No.: Production Function with 1 Input, Production Function with 2 Inputs

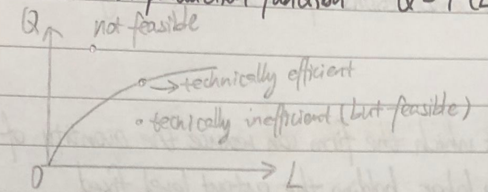
Date:

(I) Production function

- tells us the maximum quantity (Q) of output the firm can produce given the amount of L and K.

  $Q = F(L, K)$

- short-run production function : $Q = F(L)$



(II) Marginal product

- Marginal product of labor : rate at which output level changes as quantity of labor changes.

  $MP_L = \frac{dQ}{dL} = \frac{\Delta Q}{\Delta L}$

  → slope of the production function

(III) Law of diminishing marginal returns

- Suppose capital is fixed, MP_L will eventually decline as the quantity of labor ↑.

(IV) Diminishing total returns

- $Q \downarrow$ as $L \uparrow$
- MP_L is -ve

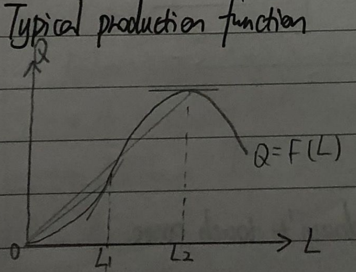
(V) Average product

- Average product of labor : output per unit of labor

  $AP_L = \frac{Q}{L}$

  → ray connecting origin and point $(L, F(L))$

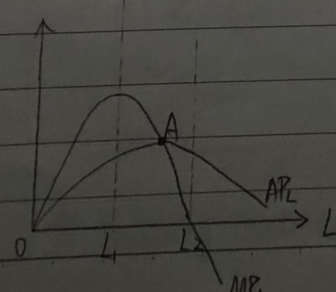
(VI) Typical production function



0 to L_1 : increasing marginal returns

L_1 to L_2 : diminishing marginal returns

L_2 above : diminishing total returns



At A : $MP_L = AP_L$

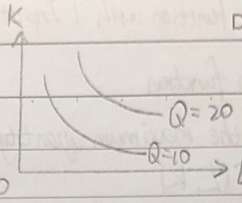
left of A : $MP_L > AP_L$

right of A : $MP_L < AP_L$

MP crosses AP at its highest point !

No.: Date:

Production function with 2 inputs



- $Q = F(L, K)$
- $MP_L = \frac{\partial Q}{\partial L}$, $MP_K = \frac{\partial Q}{\partial K}$
- isoquant: a curve that connects all combinations of L and K that generate same level of output.

Marginal rate of technical substitution

- MRTS of labor for capital: rate at which the firm can reduce the quantity of capital for more labor, holding the output level fixed.

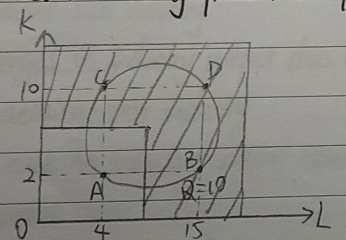
$$MRTS_{L,K} = -\frac{dK}{dL}\Big|_{\text{same } Q} = -\frac{\Delta K}{\Delta L}\Big|_{\text{same } Q}$$

→ negative of the slope of isoquant.

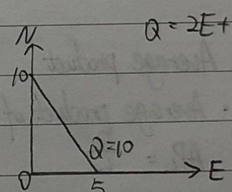
- Diminishing MRTS: isoquant convex to origin
- $MRTS_{L,K} = \frac{MP_L}{MP_K}$

Uneconomic region of production

- In uneconomic region of production, at least 1 marginal product is −ve.
- Cost-minimizing firms never produce in the uneconomic region of production.

