LECTURE 2
PREFERENCE
BUDGET CONSTRAINT
CONSUMER CHOICE

Makeup Tutorials

- W1-W6 on 27 Jan are canceled due to CNY
- Makeup Tutorials
  - 28 Jan (Tue): 12 pm to 1 pm, AS2-0510
  - 28 Jan (Tue): 1 pm to 2 pm, AS2-0510
  - 29 Jan (Wed): 11 am to 12 pm, AS2-0510
- If you are from W1-W6, try to attend one of the above makeups if possible
  - If not, try to go to any other tutorial that fits your schedule

Where are we?

- Preference
  - Indifference curves
    - Do not cross
    - Downward sloping if “more is better” is satisfied for both goods
    - Slope of indifference curve
  - Utility functions
  - Special preferences
- Budget constraint
- Consumer choice
  - Which basket will the consumer choose to buy?

Part 1

Preference

Marginal Rate of Substitution

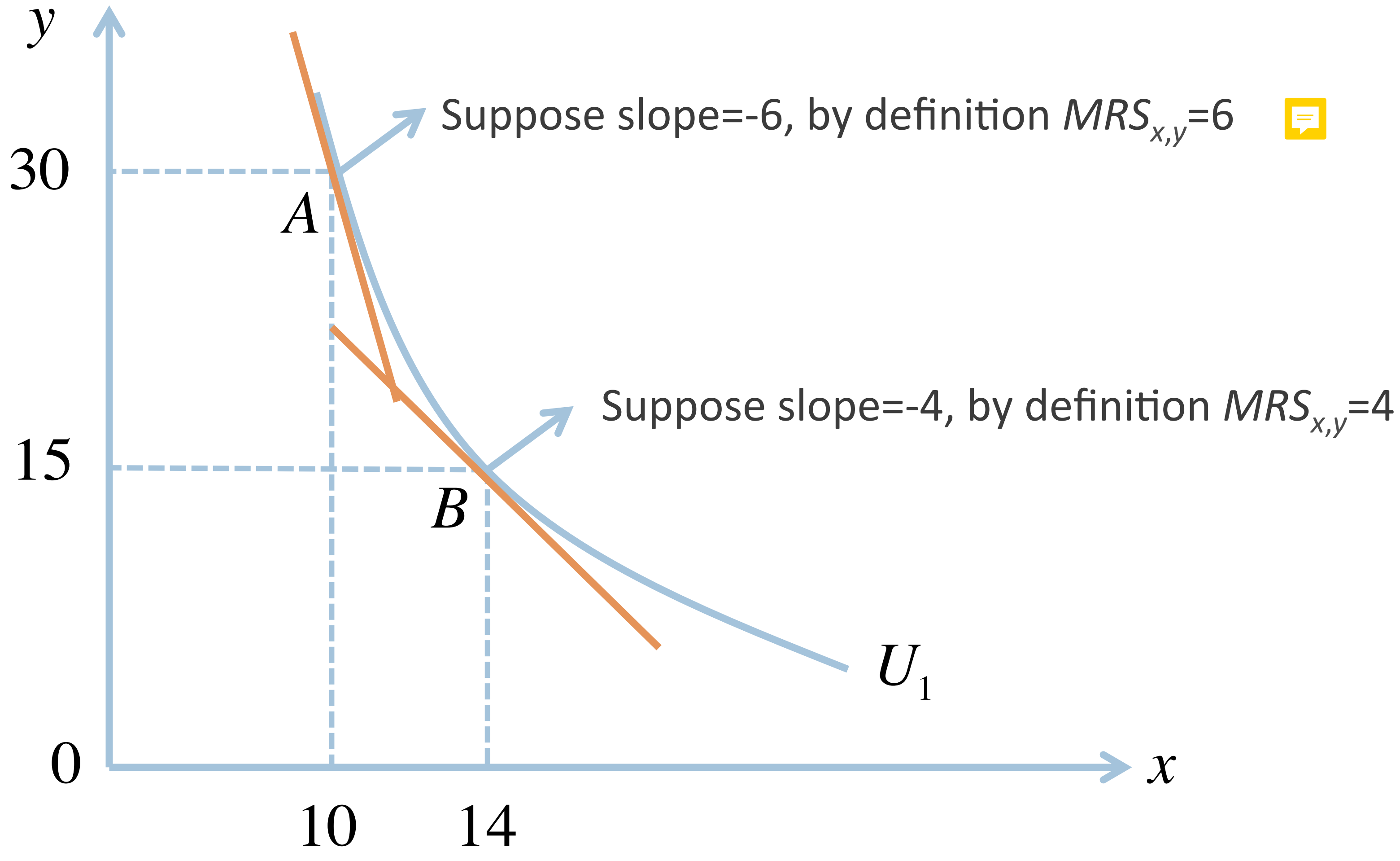
- Definition 2.1 *Marginal rate of substitution* of *x* for *y* is the rate at which the consumer is willing to give up *y* to get more of *x*, maintaining the same level of satisfaction

$$MRS_{x,y} = -\frac{dy}{dx}\bigg|_{Same \quad U} = -\frac{\Delta y}{\Delta x}\bigg|_{Same \quad U}$$

  where Δx is extremely small

- $MRS_{x,y}$ is the negative of the slope of the indifference curve (with *x* on the horizontal axis and *y* on the vertical axis)

