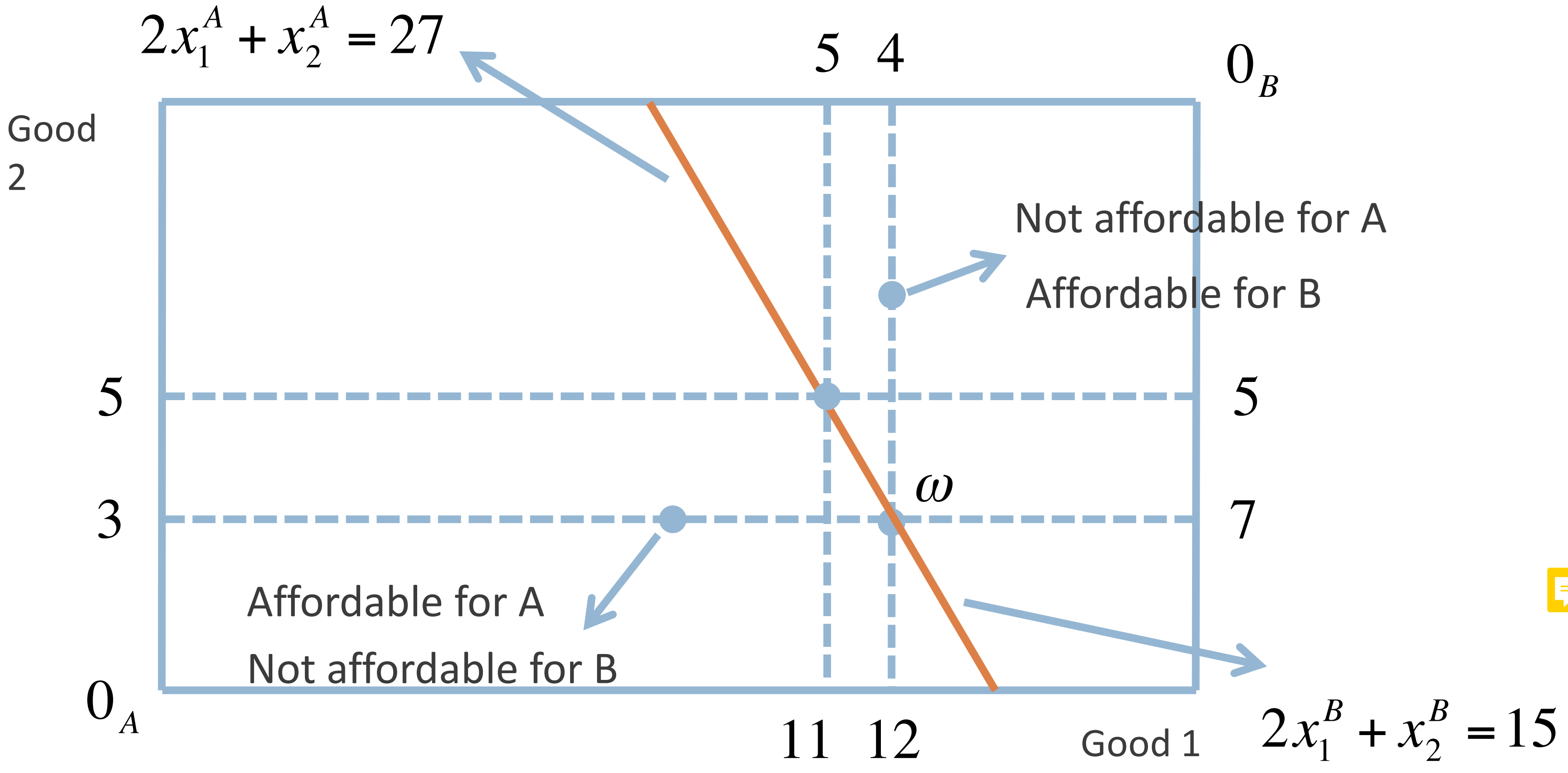
Example: Budget Constraints

- Suppose the price of good 1 is \$2 and the price of good 2 is \$1
- Consumer A's endowment is (12, 3)
  - The endowment is worth 12*\$2+3*\$1=\$27
  - Equivalent to having \$27 of income
- Consumer B's endowment is (4, 7)
  - The endowment is worth 4*\$2+7*\$1=\$15
  - Equivalent to having \$15 of income
- The budget constraints are