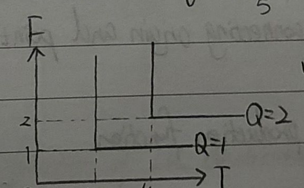


Perfect substitutes



- Linear production function, linear isoquant, MRTS constant

Perfect complements

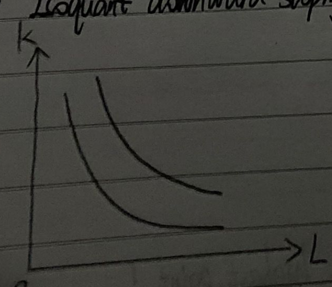


- Min production function, L-shaped isoquant

Cobb-Douglas Production function

- $Q = AL^{\alpha}K^{\beta}$, $A > 0$, $\alpha > 0$, $\beta > 0$
- Isoquant downward sloping, smooth, convex & doesn't touch axes.

$$MRTS_{L,K} = \frac{\alpha K}{\beta L}.$$



No.: Returns to Scale, Technological Progress

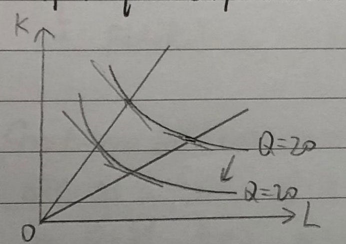
Date:

(i) Returns to scale

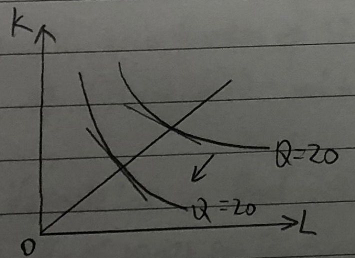
- measures the rate at which output ↑ when all inputs ↑ proportionately.
- Suppose when L increases to aL and K increases to aK. $(a>1)$, output increases to bQ.
  → $b>a$: increasing returns to scale
  $b=a$: constant ...
  $b<a$: decreasing
- increasing returns to scale → isoquants closer to each other, compared to constant returns to scale.

(ii) Technological progress

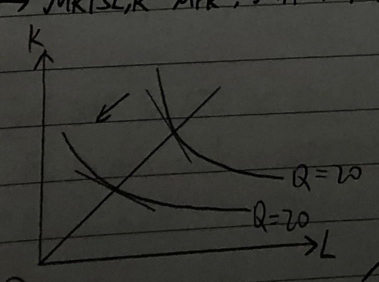
- We have tech progress if for any given combination of inputs, firm produces higher Q.
  Or, to produce any Q, the firm uses less input.
- Neutral tech progress
  → if isoquant shifts inward + $MRTS_{L,K}$ along any ray from origin remains the same.



- Capital-saving tech progress (labor more productive)
  → if isoquant shifts inward + $MRTS_{L,K}$ along any ray from origin increases.
  → $MRTS_{L,K} = \frac{MP_L}{MP_K}$, MP_L ↑ relative to MP_K



- Labor-saving tech progress (capital more productive)
  → if isoquant shifts inward + $MRTS_{L,K}$ along any ray from origin ↓.
  → $MRTS_{L,K} = \frac{MP_L}{MP_K}$, MP_K ↑ relative to MP_L.



We can just compare MRTS for Cobb-Douglas Production Functions

☆ Comparing equations of MRTS ≠ Comparing MRTS along a ray from origin!

Concepts of Costs, Cost in the Short Run

Opportunity cost
- cost associated with the best alternative that is not chosen.
- = economic cost = explicit + implicit costs

Sunk cost
- cost that can never be recovered no matter what you do.

  → sunk costs are irrelevant for future decisions

Short-run total cost
- capital fixed at k_0

  $STC = wL + rk_0$
- Firm chooses L to minimize total cost of production

  $\min_{L} wL + rk_0$

  s.t. $F(L, k_0) = Q_0$ → tells how much L to use (cost-minimizing quantity of labor)

Short-run total cost curve
- STC as a function of Q, holding w & r fixed

  eg. $Q = KL$, $K=2$, $w=2$, $r=3$

  $STC(Q) = wL + rk = 2L + 3k = 2(\frac{Q}{2}) + 6 = Q + 6$

Short-run total cost function
- STC as a function of Q, w and r.

  eg. $STC(Q, w, r) = wL + rk = w(\frac{Q}{2}) + 2r$

Variable Cost vs Fixed cost
- VC: varies as Q changes; $= 0$ when $Q = 0$
- FC — doesn't vary with Q as long as $Q > 0$
- In SR, for any $Q > 0$, $STC(Q) = wL + rk_0 = VC(Q) + FC$