MRS and the Shape of Indifference Curve



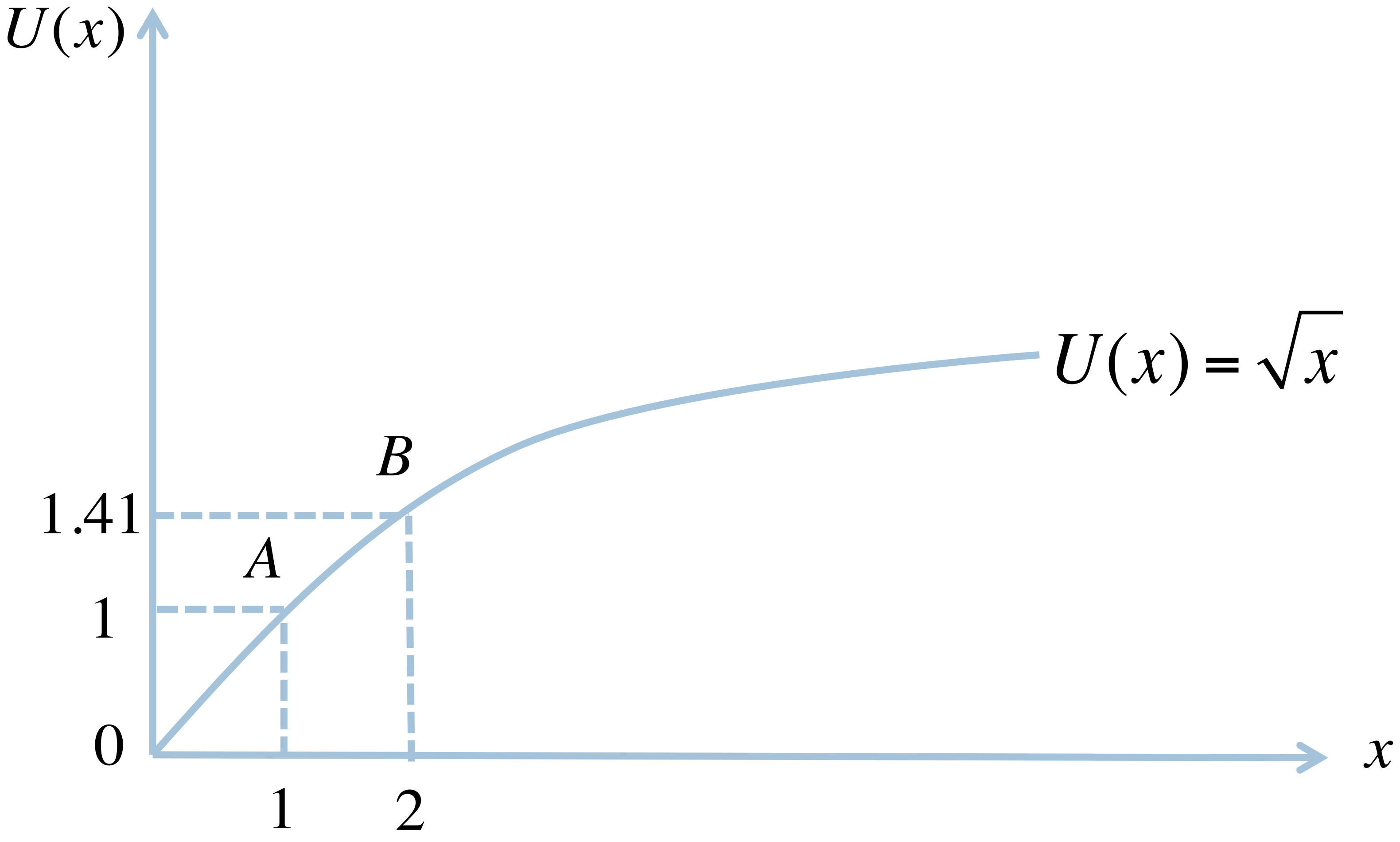
Diminishing Marginal Rate of Substitution

- *Diminishing marginal rate of substitution* means $MRS_{x,y}$ decreases as the consumer gets more *x* and less *y* along the same indifference curve
  - Holding satisfaction level fixed, as the consumer gets more of *x*, the willingness to give up *y* and get additional *x* reduces
- If diminishing marginal rate of substitution holds
  - And if the three assumptions hold
  - Then indifference curves are convex to the origin

Utility Function

- *Utility* is a numeric value indicating the consumer's level of satisfaction
- Definition 2.2 *Utility function* assigns a level of utility to each consumption basket so that if $A \succ B$, $U(A)>U(B)$
  - Utility function represents preference
  - Higher utility = higher the level of satisfaction

Utility Function with One Good: An Example



Marginal Utility

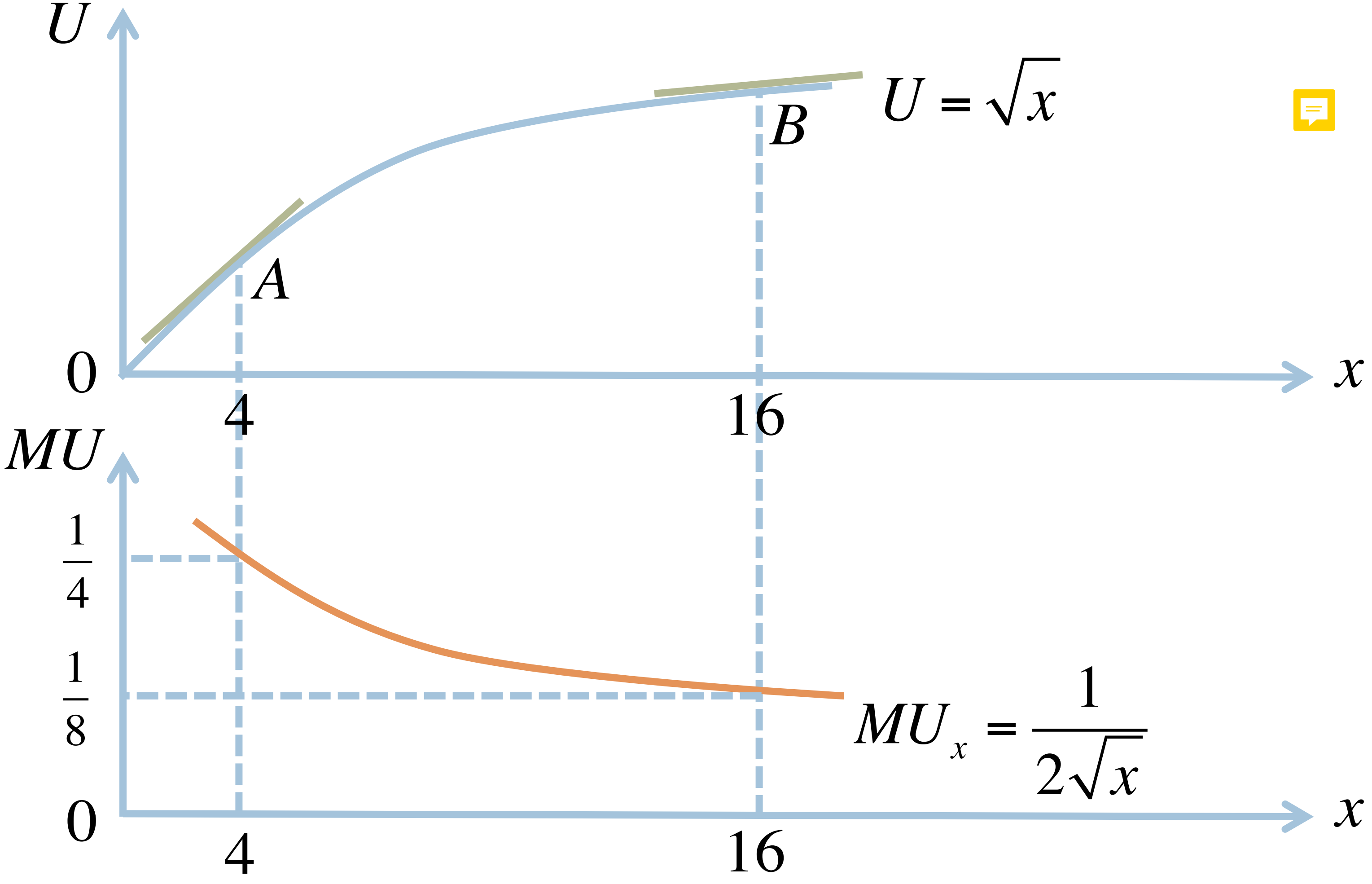
- Definition 2.3 *Marginal utility* is the rate at which utility changes as the level of consumption of a good changes

$$MU_x = \frac{dU}{dx} = \frac{\Delta U}{\Delta x}$$

where Δx is extremely small

- *MU* is the slope of the utility function
- What does the sign of marginal utility tell us?