$$2x_1^A + x_2^A \le 27 \quad (A)$$

$$2x_1^B + x_2^B \le 15 \quad (B)$$

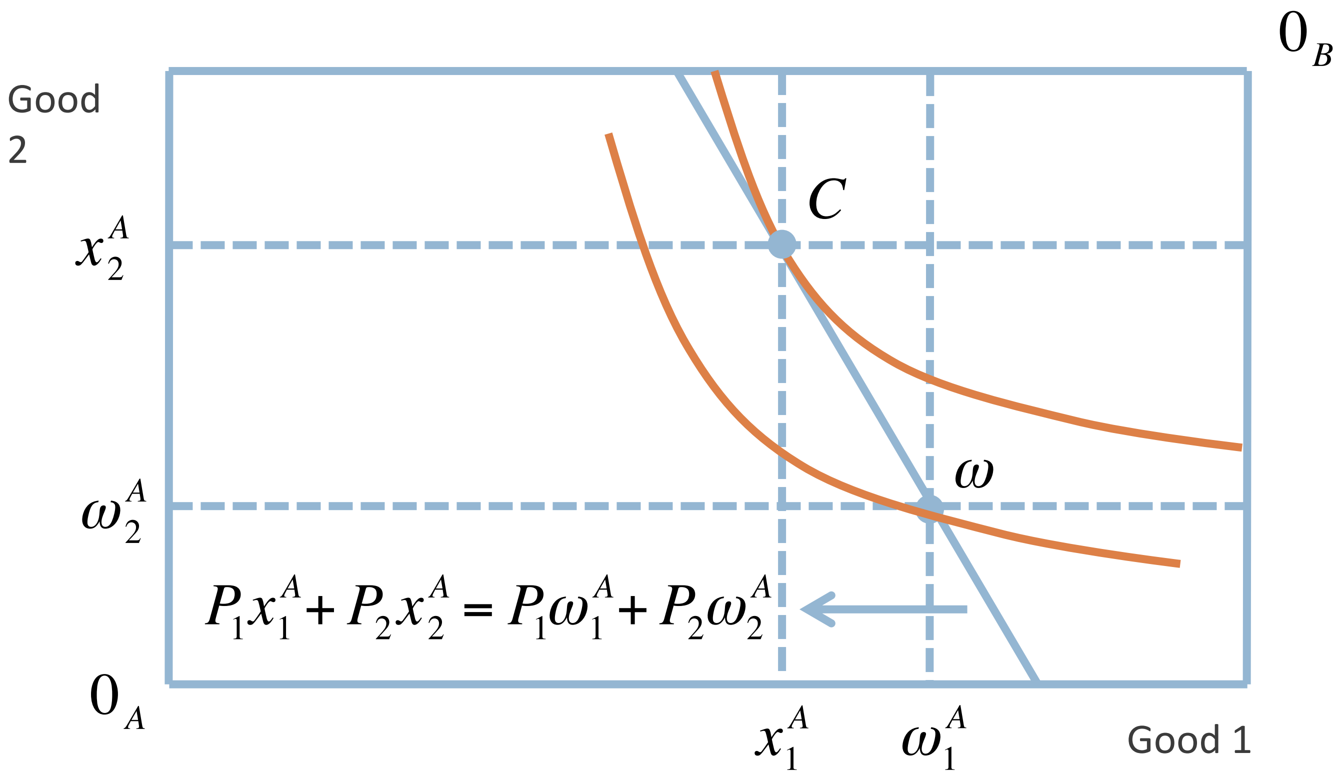
Example: Budget Constraints in Graph



