# Bits, Bytes, and Integers

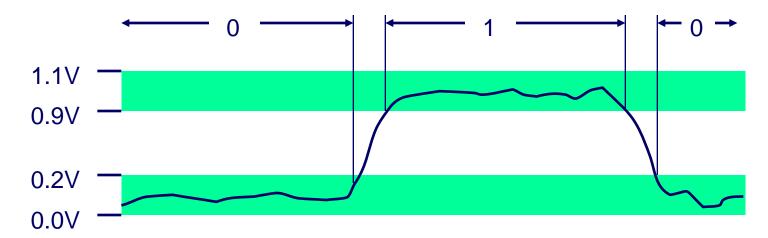
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# Bits, Bytes, and Integers

- Representing information as bits
- Bit-level manipulations
- Integers
  - Representation: unsigned and signed
  - Conversion, casting
  - Expanding, truncating
  - Addition, negation, multiplication, shifting
  - Summary
- Representations in memory, pointers, strings

# **Everything is bits**

- Each bit is 0 or 1
- By encoding/interpreting sets of bits in various ways
  - Computers determine what to do (instructions)
  - ... and represent and manipulate numbers, sets, strings, etc...
- Why bits? Electronic Implementation
  - Easy to store with bistable elements
  - Reliably transmitted on noisy and inaccurate wires



## For example, can count in binary

## Base 2 Number Representation

- Represent 15213<sub>10</sub> as 11101101101101<sub>2</sub>
- Represent 1.20<sub>10</sub> as 1.0011001100110011[0011]...<sub>2</sub>
- Represent 1.5213 X 10<sup>4</sup> as 1.1101101101101<sub>2</sub> X 2<sup>13</sup>

# **Encoding Byte Values**

- Byte = 8 bits
  - Binary 000000002 to 111111112
  - Decimal: 0<sub>10</sub> to 255<sub>10</sub>
  - Hexadecimal 00<sub>16</sub> to FF<sub>16</sub>
    - Base 16 number representation
    - Use characters '0' to '9' and 'A' to 'F'
    - Write FA1D37B<sub>16</sub> in C as
      - 0xFA1D37B
      - 0xfa1d37b
  - Why: Binary values might not represent numbers
    - Binary: 11110000
    - Hex: 0xF0
    - Decimal: 240

#### Hex Decimal Binary 0001 0010 0011 0100 0101 6 0110 0111 8 1000 1001 10 1010 11 1011 B 12 1100 13 1101 D E 14 1110 15 1111

# **Example Data Representations**

## **ISA Word Size:**

C Data Type	Typical 32-bit	Typical 64-bit	x86-64
char	1	1	1
short	2	2	2
int	4	4	4
long	4	8	8
float	4	4	4
double	8	8	8
long double	-	-	10/16
pointer	4	8	8

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# **Boolean Algebra**

- **Developed by George Boole in 19th Century** 
  - Algebraic representation of logic
    - Encode "True" as 1 and "False" as 0

And

Or

■ A&B = 1 when both A=1 and B=1

&	0	1
0	0	0
1	0	1

ı	0	1
0	0	1
1	1	1

Not

**Exclusive-Or (Xor)** 

~A = 1 when A=0

■ A^B = 1 when either A=1 or B=1, but not both

■ A | B = 1 when either A=1 or B=1

٨	0	1
0	0	1
1	1	0

## **General Boolean Algebras**

- Operate on Bit Vectors
  - Operations applied bitwise

```
01101001 01101001 01101001

& 01010101 | 01010101 ^ 01010101 ~ 01010101

01000001 01111101 00111100 1010101
```

All of the Properties of Boolean Algebra Apply

# **Example: Representing & Manipulating Sets**

## Representation

- Width w bit vector represents subsets of {0, ..., w-1}
- $a_j = 1$  if  $j \in A$ 
  - 01101001 { 0, 3, 5, 6 }
  - **76543210**
  - 01010101 { 0, 2, 4, 6 }
  - **76543210**

## Operations

<b>8</b>	Intersection	01000001	{ 0, 6 }
•	Union	01111101	{ 0, 2, 3, 4, 5, 6 }
^	Symmetric difference	00111100	{ 2, 3, 4, 5 }
~	Complement	10101010	{ 1, 3, 5, 7 }

# **Bit-Level Operations in C**

## ■ Operations &, |, ~, ^ Available in C

- Apply to any "integral" data type
  - long, int, short, char, unsigned
- View arguments as bit vectors
- Operations applied bit-wise