  $FC = rk_0$

Sunk cost & STC at $Q=0$ ($STC(Q) = VC(Q) + FC$)
- FC is non-sunk : $STC(0) = 0$

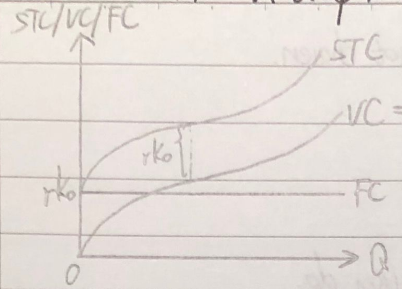
  FC is sunk : $STC(0) = FC$

  part of FC is sunk : $STC(0)$ = sunk part of FC

☆ Whether FC is sunk or not only affects STC when you don't produce anything!

No.: Date:

⑪ STC, VC, FC in Graph (typical)



$\because STC(0) = rk_0$ here

$VC = STC - FC$ $\because$ FC is sunk for this firm.

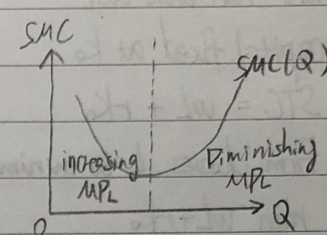
⑫ Short-run marginal cost

- rate at which STC changes with output

$$SMC(Q) = \frac{d\,STC(Q)}{dQ} = \frac{\Delta STC(Q)}{\Delta Q}$$

→ slope of STC curve OR VC curve

↑ slope of STC



⑬ Diminishing marginal return (of labor) and SMC

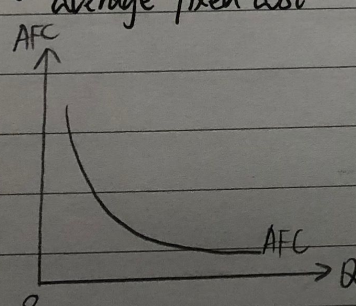
- $SMC = \frac{\Delta VC}{\Delta Q} = \frac{w\Delta L}{\Delta Q} = \frac{w}{MP_L}$

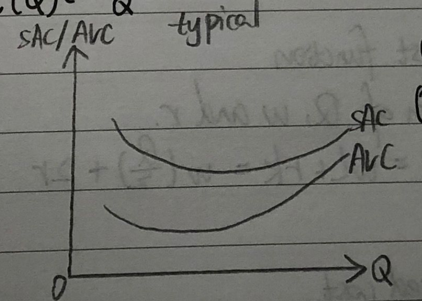
→ If we have diminishing marginal returns (assume $MP_L > 0$), then SMC ↑ as Q ↑.

↳ SMC upward sloping (vice versa)

⑭ Short-run average costs

- short-run average total cost : $SAC(Q) = \frac{STC(Q)}{Q}$
- average variable cost : $AVC(Q) = \frac{VC(Q)}{Q}$
- average fixed cost : $AFC(Q) = \frac{FC}{Q}$

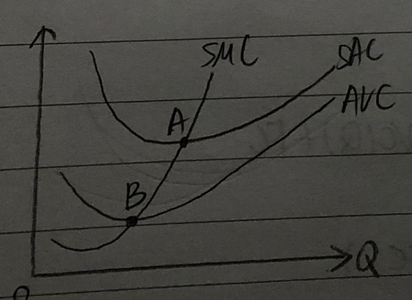




① SAC higher than AVC

② they get closer as Q ↑

⑮ SMC crosses SAC & AVC at the minimum points of SAC & AVC



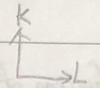
No.: Long-Run Cost Minimizing Input Choice, Long-Run cost Curves

Date:

[II] Isocost

- connects all combinations of L and K that cost the firm same amount of money

  $wL + rK = TC$

  → slope of isocost $= -\frac{w}{r}$

  → higher isocost, higher total cost

- 2 pts on the same isoquant are not necessarily on the same isocost. (vice versa)

[III] For any output level Q_0, firm chooses L & K to minimize total cost of production.

