LECTURE 6
FIRST WELFARE THEOREM
WALRAS’ LAW
MIDTERM REVIEW

Where are we?

- Edgeworth box
- Pareto efficiency
- Competitive equilibrium
- First (and second) welfare theorems
  - What is the relationship between Pareto efficiency and competitive equilibrium?
- Walras' Law
  - A result derived from budget lines and optimal baskets
- Midterm Review

Part 1

First Welfare Theorem

General Competitive Equilibrium

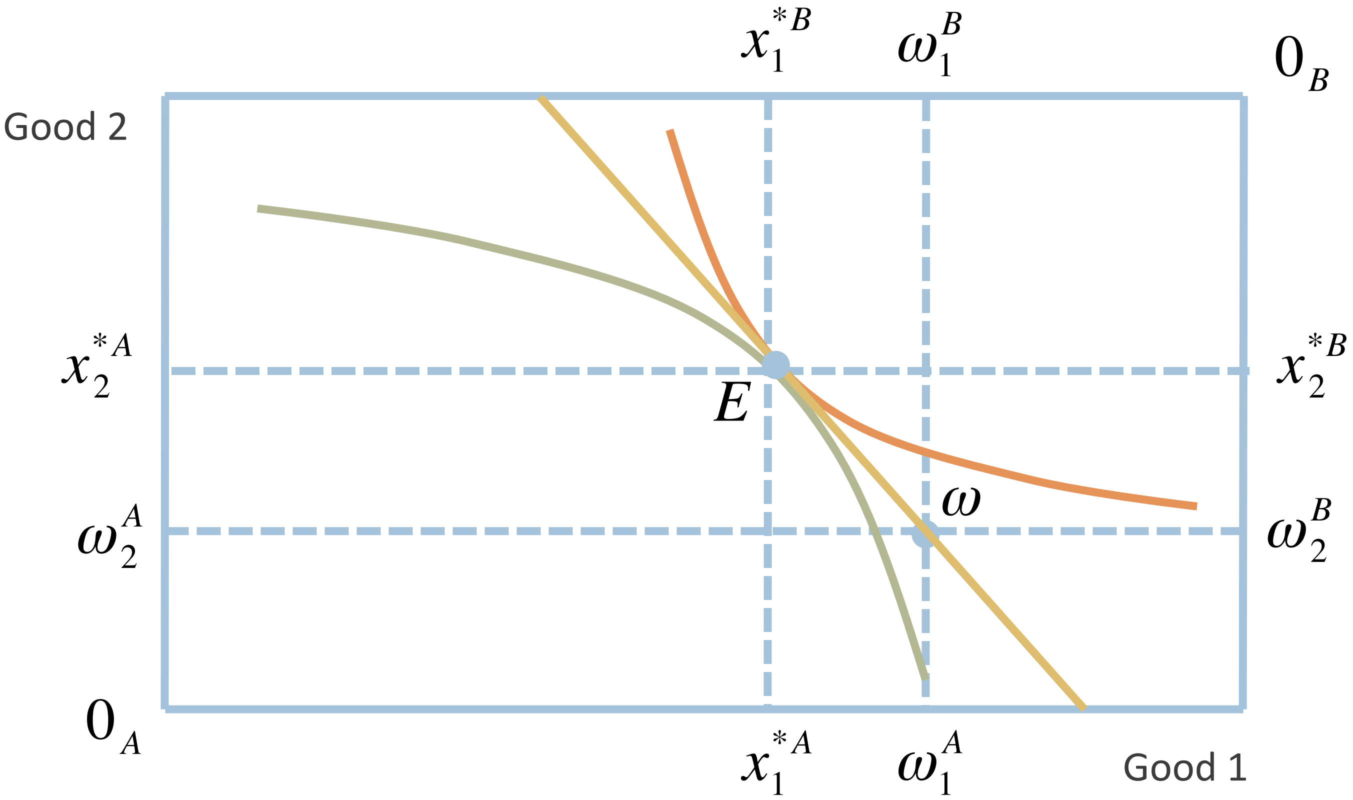
- A pair of prices (P_1, P_2) constitutes a (general) *competitive equilibrium* if at the prices
  - Each consumer maximizes his/her utility given the budget constraint

$$x_1^{*A}, x_2^{*A}, x_1^{*B}, x_2^{*B}$$

  denotes the optimal consumption for each consumer given the equilibrium prices
  - Markets for both goods clear

$$x_1^{*A} + x_1^{*B} = \omega_1^A + \omega_1^B$$
$$x_2^{*A} + x_2^{*B} = \omega_2^A + \omega_2^B$$

