Intelligent Systems - Assignment 1

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Corresponding GitHub: https://github.com/joeljanson19/intelligent_systems.git

In [455…
```
import numpy as np
from sklearn import datasets
from sklearn.preprocessing import StandardScaler
from sklearn.model_selection import train_test_split
from sklearn.metrics import mean_squared_error,accuracy_score,classification_report
import skfuzzy as fuzz
import matplotlib.pyplot as plt
import torch
import torch.nn as nn
import torch.optim as optim
import pandas
```

We load the two different datasets. For dataset 2 (classification), the target y contains either "tested_positive" or "tested_negative". Because we want numerical values, we instead convert these into 1 or 0 respectively.

In [456…
```
# CHOOSE DATASET

# 1. Regression
diabetes = datasets.load_diabetes(as_frame=True)
X = diabetes.data.values
y = diabetes.target.values

# 2. Classification
# diabetes = datasets.fetch_openml(name="diabetes", version=1, as_frame=True)
# X = diabetes.data.values
# y = diabetes.target
# y = y.replace({'tested_positive': 1, 'tested_negative': 0}).values
```

In [457…
```
#train test spliting
test_size = 0.2
Xtr, Xte, ytr, yte = train_test_split(X, y, test_size=test_size, random_state=42)
```

In [458…
```
# Standardize features
scaler=StandardScaler()
Xtr= scaler.fit_transform(Xtr)
Xte= scaler.transform(Xte)
```

We want to include the target (y) in clustering.

In pure unsupervised clustering, the target is not included. But in Takagi–Sugeno–Kang (TSK) fuzzy systems, clustering is applied not only on the input features X but also on the target y. This enables clusters to align with the labels and fuzzy rules that map X to y (input-output relationship).

In order to optimize the model, we can change the number of clusters and m. (A change in m basically corresponds to the membership function become "wider" or "narrower")

In [459…
```
 # Number of clusters
n_clusters = 2
m = 2

# Concatenate target for clustering
Xexp=np.concatenate([Xtr, ytr.reshape(-1, 1)], axis=1)

# Transpose data for skfuzzy (expects features x samples)
Xexp_T = Xexp.T

# Fuzzy C-means clustering
centers, u, u0, d, jm, p, fpc = fuzz.cluster.cmeans(
    Xexp_T, n_clusters, m=m, error=0.005, maxiter=1000, init=None,
)
```

In [460…
```
centers.shape
```

Out[460]:
```
(2, 11)
```

In [461…
```
# Compute sigma (spread) for each cluster
sigmas = []
for j in range(n_clusters):
    # membership weights for cluster j, raised to m
    u_j = u[j, :] ** m
    # weighted variance for each feature
    var_j = np.average((Xexp - centers[j])**2, axis=0, weights=u_j)
    sigma_j = np.sqrt(var_j)
```

```
    sigmas.append(sigma_j)
sigmas=np.array(sigmas)
```

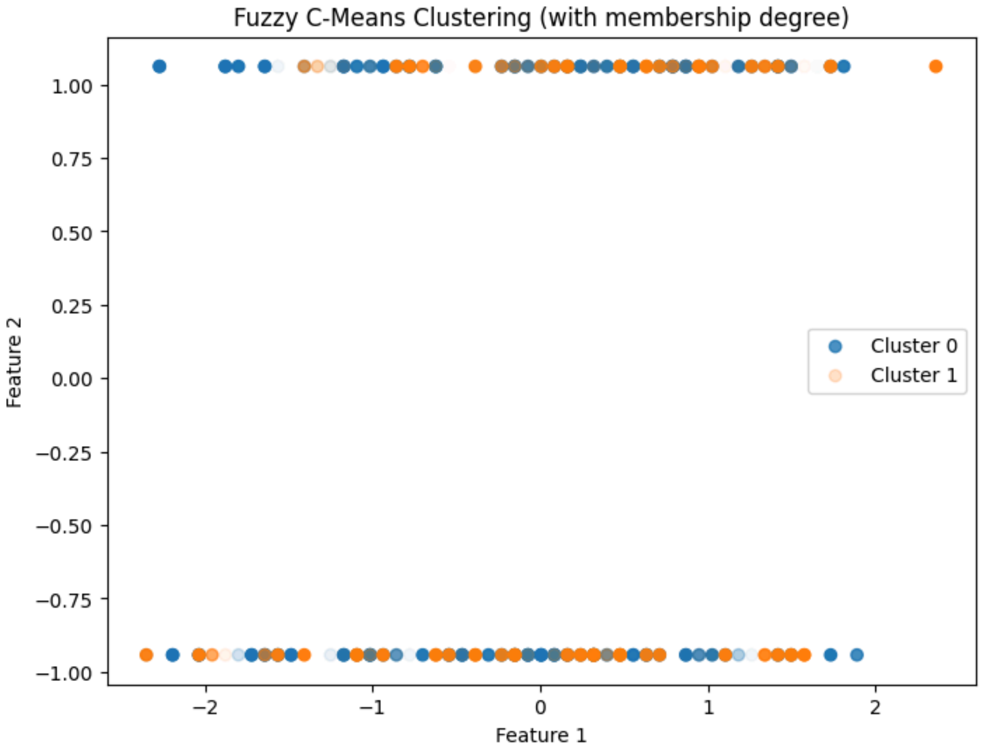
In [462…

```
# Hard clustering from fuzzy membership
cluster_labels = np.argmax(u, axis=0)
print("Fuzzy partition coefficient (FPC):", fpc)

# Plot first two features with fuzzy membership
plt.figure(figsize=(8,6))
for j in range(n_clusters):
    plt.scatter(
        Xexp[cluster_labels == j, 0],             # Feature 1
        Xexp[cluster_labels == j, 1],             # Feature 2
        alpha=u[j, :],          # transparency ~ membership
        label=f'Cluster {j}'
    )

plt.title("Fuzzy C-Means Clustering (with membership degree)")
plt.xlabel("Feature 1")
plt.ylabel("Feature 2")
plt.legend()
plt.show()
```

