LECTURE 9
COST IN THE LONG RUN
SHORT-RUN COST VS. LONG-RUN COST

Where are we?

- Production function
  - How firms turn L and K into Q
- Optimal choice of L and K in the short run
  - Cost curves in the short run
- Optimal choice of L and K in the long run
  - To produce a certain amount of output Q_0, how much L and K should the firm use?
  - How much does it cost to produce Q_0?
  - Cost curve: cost as a function of Q
- Short-run cost vs. long-run cost

Part 1

Long-Run Cost Minimizing Input Choice

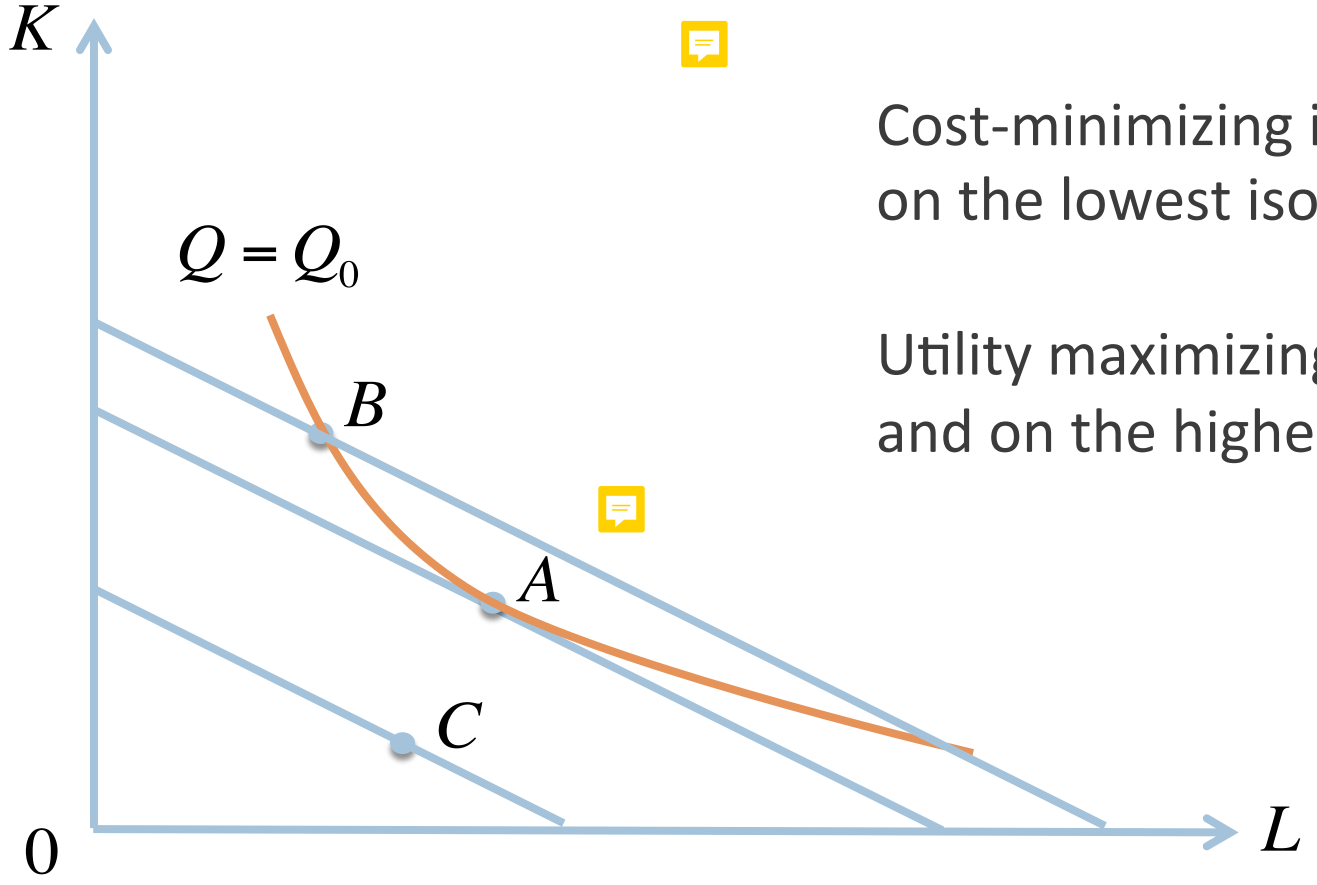
How much labor and capital should the firm use?

- Recall
  - price of labor is *w* per unit
  - price of capital is *r* per unit
  - In the long run, both *L* and *K* are variable
- Assume the firm maximizes profit
- For any output level Q_0, the firm chooses *L* and *K* to *minimize* the total cost of production

$$\min_{L,K} \quad wL + rK$$

$$s.t. \quad F(L,K) = Q_0$$

Which combination is cost-minimizing?



Cost-minimizing input: on the isoquant and on the lowest isocost

Utility maximizing basket: on the budget line and on the highest indifference curve

Cost-Minimizing Input Choice

- The cost minimizing input combination
  - must be on the isoquant
  - must be on the lowest isocost
- On the isoquant

$$F(L,K) = Q_0$$

- Tangency condition

$$MRTS_{L,K} = \frac{w}{r}$$

- Equivalently

$$MRTS_{L,K} = \frac{MP_L}{MP_K} = \frac{w}{r} \Rightarrow \frac{MP_L}{w} = \frac{MP_K}{r}$$

Example: Solving for the Cost-Minimizing Choice of Inputs

- Suppose the production function is

$$Q = KL$$

- Input prices are w=1 and r=2
- What is the cost-minimizing choice of inputs if the firm wants to produce 8 units?
- To minimize cost, the firm chooses K and L such that

$$\frac{K}{L} = \frac{1}{2}$$

Example: Solving for the Cost-Minimizing Choice of Inputs Cont’