Competitive Equilibrium in Graph



At point E, the two consumers' indifference curves are tangent to each other

First Welfare Theorem

- Definition 6.1 The *First Fundamental Theorem of Welfare Economics* states that a competitive equilibrium allocation is Pareto efficient
  - Suppose the equilibrium prices are (P_1, P_2) and the allocation

$$x_1^{*A}, x_2^{*A}, x_1^{*B}, x_2^{*B}$$

  is the allocation given the equilibrium prices
  - Then the allocation

$$x_1^{*A}, x_2^{*A}, x_1^{*B}, x_2^{*B}$$

  is Pareto efficient

Proof of First Welfare Theorem

- Suppose at the equilibrium prices P_1 and P_2, the equilibrium allocation is $x_1^{*A}, x_2^{*A}, x_1^{*B}, x_2^{*B}$
- Proof by contradiction: suppose this allocation is not Pareto efficient
- Then there must exist another feasible allocation

$$y_1^A, y_2^A, y_1^B, y_2^B$$

  - where at least one consumer is better off
  - and no one is worse off
  - compared to the equilibrium allocation

Proof of First Welfare Theorem Cont'

- Suppose consumer A strictly prefers (y^A_1, y^A_2) to (x^{*A}_1, x^{*A}_2) while consumer B weakly prefers (y^B_1, y^B_2) to (x^{*B}_1, x^{*B}_2)
- By definition, the equilibrium allocation is the utility-maximizing basket for each consumer given the budget constraint, thus by revealed preference,

$$P_1 y_1^A + P_2 y_2^A > P_1 \omega_1^A + P_2 \omega_2^A \qquad (1)$$

Proof of First Welfare Theorem Cont'

- Add up (1) and (2), we have

$$P_1(y_1^A + y_1^B) + P_2(y_2^A + y_2^B) > P_1(\omega_1^A + \omega_1^B) + P_2(\omega_2^A + \omega_2^B) \quad (3)$$

- Allocation $y_1^A, y_2^A, y_1^B, y_2^B$ must also be feasible

$$y_1^A + y_1^B = \omega_1^A + \omega_1^B$$

$$y_2^A + y_2^B = \omega_2^A + \omega_2^B$$

- Substituting into (3), we have a contradiction

$$P_1(\omega_1^A + \omega_1^B) + P_2(\omega_2^A + \omega_2^B) > P_1(\omega_1^A + \omega_1^B) + P_2(\omega_2^A + \omega_2^B)$$

The Invisible Hand

- Each consumer maximizes his/her own utility
- No central planner
- Yet competitive market leads to a Pareto efficient allocation

Implication of First Welfare Theorem

- How should we allocate limited resources in the economy?
  - E.g., land, masks, hand sanitizers
- What is a "good" way to allocate resources?
  - There are many ways to define "good"
  - Let's suppose "good" means Pareto efficient
- Is there a mechanism we can rely on to allocate resources efficiently?
  - Yes, FWT tells us that we just need to create a competitive market and the market will allocate resources efficiently