## Examples (Char data type)

- ~0x41 -> 0xBE
  - ~01000001₂ → 101111110₂
- ~0x00 -> 0xFF
  - ~000000002 → 1111111112
- 0x69 & 0x55 -> 0x41
  - 011010012 & 010101012 -> 010000012
- 0x69 | 0x55 -> 0x7D
  - 01101001<sub>2</sub> | 01010101<sub>2</sub> -> 01111101<sub>2</sub>

## **Contrast: Logic Operations in C**

## Contrast to Logical Operators

- **&&**, ||, !
  - View 0 as "False"
  - Anything nonzero as "True"
  - Always return 0 or 1
  - Early termination

## Examples (char data type)

- !0x41 -> 0x00
- !0x00 -> 0x01
- !!0x41 -> 0x01
- 0x69 && 0x55 -> 0x01
- 0x69 || 0x55 -> 0x01
- p && \*p (avoids null pointer access)

## **Contrast: Logic Operations in C**

- Contrast to Logical Operators
  - **&**&, ||, !
    - View 0 as "Fall
    - Anything ponzo
    - Alway
    - Early
- Example
  - !0x41
  - !0x00
  - !!0x41
- Watch out for && vs. & (and || vs. |)... one of the more common oopsies in
- **C** programming
- 0x69 && 0x55 -> 0x01
- 0x69 || 0x55 -> 0x01
- p && \*p (avoids null pointer access)

# **Shift Operations**

- Left Shift: x << y</p>
  - Shift bit-vector x left y positions
    - Throw away extra bits on left
    - Fill with 0's on right
- Right Shift: x >> y
  - Shift bit-vector x right y positions
    - Throw away extra bits on right
  - Logical shift
    - Fill with 0's on left
  - Arithmetic shift
    - Replicate most significant bit on left

	•		•	
ndefi	INAM	Ror	121/10	r
HUCH	IIICU	DCI	ıavıv	/∎

Shift amount < 0 or ≥ word size</p>

Argument x	01100010
<< 3	00010 <i>000</i>
Log. >> 2	00011000
<b>Arith.</b> >> 2	00011000

Argument x	10100010
<< 3	00010 <i>000</i>
Log. >> 2	<i>00</i> 101000
<b>Arith.</b> >> 2	<i>11</i> 101000

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# **Encoding Integers**

## **Unsigned**

$$B2U(X) = \sum_{i=0}^{w-1} x_i \cdot 2^i$$

## Two's Complement

$$B2T(X) = -x_{w-1} \cdot 2^{w-1} + \sum_{i=0}^{w-2} x_i \cdot 2^i$$

short int 
$$x = 15213$$
;  
short int  $y = -15213$ ;

## Sign Bit

## In C, a short is 2 bytes long

	Decimal	Hex	Binary
x	15213	3B 6D	00111011 01101101
У	-15213	C4 93	11000100 10010011

## Sign Bit

- For 2's complement, most significant bit indicates sign
  - 0 for nonnegative
  - 1 for negative

# **Two-complement Encoding Example (Cont.)**

x = 15213: 00111011 01101101y = -15213: 11000100 10010011

Weight	152	13	-152	213
1	1	1	1	1
2	0	0	1	2
4	1	4	0	0
8	1	8	0	0
16	0	0	1	16
32	1	32	0	0
64	1	64	0	0
128	0	0	1	128
256	1	256	0	0
512	1	512	0	0
1024	0	0	1	1024
2048	1	2048	0	0
4096	1	4096	0	0
8192	1	8192	0	0
16384	0	0	1	16384
-32768	0	0	1	-32768

## **Numeric Ranges**

### Unsigned Values

$$UMax = 2^w - 1$$

$$111...1$$

## **■ Two's Complement Values**

■ 
$$TMin = -2^{w-1}$$
100...0

■ 
$$TMax = 2^{w-1} - 1$$
  
011...1

#### Other Values

Minus 1111...1

#### Values for W = 16

	Decimal	Hex	Binary
UMax	65535	FF FF	11111111 11111111
TMax	32767	7F FF	01111111 11111111
TMin	-32768	80 00	10000000 000000000
-1	-1	FF FF	11111111 11111111
0	0	00 00	00000000 00000000

## Values for Different Word Sizes

		W		
	8	16	32	64
UMax	255	65,535	4,294,967,295	18,446,744,073,709,551,615
TMax	127	32,767	2,147,483,647	9,223,372,036,854,775,807
TMin	-128	-32,768	-2,147,483,648	-9,223,372,036,854,775,808