$$\min_{L,K} wL + rK$$
$$\text{s.t. } F(L,K) = Q_0$$

[IIII] Cost-minimizing input choice

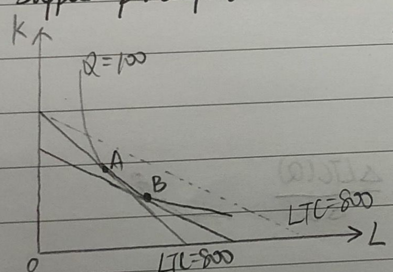
- On the isoquant & on the lowest isocost
- ① on isoquant: $F(L,K) = Q_0$

  ② Tangency condition: $MRTS_{L,K} = \frac{w}{r}$

  ↳ equivalently, $MRTS_{L,K} = \frac{MP_L}{MP_K} = \frac{w}{r} \Rightarrow \frac{MP_L}{w} = \frac{MP_K}{r}$

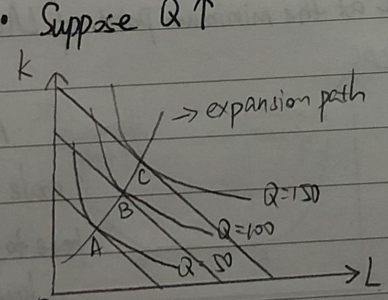
[IV] Comparative statics

- Suppose price of labor ↓



From A to B: the firm uses more labor & less capital

- Suppose Q ↑



The firm uses more labor & more capital

expansion path: a curve that ~~ge~~goes through cost-minimizing choice of inputs when the firm changes output level without changing input prices.

- Normal input

  Cost-minimizing quantity of the input ↑ when output ↑, holding input prices fixed.

- Inferior input

  Cost-minimizing quantity of the input ↓ when output ↑, holding input prices fixed.

No.: Date:

II Input demand function

- As input prices / output level change, firm's cost-minimizing choice of labor & capital may change.
- Demand function of an input:

  Cost-minimizing choice of input as a function of w, r, Q.

  eg. $Q = KL$, w, r input prices

  → to minimize cost, firm chooses K & L such that

  $$\frac{K}{L} = \frac{w}{r} \Rightarrow K = \frac{w}{r}L,\ L = \frac{r}{w}K$$

  subst $Q = KL = \left(\frac{w}{r}L\right)L = \frac{w}{r}L^2$

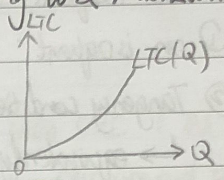
  → demand function of labor : $L(w,r,Q) = \sqrt{\frac{rQ}{w}}$ > normal inputs

  ⋯⋯ capital : $K(w,r,Q) = \sqrt{\frac{wQ}{r}}$

III Long-run total cost curve / function

- Long-run total cost curve : LTC as a function of Q, holding w & r constant

  → $LTC = 0$ when $Q = 0$ (no fixed cost in the LR)



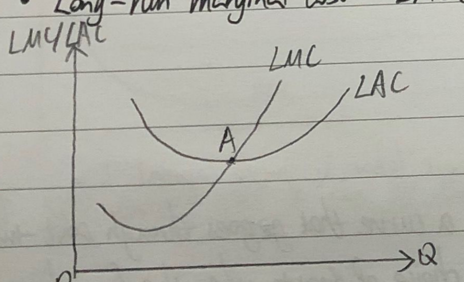
- Long-run total cost function : LTC as a function of Q, w, r.

  eg. cost-minimizing choice of labor & capital : $L(w,r,Q) = \sqrt{\frac{rQ}{w}}$, $K(w,r,Q) = \sqrt{\frac{wQ}{r}}$

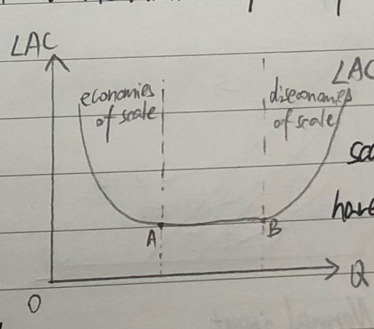
  $$LTC(Q,w,r) = wL + rK = w\sqrt{\frac{rQ}{w}} + r\sqrt{\frac{wQ}{r}} = 2\sqrt{wrQ}$$

IV Average cost & marginal cost

- Long-run average cost : $LAC(Q) = \frac{LTC(Q)}{Q}$
- Long-run marginal cost : $LMC(Q) = \frac{d\,LTC(Q)}{dQ} = \frac{\Delta LTC(Q)}{\Delta Q}$



LMC crosses LAC at the minimum point of LAC.