Marginal Utility in Graph



Principle of Diminishing Marginal Utility

- *Principle of diminishing marginal utility*
  - Marginal utility decreases as consumption level rises
  - Utility increases slower as consumption level rises
  - Utility function becomes flatter as consumption level rises

Marginal Utility with Two Goods

- More often, we will deal with utility functions with two goods $U(x, y)$
- Marginal utility of x

$$MU_x = \frac{\partial U}{\partial x}$$

- Marginal utility of y

$$MU_y = \frac{\partial U}{\partial y}$$

- Diminishing marginal utilities
  - MU_x decreases with x, holding y constant
  - MU_y decreases with y, holding x constant

Utility Function with Two Goods: An Example

- Suppose the utility function of the consumer is

$$U(x,y) = \sqrt{xy}$$

- "More is better" is satisfied for both goods

$$MU_x = \frac{\sqrt{y}}{2\sqrt{x}} > 0, \quad MU_y = \frac{\sqrt{x}}{2\sqrt{y}} > 0$$

- Diminishing marginal utility is satisfied for both goods

From Utility Function to Indifference Curves

- Indifference curves can be drawn from the utility function
- (2,2) indifferent to (1,4) and (4,1)

$$U(2,2) = U(1,4) = U(4,1) = \sqrt{4} = 2$$

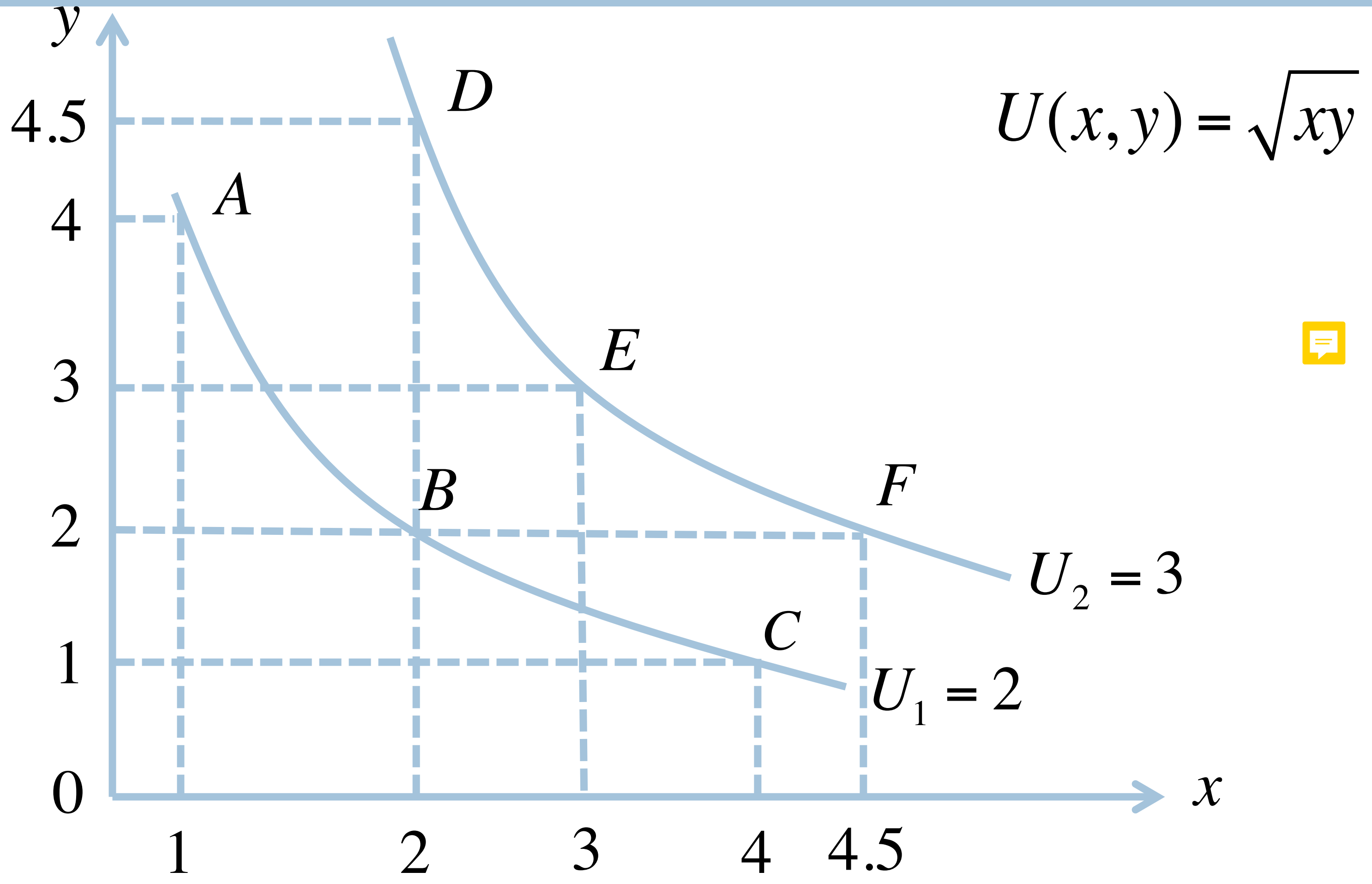
- (3,3) preferred to (2,2)