Slope of budget line = $-P_1/P_2$

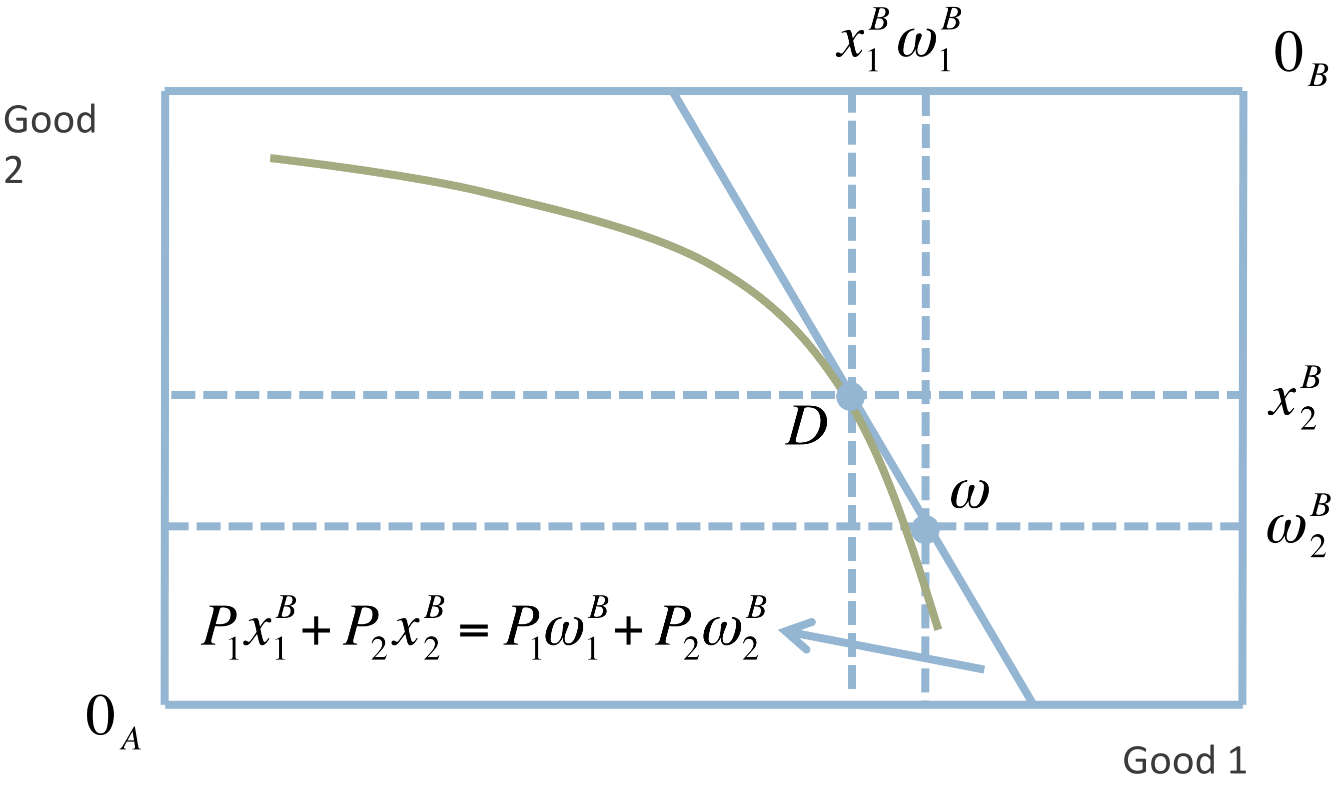
The endowment allocation is on the budget line

