Confidential

basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

SENIOR CERTIFICATE EXAMINATIONS/ NATIONAL SENIOR CERTIFICATE EXAMINATIONS

MATHEMATICS P2

MAY/JUNE 2024

MARKS: 150

TIME: 3 hours

This question paper consists of 12 pages, 1 information sheet and an answer book of 23 pages.

INSTRUCTIONS AND INFORMATION

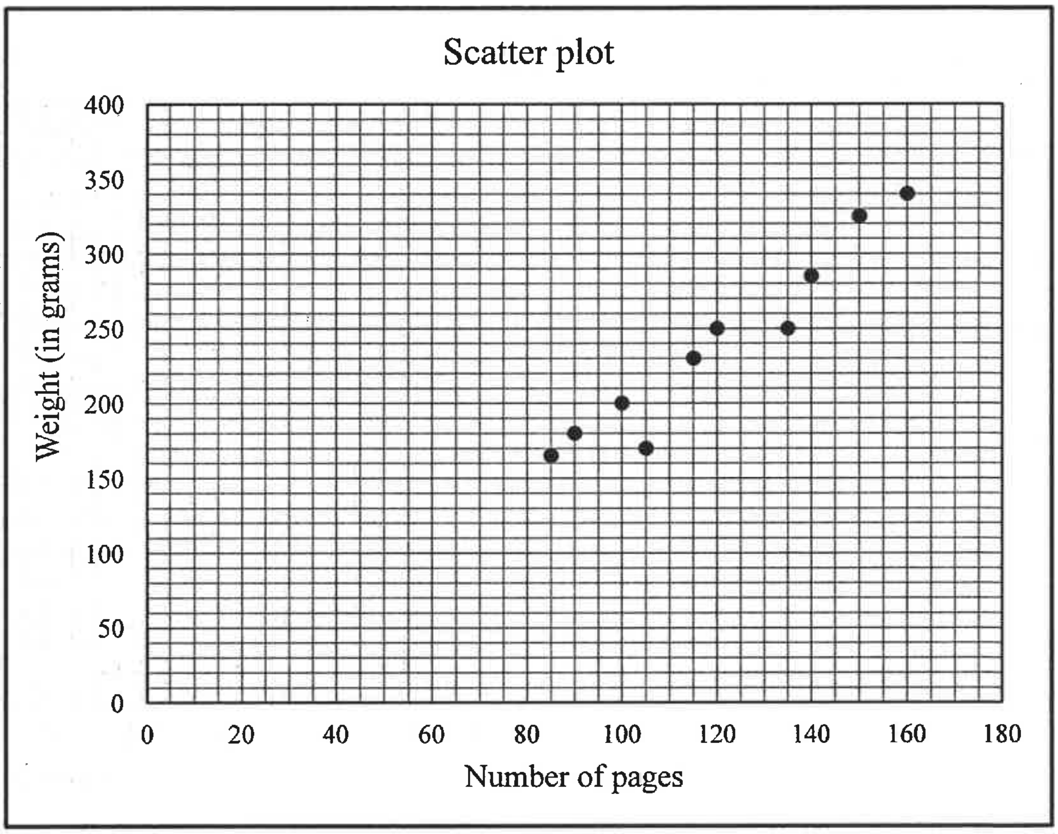
Read the following instructions carefully before answering the questions.

1. This question paper consists of 10 questions.
2. Answer ALL the questions in the SPECIAL ANSWER BOOK provided.
3. Clearly show ALL calculations, diagrams, graphs, etc. which you have used in determining your answers.
4. Answers only will NOT necessarily be awarded full marks.
5. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
6. If necessary, round off answers correct to TWO decimal places, unless stated otherwise.
7. Diagrams are NOT necessarily drawn to scale.
8. An information sheet with formulae is included at the end of the question paper.
9. Write neatly and legibly.

QUESTION 1

The number of pages in ten A4 books and their corresponding weights (in grams) are given in the table below. The data is also represented in the scatter plot.