$$U(3,3) = 3 > U(2,2) = 2$$

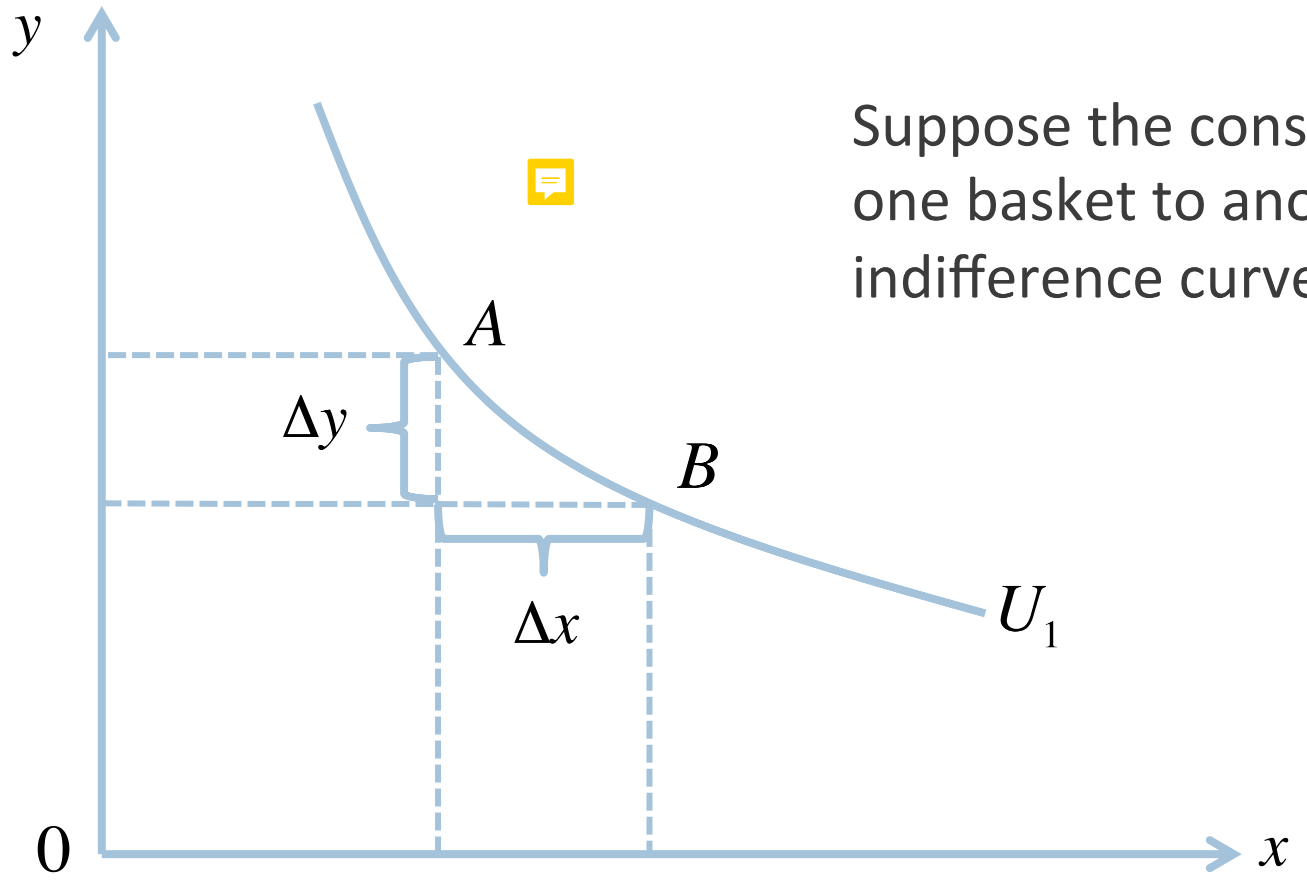
- (3,3) indifferent to (2,4.5) and (4.5,2)

$$U(3,3) = U(2,4.5) = U(4.5,2) = 3$$

Plotting Indifference Curves



MRS and *MU*



Suppose the consumer moves from one basket to another along the same indifference curve

MRS and *MU* Cont'

- The total change in utility is

$$\Delta U = MU_x(\Delta x) + MU_y(\Delta y)$$

- The total change in utility must be 0

$$0 = MU_x(\Delta x) + MU_y(\Delta y)$$

- Thus

$$\frac{MU_x}{MU_y} = -\frac{\Delta y}{\Delta x} = MRS_{x,y}$$

  - The rate at which the consumer is willing to substitute between the two goods holding utility constant is equal to the ratio of the marginal utilities of the two goods

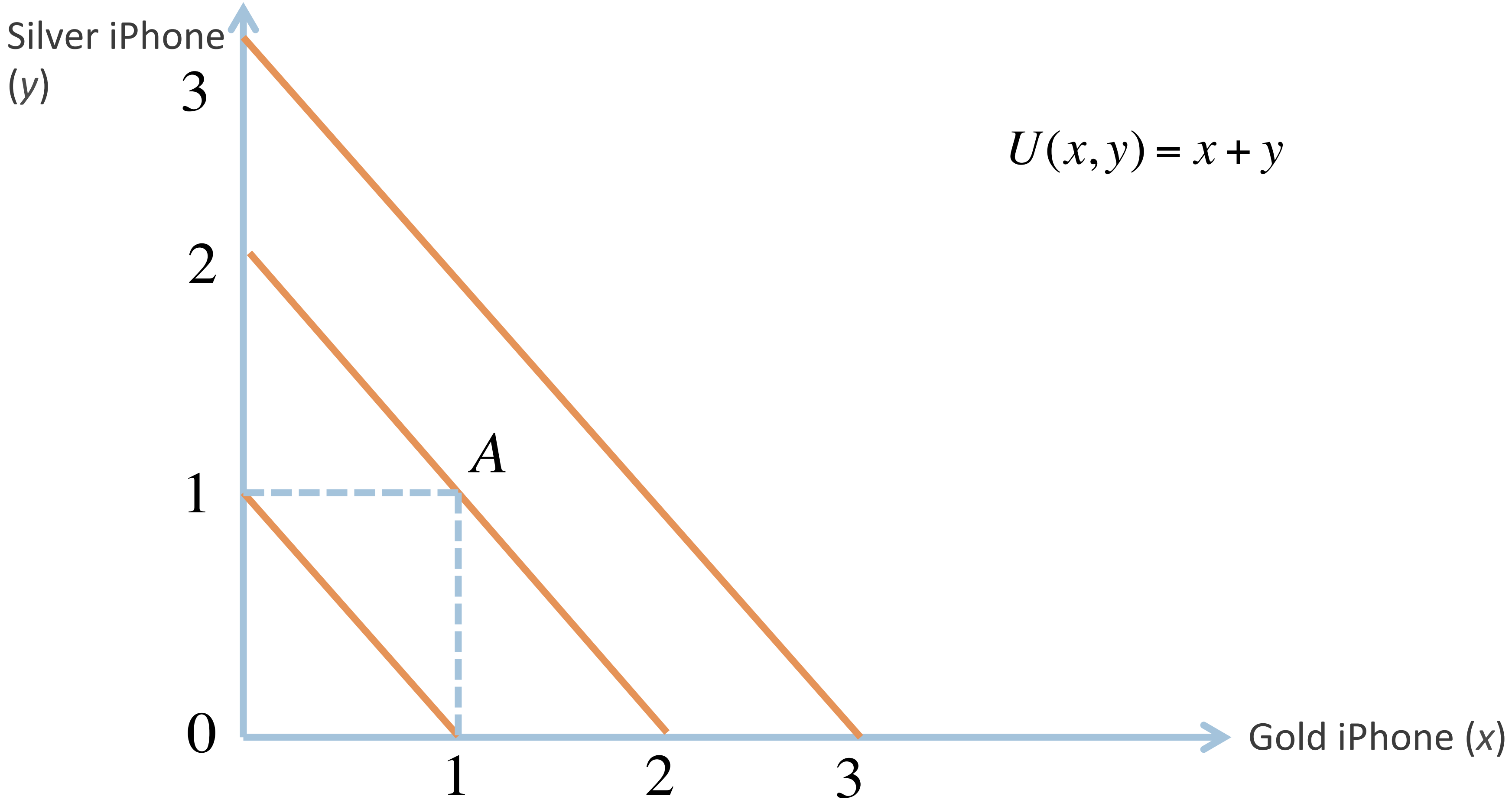
Special Preferences

- Definition 2.4 Perfect substitutes
  - Two goods are *perfect substitutes* if *MRS* is constant
  - Indifference curves are linear
- Definition 2.5 Perfect complements
  - Two goods are *perfect complements* if *MRS* is 0 or infinity
  - Indifference curves are L-shaped