- The firm must produce 8 units of output

$$KL = 8$$

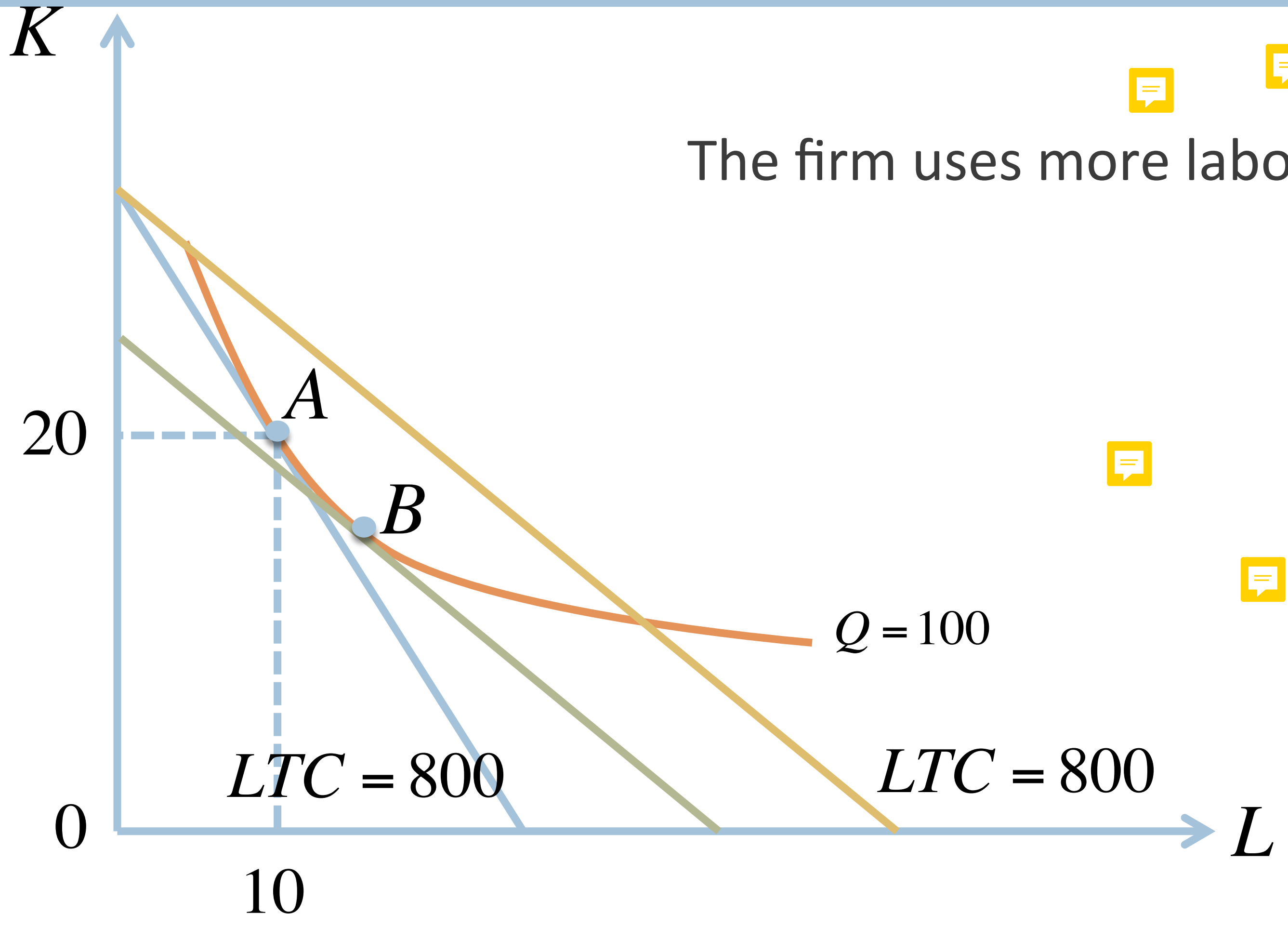
- Solving the two equations we get

$$L = 4, \quad K = 2$$

Comparative Statics: Changes in Input Prices and Output Level

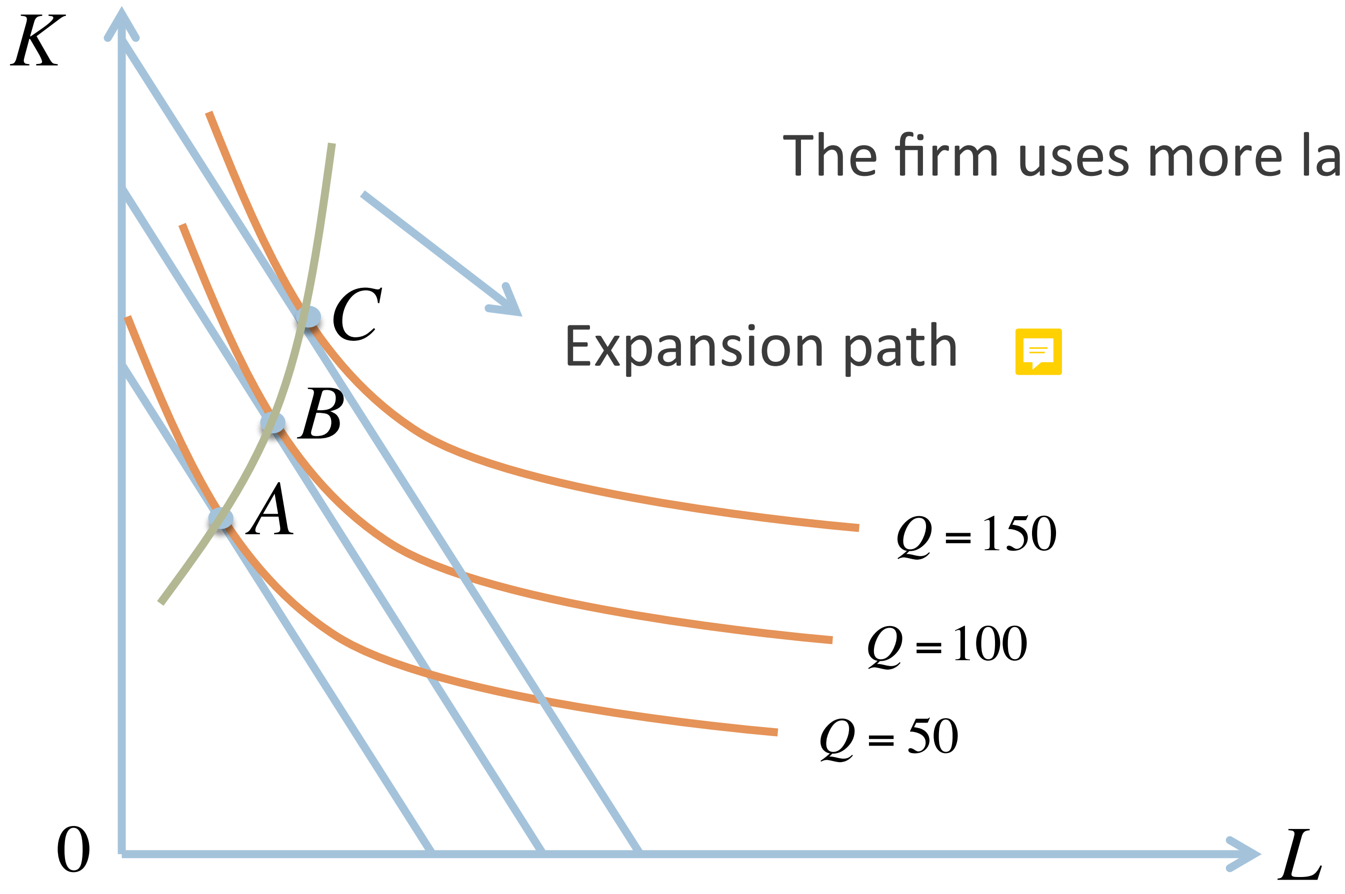
- When input prices change
  - How does the cost-minimizing choice of *L* and *K* change?
- When output level changes
  - How does the cost-minimizing choice of *L* and *K* change?
- The above analysis is called *comparative statics*

Suppose price of labor drops



The firm uses more labor and less capital

Suppose Q increases



The firm uses more labor and more capital

Expansion path

Normal vs. Inferior Input

- Definition 9.1 *Normal input*
  - The cost-minimizing quantity of the input increases when output increases
  - Holding input prices fixed
- Definition 9.2 *Inferior input*
  - The cost-minimizing quantity of the input decreases when output increases
  - Holding input prices fixed

Input Demand Function

- As the input prices or the output level change, firm's cost-minimizing choice of labor and capital may also change
- Definition 9.3 The *demand function of an input* is the cost-minimizing choice of input as a function of w, r, and Q
  - Demand function of labor
  - Demand function of capital

