Mathematical Modeling on the Great Plague Of London

Math 170: Mathematical Modeling for Life Sciences **Final Project**

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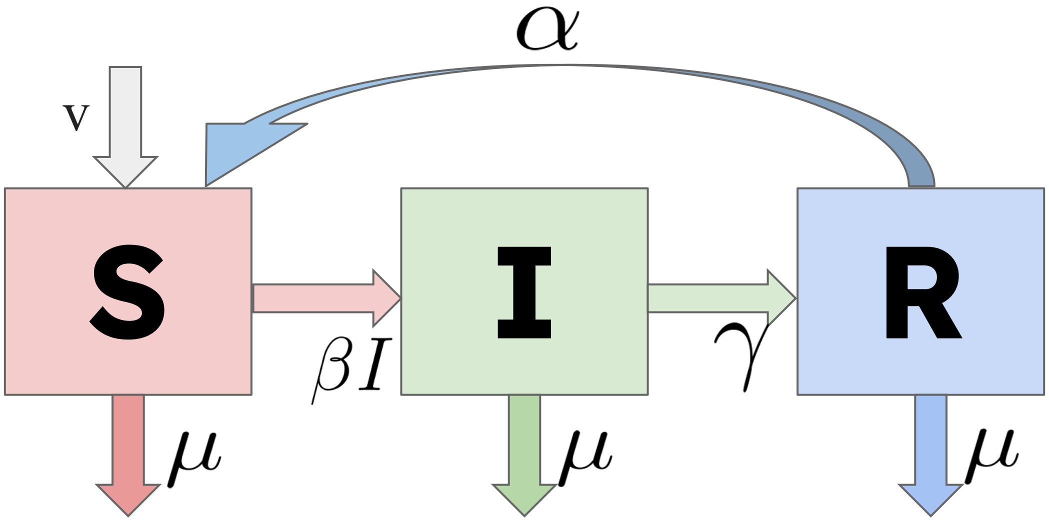
Epidemic Description

From 1665 to 1666

- 68,596 people died
- the actual number of deaths is suspected to have exceeded 100,000 out of a total population estimated at 460,000.
- The Great Plague was not an isolated event—40,000 Londoners had died of the plague in 1625—but it was the last and worst of the epidemics.
- The disappearance of plague from London has been attributed to the **Great Fire of London** in September 1666, but it also subsided in other cities without such cause.

SIRS Model

v = population growth rate
μ = mortality rate
α = rate of people who become susceptible after recovered from the disease
β = average number of infectious contacts per person per time
γ = coverage rate



Assumption: Anyone could die regardless of being infected or not.

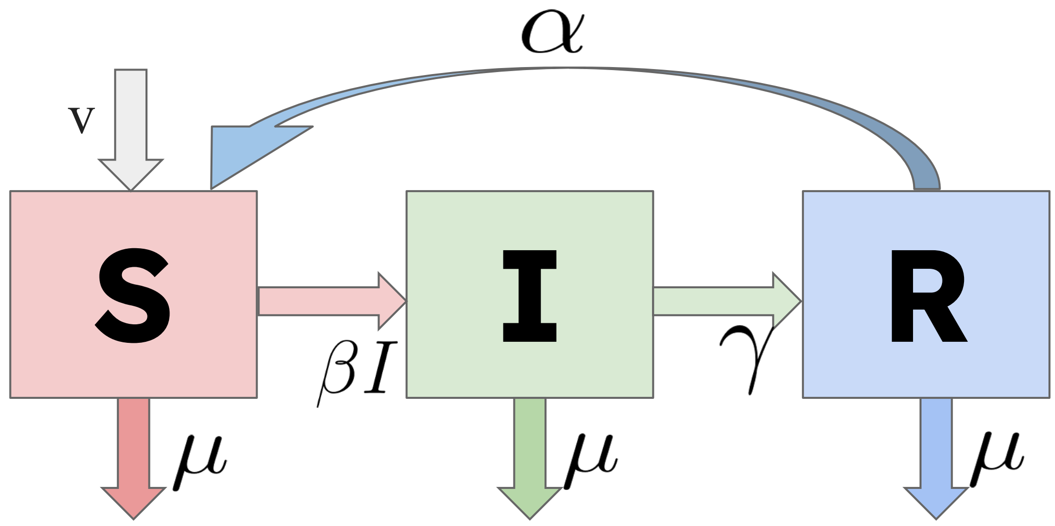
3 differential Equations

$$\frac{dS}{dt} = VN - S\beta I - S\mu + R\alpha$$

$$\frac{dI}{dt} = S\beta I - I\mu + I\alpha$$

$$\frac{dR}{dt} = I\gamma - R\mu - R\alpha$$

$$\frac{dN}{dt} = VN$$



Steady States

$$0 = VN - S\beta I - S\mu + R\alpha$$

$$0 = S\beta I - I\mu + I\alpha$$

$$0 = I\gamma - R\mu - R\alpha$$

Disease Free Equilibrium (DFE)

`Setting I = 0`

$$(S, I, R) = (\frac{VN}{\mu}, 0, 0)$$

Endemic Equilibrium (EE)

`Setting I ≠ 0,`

$$(S, I, R) = (\frac{\gamma + \mu}{\beta}, \frac{VN}{\gamma + \mu} + \frac{I\gamma\alpha}{(\gamma + \mu)(\mu + \alpha)} - \frac{\mu}{\beta}, \frac{I\gamma}{\mu + \alpha})$$

$\neq$

Determine the stability

J =

$$\begin{bmatrix} -\beta I - \mu & -S\beta & \alpha \\ \beta I & S\beta - \gamma - \mu & 0 \\ 0 & \gamma & -\mu - \alpha \end{bmatrix}$$

,

$$(S, I, R) = (\frac{VN}{\mu}, 0, 0)$$

J|DFE =

$$\begin{bmatrix} -\mu & vN\beta/\mu & \alpha \\ 0 & vN\beta/\mu - \gamma - \mu & 0 \\ 0 & \gamma & -\mu - \alpha \end{bmatrix}$$

det(J - λI) =

$$\begin{vmatrix} -\mu-\lambda & vN\beta/\mu & \alpha \\ 0 & vN\beta/\mu - \gamma - \mu - \lambda & 0 \\ 0 & \gamma & -\mu - \alpha - \lambda \end{vmatrix}$$

Stability of DFE

$$\lambda_{1,2,3} = \{-\mu, \frac{VN\beta}{\mu} - \gamma - \mu, -\mu - \alpha\}$$

$$\frac{vN\beta}{\mu} - \gamma - \mu < 0$$

$$\frac{vN\beta}{\mu(\gamma + \mu)} < 1$$

$$\frac{v}{\mu} r0 < 1$$

$$\frac{v}{\mu} < \frac{1}{r0}$$

$$\frac{\mu}{v} > r0$$

Conclusion

- Our model shows that 2 parameters relate to the r0 and subsequently the Great plagues pandemic status.

$$\frac{\mu}{\nu} > r0 \left(\quad r0 = \frac{N\beta}{(\gamma+\mu)} \quad 1 \right) \qquad \frac{\mu}{\nu} > 1$$

- ν, population growth rate μ, mortality rate

- London’s population decreased by approximately 15%.

- Long-term behaviour depends on these parameters.