A: minimum efficient scale (min quantity you have to produce to reach the lowest part of LAC)

- Economies of scale : LAC is decreasing in Q

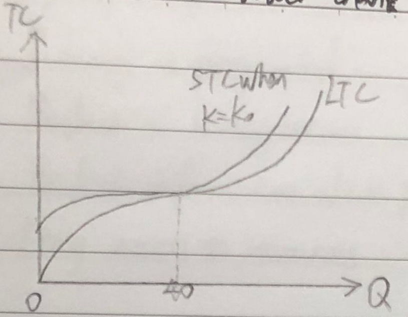
  Diseconomies of scale : LAC is increasing in Q.
- Source of economies of scale ⟨ indivisible input / returns to specialization
- Source of diseconomies of scale — managerial diseconomies of scale

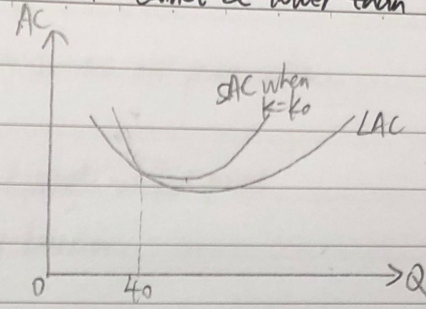
  An a% ↑ in Q requires a more than a% ↑ in firm's spending on managers.

No.: Short-run Cost vs. Long-Run Cost

Date:

Ⅰ STC cannot be lower than LTC → SAC cannot be lower than LAC





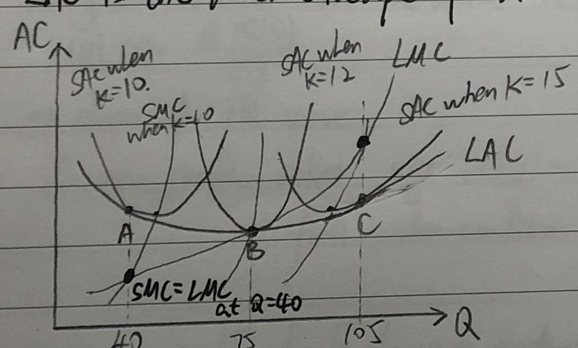
↳ As long as STC & LTC are smooth (MC defined), then the point where STC = LTC will be a tangency point and SMC = LMC.

- Suppose in SR, $K = K_0$.

Suppose when firm produces Q_0, K_0 is the cost-minimizing capital choice in LR.

→ When $Q = Q_0$, input choices are the same in SR and LR. ⟨ STC = LTC; SAC = LAC; SMC = LMC (if MC defined)

Ⅱ LAC is the lower envelope of SAC



No.: Short-Run Supply Curve, Short-Run Market Equilibrium, Producer Surplus in the Short Run

Date:

Ⅰ Perfectly competitive market

- Industry is fragmented – firms & consumers are price takers.
- Product is homogeneous
- Perfect information about prices – 1 single market price
- Easy access to resources – no entry barrier.

Ⅱ SR vs. LR

- SR < at least 1 input fixed; firms choose output to max profit

  LR < all inputs adjustable; firms choose output to max profit; firms decide whether to enter/exit market.

Ⅲ Profit & Revenue

- $\pi(Q) = TR(Q) - TC(Q)$
- marginal revenue: rate at which TR changes with output.

  $MR(Q) = \frac{d\,TR(Q)}{dQ}$

  → slope of TR curve
- To max profit, solve $\max_{Q} TR(Q) - TC(Q)$

  first-order condition: $MR(Q) - MC(Q) = 0$

  $MR(Q) = MC(Q)$ → apply to any profit-max firm, no matter market.