Number of pages (x)	85	150	100	120	90	140	135	105	115	160
Weight (in grams) (y)	165	325	200	250	180	285	250	170	230	340



1.1 Determine the equation of the least squares regression line. (3)

1.2 Draw the least squares regression line on the scatter plot in the ANSWER BOOK. (2)

1.3 Predict the weight of an A4 book that has 110 pages. (2)

1.4 Calculate the percentage weight increase between a book with 110 pages and a book with 130 pages. (3)

[10]

QUESTION 2

Fifty athletes need to access suitable training facilities. The table below shows the distances, in km, that they need to travel to obtain access to suitable training facilities.

DISTANCE (x km)	NUMBER OF ATHLETES
$0 \leq x < 5$	3
$5 \leq x < 10$	7
$10 \leq x < 15$	20
$15 \leq x < 20$	12
$20 \leq x < 25$	5
$25 \leq x < 30$	3

2.1 Complete the cumulative frequency column provided in the table in the ANSWER BOOK. (2)

2.2 On the grid provided in the ANSWER BOOK, draw a cumulative frequency graph (ogive) to represent the above data. (3)

2.3 Calculate the interquartile range (IQR) of the above data. (2)

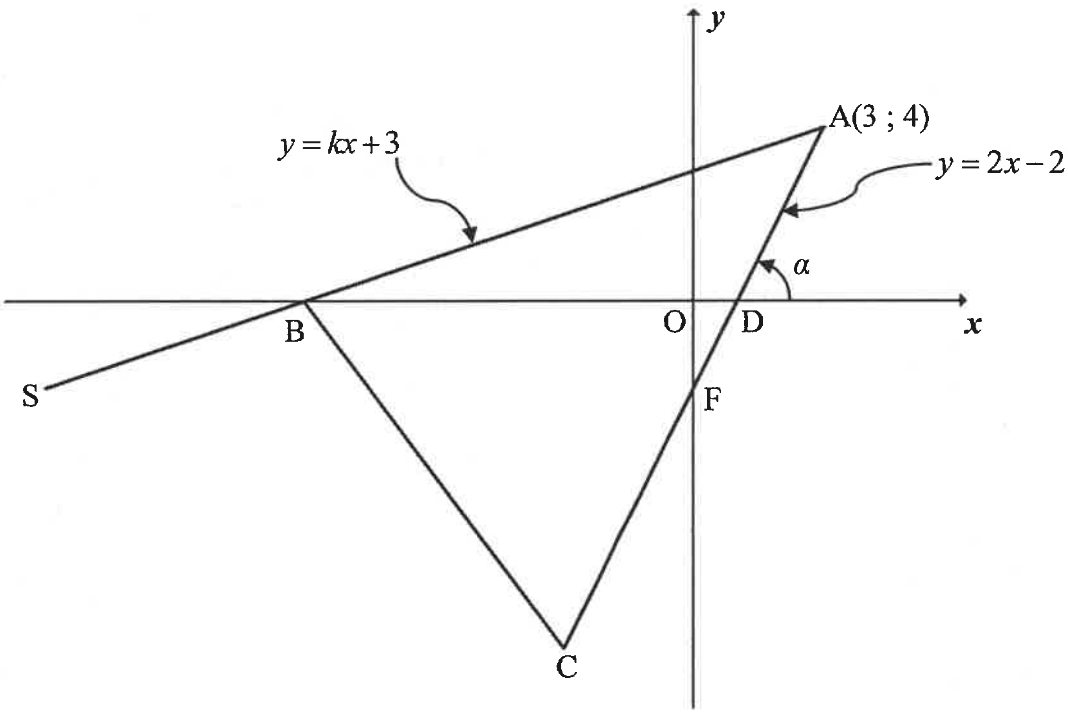
2.4 The families of 4 of the athletes above who stay between 15 and 20 km from a suitable training facility, decide to move 10 kilometres closer to the facility. In which interval will the number of athletes increase? (1)

2.5 Calculate the estimated mean distance that the fifty athletes need to travel after the 4 families have moved 10 kilometres closer to the facility. Clearly show ALL working. (3)

[11]

QUESTION 3

In the diagram, A(3 ; 4), B and C are vertices of ΔABC. AB is produced to S. D and F are the x- and y-intercepts of AC respectively. F is the midpoint of AC and the angle of inclination of AC is α. The equation of AB is $y = kx + 3$ and the equation of AC is $y = 2x - 2$.