#### Observations

- $\blacksquare$  |TMin | = TMax + 1
  - Asymmetric range
- UMax = 2 \* TMax + 1

## C Programming

- #include imits.h>
- Declares constants, e.g.,
  - ULONG\_MAX
  - LONG\_MAX
  - LONG\_MIN
- Values platform specific

# **Unsigned & Signed Numeric Values**

Χ	B2U( <i>X</i> )	B2T( <i>X</i> )
0000	0	0
0001	1	1
0010	2	2
0011	3	3
0100	4	4
0101	5	5
0110	6	6
0111	7	7
1000	8	-8
1001	9	<b>-</b> 7
1010	10	-6
1011	11	<b>-</b> 5
1100	12	<b>-4</b>
1101	13	-3
1110	14	-2
1111	15	-1

## Equivalence

Same encodings for nonnegative values

## Uniqueness

- Every bit pattern represents unique integer value
- Each representable integer has unique bit encoding

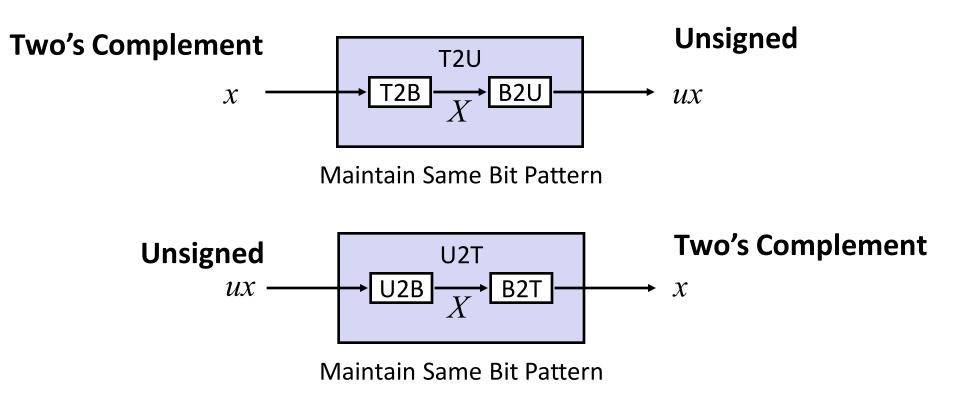
## ■ ⇒ Can Invert Mappings

- $U2B(x) = B2U^{-1}(x)$ 
  - Bit pattern for unsigned integer
- $T2B(x) = B2T^{-1}(x)$ 
  - Bit pattern for two's comp integer

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# **Mapping Between Signed & Unsigned**

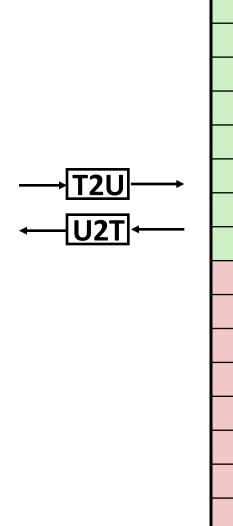


Mappings between unsigned and two's complement numbers: Keep bit representations and reinterpret

# Mapping Signed ↔ Unsigned

Bits
0000
0001
0010
0011
0100
0101
0110
0111
1000
1001
1010
1011
1100
1101
1110
1111

Signed	
0	
1	
2	
3	
4	
5	
6	
7	
-8	
-7	
-6	
-5	
-4	
-3	
-2	
-1	

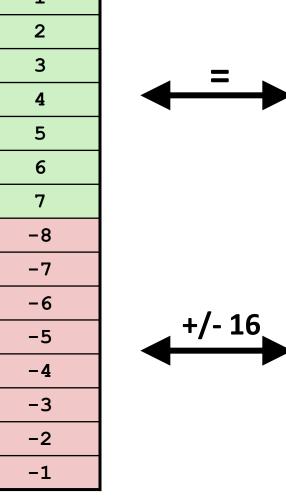


Unsigned
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15

# Mapping Signed ↔ Unsigned

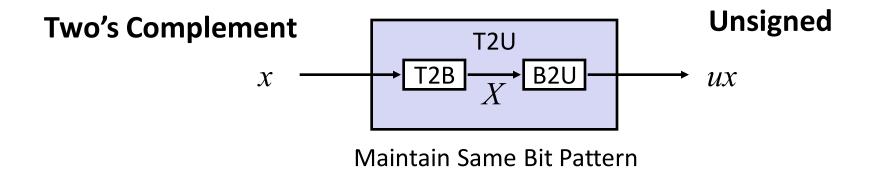
Bits
0000
0001
0010
0011
0100
0101
0110
0111
1000
1001
1010
1011
1100
1101
1110
1111