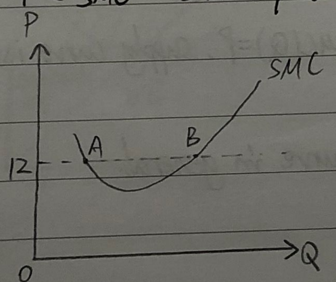
  → to max profit, should choose to produce at quantity where $MR = MC$.
- In perfectly competitive market,

  $TR(Q) = PQ \rightarrow MR(Q) = P$

  To max profit, $P = MC(Q)$

Ⅳ Profit-maximizing optimal output choice : $P = SMC$

- $P > SMC$: should produce more to ↑ profit

  $P < SMC$: should produce less to ↑ profit
- 

  A: profit-minimizing point

  B: profit-maximizing point

  ☆ To max profit, shouldn't produce at the part of MC curve that is downward-sloping!

No.: Date:

Non-sunk cost vs. Sunk cost

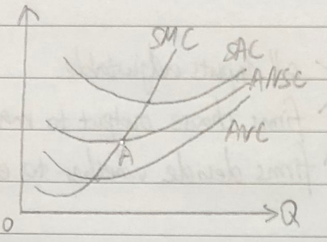
SR total cost

- FC may / may not be sunk
- Total non-sunk cost : TNSC = Total variable cost + total non-sunk fixed cost.
- Total sunk cost : TSC = total sunk fixed cost

→ If all FC is non-sunk : TNSC = STC

→ If all FC is sunk : TNSC = VC

> TNSC in between VC & STC



SMC crosses ANSC at the min point of ANSC

Should the firm produce at all ?

- if doesn't produce : $\pi = -TSC$

  if produces : $\pi = TR(Q) - TNSC(Q) - TSC$

  $\underbrace{TR(Q)}_{PQ} \geq \underbrace{TNSC(Q)}_{ANSC(Q) \times Q}$

- Firm only produces when $P \geq ANSC(Q)$

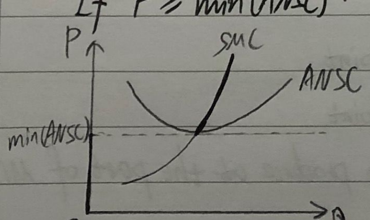
Profit-maximizing conditions in SR

- When $P \geq \min(ANSC)$, each firm should choose a level of Q such that
  ① at that output level, P = SMC
  ② SMC is non-decreasing in Q.
- When $P < \min(ANSC)$, each firm should set $Q = 0$.

Individual firm's supply curve SR

- SR supply curve for an individual firm is the profit-maximizing quantity for the firm as a function of the market price.
- If $P < \min(ANSC)$: $Q = 0$, supply curve is the vertical axis

  If $P \geq \min(ANSC)$: Firm chooses Q such that $SMC(Q) = P$, supply curve is the SMC curve.



— : firm's SR supply curve in general.

Short-run market supply curve

- horizontal sum of all individual firm's supply curve.

  eg. 100 firms, $Q_f = \frac{P}{2}$

  $S(P) = 100\left(\frac{P}{2}\right) = 50P$.

No.:

Date:

SR market equilibrium (in a competitive market)

① Total quantity demanded = total quantity supplied

② Each firm produces at the profit-maximizing output level given the equilibrium price.

③ Each consumer buys at the utility-maximizing quantity given the equilibrium price.

→ $S(P) = D(P)$

Profit at SR Market equilibrium

- $\pi = TR - STC = P \times Q_f - SAC(Q_f) \times Q_f$

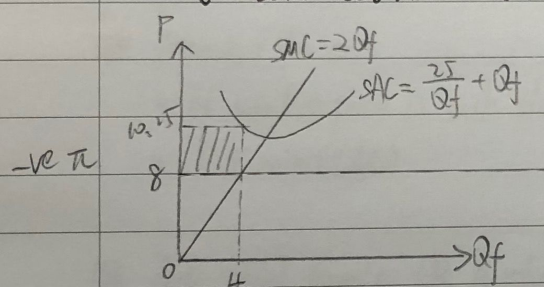
  $= [P - SAC(Q_f)]\, Q_f$

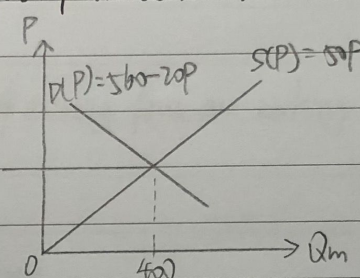
→ $P > SAC(Q_f)$, firm's profit is +ve at output level Q_f.