3.1 Show that $k = \frac{1}{3}$. (1)

3.2 Calculate the coordinates of B, the x-intercept of line AS. (2)

3.3 Calculate the coordinates of C. (4)

3.4 Determine the equation of the line parallel to BC and passing through S(–15 ; –2). Write your answer in the form $y = mx + c$. (5)

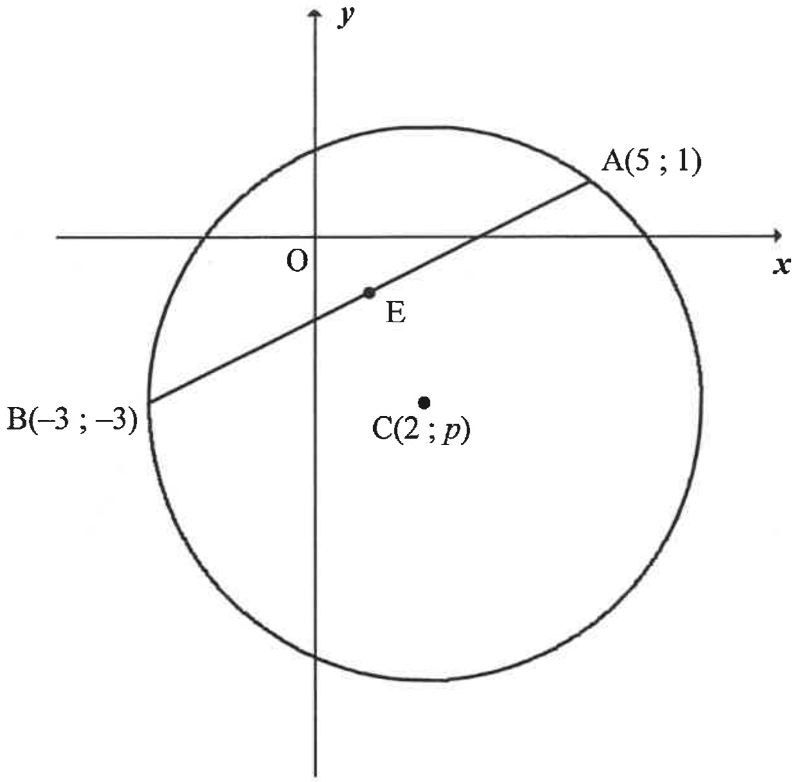
3.5 Calculate the size of $\hat{BAC}$. (5)

3.6 If it is further given that the length of AC is $6\sqrt{5}$ units, calculate the value of $\frac{\text{Area of } \Delta ABD}{\text{Area of } \Delta ASC}$. (5)

[22]

QUESTION 4

In the diagram, the circle centred at $C(2\,;p)$ is drawn. $A(5\,;1)$ and $B(-3\,;-3)$ are points on the circle. E is the midpoint of AB.



4.1 Calculate the coordinates of E, the midpoint of AB. (2)

4.2 Calculate the length of AB. Leave your answer in surd form. (1)

4.3 Determine the equation of the perpendicular bisector of AB in the form $y = mx + c$. (4)

4.4 Show that $p = -3$. (1)

4.5 Show, by calculation, that the equation of the circle is $x^2 + y^2 - 4x + 6y - 12 = 0$ (4)

4.6 Calculate the values of t for which the straight line $y = tx + 8$ will not intersect the circle. (6)

[18]

QUESTION 5

5.1 If $\sin 40° = p$, write EACH of the following in terms of p.

5.1.1 $\sin 220°$ (2)

5.1.2 $\cos^2 50°$ (2)

5.1.3 $\cos(-80°)$ (3)