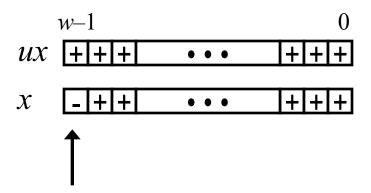
Signed
0
1
2
3
4
5
6
7
-8
-7
-6
-5
-4
-3
-2
-1



Unsigned
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15

# **Relation between Signed & Unsigned**





Large negative weight becomes

Large positive weight

## **Conversion Visualized**

2's Comp.  $\rightarrow$  Unsigned **UMax** Ordering Inversion UMax - 1Negative → Big Positive TMax + 1Unsigned TMax **TMax** Range 2's Complement Range

# Signed vs. Unsigned in C

#### Constants

- By default are considered to be signed integers
- Unsigned if have "U" as suffixOU, 4294967259U

## Casting

Explicit casting between signed & unsigned same as U2T and T2U

```
int tx, ty;
unsigned ux, uy;
tx = (int) ux;
uy = (unsigned) ty;
```

Implicit casting also occurs via assignments and procedure calls

```
tx = ux;

uy = ty;
```

## **Casting Surprises**

## Expression Evaluation

- If there is a mix of unsigned and signed in single expression, signed values implicitly cast to unsigned
- Including comparison operations <, >, ==, <=, >=
- **Examples for** W = 32: **TMIN = -2,147,483,648**, **TMAX = 2,147,483,647**

Constant <sub>1</sub>	Constant <sub>2</sub>	Relation	<b>Evaluation</b>
0	0U	==	unsigned
-1	0	<	signed
-1	0U	>	unsigned
2147483647	-2147483647-1	>	signed
2147483647U	-2147483647-1	<	unsigned
-1	-2	>	signed
(unsigned)-1	-2	>	unsigned
2147483647	2147483648U	<	unsigned
2147483647	(int) 2147483648U	>	signed

# Summary Casting Signed ↔ Unsigned: Basic Rules

- Bit pattern is maintained
- But reinterpreted
- Can have unexpected effects: adding or subtracting 2<sup>w</sup>
- Expression containing signed and unsigned int
  - int is cast to unsigned!!

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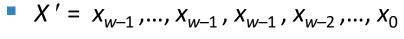
## **Sign Extension**

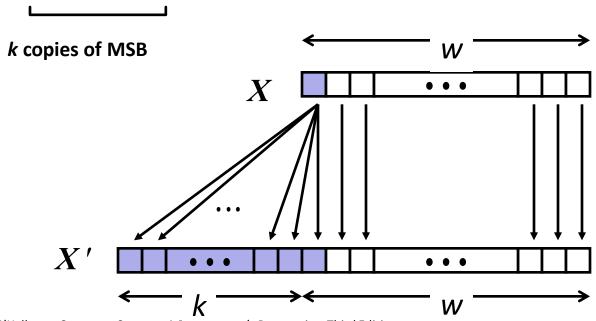
## Task:

- Given w-bit signed integer x
- Convert it to w+k-bit integer with same value

## Rule:

Make k copies of sign bit:





# **Sign Extension Example**

```
short int x = 15213;
int        ix = (int) x;
short int y = -15213;
int        iy = (int) y;
```

	Decimal	Hex	Binary
x	15213	3B 6D	00111011 01101101
ix	15213	00 00 3B 6D	00000000 00000000 00111011 01101101
У	-15213	C4 93	11000100 10010011
iy	-15213	FF FF C4 93	11111111 11111111 11000100 10010011

- Converting from smaller to larger integer data type
- C automatically performs sign extension

# Summary: Expanding, Truncating: Basic Rules

- Expanding (e.g., short int to int)
  - Unsigned: zeros added
  - Signed: sign extension
  - Both yield expected result
- Truncating (e.g., unsigned to unsigned short)
  - Unsigned/signed: bits are truncated
  - Result reinterpreted
  - Unsigned: mod operation
  - Signed: similar to mod
  - For small numbers yields expected behavior

