Q1)

a)

Ljung-Box Test (10 Lags):
X-squared = 898.09, df = 10, p-value ¡ 2.2e-1658
Hence the we can reject the Null Hypothesis: First 10 lags of ACF of a_t^2 is 0.
LM ARCH Test:

```
 Lags Statistic        df pvalue
   10  802.2256 7.857143      0
```

Hence, we can reject the Null Hypothesis: There is no ARCH effects for first 10 lags.

Therefore, from the Ljung-Box Test and the LM ARCH Test, there is sufficient evidence that there are ARCH effects in the log returns of GM.

b)

ARCH(1)
p-value of Ljung-Box Test(R^2,lag 10):0
AIC: -4.242443

ARCH(2)
p-value of Ljung-Box Test(R^2,lag 10):6.008332e-10
AIC:-4.410438

GARCH(1,1)
p-value of Ljung-Box Test(R^2,lag 10):0.5261773
AIC: 1.125236

Since the Ljung-Box test on the first 2 models is significant, GARCH(1,1) is the most suitable. Hence,

$$
\begin{aligned}
r_t &= -0.006190 + a_t \\
a_t &= \sigma_t \epsilon_t \\
\sigma_t^2 &= 0.010761 + 0.153134 a_{t-1}^2 + 0.805974 \sigma_{t-1}^2 \\
\epsilon_t &\sim N(0,1)
\end{aligned}
$$

c)

GARCH-M(1,1) Model:

$$r_t = 0.000775 +$$

2)

a)

We fit an ARMA(1,5) Model from the auto.arima function. Then we conduct the test on the residuals of the model:
Ljung-Box Test on R^2: X-squared = 2442.1, df = 10, p-value ¡ 2.2e-16
LM ARCH Test:

```
 Lags Statistic       df pvalue
   10   1042.24 7.857143      0
```

Since both test statistics are significant, we can reject the NULL Hypothesis: there is no ARCH effects for first 10 lags.

b)

We try to fit the follow models:
ARMA(1,0)+GARCH(1,1)

```
Ljung-Box Test     R^2  Q(10)  13.94584  0.1754764
Log Likelihood:
 7855.843    normalized:  3.123596
      AIC       BIC       SIC      HQIC
-6.243215 -6.231625 -6.243223 -6.239009
```

ARMA(0,1)+GARCH(1,1):

```
Ljung-Box Test     R^2  Q(10)  14.03401  0.171446
Log Likelihood:
 7856.114    normalized:  3.123704
      AIC       BIC       SIC      HQIC
-6.243431 -6.231840 -6.243439 -6.239224
```

ARMA(1,1)+GARCH(1,1):

```
Ljung-Box Test     R^2  Q(10)  14.06938  0.1698506
Log Likelihood:
 7859.965    normalized:  3.125234
       AIC       BIC       SIC      HQIC
-6.245697 -6.231789 -6.245709 -6.240650
```

ARMA(2,1)+GARCH(1,1):

```
Ljung-Box Test     R^2  Q(10)  13.95878  0.17488
Log Likelihood:
 7860.663    normalized:  3.125512
      AIC       BIC       SIC      HQIC
-6.245458 -6.229231 -6.245473 -6.239568
ar2 not significant
```

ARMA(1,2)+GARCH(1,1):

```
Ljung-Box Test     R^2  Q(10)  13.95475  0.1750656
Log Likelihood:
 7860.663    normalized:  3.125512
      AIC       BIC       SIC      HQIC
-6.245458 -6.229231 -6.245473 -6.239568
ma2 not significant
```

ARMA(2,2)+GARCH(1,1):

```
Ljung-Box Test     R^2  Q(10)  13.99351  0.1732879
Log Likelihood:
 7860.697    normalized:  3.125526
      AIC       BIC       SIC      HQIC
-6.244689 -6.226145 -6.244709 -6.237959
ar1,ar2,ma1,ma2 not significant
```

In all cases, the mu is not significant. Since the last 3 models are misspecified, we will choose ARMA(1,1)+GARCH(1,1) since it has the highest Log-likelihood and lowest AIC. After removing the mu, we get:

$$
\begin{aligned}
r_t &= 7.130 \times 10^{-1} r_{t-1} + a_t - 7.677 \times 10^{-1} a_{t-1} \\
a_t &= \sigma_t \epsilon_t \\
\sigma_t^2 &= 1.000 \times 10^{-6} + 6.996 \times 10^{-2} a_{t-1}^2 + 9.248 \times 10^{-1} \sigma_{t-1}^2 \\
\epsilon_t &\sim N(0,1)
\end{aligned}
$$

c)

```
   meanForecast  meanError standardDeviation
1 -0.0015129348 0.02770432        0.02770432
2 -0.0010787226 0.02769177        0.02765025
3 -0.0007691292 0.02765891        0.02759635
4 -0.0005483892 0.02761582        0.02754263
```

3)

a)

We test $H_0 : \mu = 0$ vs $H_1 : \mu = 0$
T-test: t = 4.2198, df = 608, p-value = 2.819e-05
Hence, we reject the Null Hypothesis that expected monthly log return is zero and the mean is 0.01057841.
Ljung-Box Test(12 lags):X-squared = 14.821, df = 12, p-value = 0.2514
Hence we cannot reject the Null Hypothesis that there are no serial correlations in the series.
Next, we test for ARCH effects in the log returns:
Ljung-Box Test (R^2,12 Lags): X-squared = 184.93, df = 12, p-value $\leq 2.2e-16$
LM ARCH Test:

```
 Lags Statistic   df pvalue
   12   145.429 9.36      0
```

Therefore, we can reject the Null Hypothesis:there is no ARCH effect on the first 12 lags.

b)

GARCH(1,1):

```
Ljung-Box Test     R^2  Q(10)  12.54335  0.2503354
Ljung-Box Test     R^2  Q(15)  13.04873  0.5985339
Log Likelihood:
 869.3329    normalized:  1.427476
      AIC       BIC       SIC      HQIC
-2.841816 -2.812838 -2.841901 -2.830543
```

GARCH(2,1):

```
alpha1 0.0503265   0.0297111    1.694  0.09029 .  *
alpha2 0.0771708   0.0444357    1.737  0.08244 .  *

Ljung-Box Test     R^2  Q(10)  10.34415  0.410836
Ljung-Box Test     R^2  Q(15)  10.78324  0.7678034
Log Likelihood:
 870.7469    normalized:  1.429798
      AIC       BIC       SIC      HQIC
-2.843176 -2.806954 -2.843309 -2.829084
```

Despite having a higher log-likelihood and lower AIC, the alpha1 and alpha2 parameters in the GARCH(2,1) is not significant at 0.05. Hence, we will use GARCH(1,1) as the model

$$= 1.237 \times 10^{-2} + a_t$$

a$_t = \sigma_t \epsilon_t$
a$_t^2 = 2.592 \times 10^{-4} + 9.878 \times 10^{-2} a_{t-1}^2 + 0.8298 \sigma_{t-1}^2$
$\epsilon_t \sim N(0, 1)$