Example: Deriving Input Demand Functions

- Suppose the production function is

$$Q = KL$$

- Input prices are *w* and *r*
- To minimize cost, the firm chooses *K* and *L* such that

$$\frac{K}{L} = \frac{w}{r}$$

- This gives us

$$K = \frac{w}{r}L, \quad L = \frac{r}{w}K$$

Example: Deriving Input Demand Functions Cont'

- Substituting

$$Q = KL = (\frac{wL}{r})L = \frac{w}{r}L^2$$

- The demand function of labor is

$$L(w,r,Q) = \sqrt{\frac{rQ}{w}}$$

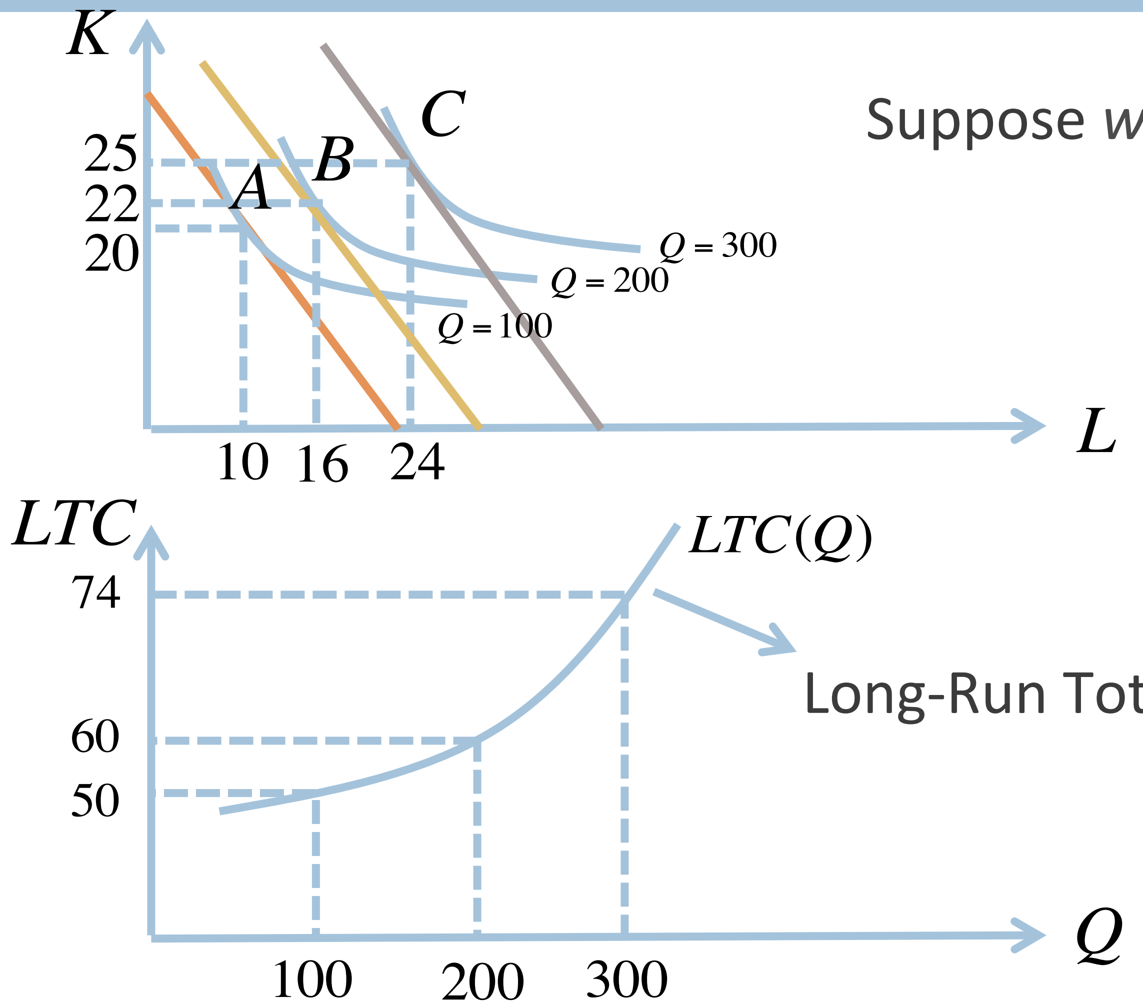
- The demand function of capital is

$$K(w,r,Q) = \sqrt{\frac{wQ}{r}}$$

Part 2

Long-Run Cost Curves

Long-Run Total Cost Curve in Graph



Suppose w=1 and r=2

Long-Run Total Cost Curve

Long-run Total Cost Curve/Function

- Definition 9.4 *Long-run total cost curve* is total cost in the long run as a function of *Q*
  - Holding *w* and *r* constant
- Every point on the long-run total cost curve represents the firm's *minimized* total cost for a given level of output, holding input prices fixed
- No fixed cost in the long run
  - *LTC*=0 when *Q*=0
- Definition 9.5 *Long-run total cost function* is total cost in the long run as a function of *Q*, *w*, and *r*

Example: Deriving Long-Run Total Cost Function

- Suppose the production function is

$$Q = KL$$

- Input prices are *w* and *r*
- We have already derived the cost-minimizing choice of labor and capital

$$L(w,r,Q) = \sqrt{\frac{rQ}{w}}$$

$$K(w,r,Q) = \sqrt{\frac{wQ}{r}}$$

Example: Deriving Long-Run Total Cost Function Cont'

- The long-run total cost function is

$$LTC(Q,w,r) = wL + rK = w\sqrt{\frac{rQ}{w}} + r\sqrt{\frac{wQ}{r}}$$

- Simplifying, we get

$$LTC(Q,w,r) = 2\sqrt{wrQ}$$

Average Cost and Marginal Cost

- Definition 9.6 *Long-run average cost* (*LAC*)
  - Total cost per unit of output