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# **Unsigned Addition**

Operands: w bits

u

True Sum: w+1 bits



Discard Carry: w bits

 $UAdd_{w}(u, v)$ 



### Standard Addition Function

- Ignores carry output
- Implements Modular Arithmetic

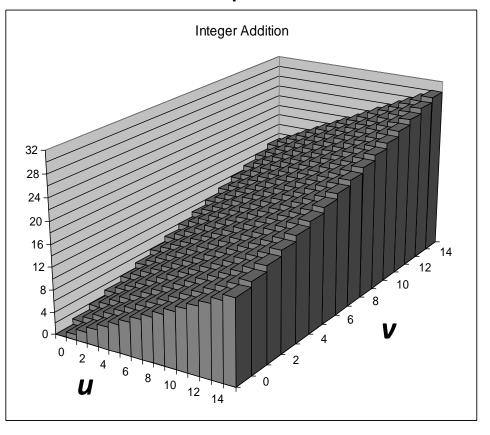
$$s = UAdd_w(u, v) = u + v \mod 2^w$$

# Visualizing (Mathematical) Integer Addition

## Integer Addition

- 4-bit integers u, v
- Compute true sum  $Add_4(u, v)$
- Values increase linearly with u and v
- Forms planar surface

## $Add_4(u, v)$

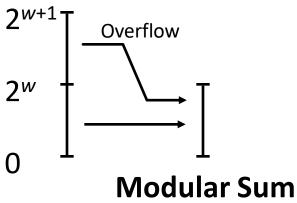


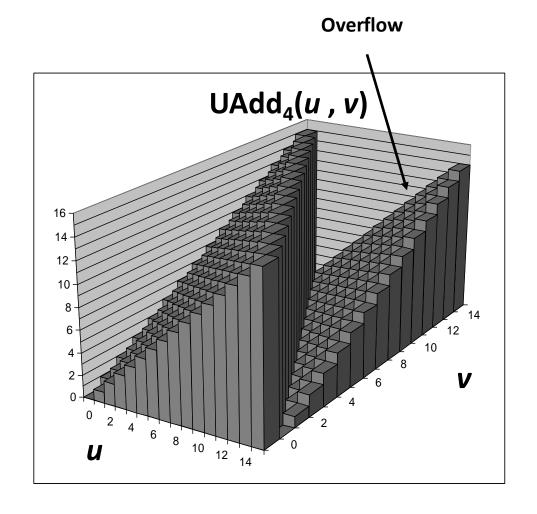
## **Visualizing Unsigned Addition**

#### Wraps Around

- If true sum  $\ge 2^w$
- At most once

#### **True Sum**



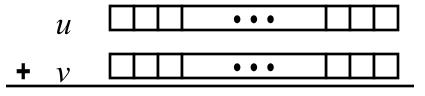


# **Two's Complement Addition**

Operands: w bits

True Sum: w+1 bits

Discard Carry: w bits



 $TAdd_{w}(u, v)$ 

#### TAdd and UAdd have Identical Bit-Level Behavior

Signed vs. unsigned addition in C:

```
int s, t, u, v;
s = (int) ((unsigned) u + (unsigned) v);
t = u + v
```

Will give s == t

#### **TAdd Overflow**

#### Functionality

- True sum requires w+1 bits
- Drop off MSB
- Treat remaining bits as 2's comp. integer

#### **True Sum**

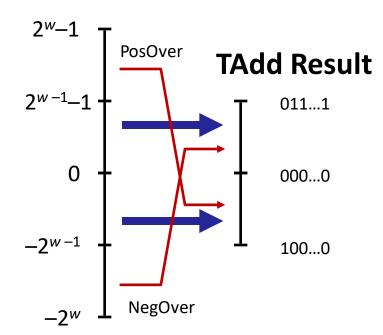


**0** 100...0

**0** 000...0

**1** 011...1

1 000...0



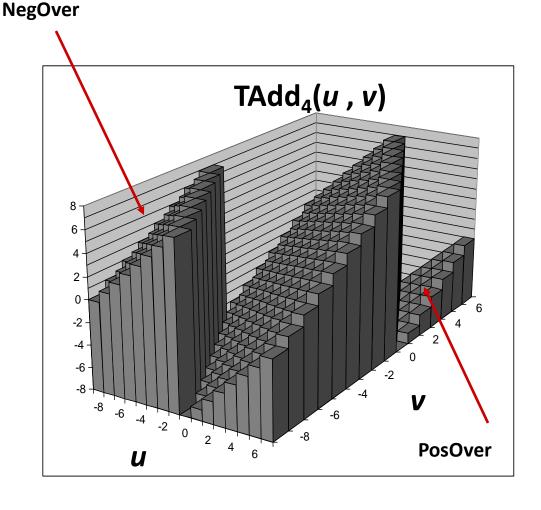
## **Visualizing 2's Complement Addition**