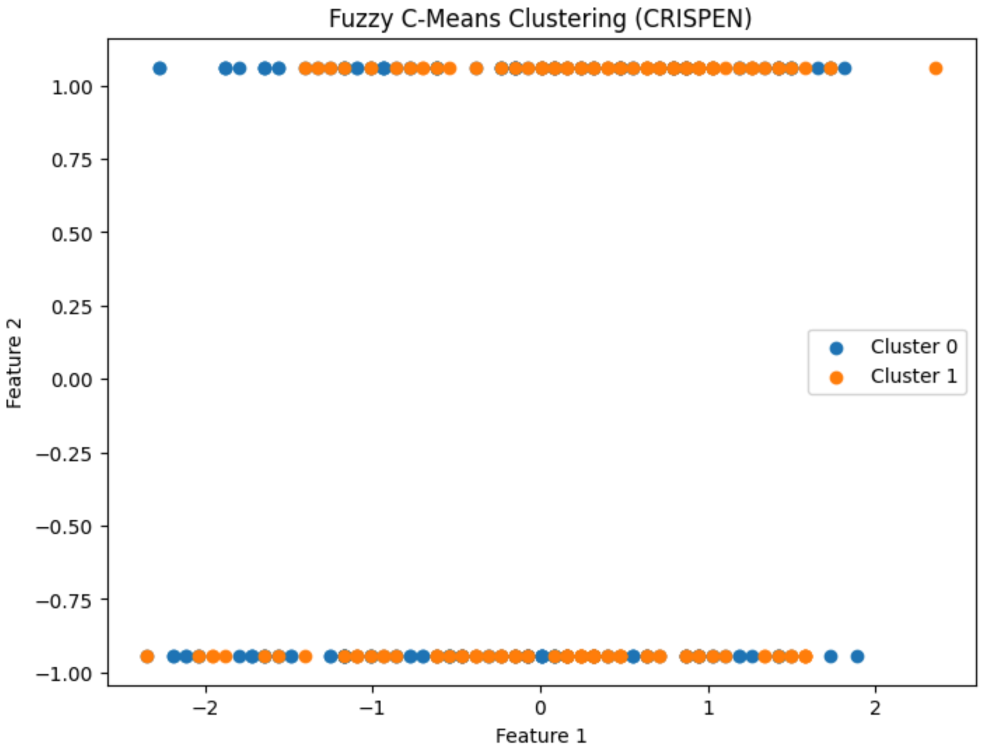
```
Fuzzy partition coefficient (FPC): 0.8556195015117771
```



In [463…

```
# Plot first two features with cluster assignments
plt.figure(figsize=(8,6))
for j in range(n_clusters):
    plt.scatter(
        Xexp[cluster_labels == j, 0],
        Xexp[cluster_labels == j, 1],
        label=f'Cluster {j}'
    )

plt.title("Fuzzy C-Means Clustering (CRISPEN)")
plt.xlabel("Feature 1")
plt.ylabel("Feature 2")
plt.legend()
plt.show()
```