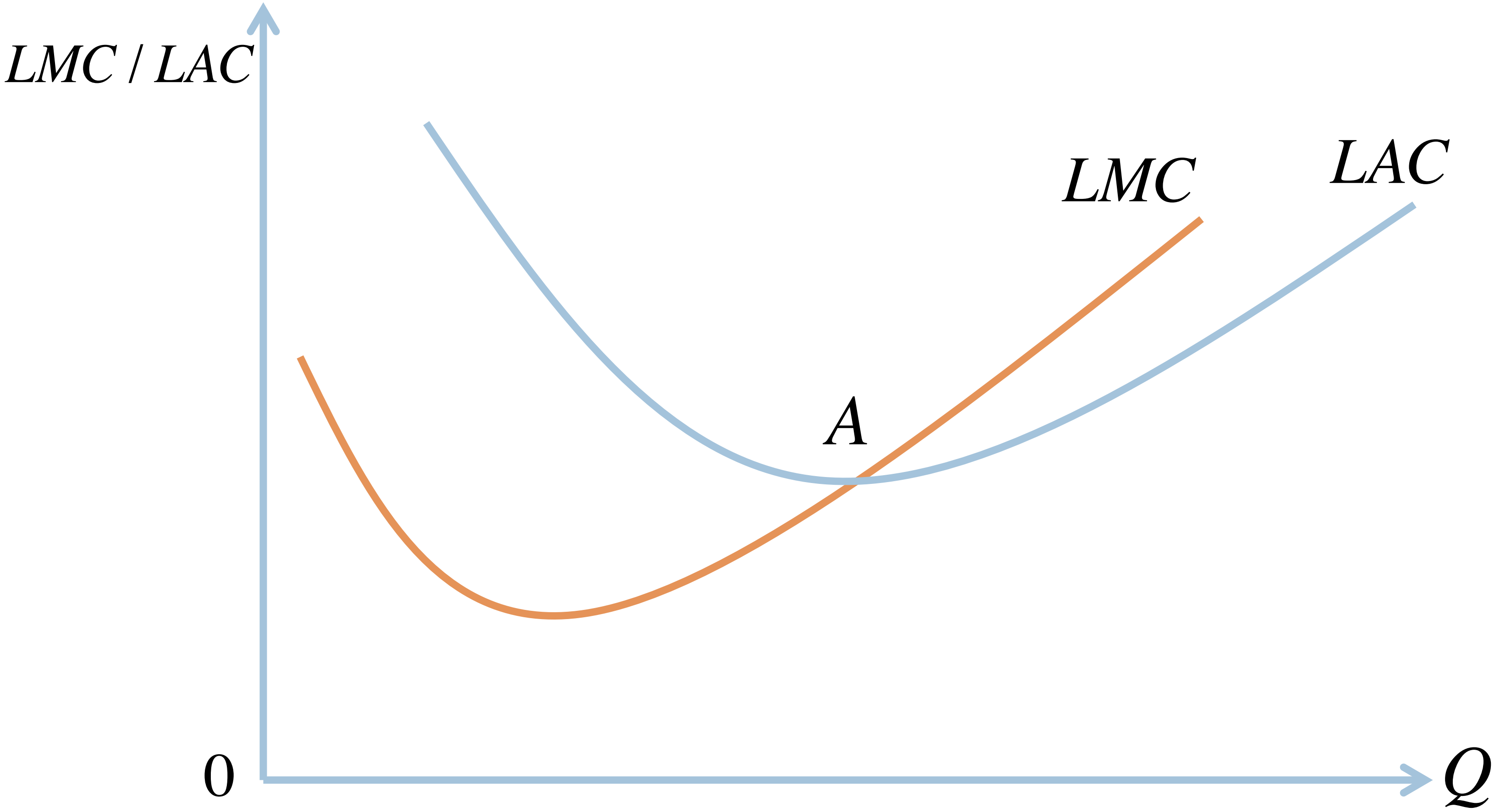
$$LAC(Q) = \frac{LTC(Q)}{Q}$$

- Definition 9.7 *Long-run marginal cost* (*LMC*)

$$LMC(Q) = \frac{dLTC(Q)}{dQ} = \frac{\Delta LTC(Q)}{\Delta Q}$$

where ΔQ is extremely small

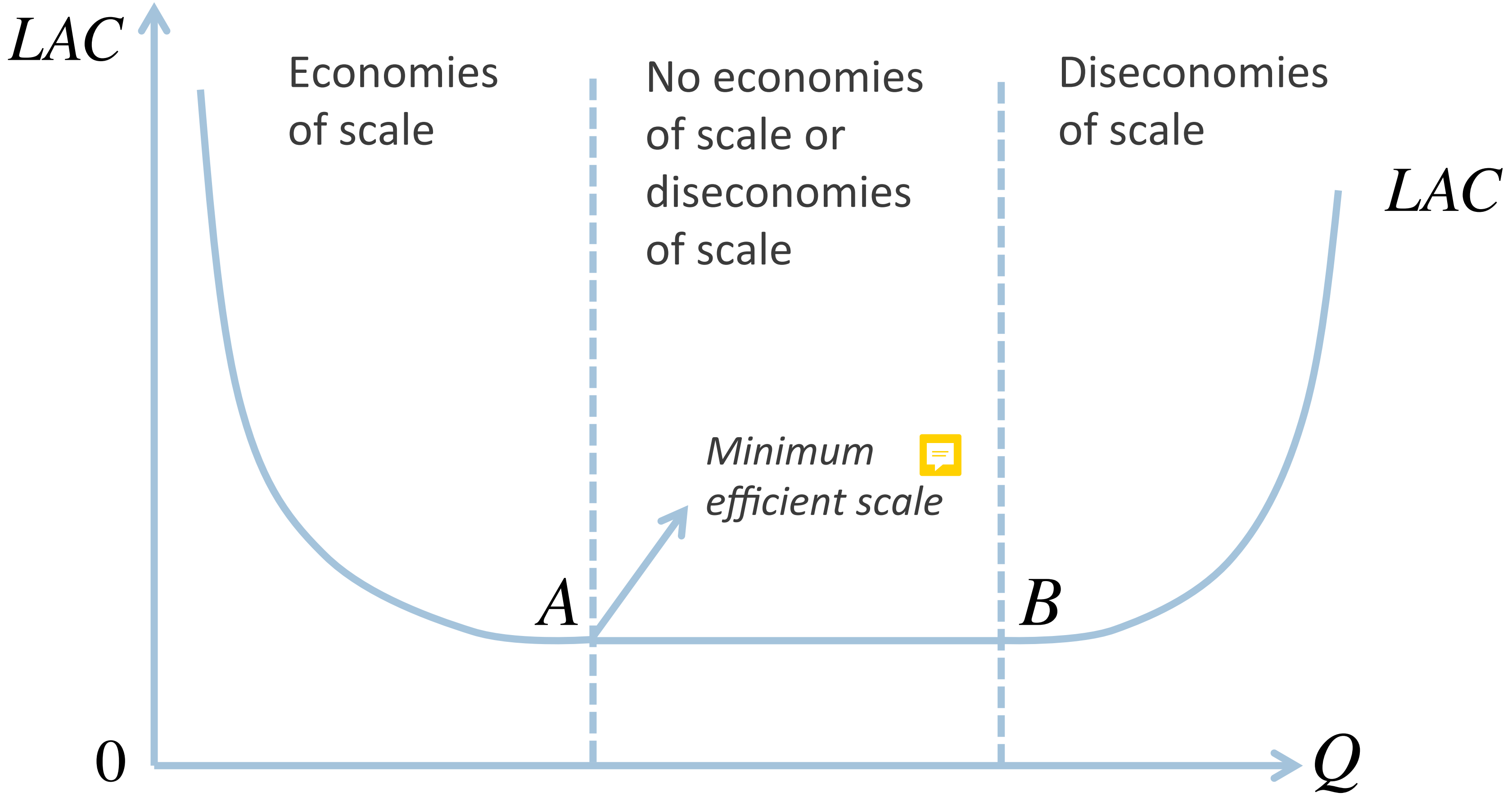
Relationship between *LMC* and *LAC*



Economies of Scale

- Definition 9.8 *Economies of scale*
  - If *LAC* is decreasing in *Q*
- Definition 9.9 *Diseconomies of scale*
  - If *LAC* is increasing in *Q*

Economies of Scale in Graph



Source of Economies of Scale

- Indivisible input
  - The size of some input cannot be scaled down
  - The cost of the input gets spread out as quantity of output increases
- Returns to specialization
  - More workers can lead to better specialization
  - Specialization improves productivity
  - Example
    - When L=2, K=1, Q=2, suppose w=r=1, LTC(2)=3, LAC(2)=1.5
    - When L=3, K=1, Q=4 because of better specialization of labor
    - LTC(4)=4, LAC(4)=1