#### Values

- 4-bit two's comp.
- Range from -8 to +7

#### Wraps Around

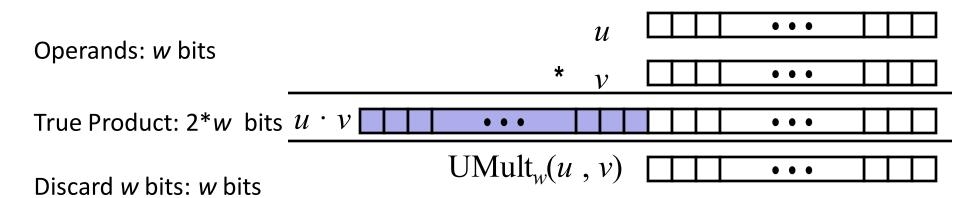
- If sum  $\geq 2^{w-1}$ 
  - Becomes negative
  - At most once
- If sum  $< -2^{w-1}$ 
  - Becomes positive
  - At most once



## Multiplication

- Goal: Computing Product of w-bit numbers x, y
  - Either signed or unsigned
- But, exact results can be bigger than w bits
  - Unsigned: up to 2w bits
    - Result range:  $0 \le x * y \le (2^w 1)^2 = 2^{2w} 2^{w+1} + 1$
  - Two's complement min (negative): Up to 2w-1 bits
    - Result range:  $x * y \ge (-2^{w-1})*(2^{w-1}-1) = -2^{2w-2} + 2^{w-1}$
  - Two's complement max (positive): Up to 2w bits, but only for  $(TMin_w)^2$ 
    - Result range:  $x * y \le (-2^{w-1})^2 = 2^{2w-2}$
- So, maintaining exact results...
  - would need to keep expanding word size with each product computed
  - is done in software, if needed
    - e.g., by "arbitrary precision" arithmetic packages

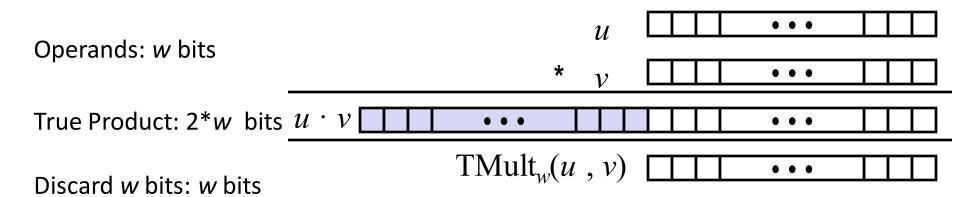
# **Unsigned Multiplication in C**



- Standard Multiplication Function
  - Ignores high order w bits
- Implements Modular Arithmetic

$$UMult_w(u, v) = u \cdot v \mod 2^w$$

# Signed Multiplication in C



#### Standard Multiplication Function

- Ignores high order w bits
- Some of which are different for signed vs. unsigned multiplication
- Lower bits are the same

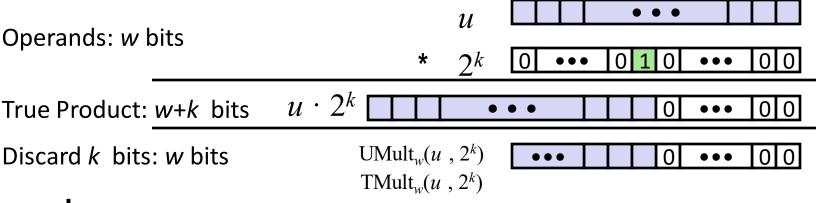
k

## Power-of-2 Multiply with Shift

#### **Operation**

- u << k gives u \* 2<sup>k</sup>
- Both signed and unsigned

Operands: w bits

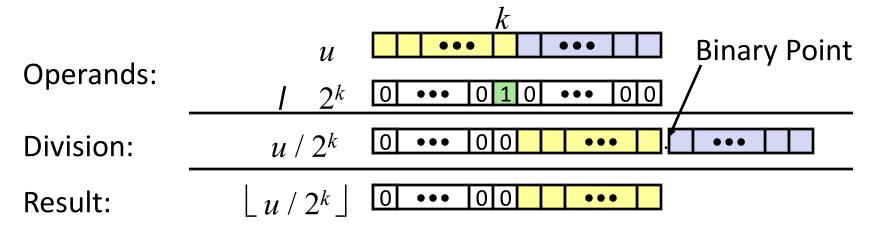


#### **Examples**

- u << 3</li>
- (u << 5) (u << 3) == u \* 24
- Most machines shift and add faster than multiply
  - Compiler generates this code automatically