Consumer A's Optimal Choice



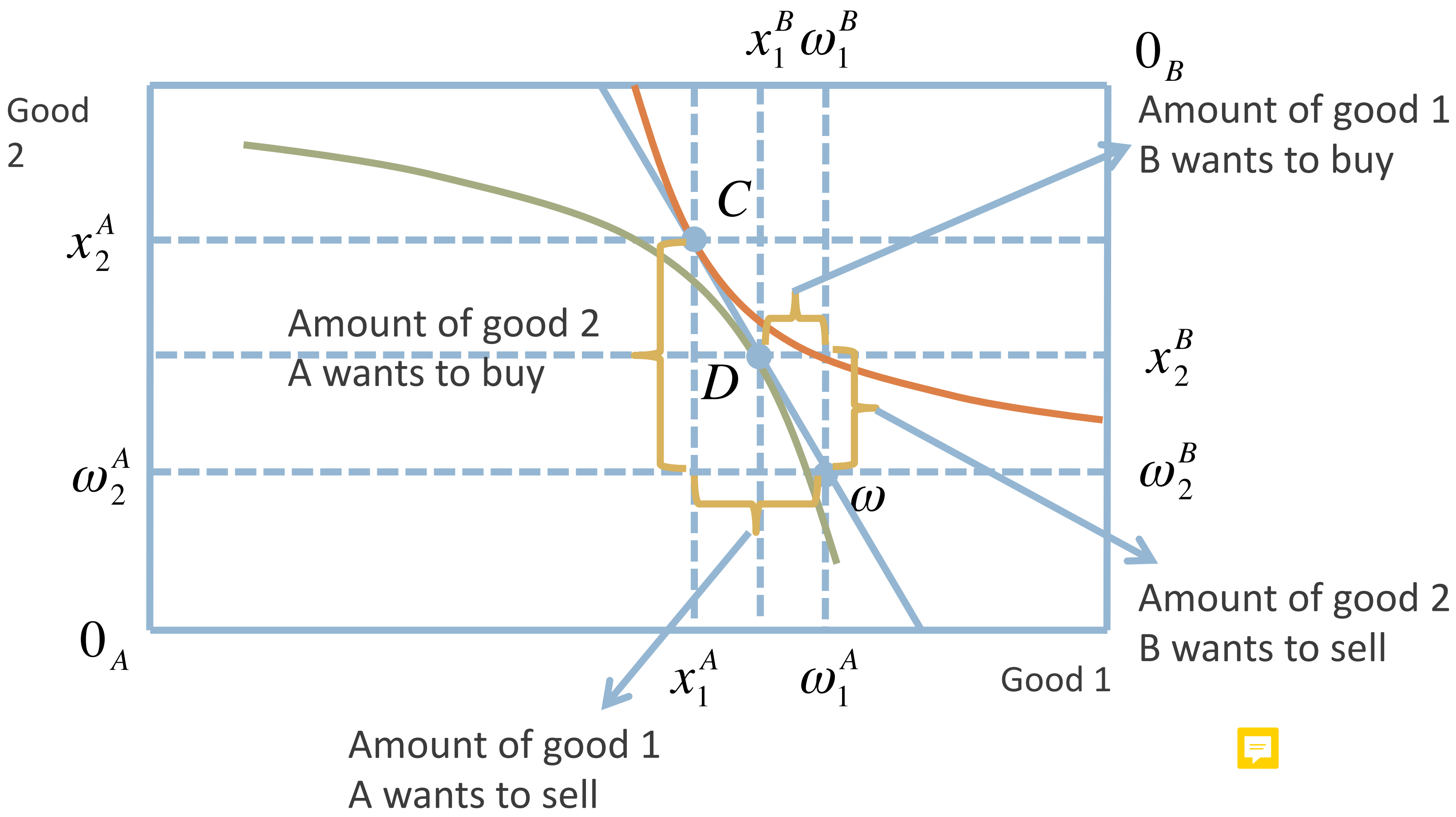
Given the endowment, consumer A wants to sell some in exchange for some

Consumer B's Optimal Choice



Given the endowment, consumer B wants to sell some in exchange for some

Can the consumers complete their desired transactions?