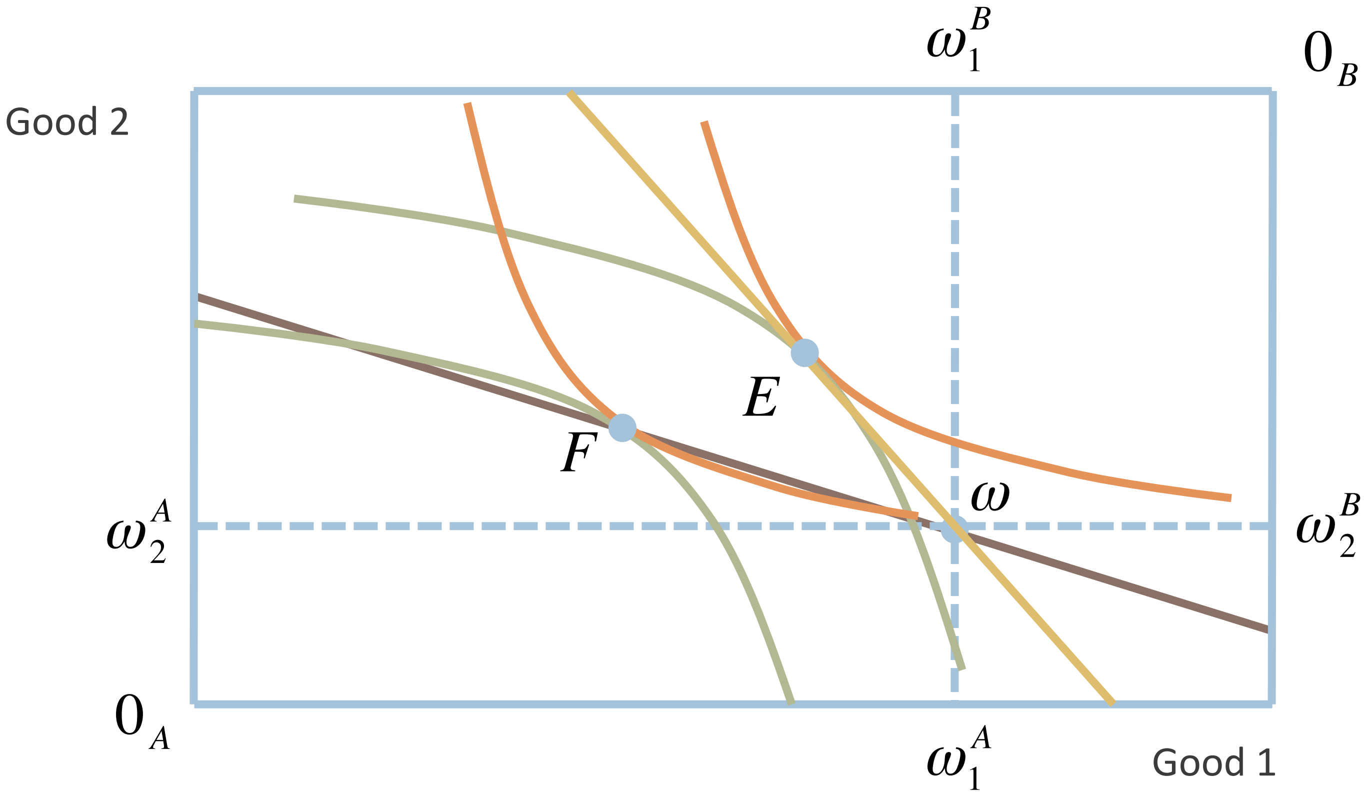
Comments on First Welfare Theorem

- It only holds in competitive markets
  - Not true if consumers or firms have price setting power
  - Not true if there is externality
  - Not true if there is asymmetric information
- Efficiency does not mean equity
  - A Pareto efficient allocation may or may not be an equitable allocation
  - E.g., one consumer has everything and the other consumer has nothing can be Pareto efficient