Example of Perfect Substitutes

- Suppose the consumer has the following preference
  - "Gold iPhones and silver iPhones are equivalent"
  - 1 gold iPhone always brings the same utility as 1 silver iPhone
- To the consumer, gold iPhone and silver iPhone are perfect substitutes

Example of Perfect Substitutes: iPhones



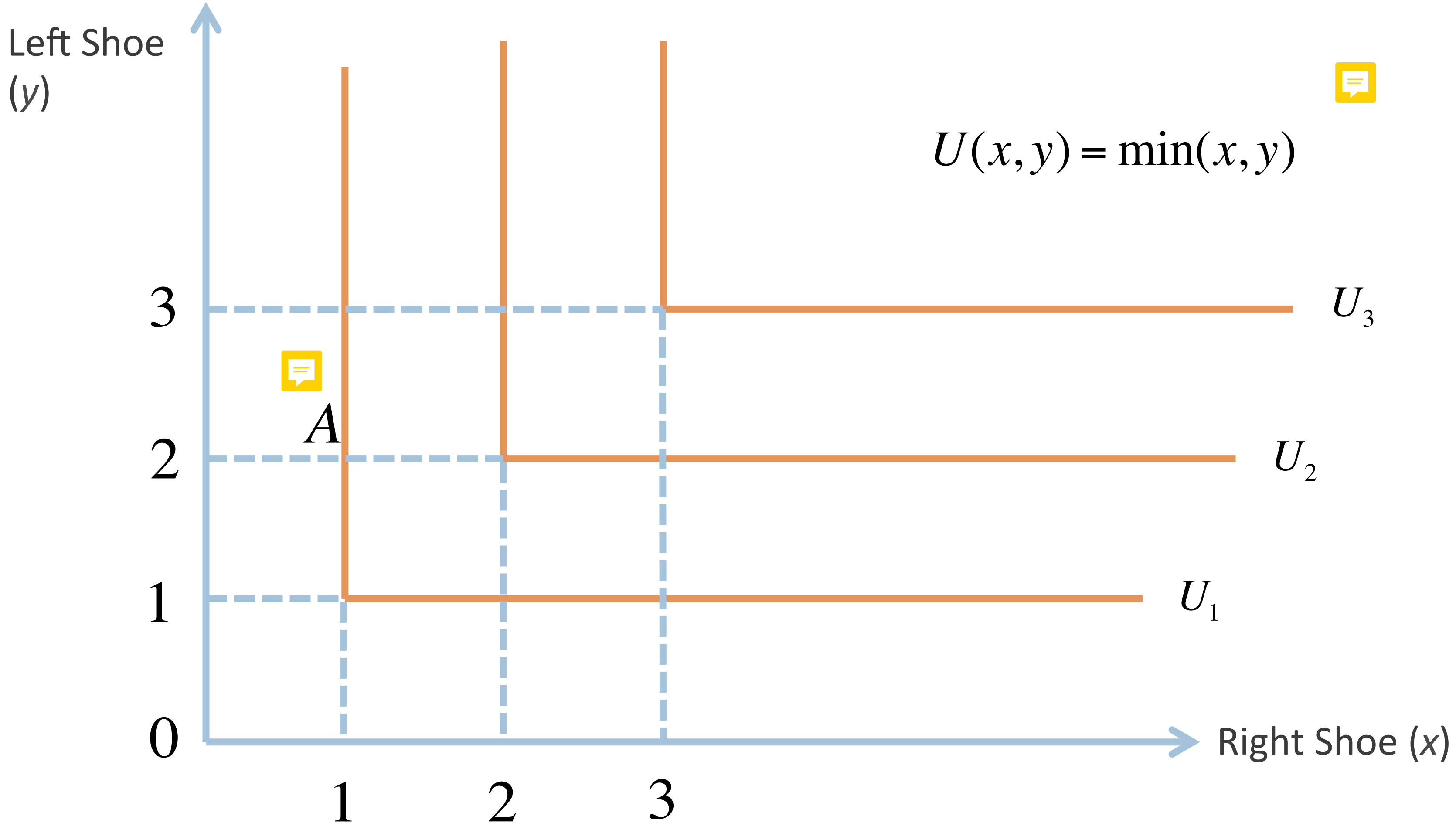
Example of Perfect Substitutes: iPhones Cont'

- We know from the utility function for iPhones
  - Marginal utility of gold iPhone: $MU_x = 1$
  - Marginal utility of silver iPhone: $MU_y = 1$
  - $MRS_{x,y}=1/1=1$
    - Verify from graph, slope of indifference curve is -1
- For perfect substitutes
  - Utility functions are linear

Example of Perfect Complements

- Suppose the consumer has the following preference
  - "For every right shoe, I need exactly one left shoe"
  - 2 right shoes and 1 left shoe brings the same utility as 1 right shoe and 1 left shoe
- To the consumer, right shoes and left shoes are perfect complements

Example of Perfect Complements: Shoes



Example of Perfect Complements: Shoes Cont’

- How does the “min” function work?
  - min (x,y) = the smaller of x and y
  - E.g. if x=1 and y=2, $U(x,y)$ = min (1,2)=1
  - E.g. if x=1 and y=1, $U(x,y)$ = min (1,1)=1
- $MRS_{x,y}$ is
  - Infinity in the vertical part
  - 0 in the horizontal part
- For perfect complements
  - Utility functions are the “min” functions

Part 2

Budget Constraint

Budget Constraint

- Suppose consumer chooses *F* units of food and *C* units of clothing
- The price of food is P_F
- The price of clothing is P_C
- Consumer has limited income *I*
- *Budget constraint* is

$$P_F F + P_C C \leq I$$

Budget Set vs. Budget Line

- Budget set
  - The set of all baskets the consumer can afford, that is, all baskets that satisfy the budget constraint
- Budget line
  - The set of all baskets consumer can afford by spending all income