## **Unsigned Power-of-2 Divide with Shift**

- Quotient of Unsigned by Power of 2
  - $\mathbf{u} \gg \mathbf{k}$  gives  $\lfloor \mathbf{u} / 2^k \rfloor$
  - Uses logical shift



	Division	Computed	Hex	Binary		
x	15213	15213	3B 6D	00111011 01101101		
x >> 1	7606.5	7606	1D B6	00011101 10110110		
x >> 4	950.8125	950	03 B6	00000011 10110110		
x >> 8	59.4257813	59	00 3B	00000000 00111011		

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#### **Arithmetic: Basic Rules**

#### Addition:

- Unsigned/signed: Normal addition followed by truncate, same operation on bit level
- Unsigned: addition mod 2<sup>w</sup>
  - Mathematical addition + possible subtraction of 2<sup>w</sup>
- Signed: modified addition mod 2<sup>w</sup> (result in proper range)
  - Mathematical addition + possible addition or subtraction of 2<sup>w</sup>

#### Multiplication:

- Unsigned/signed: Normal multiplication followed by truncate, same operation on bit level
- Unsigned: multiplication mod 2<sup>w</sup>
- Signed: modified multiplication mod 2<sup>w</sup> (result in proper range)

## Why Should I Use Unsigned?

- Don't use without understanding implications
  - Easy to make mistakes

```
unsigned i;
for (i = cnt-2; i >= 0; i--)
a[i] += a[i+1];
```

Can be very subtle

```
#define DELTA sizeof(int)
int i;
for (i = CNT; i-DELTA >= 0; i-= DELTA)
. . . .
```

### **Counting Down with Unsigned**

Proper way to use unsigned as loop index

```
unsigned i;
for (i = cnt-2; i < cnt; i--)
  a[i] += a[i+1];</pre>
```

- See Robert Seacord, Secure Coding in C and C++
  - C Standard guarantees that unsigned addition will behave like modular arithmetic
    - $0-1 \rightarrow UMax$
- Even better

```
size_t i;
for (i = cnt-2; i < cnt; i--)
   a[i] += a[i+1];</pre>
```

- Data type size\_t defined as unsigned value with length = word size
- Code will work even if cnt = UMax
- What if cnt is signed and < 0?</p>

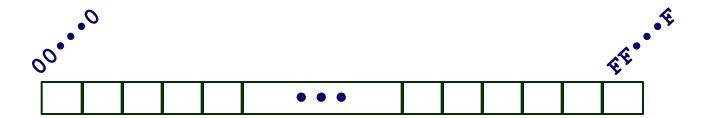
## Why Should I Use Unsigned? (cont.)

- Do Use When Performing Modular Arithmetic
  - Multiprecision arithmetic
- Do Use When Using Bits to Represent Sets
  - Logical right shift, no sign extension

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### **Byte-Oriented Memory Organization**



#### Programs refer to data by address

- Conceptually, envision it as a very large array of bytes
  - In reality, it's not, but can think of it that way
- An address is like an index into that array
  - and, a pointer variable stores an address

#### Note: system provides private address spaces to each "process"

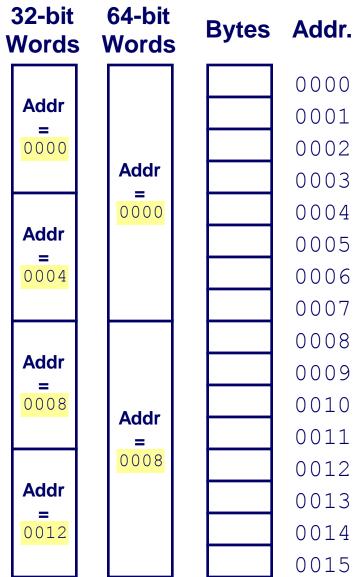
- Think of a process as a program being executed
- So, a program can clobber its own data, but not that of others

#### **Machine Words**

- Any given computer has a "Word Size"
  - Nominal size of integer-valued data
    - and of addresses
  - Until recently, most machines used 32 bits (4 bytes) as word size
    - Limits addresses to 4GB (2<sup>32</sup> bytes)
  - Increasingly, machines have 64-bit word size
    - Potentially, could have 18 EB (exabytes) of addressable memory
    - That's 18.4 X 10<sup>18</sup>
    - Machines still support multiple data formats
      - Fractions or multiples of word size
      - Always integral number of bytes