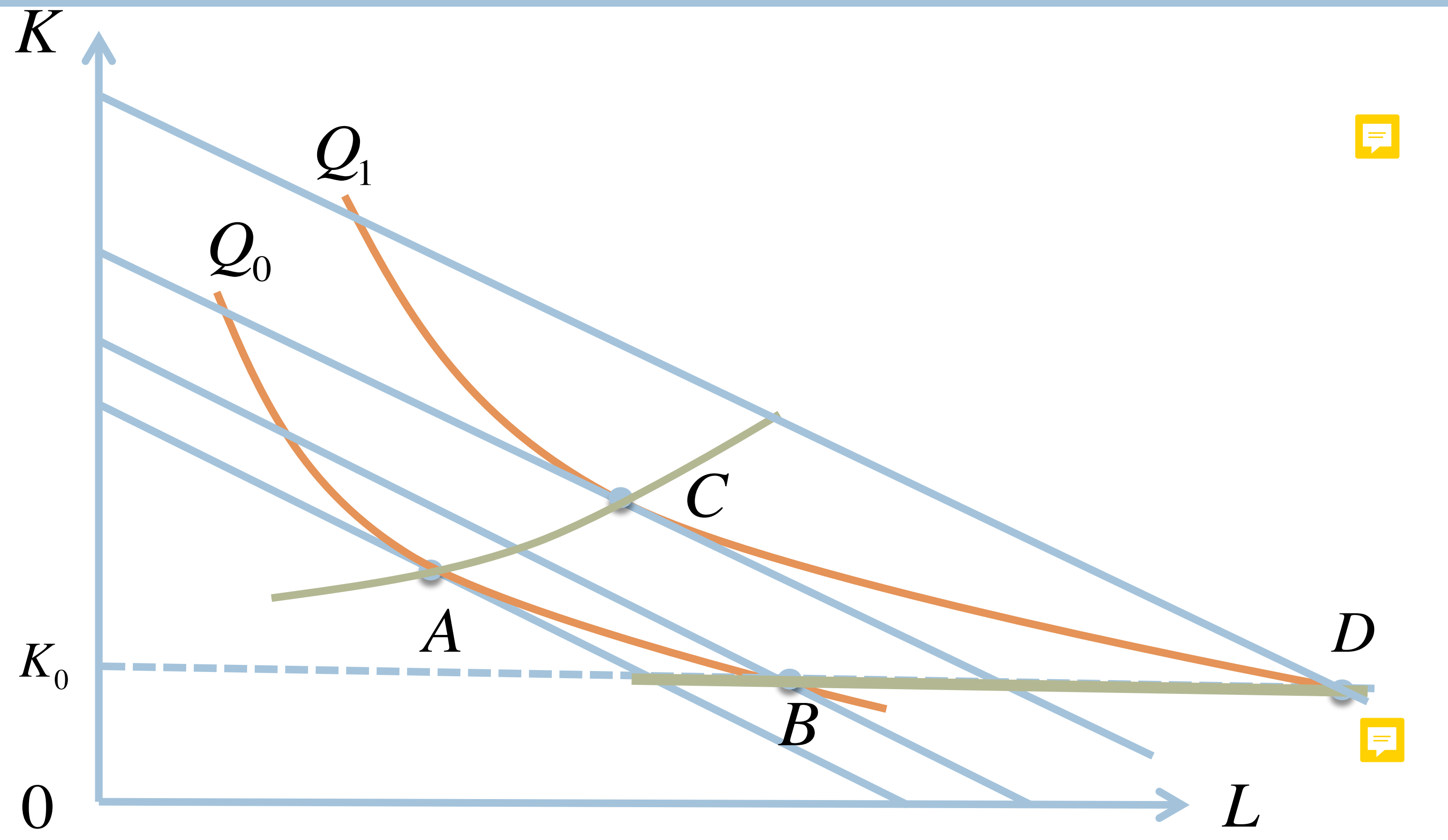
Source of Diseconomies of Scale

- Managerial diseconomies of scale
  - An *a*% increase in *Q* requires a more than *a*% increase in the firm's spending on managers