In [464…

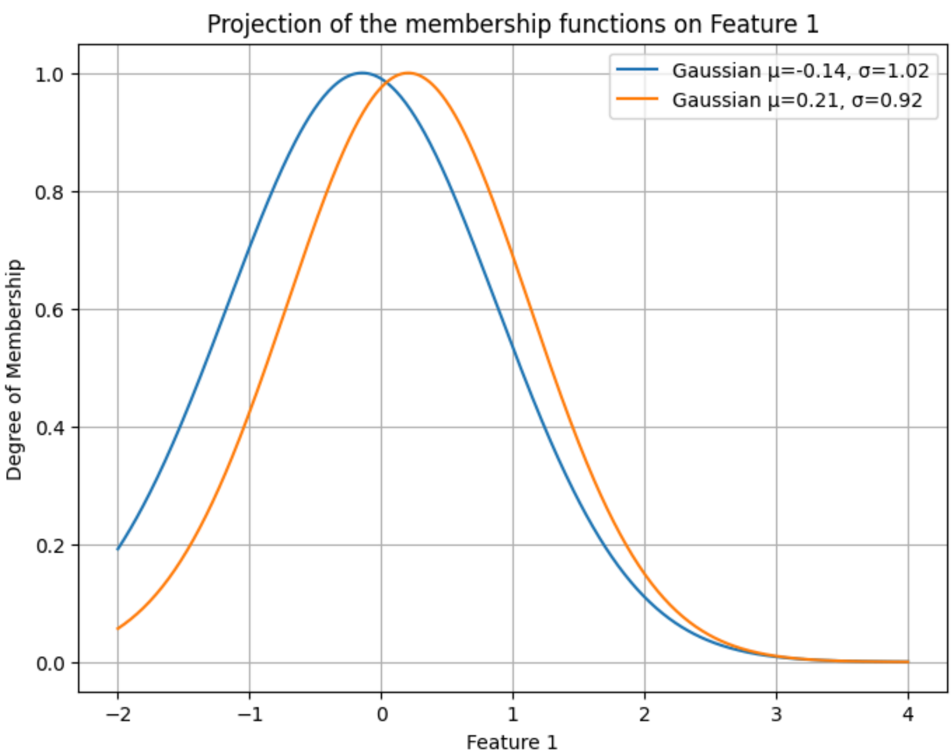
```
# Gaussian formula
def gaussian(x, mu, sigma):
    return np.exp(-0.5 * ((x - mu)/sigma)**2)

lin=np.linspace(-2, 4, 500)
plt.figure(figsize=(8,6))

y_aux=[]
for j in range(n_clusters):
# Compute curves
    y_aux.append(gaussian(lin, centers[j,0], sigmas[j,0]))

# Plot
    plt.plot(lin, y_aux[j], label=f"Gaussian μ={np.round(centers[j,0],2)}, σ={np.round(sigmas[j,0],2)}")

plt.title("Projection of the membership functions on Feature 1")
plt.xlabel("Feature 1")
plt.ylabel("Degree of Membership")
plt.legend()
plt.grid(True)
plt.show()
```



In [465…

```python
# --------------------------
# Gaussian Membership Function
# --------------------------
class GaussianMF(nn.Module):
    def __init__(self, centers, sigmas, agg_prob):
        super().__init__()
        self.centers = nn.Parameter(torch.tensor(centers, dtype=torch.float32))
        self.sigmas = nn.Parameter(torch.tensor(sigmas, dtype=torch.float32))
        self.agg_prob=agg_prob

    def forward(self, x):
        # Expand for broadcasting
        # x: (batch, 1, n_dims), centers: (1, n_rules, n_dims), sigmas: (1, n_rules, n_dims)
        diff = abs((x.unsqueeze(1) - self.centers.unsqueeze(0))/self.sigmas.unsqueeze(0)) #(batch, n_rules, n_dims)

        # Aggregation
        if self.agg_prob:
            dist = torch.norm(diff, dim=-1)  # (batch, n_rules) # probablistic intersection
        else:
            dist = torch.max(diff, dim=-1).values  # (batch, n_rules) # min intersection (min instersection of normal funtion

        return torch.exp(-0.5 * dist ** 2)


# --------------------------
# TSK Model
# --------------------------
class TSK(nn.Module):
    def __init__(self, n_inputs, n_rules, centers, sigmas,agg_prob=False):
        super().__init__()
        self.n_inputs = n_inputs
        self.n_rules = n_rules

        # Antecedents (Gaussian MFs)

        self.mfs=GaussianMF(centers, sigmas,agg_prob)

        # Consequents (linear functions of inputs)
        # Each rule has coeffs for each input + bias
        self.consequents = nn.Parameter(
            torch.randn(n_inputs + 1,n_rules)
        )

    def forward(self, x):
        # x: (batch, n_inputs)
        batch_size = x.shape[0]

        # Compute membership values for each input feature
        # firing_strengths: (batch, n_rules)
        firing_strengths = self.mfs(x)

        # Normalize memberships
        # norm_fs: (batch, n_rules)
        norm_fs = firing_strengths / (firing_strengths.sum(dim=1, keepdim=True) + 1e-9)

        # Consequent output (linear model per rule)
        x_aug = torch.cat([x, torch.ones(batch_size, 1)], dim=1)  # add bias

        rule_outputs = torch.einsum("br,rk->bk", x_aug, self.consequents)  # (batch, rules)
        # Weighted sum
        output = torch.sum(norm_fs * rule_outputs, dim=1, keepdim=True)

        return output, norm_fs, rule_outputs
```