Markets do not clear at the current prices

- There is excess supply of good 1
  - The amount B wants to buy is less than the amount A wants to sell
- There is excess demand of good 2
  - The amount A wants to buy is more than the amount B wants to sell
- Sum of the demand for each good does not equal to the total quantity available

$$x_1^A + x_1^B < \omega_1^A + \omega_1^B$$

$$x_2^A + x_2^B > \omega_2^A + \omega_2^B$$

General Competitive Equilibrium

- Definition 5.5 A pair of prices (P_1, P_2) constitutes a (general) *competitive equilibrium* if at these prices
  - Each consumer maximizes his/her utility given the budget constraint

$$x_1^{*A}, x_2^{*A}, x_1^{*B}, x_2^{*B}$$

denotes the optimal consumption for each consumer given the equilibrium prices

  - Markets for both goods clear

$$x_1^{*A} + x_1^{*B} = \omega_1^A + \omega_1^B$$

$$x_2^{*A} + x_2^{*B} = \omega_2^A + \omega_2^B$$

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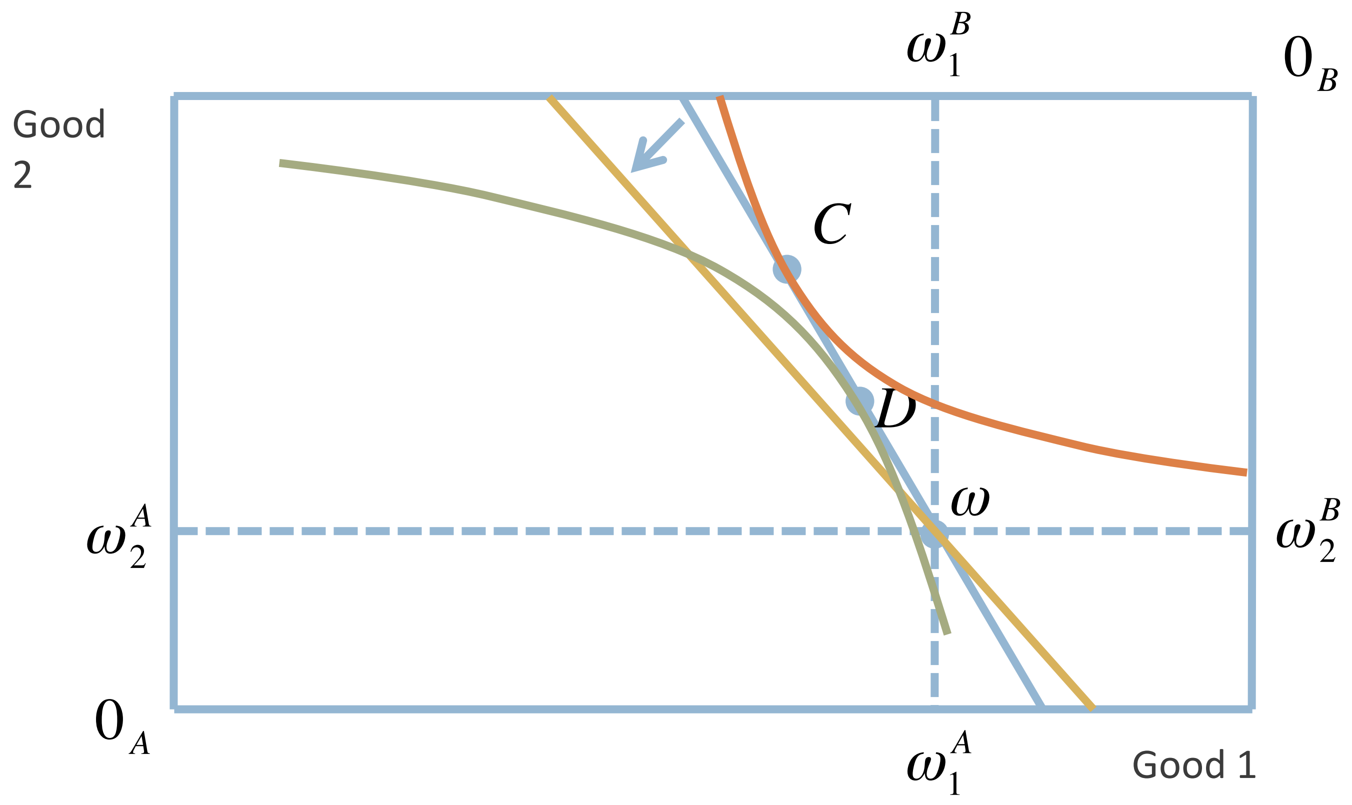
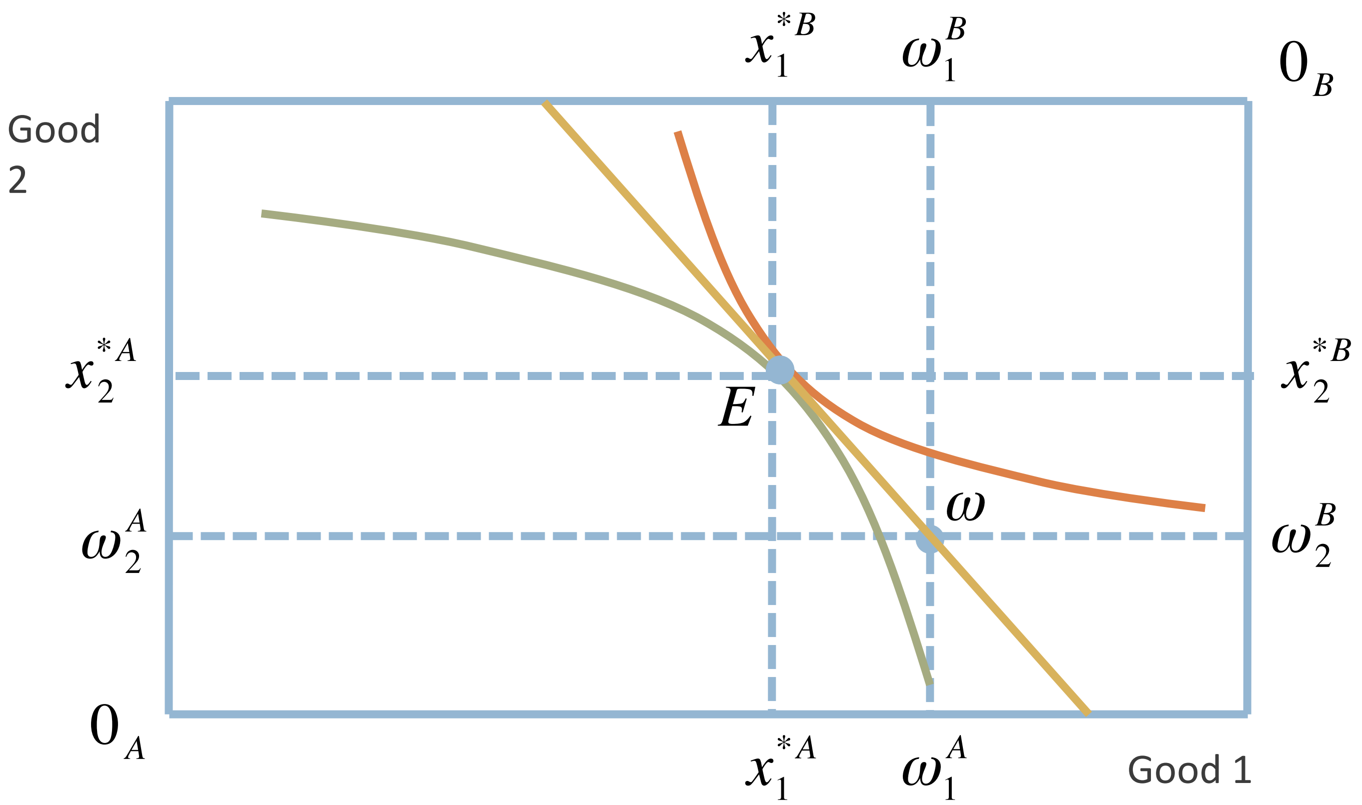
- Since there is excess supply of good 1
  - The price of good 1 will
- Since there is excess demand of good 2
  - The price of good 2 will
- Thus P_1/P_2 will decrease
- Budget line will become flatter
  - But it still goes through the endowment allocation