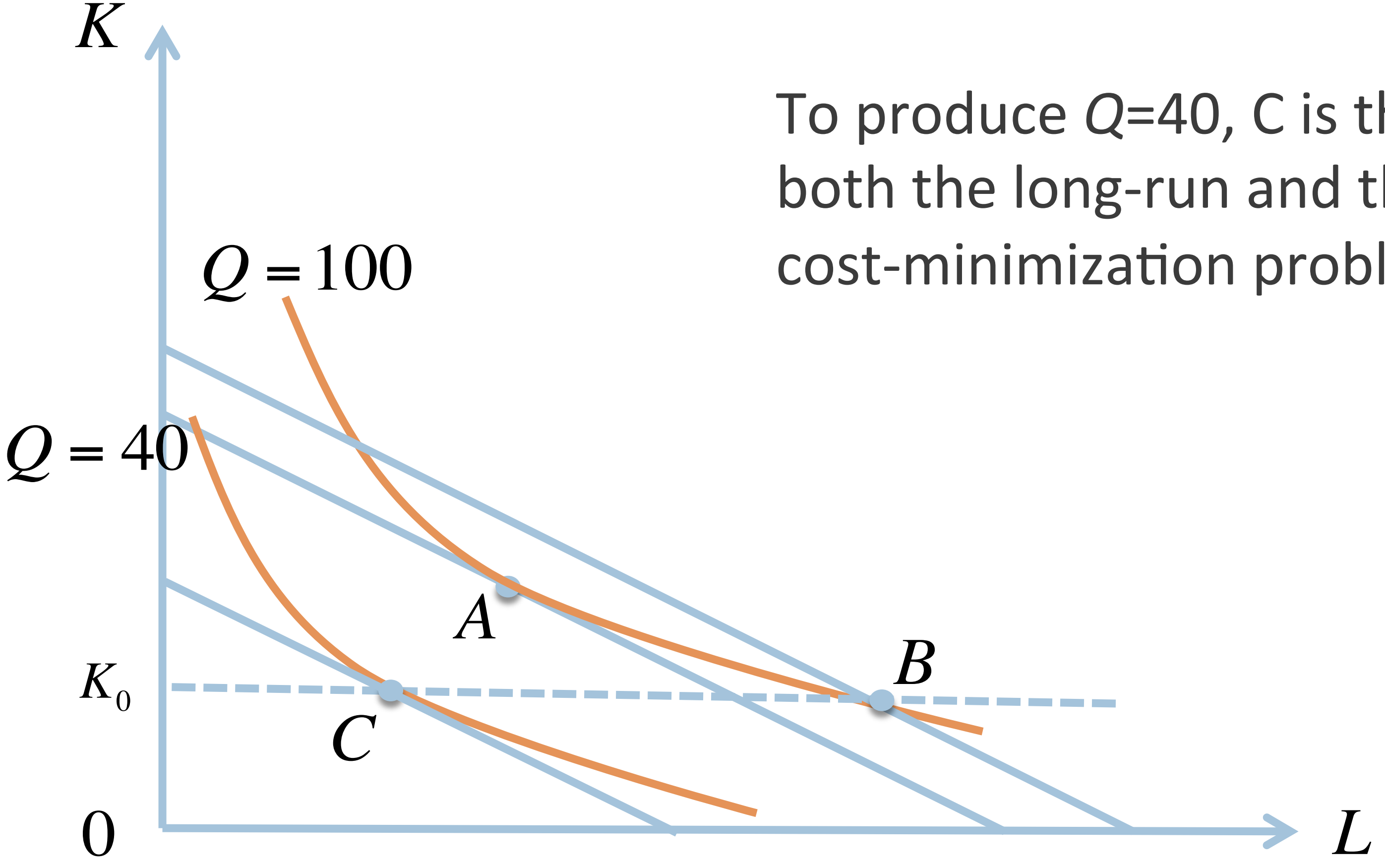
Part 3

Short-Run Cost Vs. Long-Run Cost

Short-Run Expansion Path

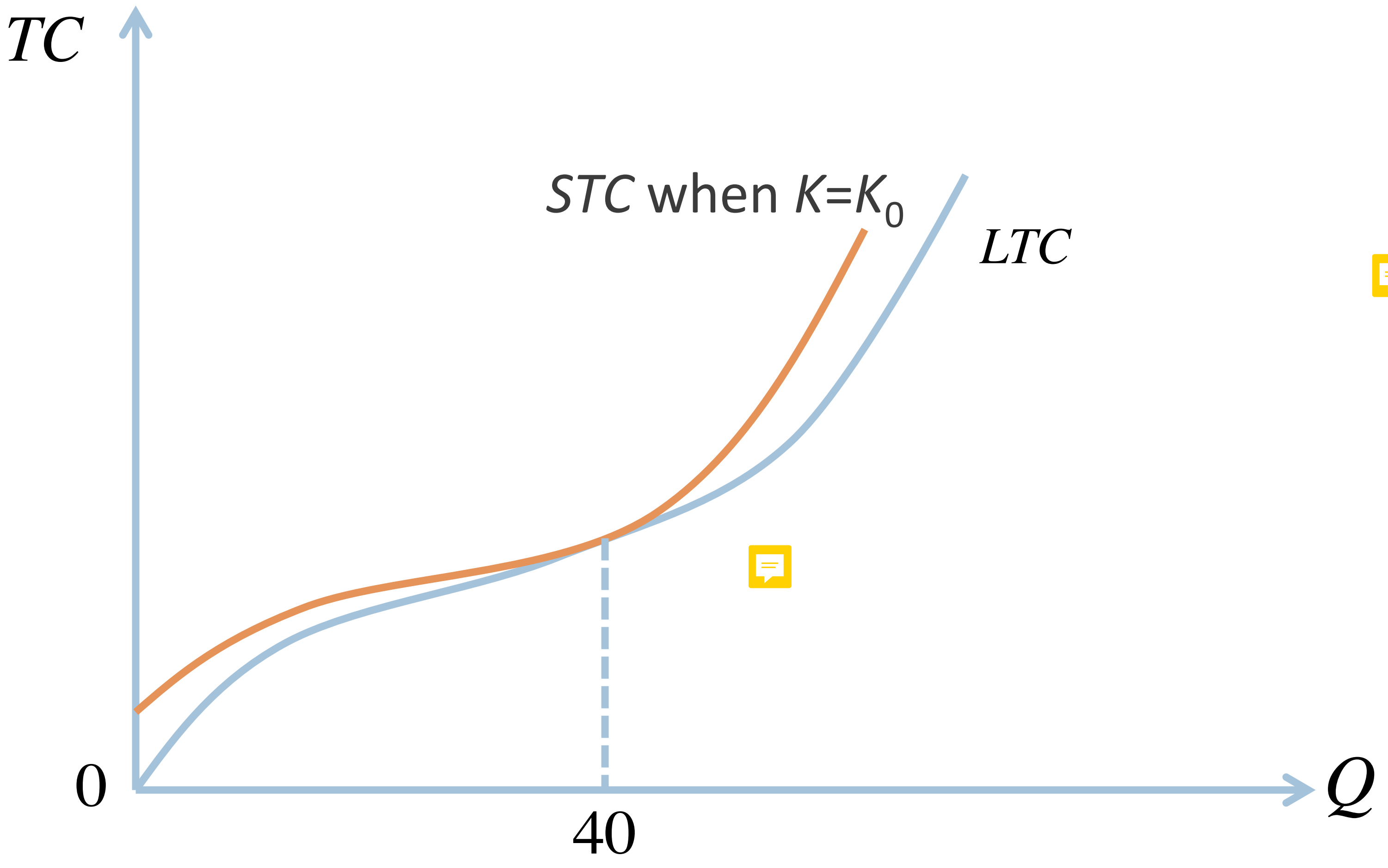


Is *STC*=*LTC* possible?