$$P_F F + P_C C = I$$

  - Rearranging the budget line

$$C = \frac{I}{P_C} - \frac{P_F}{P_C} F$$

Example of Budget Constraint

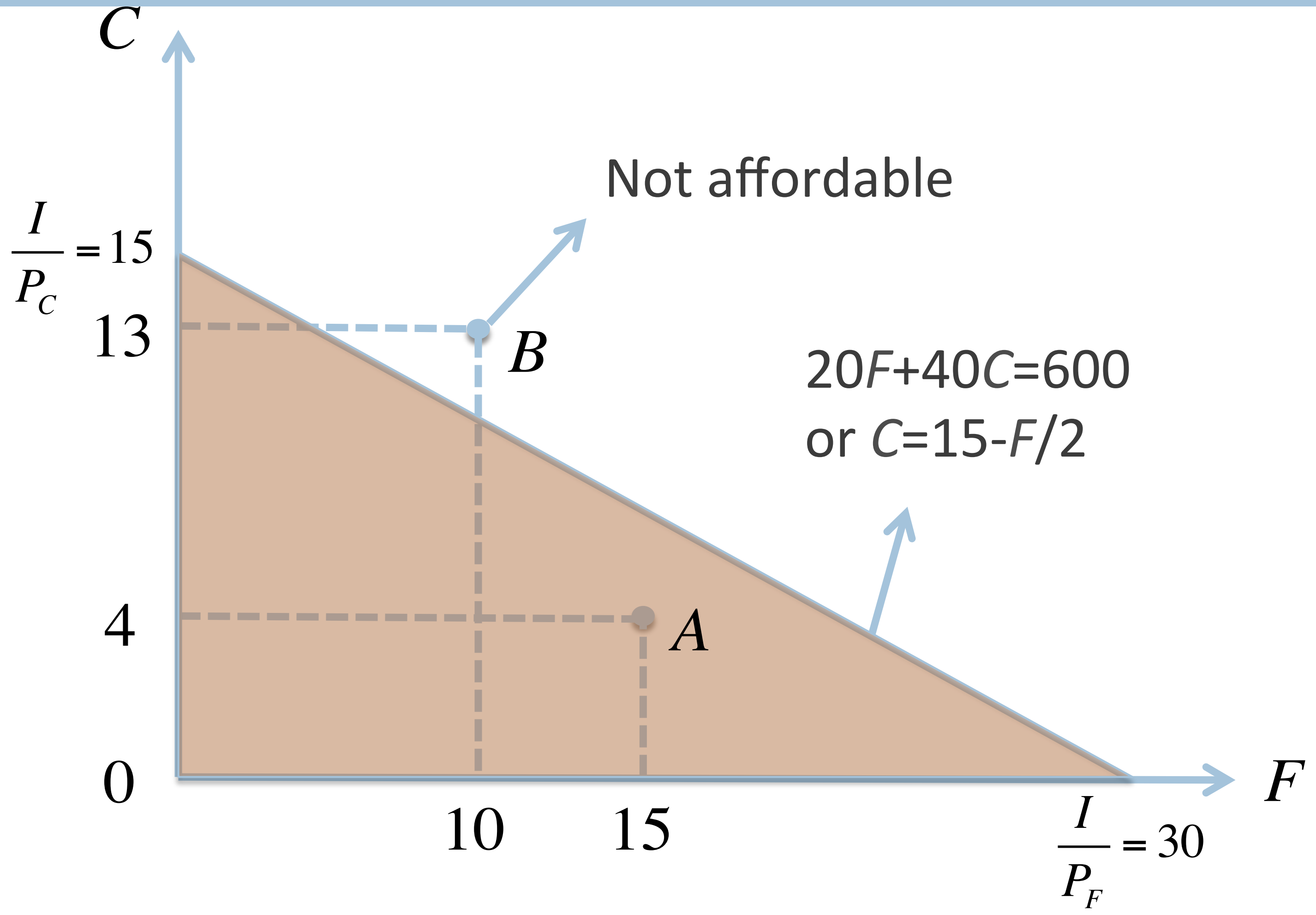
- Suppose
  - The price of food is 20
  - The price of clothing is 40
  - Consumer’s income is 600
- Budget constraint (and budget set) is

$$20F + 40C \leq 600$$

- Budget line is

$$20F + 40C = 600$$

Budget Constraint in Graph



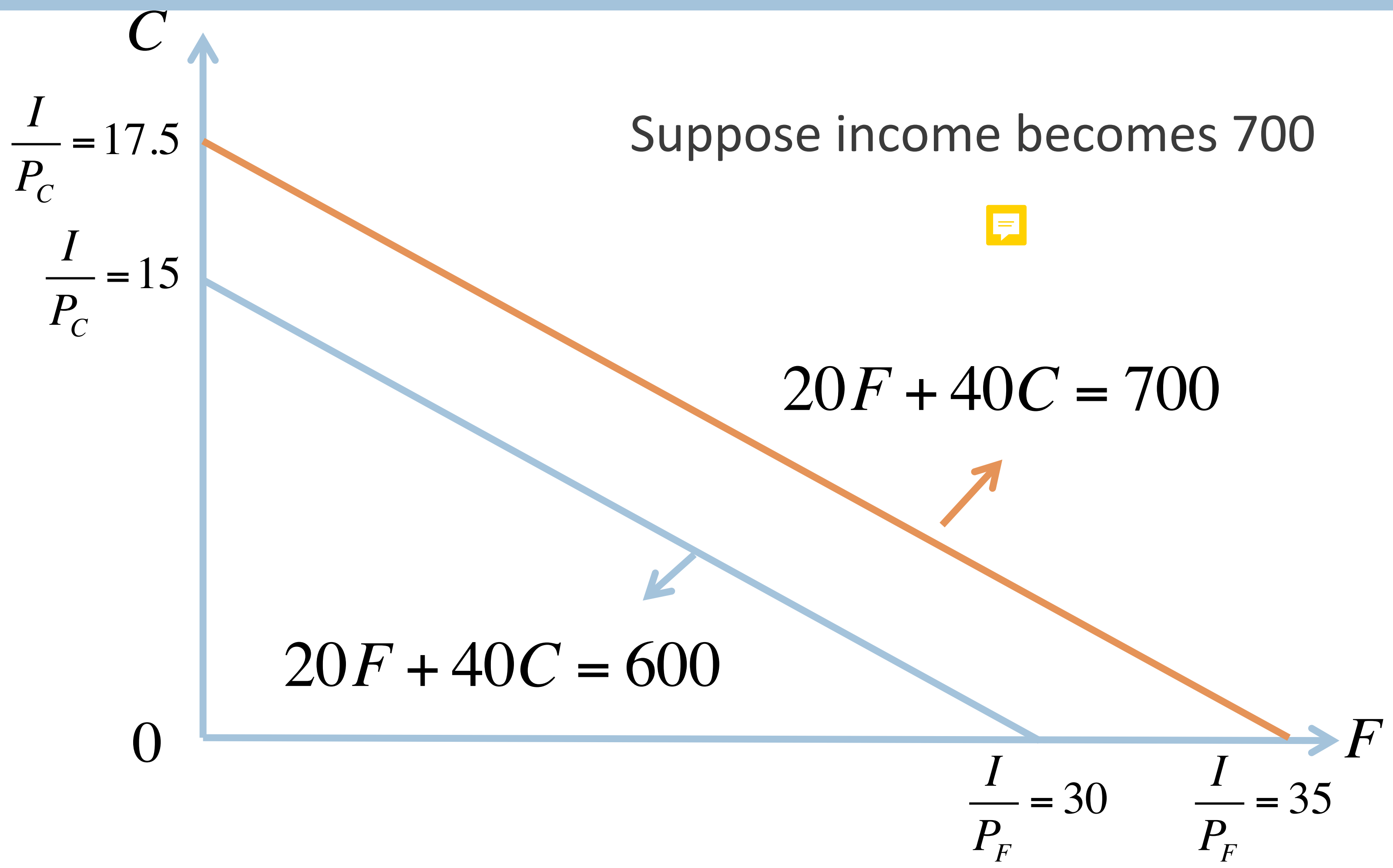
Slope of Budget Line

- The budget line has a slope of

$$-\frac{P_F}{P_C} = -\frac{20}{40} = -\frac{1}{2}$$

- Slope of budget line represents the rate at which two goods can be substituted in the market, that is, based on the prices
  - Because clothing is twice as expensive as food
  - To get 1 additional unit of food, the consumer *must* give up 0.5 unit of clothing