Pareto Efficiency vs. Competitive Equilibrium

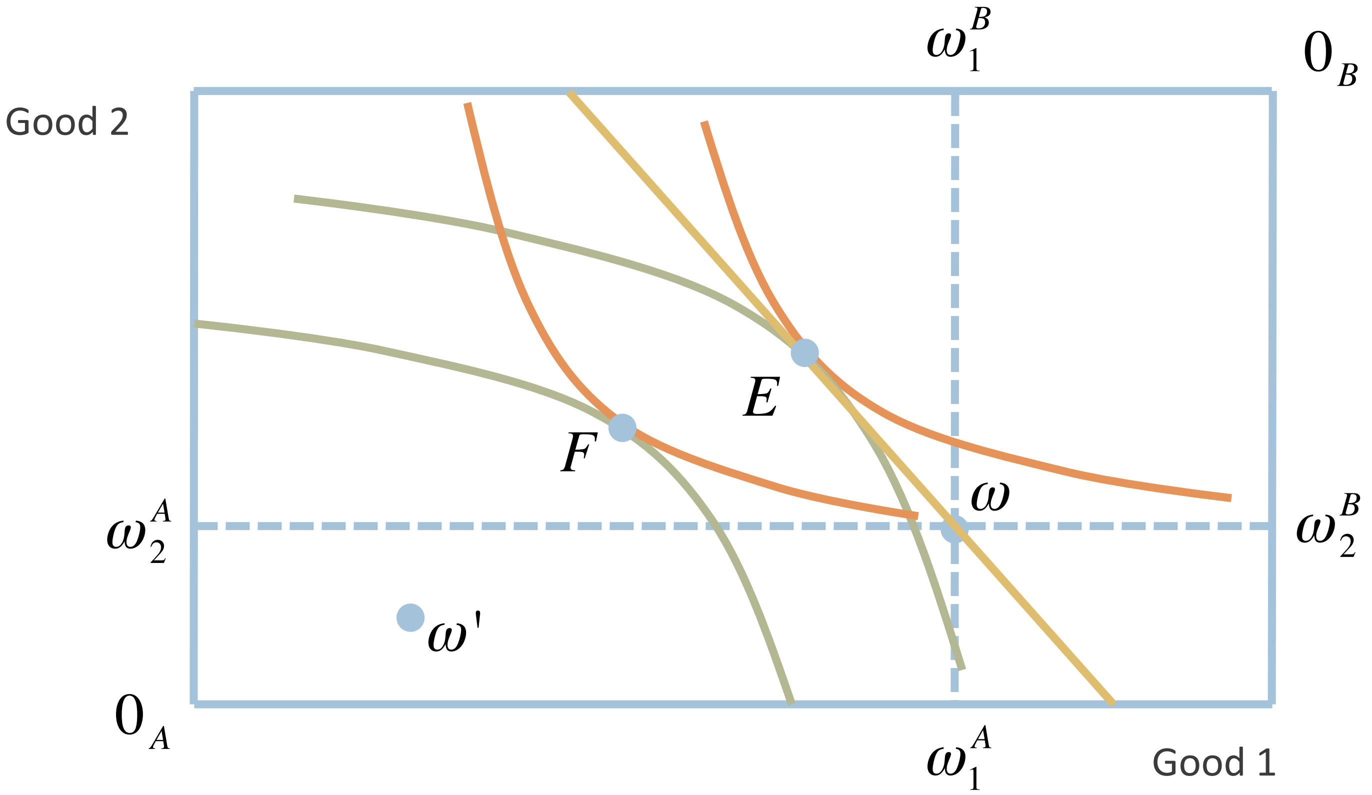
- Pareto Efficiency
  - An allocation where it is impossible to make someone better off without making someone else worse off
  - Does not depend on prices
  - Does not depend on endowment
- Competitive Equilibrium
  - A pair of prices such that
    - Markets clear
    - Everyone maximizes utility given budget constraint
  - Depend on endowment
    - Endowment allocation (and prices) determine budget constraints