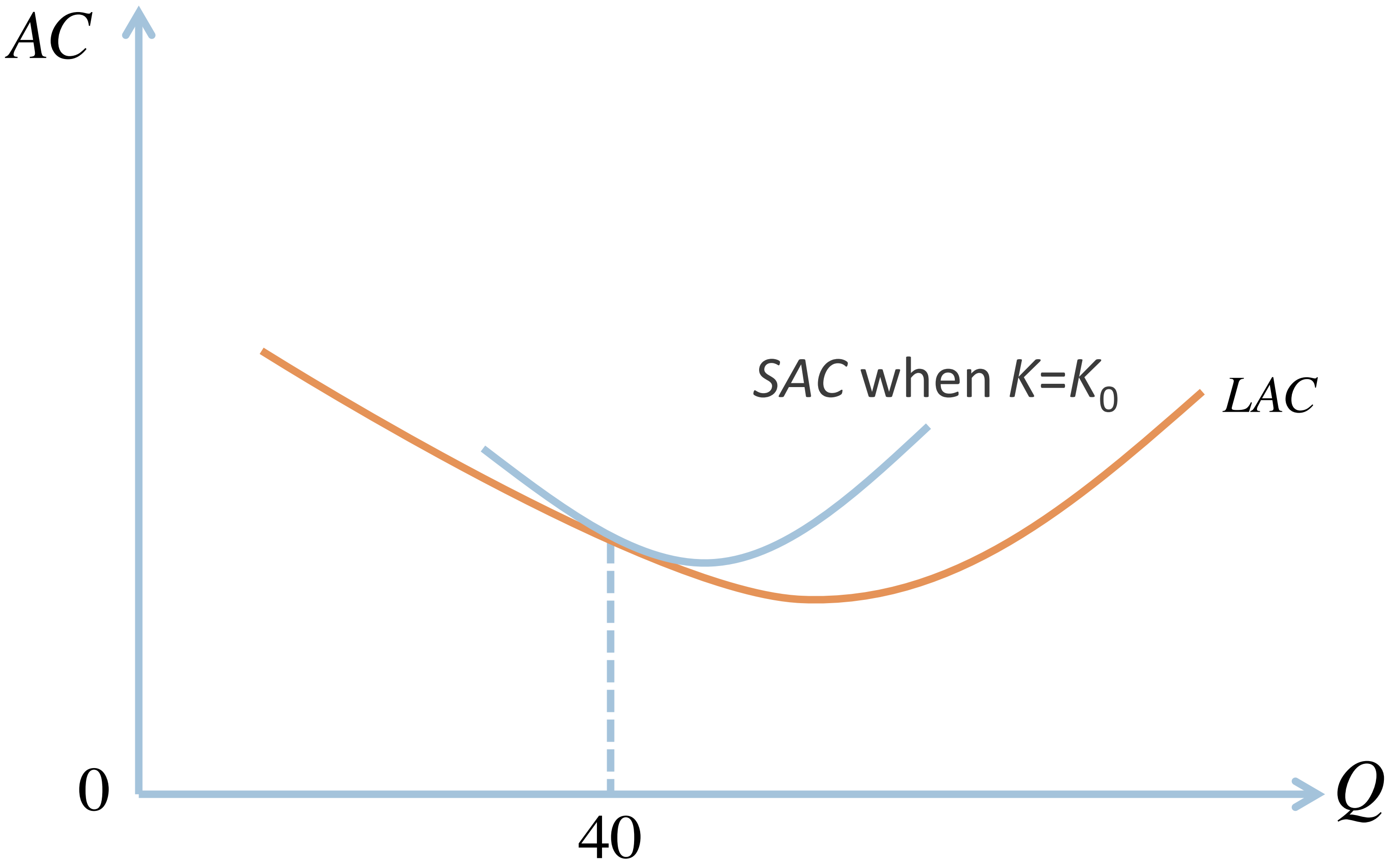


To produce *Q*=40, C is the solution to both the long-run and the short-run cost-minimization problem

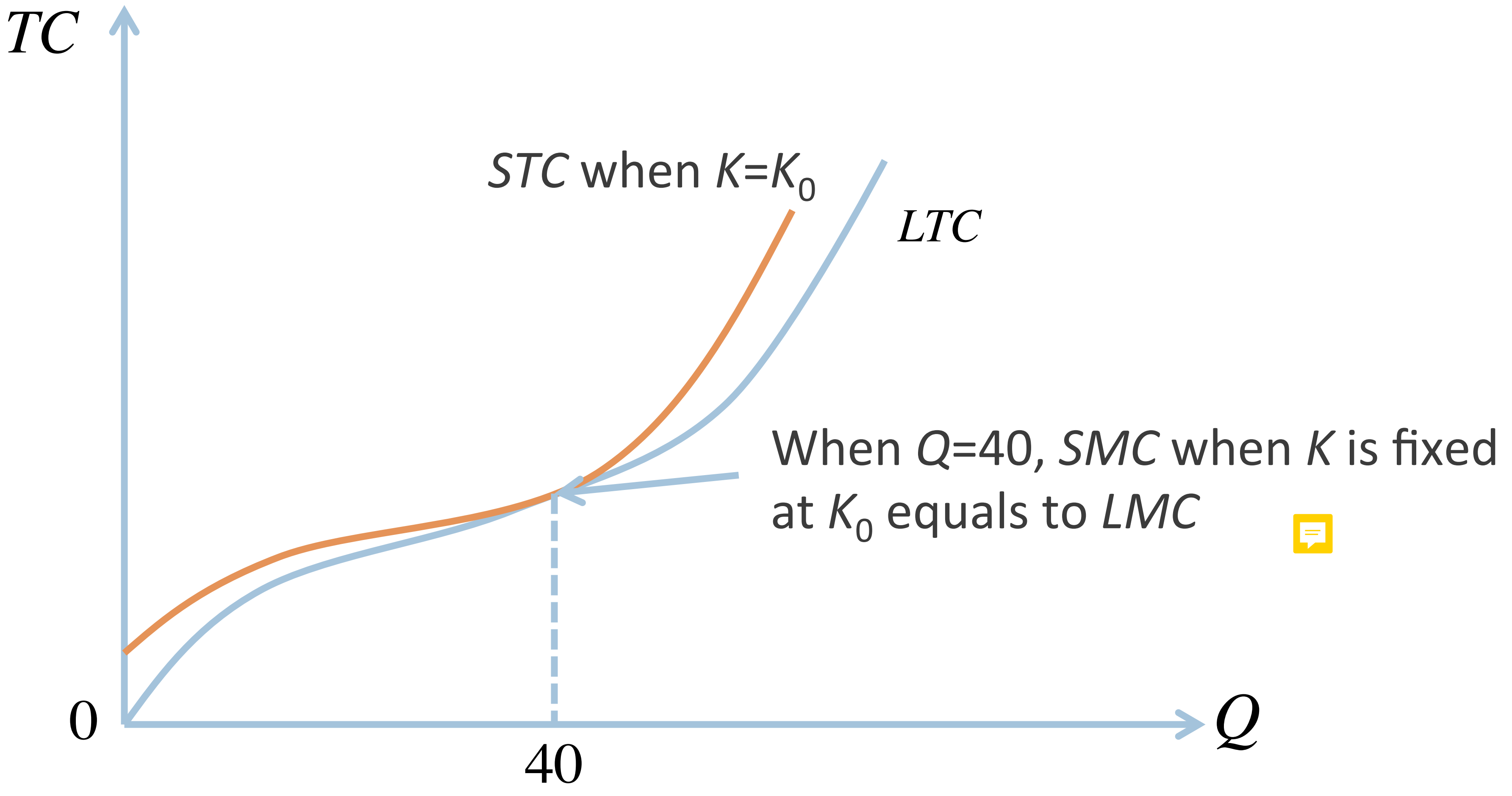
STC cannot be lower than *LTC*



SAC cannot be lower than *LAC*



How about marginal cost?



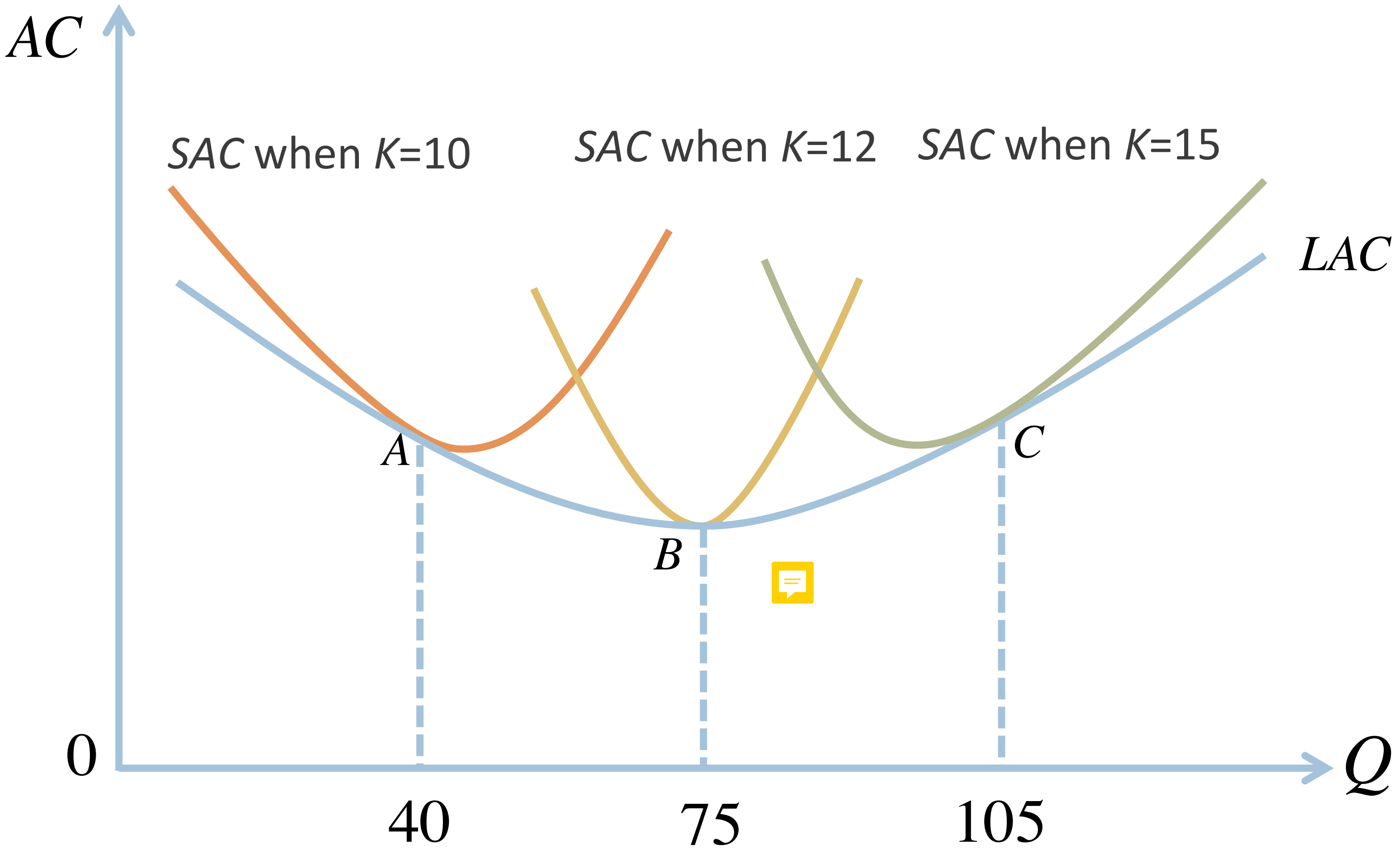
When does Long-Run Cost=Short-Run Cost?