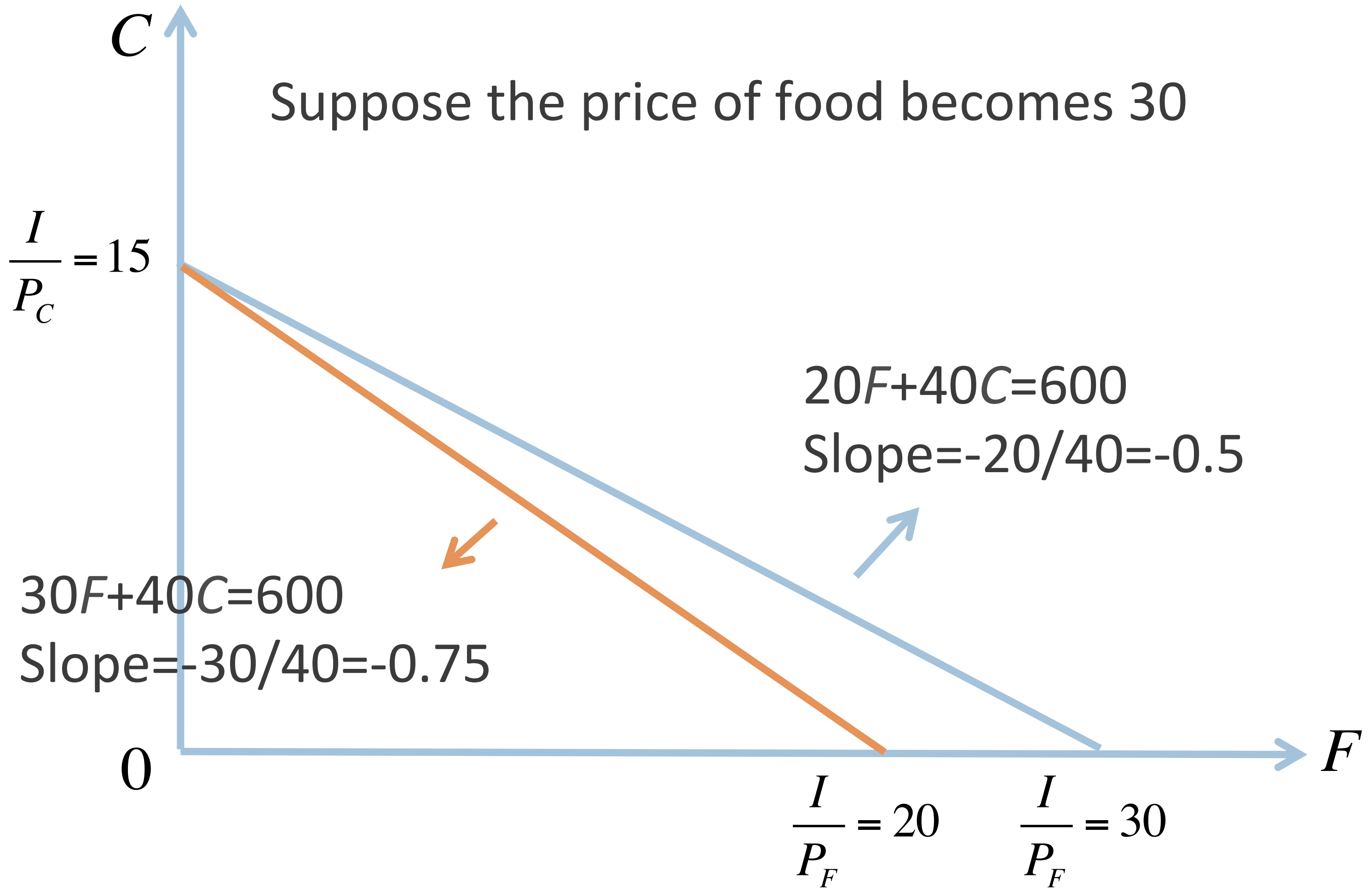
What if income increases?



Suppose income becomes 700

What if food becomes more expensive?

Suppose the price of food becomes 30



Part 3

Consumer Choice

Optimal Choice

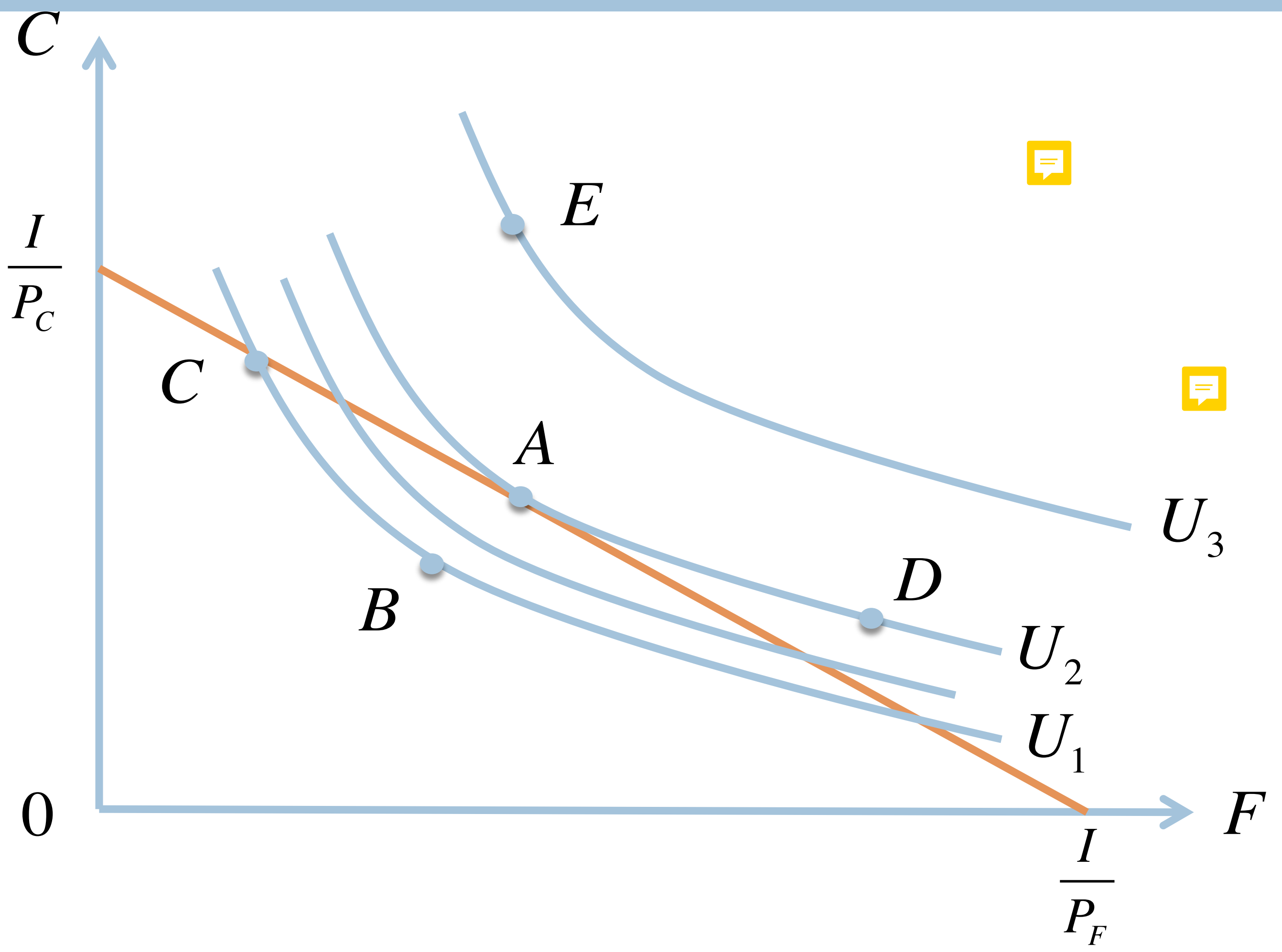
- What is the optimal basket?
  - Consumer chooses the basket that gives him/her the highest utility given the budget constraint
- Let $U(F,C)$ be the utility function of F units of food and C units of clothing, the consumer solves