Reaching an Equilibrium



Reaching an Equilibrium Cont'



At the new prices, markets for the two goods clear and each consumer maximizes utility given the budget constraint

Example: Solving for Competitive Equilibrium

- Suppose consumer A's utility function is

$$U^A(x_1^A, x_2^A) = x_1^A x_2^A$$

- Suppose consumer B's utility function is

$$U^B(x_1^B, x_2^B) = x_1^B x_2^B$$

- Consumer A's endowment is (10, 6) and consumer B's endowment is (10, 4)
- Find the equilibrium prices P_1 and P_2

Example: Solving for Competitive Equilibrium Cont'

- Consumer A's optimal choice

$$\frac{x_2^A}{x_1^A} = \frac{P_1}{P_2} \qquad (1)$$

$$P_1 x_1^A + P_2 x_2^A = 10P_1 + 6P_2 \qquad (2)$$

- Consumer B's optimal choice

$$\frac{x_2^B}{x_1^B} = \frac{P_1}{P_2} \qquad (3)$$

$$P_1 x_1^B + P_2 x_2^B = 10P_1 + 4P_2 \qquad (4)$$

- Market clearing

$$x_1^A + x_1^B = 10 + 10 = 20 \qquad (5)$$

$$x_2^A + x_2^B = 6 + 4 = 10 \qquad (6)$$

Example: Solving for Competitive Equilibrium Cont'

- (1) and (3) give us

$$\frac{x_2^A}{x_1^A} = \frac{x_2^B}{x_1^B} = \frac{P_1}{P_2} \qquad (7)$$

- Plugging (5) and (6) into (7)

$$\frac{x_2^A}{x_1^A} = \frac{10 - x_2^A}{20 - x_1^A} = \frac{P_1}{P_2} \qquad (8)$$

- Solving

$$\frac{x_2^A}{x_1^A} = \frac{10 - x_2^A}{20 - x_1^A}$$

- We get

$$x_1^A = 2x_2^A \qquad (9)$$

Example: Solving for Competitive Equilibrium Cont'

- Plugging (9) into (8)

$$\frac{P_1}{P_2} = 0.5 \quad (10)$$

- Plugging (9) and (10) into (2)

$$P_1 2x_2^A + 2P_1 x_2^A = 10P_1 + 12P_1 \quad \Rightarrow \quad x_2^A = 5.5$$

- The equilibrium allocation is

$$x_1^{*A} = 11, \quad x_2^{*A} = 5.5, \quad x_1^{*B} = 9, \quad x_2^{*B} = 4.5$$

Relative Price

- In the previous example, we can only solve for the relative price P_1/P_2
- Relative price is what matters
  - In the previous example, we just need the price ratio to be $P_1/P_2=0.5$ in equilibrium
  - It does not matter if $P_1=2$, $P_2=4$ or $P_1=3$, $P_2=6$
- It is convenient to set one of the prices to 1
  - Such a price is called a *numeraire price*, such a good is called a *numeraire*
  - If we set good 2 as a numeraire in the example, then $P_1=0.5$