E and F are Always Pareto Efficient Regardless of Prices



A Pareto efficient allocation may not be an equilibrium allocation

E and F are Always Pareto Efficient Regardless of Endowment

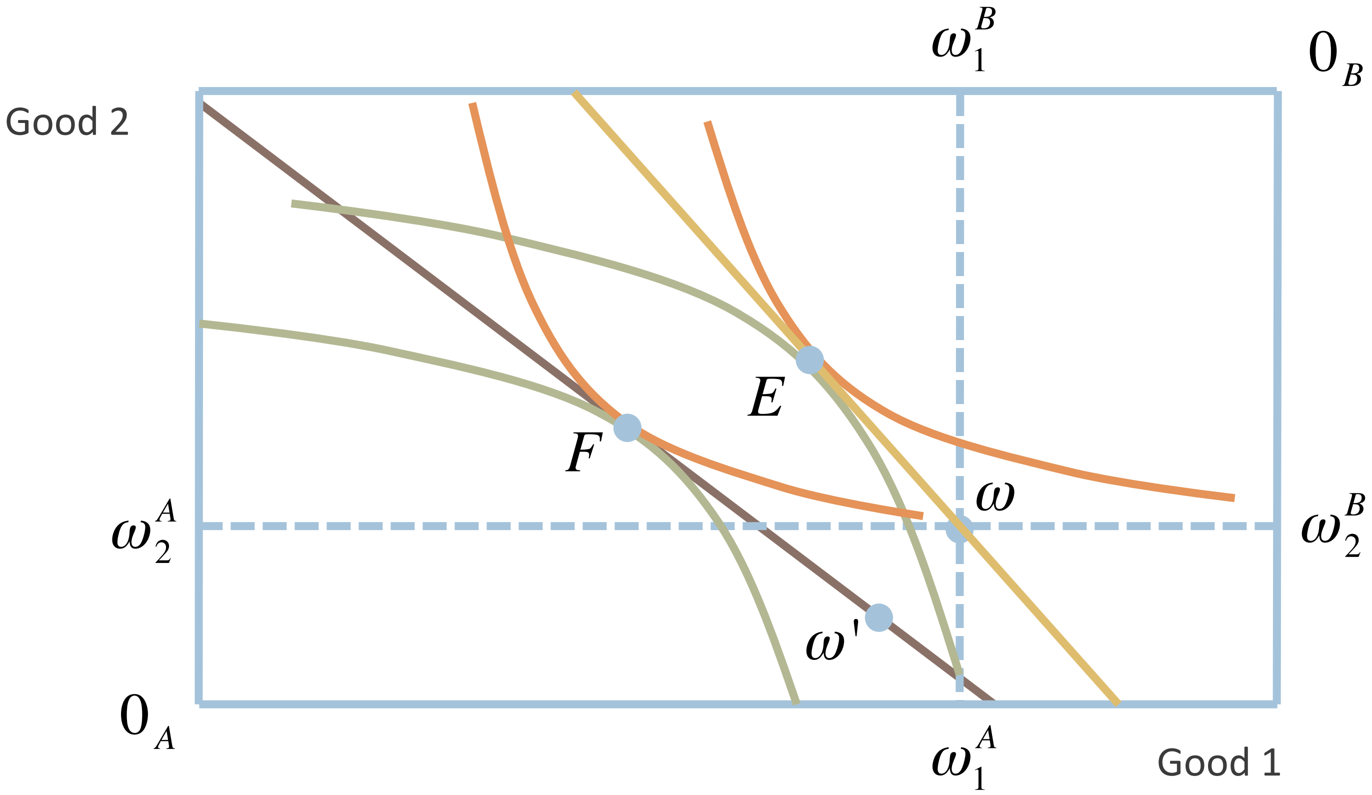


E or F is not the equilibrium allocation is the endowment is at point *w'*

Second Welfare Theorem

- First welfare theorem
  - Competitive equilibrium allocation is Pareto efficient
- How about the reverse?
  - We know that not every Pareto efficient allocation can be achieved in equilibrium given a particular endowment allocation
- The *Second Fundamental Theorem of Welfare Economics* states that any Pareto efficient allocation can be achieved in a competitive equilibrium through redistribution of endowments

Second Welfare Theorem in Graph



F will be an equilibrium allocation if the endowment is *w'*

Part 2

Walras' Law

Gross Demand at Any Given Prices

- Let P_1, P_2 be any pair of prices
  - May or may not be the equilibrium prices
- Let $(x^A{}_1, x^A{}_2)$ be A's gross demand and $(x^B{}_1, x^B{}_2)$ be B's gross demand given P_1, P_2
  - The utility-maximizing quantity of each good for each consumer at the given prices
- Since P_1, P_2 may not be the equilibrium prices, it is possible that

$$x_1^A + x_1^B \neq \omega_1^A + \omega_1^B$$

$$x_2^A + x_2^B \neq \omega_2^A + \omega_2^B$$

Net Demand

- Definition 6.2 The *net demand* of a consumer for a good is the difference between the gross demand for that good and his/her endowment for that good
- A's net demand for good 1 is

$$x_1^A - \omega_1^A$$

- A's net demand for good 2 is

$$x_2^A - \omega_2^A$$

Aggregate Net Demand

- Definition 6.3 The *aggregate net demand* for a good is the sum of the net demand for that good for the two consumers

$$x_1^A + x_1^B - \omega_1^A - \omega_1^B, \quad x_2^A + x_2^B - \omega_2^A - \omega_2^B$$

- When the aggregate net demand for a good is positive
  - There is excess for that good
- When the aggregate net demand for a good is negative
  - There is excess for that good

Value of Net Demand

- $(x^A{}_1, x^A{}_2)$ lies on consumer A's budget line

$$P_1 x_1^A + P_2 x_2^A = P_1 \omega_1^A + P_2 \omega_2^A$$

- Rearranging

$$P_1 (x_1^A - \omega_1^A) + P_2 (x_2^A - \omega_2^A) = 0$$

- The total value of consumer A's net demand for the two goods is 0
  - The value of A's net demand for good 1 is

$$P_1 (x_1^A - \omega_1^A)$$

  - The value of A's net demand for good 2 is

$$P_2 (x_2^A - \omega_2^A)$$

Walras' Law

- Similarly, the total value of consumer B's net demand for the two goods is 0

$$P_1(x_1^B - \omega_1^B) + P_2(x_2^B - \omega_2^B) = 0$$

- Adding up the equation for A and B

$$P_1(x_1^A + x_1^B - \omega_1^A - \omega_1^B) + P_2(x_2^A + x_2^B - \omega_2^A - \omega_2^B) = 0$$

- Definition 6.4 The equation above is the *Walras' Law*
  - The total value of the aggregate net demand for the two goods is 0

Implications of Walras' Law

- In the two-good exchange economy, if one market is in equilibrium, the other market must also be in equilibrium
- Suppose the market for good 1 clears

$$x_1^A + x_1^B - \omega_1^A - \omega_1^B = 0$$

- By the Walras' law

$$P_1(x_1^A + x_1^B - \omega_1^A - \omega_1^B) + P_2(x_2^A + x_2^B - \omega_2^A - \omega_2^B) = 0$$