## **Word-Oriented Memory Organization**

- Addresses Specify Byte Locations
  - Address of first byte in word
  - Addresses of successive words differ by 4 (32-bit) or 8 (64-bit)



# **Example Data Representations**

C Data Type	Typical 32-bit	Typical 64-bit	x86-64	
char	1	1	1	
short	2	2	2	
int	4	4	4	
long	4	8	8	
float	4	4	4	
double	8	8	8	
long double	-	-	10/16	
pointer	4	8	8	

## **Byte Ordering**

- So, how are the bytes within a multi-byte word ordered in memory?
- Conventions
  - Big Endian: Sun, PPC Mac, Internet
    - Least significant byte has highest address
  - Little Endian: x86, ARM processors running Android, iOS, and Windows
    - Least significant byte has lowest address

## **Byte Ordering Example**

#### Example

- Variable x has 4-byte value of 0x01234567
- Address given by &x is 0x100

Big Endian		0x100	0x101	0x102	0x103		
			01	23	45	67	
Little Endian			0x100	0x101	0x102	0x103	
			67	45	23	01	

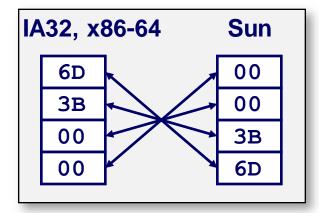
## **Representing Integers**

**Decimal: 15213** 

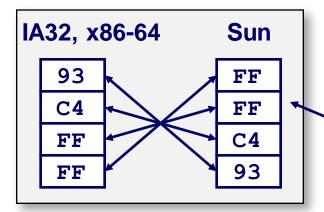
**Binary:** 0011 1011 0110 1101

**Hex:** 3 B 6 D

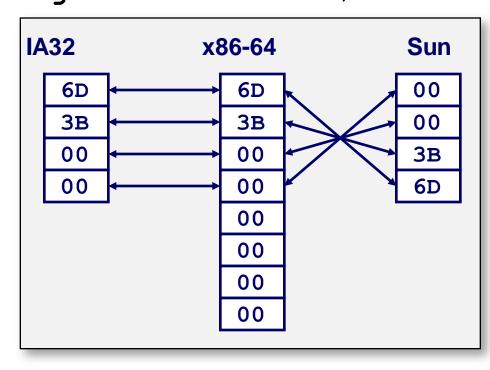




int B = -15213;



long int C = 15213;



Two's complement representation

### **Examining Data Representations**

- Code to Print Byte Representation of Data
  - Casting pointer to unsigned char \* allows treatment as a byte array

```
typedef unsigned char *pointer;

void show_bytes(pointer start, size_t len){
    size_t i;
    for (i = 0; i < len; i++)
        printf("%p\t0x%.2x\n",start+i, start[i]);
    printf("\n");
}</pre>
```

#### **Printf directives:**

%p: Print pointer

%x: Print Hexadecimal

## show bytes Execution Example

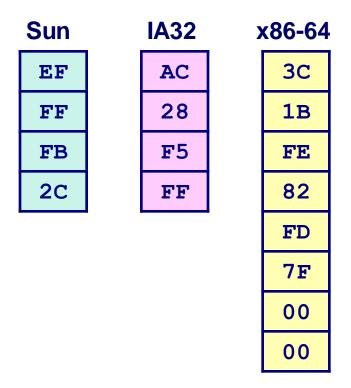
```
int a = 15213;
printf("int a = 15213;\n");
show_bytes((pointer) &a, sizeof(int));
```

#### Result (Linux x86-64):

```
int a = 15213;
0x7fffb7f71dbc 6d
0x7fffb7f71dbd 3b
0x7fffb7f71dbe 00
0x7fffb7f71dbf 00
```

### **Representing Pointers**

int 
$$B = -15213;$$
  
int \*P = &B



Different compilers & machines assign different locations to objects

## **Representing Strings**

char S[6] = "18213";

#### Strings in C

- Represented by array of characters
- Each character encoded in ASCII format
  - Standard 7-bit encoding of character set
  - Character "0" has code 0x30
    - Digit i has code 0x30+i
- String should be null-terminated
  - Final character = 0

#### Compatibility

Byte ordering not an issue