$P < SAC(Q_f)$, firm's profit is −ve at output level Q_f.

→ Negative profit is possible in SR market equilibrium.

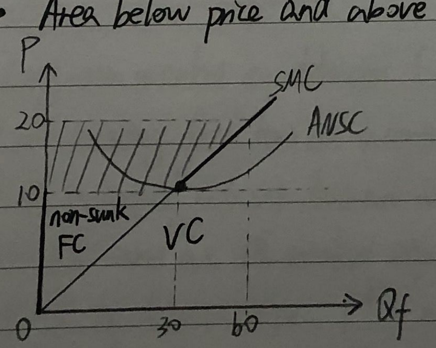
(∵ sunk cost not considered when firms decide how much to produce)





Producer Surplus (PS)

- diff between the amt producers actually receive by producing & selling a certain units and the amt producers have to receive to produce a certain units.
- PS = total revenue − total non-sunk cost
- Area below price and above supply curve



VC: area under SMC curve

TNSC = VC + non-sunk fixed cost

No.: Long-Run Equilibrium, LR Market Supply Curve

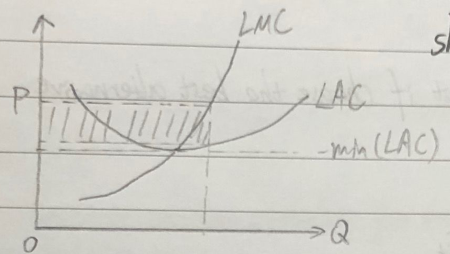
Date:

Long-Run profit-maximizing output choice

- $MR = P = LMC$
- $P > LMC$: producing too little, adjust both K & L to increase Q.
  $P < LMC$: producing too much, adjust both K & L to decrease Q.

Incentive for entry

- If market price is such that if enters, firm can make +ve profit. $TR > LTC$, ÷Q: $P > LAC$
- $P > \min(LAC)$



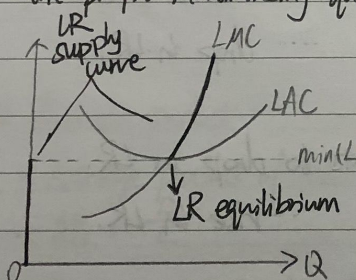
shaded area: +ve profit.

Incentive for exit

- If market price is such that existing firms are making -ve profit.
- $P < \min(LAC)$

Individual firm's LR supply curve

- the profit-maximizing quantity for the firm in the LR as a function of market price.



Long-run market equilibrium (in a competitive market)

① No existing firm has an incentive to exit the market $P \geq \min(LAC)$
② No potential entrant has an incentive to enter the market. $P \leq \min(LAC)$
③④⑤ same as SR market equilibrium.

→ LR equilibrium price $P^* = \min(LAC)$
LR equilibrium output for each firm: $P^* = LMC(Q^*) = \min(LAC) = LAC(Q^*)$
LR equilibrium profit for each firm: $[P^* - LAC(Q^*)]\,Q^* = 0$!

→ no of firms not fixed in LR
no of firms in LR equilibrium can be determined.

No.:

Date:

ⅩⅦ LR Dynamic: Entry

profit → entry → market supply curve shifts right → price falls → no more incentive for entry → entry stops (LR equilibrium)

ⅩⅧ LR Dynamic: Exit

loss → exit → market supply curve shifts left → price rises → no more incentive to exit → exit stops (LR equilibrium)

ⅩⅨ Economic profit

- 0 : getting the same return as you could get if choose the best alternative.
  +ve : .... ~~mo~~higher return than ....
  -ve : .... less return than ...
  → In LR equilibrium, all firms earn 0 profit.

ⅩⅩ Input prices in LR

- Constant-cost industry
  changes in industry output does not affect input prices in LR.
- Increasing-cost industry
  increase in industry output causes the prices of inputs to rise in LR.
  decrease .... drop in LR.
- Decreasing-cost industry
  increase in industry output causes the prices of inputs to drop in LR.
  decrease ... ... rise in LR.

ⅩⅪ LR market supply curve

- total quantity supplied in LR equilibrium as a function of LR equilibrium price
  → constant-cost industry : horizontal curve
  Increasing-cost industry : upward sloping
  decreasing-cost industry : downward sloping