- Suppose in the short run capital is fixed at K_0
- Suppose when the firm produces Q_0, K_0 is the cost-minimizing capital choice in the long run
- When $Q=Q_0$
  - The choice of inputs in the long-run and in the short-run are the same
  - $STC=LTC$
  - $SAC=LAC$
  - $SMC=LMC$

Long-run Average Cost Curve vs. Short-run Average Cost Curves

- Suppose if the firm produces 40 units
  - Its optimal choice of capital in the long run is 10
- Suppose if the firm produces 75 units
  - Its optimal choice of capital in the long run is 12
- Suppose if the firm produces 105 units
  - Its optimal choice of capital in the long run is 15

LAC is the lower envelope of *SAC*



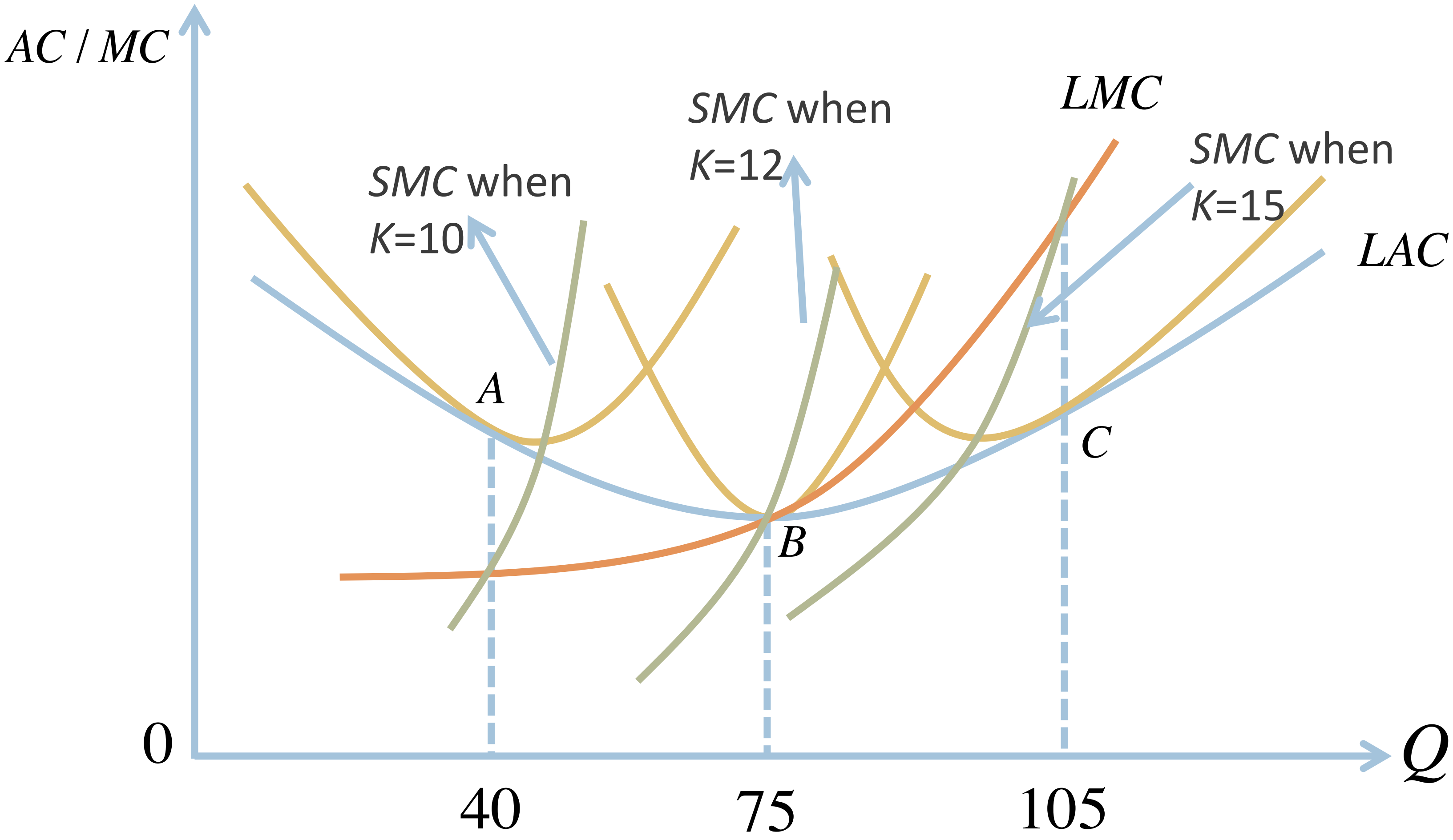
When *LAC* is at its minimum

- When the firm produces 75 units its *LAC* is the lowest across all possible output levels
- At this output level, the *SAC* when *K*=12 must also reach its minimum
  - When *LAC* is at its minimum, its slope is 0
  - At the point where the *SAC* is tangent to *LAC*, they have the same slope
  - The slope of the *SAC* at the point where it is tangent to *LAC* is also 0
  - Thus *SAC* is at its minimum

When *LAC* is not at its minimum

- *SAC* is not tangent to *LAC* at *SAC*'s minimum point
  - When *LAC* is not at its minimum, it is either decreasing or increasing, i.e., its slope is either negative or positive
  - At the point where *SAC* is tangent to *LAC*, they have the same slope
  - The slope of the *SAC* at the point where it is tangent to *LAC* is also either negative or positive
  - Thus *SAC* is not at its minimum

LMC vs. *SMC*


AC / MC
SMC when
K=12
LMC
SMC when
K=10
SMC when
K=15
LAC
A
C
B
0
Q
40
75
105

The Minimum Point of *LAC*