5.2 Given: $\tan x(1-\cos^2 x)+\cos^2 x = \dfrac{(\sin x+\cos x)(1-\sin x\cos x)}{\cos x}$

5.2.1 Prove the above identity. (5)

5.2.2 For which values of x, in the interval $x \in [-180°; 180°]$, will the identity be undefined? (3)

5.3 Given the expression: $\dfrac{\sin 150° + \cos^2 x - 1}{2}$

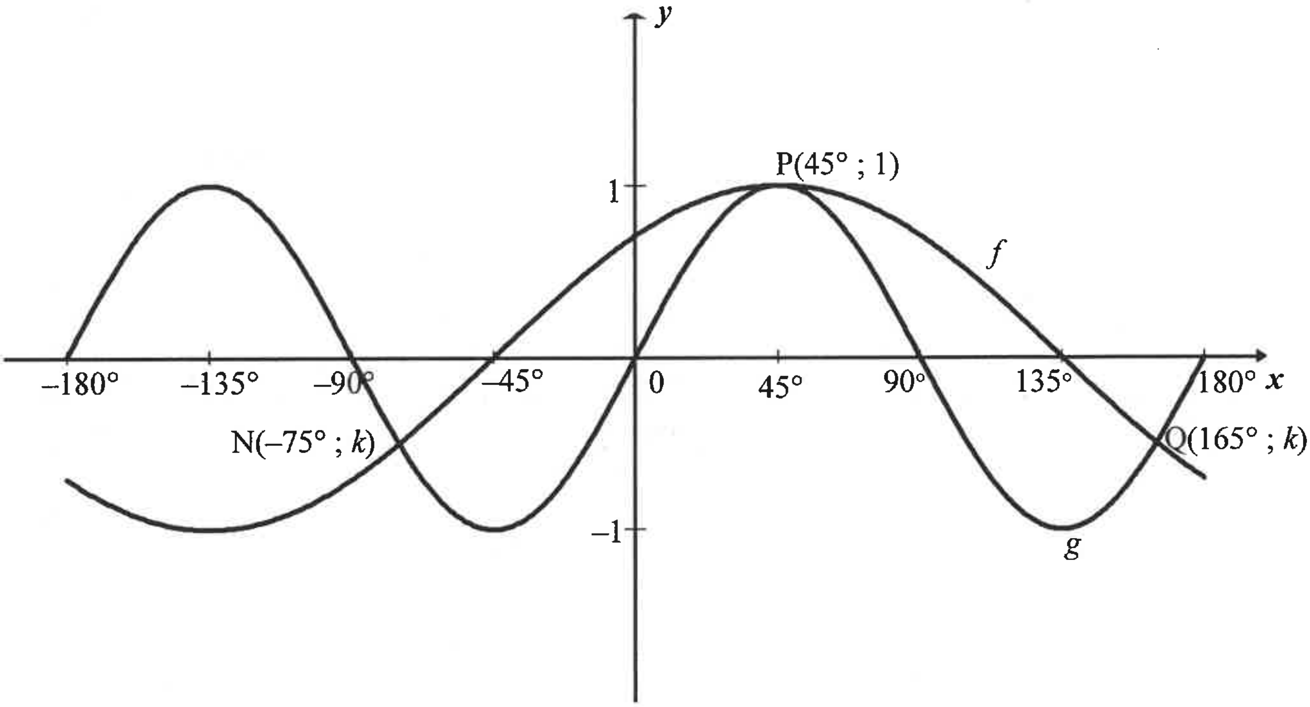
5.3.1 **Without using a calculator**, simplify the expression given above to a single trigonometric term in terms of $\cos 2x$. (6)

5.3.2 Hence, determine the general solution of $\dfrac{\sin 150° + \cos^2 x - 1}{2} = \dfrac{1}{25}$ (5)

[26]

QUESTION 6

In the diagram, the graphs of $f(x) = \cos(x + a)$ and $g(x) = \sin 2x$ are drawn for the interval $x \in [-180°; 180°]$. The graphs intersect at N(–75° ; k), P(45° ; 1) and Q(165° ; k). P is also a turning point of both graphs.



6.1 Write down the period of f. (1)

6.2 Write down the amplitude of g. (1)

6.3 Write down the value of a. (1)

6.4 Calculate the value of k, the y-coordinate of N and Q, **without the use of a calculator**. (2)

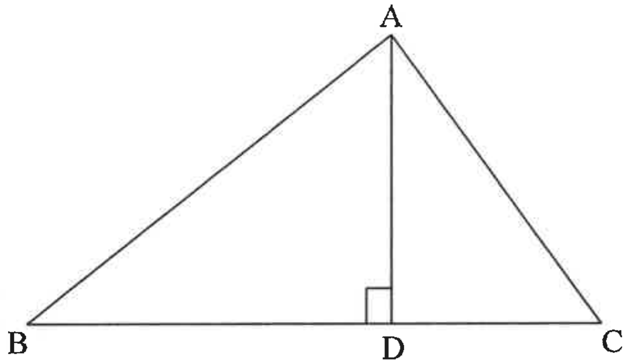
6.5 Calculate the value of x if $g(x + 60°) = f(x + 60°)$ and $x \in [-45° ; 0°]$. (1)