$$\max_{F,C} \quad U(F,C)$$

$$s.t. \quad P_F F + P_C C \leq I$$

  - F and C are the choice variables
  - P_F, P_C, and I are the parameters

Finding the Optimal Basket from the Graph



Two Conditions for Optimal Choice

- Consumer chooses the basket
  - On the budget line
  - On the highest indifference curve
- At the optimal basket
  - Consumer spends all the money

$$P_F F + P_C C = I$$

  - Budget line tangent to indifference curve

$$-\frac{P_F}{P_C} = -MRS_{F,C} \Rightarrow \frac{P_F}{P_C} = MRS_{F,C}$$

The Tangency Condition

$$MRS_{F,C} = \frac{P_F}{P_C}$$

- To maximize utility, the amount of *F* and *C* consumed should be such that
  - The rate at which the consumer is willing to substitute between the two goods holding utility constant is equal to the rate at which the two goods are exchanged in the market
  - Otherwise the consumer is not maximizing utility

The Equal Marginal Principle

- Since

$$MRS_{F,C} = \frac{MU_F}{MU_C}$$

- The tangency condition can be rewritten as

$$\frac{MU_F}{MU_C} = \frac{P_F}{P_C} \Rightarrow \frac{MU_F}{P_F} = \frac{MU_C}{P_C}$$

- To maximize utility, consumer sets marginal utility per dollar of expenditure equal for both goods
  - Extra utility per dollar spent on food is the same as the extra utility per dollar spent on clothing

What if *MU* per dollar are not the same?

- The equal marginal principle requires

$$\frac{MU_F}{P_F} = \frac{MU_C}{P_C}$$

- Suppose

$$\frac{MU_F}{P_F} < \frac{MU_C}{P_C}$$

- Assume the current basket is on the budget line
- To maximize utility, the consumer should buy
  - \$1 spent on *C* brings higher extra utility than \$1 spent on *F*

Why is basket C not optimal?

- At point C on slide 36

$$MRS_{F,C} > \frac{P_F}{P_C} \Rightarrow \frac{MU_F}{MU_C} > \frac{P_F}{P_C} \Rightarrow \frac{MU_F}{P_F} > \frac{MU_C}{P_C}$$

- Per dollar marginal utility of food higher than per dollar marginal utility of clothing
- Consumer’s utility will increase if more money spent on food
- Consumer should buy more food

Example: Finding the Optimal Basket

- Suppose the consumer's utility function is

$$U(F,C) = FC$$

- Suppose the price of food is 20, price of clothing is 40 and the consumer's income is 600
- The marginal utility is

$$MU_F = C, \quad MU_C = F$$

- The tangency condition requires

$$\frac{P_F}{P_C} = MRS_{F,C} = \frac{MU_F}{MU_C}$$

Example: Finding the Optimal Basket Cont'

- Thus

$$\frac{20}{40} = \frac{C}{F}$$

- Simplifying

$$F = 2C \quad (1)$$

- The optimal basket must also lie on the budget line

$$20F + 40C = 600 \quad (2)$$

- Solving (1) and (2) together we have *F*=15, *C*=7.5

Example: Finding the Optimal Basket Using Langrange Multiplier Method

- This is a constrained maximization problem

$$\max_{F,C} \quad FC$$

$$s.t. \quad 20F + 40C \leq 600$$

- To maximize utility, consumer spends all the money

$$\max_{F,C} \quad FC$$

$$s.t. \quad 20F + 40C = 600$$

Example: Finding the Optimal Basket Using Langrange Multiplier Method Cont'

- Rewriting the budget constraint

$$\max_{F,C} \quad FC$$

$$s.t. \quad 600 - 20F - 40C = 0$$

- The Lagrangian function is

$$\Lambda(F, C, \lambda) = FC + \lambda(600 - 20F - 40C)$$

Example: Finding the Optimal Basket Using Langrange Multiplier Method Cont'

- The first-order conditions are

$$\frac{\partial \Lambda}{\partial F} = C - 20\lambda = 0$$

$$\frac{\partial \Lambda}{\partial C} = F - 40\lambda = 0$$

$$\frac{\partial \Lambda}{\partial \lambda} = 600 - 20F - 40C = 0$$

- Solving the three equations, we get

$$F = 15, \quad C = 7.5, \quad \lambda = 0.375$$

What is the meaning of the multiplier?

- Recall the general form of the constrained maximization problem

$$\max_{x,y} \quad f(x,y)$$

$$s.t. \quad g(x,y) = 0$$

- In consumer theory, it is

$$\max_{x,y} \quad U(x,y)$$

$$s.t. \quad I - P_x x - P_y y = 0$$

The Multiplier is the Extra Utility of One Extra Dollar of Consumption

- The Lagrangian function is

$$\Lambda(x,y,\lambda) = U(x,y) + \lambda(I - P_x x - P_y y)$$

- The first-order conditions w.r.t. *x* and *y* are

$$\frac{\partial \Lambda}{\partial x} = MU_x - \lambda P_x = 0$$

$$\frac{\partial \Lambda}{\partial y} = MU_y - \lambda P_y = 0$$

- Rearranging the equations we have

$$\lambda = \frac{MU_x}{P_x} = \frac{MU_y}{P_y}$$

Price and Willingness to Pay

- Rearranging the equation

$$\lambda = \frac{MU_x}{P_x} = \frac{MU_y}{P_y}$$

- We have

$$P_x = \frac{MU_x}{\lambda}, \quad P_y = \frac{MU_y}{\lambda}$$

- Price of a good is equal to the extra utility from consuming one more unit of the good divided by the extra utility of one dollar spending
- Price represents consumer’s willingness to pay for one more unit of the good

The Sign of the Multiplier

- If you write down the Lagrangian function as

$$\Lambda(x,y,\lambda) = U(x,y) + \lambda(P_x x + P_y y - I)$$

- The first-order conditions w.r.t. *x* and *y* are
- The multiplier now becomes