- Market for good 2 clears as well

$$x_2^A + x_2^B - \omega_2^A - \omega_2^B = 0$$

Implications of Walras' Law Cont'

- In the two-good exchange economy, an excess supply in one market implies an excess demand in the other market
- Suppose there is excess supply of good 1

$$x_1^A + x_1^B - \omega_1^A - \omega_1^B < 0$$

- By the Walras' law

$$P_1(x_1^A + x_1^B - \omega_1^A - \omega_1^B) + P_2(x_2^A + x_2^B - \omega_2^A - \omega_2^B) = 0$$

- There will be excess demand of good 2

Walras' Law vs. Competitive Equilibrium

- Walras' law holds for ANY prices
  - Not just the equilibrium prices
- At the equilibrium prices, the aggregate net demand for each good is 0

$$P_1(\underbrace{x_1^A + x_1^B - \omega_1^A - \omega_1^B}_{=0}) + P_2(\underbrace{x_2^A + x_2^B - \omega_2^A - \omega_2^B}_{=0}) = 0$$

- At non-equilibrium prices, the aggregate net demand is not 0

$$P_1(\underbrace{x_1^A + x_1^B - \omega_1^A - \omega_1^B}_{\neq 0}) + P_2(\underbrace{x_2^A + x_2^B - \omega_2^A - \omega_2^B}_{\neq 0}) = 0$$

Part 3

Midterm Review

Basic Information

- 3 Mar, 6:30 pm to 7:45 pm
  - Split into multiple venues: MPSH + faculty venues
- Format
  - MCQ + structured questions
- Coverage
  - Lecture 1 – Lecture 5
- Bring
  - Student ID
  - Non-programmable calculator
- Do not write in pencil
- Memorize your tutorial number

Logistics

- Topics in the textbook that are not covered in lectures or tutorials are not required
- There will be no lecture or tutorial in week 7
- Midterm practice problems will be uploaded on LumiNUS
- My consultation hours
  - 25 Feb – 27 Feb: Zoom consultation
    - L1: 2 pm to 3:30 pm
    - L2: 3:30 pm to 5 pm
  - 28 Feb and 2 Mar: walk-in consultation
    - 2:30 pm to 5 pm

Our Topics

- Consumer Theory
  - Optimal choice
  - Revealed preference
  - Demand curve/demand function
  - Substitution and income effects
  - Consumer welfare
- Exchange
  - Pareto efficiency
  - Competitive equilibrium

Common Utility Functions

- Cobb Douglas
- Perfect substitutes
- Perfect complements
- Quasi linear

Feasible Allocation

- Recall an allocation is feasible if

$$x_1^A + x_1^B = \omega_1^A + \omega_1^B$$
$$x_2^A + x_2^B = \omega_2^A + \omega_2^B$$

- An alternative definition says that an allocation is feasible if

$$x_1^A + x_1^B \le \omega_1^A + \omega_1^B$$
$$x_2^A + x_2^B \le \omega_2^A + \omega_2^B$$

  - The total amount of each good consumed does not exceed the total amount available

Does it matter?

- Given the alternative definition, when you solve for the contract curve, you will have
- Tangency condition