In [466…

```python
# --------------------------
# Least Squares Solver for Consequents (TSK)
# --------------------------
def train_ls(model, X, y):
    with torch.no_grad():
        _, norm_fs, _ = model(X)

        # Design matrix for LS: combine normalized firing strengths with input
        X_aug = torch.cat([X, torch.ones(X.shape[0], 1)], dim=1)

        Phi = torch.einsum("br,bi->bri", X_aug, norm_fs).reshape(X.shape[0], -1)

        # Solve LS: consequents = (Phi^T Phi)^-1 Phi^T y

        theta= torch.linalg.lstsq(Phi, y).solution


        model.consequents.data = theta.reshape(model.consequents.shape)
```

In [467…

```
# --------------------------
# Gradient Descent Training
# --------------------------
def train_gd(model, X, y, epochs=100, lr=1e-3):
    optimizer = optim.Adam(model.parameters(), lr=lr)
    criterion = nn.MSELoss()
    for _ in range(epochs):
        optimizer.zero_grad()
        y_pred, _, _ = model(X)
        loss = criterion(y_pred, y)
        print(loss)
        loss.backward()
        optimizer.step()
```

In [468…

```
# --------------------------
# Hybrid Training (Classic ANFIS)
# --------------------------
def train_hybrid_alternating(model, X, y, max_iters=10, gd_epochs=20, lr=1e-3):
    for _ in range(max_iters):
        # Step A: GD on antecedents (freeze consequents)
        for p in model.consequents.parameters():
            p.requires_grad = False
        train_gd(model, X, y, epochs=gd_epochs, lr=lr)

        # Step B: LS on consequents (freeze antecedents)
        for p in model.consequents.parameters():
            p.requires_grad = True
        for p in model.mfs.parameters():
            p.requires_grad = False
        train_ls(model, X, y)

        # Re-enable antecedents
        for p in model.mfs.parameters():
            p.requires_grad = True
```

In [469…

```
# --------------------------
# Alternative Hybrid Training (LS+ gradient descent on all)
# --------------------------
def train_hybrid_classic(model, X, y, epochs=100, lr=1e-4):
    # Step 1: LS for consequents
    train_ls(model, X, y)
    # Step 2: GD fine-tuning
    train_gd(model, X, y, epochs=epochs, lr=lr)
```

In [470…

```
# Build model
model = TSK(n_inputs=Xtr.shape[1], n_rules=n_clusters, centers=centers[:,:-1], sigmas=sigmas[:,:-1])

Xtr = torch.tensor(Xtr, dtype=torch.float32)
ytr = torch.tensor(ytr, dtype=torch.float32)
Xte = torch.tensor(Xte, dtype=torch.float32)
yte = torch.tensor(yte, dtype=torch.float32)
```

We train the model using the least squares solver in this assignment

In [471…

```
# Training with LS:
train_ls(model, Xtr, ytr.reshape(-1,1))
```

We print `accuracy_score` and `mean_squared_error` as indications of the performance of the model.

In [472…

```
y_pred, _, _=model(Xte)
# print(f'ACC:{accuracy_score(yte.detach().numpy(),y_pred.detach().numpy()>0.5)}') #classification
print(f'MSE:{mean_squared_error(yte.detach().numpy(),y_pred.detach().numpy())}') #regression
```

```
MSE:2545.2890625
```