Q1)

a)

Ljung-Box Test (10 Lags):
X-squared = 898.09, df = 10, p-value ¡ 2.2e-1658
Hence the we can reject the Null Hypothesis: First 10 lags of ACF of a_t^2 is 0.
LM ARCH Test:

```
 Lags Statistic        df pvalue
   10  802.2256 7.857143      0
```

Hence, we can reject the Null Hypothesis: There is no ARCH effects for first 10 lags.

Therefore, from the Ljung-Box Test and the LM ARCH Test, there is sufficient evidence that there are ARCH effects in the log returns of GM.

b)

ARCH(1)
p-value of Ljung-Box Test(R^2,lag 10):0
AIC: -4.242443

ARCH(2)
p-value of Ljung-Box Test(R^2,lag 10):6.008332e-10
AIC:-4.410438

GARCH(1,1)
p-value of Ljung-Box Test(R^2,lag 10):0.5261773
AIC: 1.125236

Since the Ljung-Box test on the first 2 models is significant, GARCH(1,1) is the most suitable. Hence,

$$\begin{aligned} r_t &= -0.006190 + a_t \\ a_t &= \sigma_t \epsilon_t \\ \sigma_t^2 &= 0.010761 + 0.153134 a_{t-1}^2 + 0.805974 \sigma_{t-1}^2 \\ \epsilon_t &\sim N(0,1) \end{aligned}$$

c)

GARCH-M(1,1) Model:

$$r_t = 0.000775+$$

2)

a)

We fit an ARMA(1,5) Model from the auto.arima function. Then we conduct the test on the residuals of the model:
Ljung-Box Test on R^2: X-squared = 2442.1, df = 10, p-value ¡ 2.2e-16
LM ARCH Test:

```
 Lags Statistic       df pvalue
   10   1042.24 7.857143      0
```

Since both test statistics are significant, we can reject the NULL Hypothesis: there is no ARCH effects for first 10 lags.

b)

We try to fit the follow models:
ARMA(1,0)+GARCH(1,1)

```
Ljung-Box Test     R^2  Q(10)  13.94584  0.1754764
Log Likelihood:
 7855.843    normalized:  3.123596
      AIC       BIC       SIC      HQIC
-6.243215 -6.231625 -6.243223 -6.239009
```

ARMA(0,1)+GARCH(1,1):

```
Ljung-Box Test     R^2  Q(10)  14.03401  0.171446
Log Likelihood:
 7856.114    normalized:  3.123704
      AIC       BIC       SIC      HQIC
-6.243431 -6.231840 -6.243439 -6.239224
```

ARMA(1,1)+GARCH(1,1):

```
Ljung-Box Test     R^2  Q(10)  14.06938  0.1698506
Log Likelihood:
 7859.965    normalized:  3.125234
       AIC       BIC       SIC      HQIC
-6.245697 -6.231789 -6.245709 -6.240650
```

ARMA(2,1)+GARCH(1,1):

```
Ljung-Box Test     R^2  Q(10)  13.95878  0.17488
Log Likelihood:
 7860.663    normalized:  3.125512
      AIC       BIC       SIC      HQIC
-6.245458 -6.229231 -6.245473 -6.239568
ar2 not significant
```

ARMA(1,2)+GARCH(1,1):

```
Ljung-Box Test     R^2  Q(10)  13.95475  0.1750656
Log Likelihood:
 7860.663    normalized:  3.125512
      AIC       BIC       SIC      HQIC
-6.245458 -6.229231 -6.245473 -6.239568
ma2 not significant
```

ARMA(2,2)+GARCH(1,1):

```
Ljung-Box Test     R^2  Q(10)  13.99351  0.1732879
Log Likelihood:
 7860.697    normalized:  3.125526
      AIC       BIC       SIC      HQIC
-6.244689 -6.226145 -6.244709 -6.237959
ar1,ar2,ma1,ma2 not significant
```

In all cases, the mu is not significant. Since the last 3 models are misspecified, we will choose ARMA(1,1)+GARCH(1,1) since it has the highest Log-likelihood and lowest AIC. After removing the mu, we get:

$$
\begin{aligned}
r_t &= 7.130 \times 10^{-1} r_{t-1} + a_t - 7.677 \times 10^{-1} a_{t-1} \\
a_t &= \sigma_t \epsilon_t \\
\sigma_t^2 &= 1.000 \times 10^{-6} + 6.996 \times 10^{-2} a_{t-1}^2 + 9.248 \times 10^{-1} \sigma_{t-1}^2 \\
\epsilon_t &\sim N(0,1)
\end{aligned}
$$

c)

```
   meanForecast  meanError standardDeviation
1 -0.0015129348 0.02770432        0.02770432
2 -0.0010787226 0.02769177        0.02765025
3 -0.0007691292 0.02765891        0.02759635
4 -0.0005483892 0.02761582        0.02754263
```