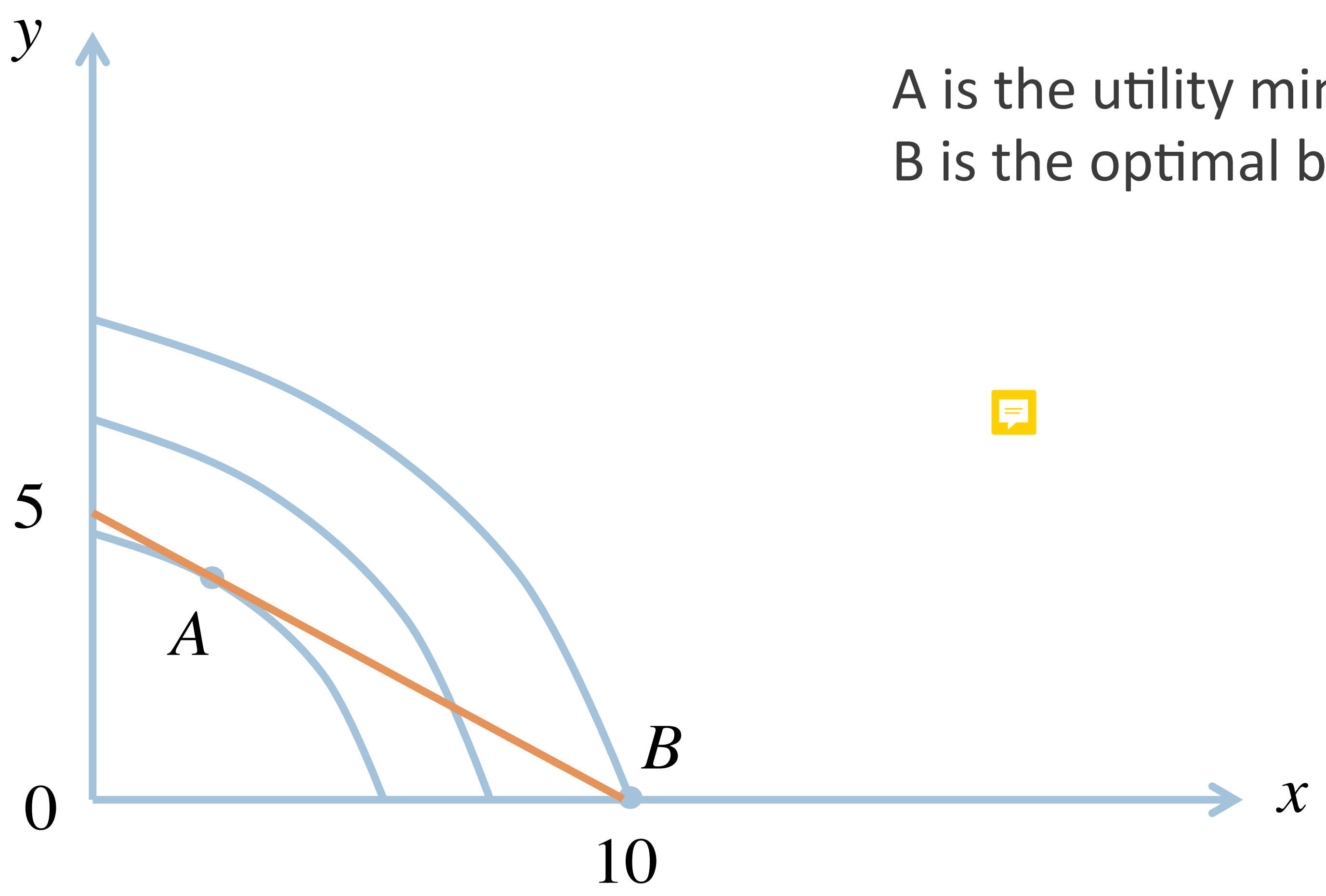
$$MRS^{A}_{1,2} = MRS^{B}_{1,2} \quad (1)$$

- The allocation must be feasible

$$x_1^A + x_1^B \leq \omega_1^A + \omega_1^B \quad (2)$$

$$x_2^A + x_2^B \leq \omega_2^A + \omega_2^B \quad (3)$$

Homework 1 Question 1 d)



A is the utility minimizing basket
B is the optimal basket

Homework 1 Question 2 b)

- This is about the direction of substitution effect when there is a price increase
- Given the initial price ($1), A is optimal and the consumer is indifferent between A and C, this means

$$x_c + 3y_c \geq x_A + 3y_A = 12$$

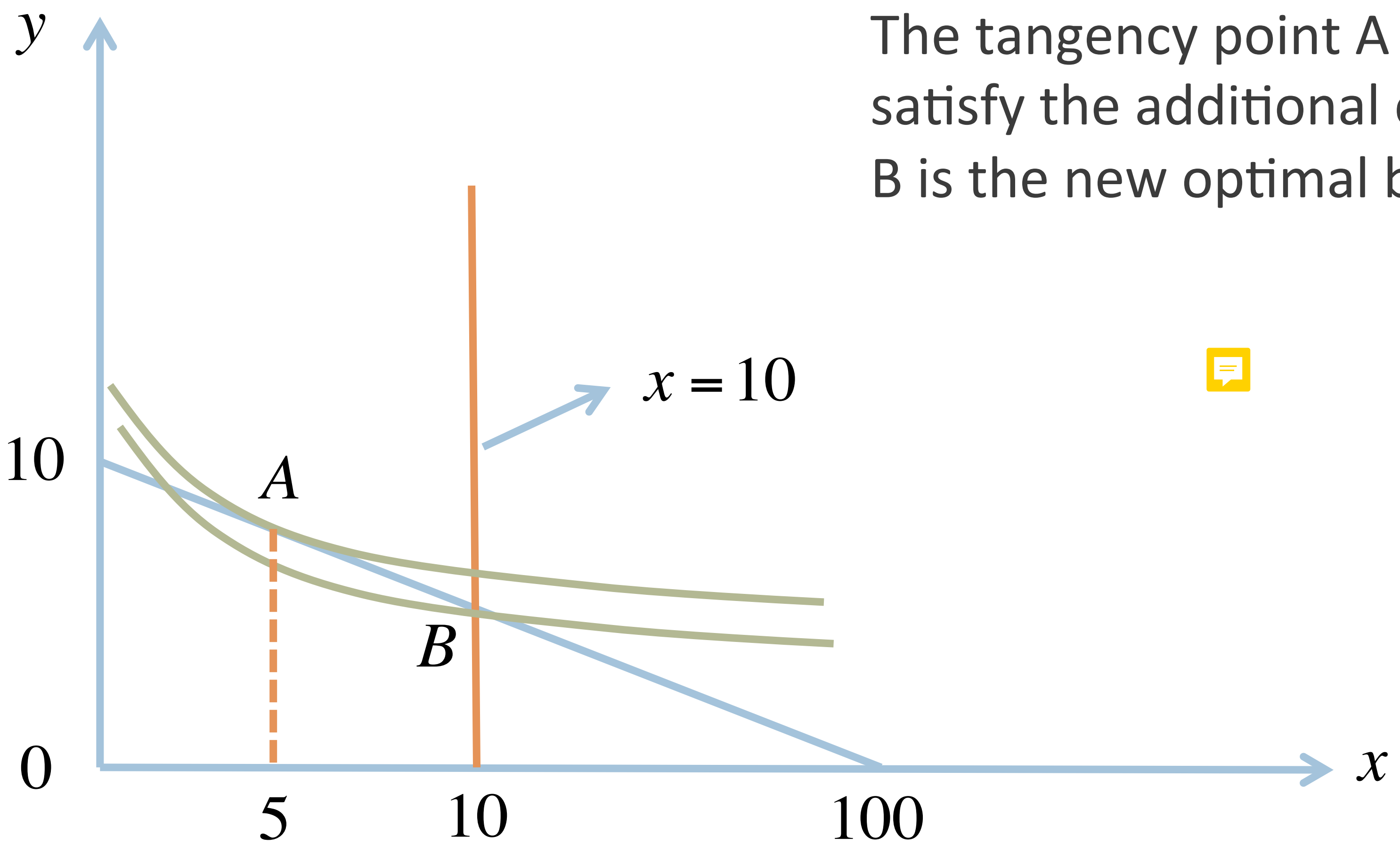
- Given the new price ($2), C is optimal and the consumer is indifferent between A and C, this means

$$18 = 2x_A + 3y_A \geq 2x_c + 3y_c$$

- Thus

$$x_c \leq x_A = 6$$

Homework 1 Question 4 c)



The tangency point A does not satisfy the additional constraint
B is the new optimal basket

MCQ: Example 1

- A consumer buys two goods, rice and housing. At the initial optimal basket, the consumer buys both goods. When the price of rice decreases while the price of housing and the consumer's income remain constant, the consumption of rice increases by 4 units. If rice is an inferior good, regarding the substitution effect (*SE*) and income effect (*IE*) with respect to rice, which of the following is true?
  - A. $0<=SE<=4$, $0<=IE<=4$
  - B. $-4<=SE<0$, $-4<=IE<=0$
  - C. $SE>4$, $IE<0$
  - D. $SE<0$, $IE>4$

Solution for MCQ 1

MCQ: Example 2

- Suppose a consumer has utility function $U(x,y)=\min(ax,y)$, where $a>0$ .Which of the following statements is true?
  - A. The consumer always buys the same amount of *x* and *y*.
  - B. The consumer's expenditure on *y* is always greater than the expenditure on *x*.
  - C. When income doubles, the consumer doubles his consumption of both *x* and *y*.
  - D. When *x* becomes more expensive, the consumer buys more *y*.

Solution for MCQ 2

MCQ: Example 3

- Suppose a consumer's preference satisfies the three assumptions. The consumer has an income of \$18. When the price of *x* is \$2 and the price of *y* is \$1, the consumer's optimal choice is 6 units of *x* and 6 units of *y*. When the price of *x* becomes \$1 and the price of *y* becomes \$2, assuming income does not change, the optimal choice <u>CANNOT</u> be
  - A. 10 units of *x* and 4 units of *y*
  - B. 8 units of *x* and 5 units of *y*
  - C. 6 units of *x* and 6 units of *y*
  - D. 4 units of *x* and 7 units of *y*

Solution for MCQ 3