6.6 **Without using a calculator**, determine the number of solutions the equation $\sqrt{2}\sin 2x = \sin x + \cos x$ has in the interval $x \in [-90° ; 90°]$. Clearly show ALL working. (4)

[10]

QUESTION 7

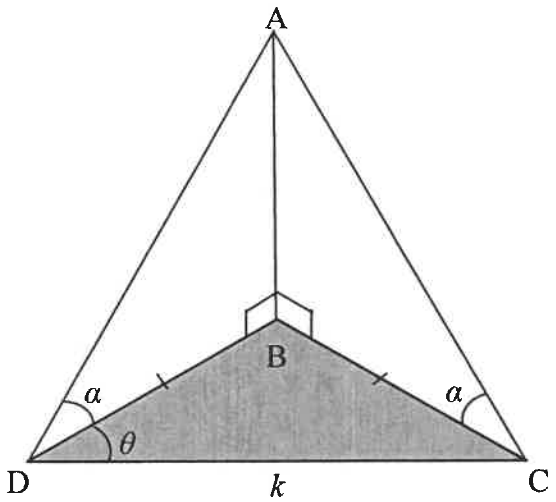
7.1 In the diagram, ΔABC is drawn. AD is drawn such that $AD \perp BC$.



7.1.1 Use the diagram above to determine AD in terms of $\sin \hat{B}$ (2)

7.1.2 Hence, prove that the area of $\Delta ABC = \frac{1}{2}(BC)(AB)\sin \hat{B}$ (1)

7.2 In the diagram, points B, C and D lie in the same horizontal plane.
$A\hat{D}B = A\hat{C}B = \alpha$, $C\hat{D}B = \theta$ and $DC = k$ units. BD = BC.



7.2.1 Prove that AD = AC (2)

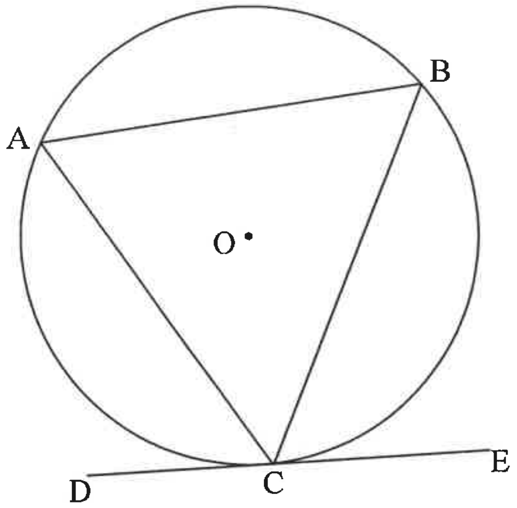
7.2.2 Prove that $BD = \dfrac{k}{2\cos\theta}$ (3)

7.2.3 Determine the area of ΔBCD in terms of k and a single trigonometric ratio of θ. (3)

[11]

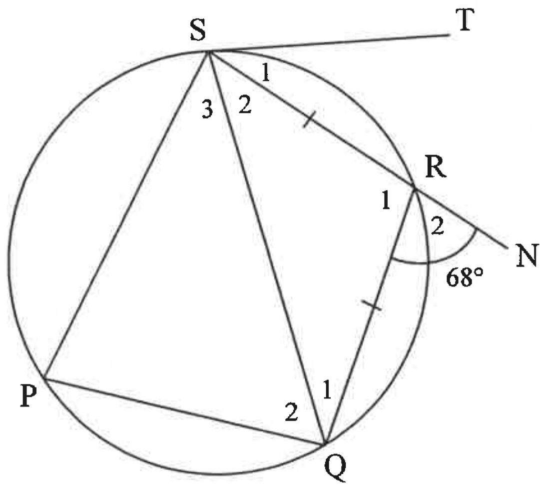
QUESTION 8

8.1 In the diagram, chords AB, BC and AC are drawn in the circle with centre O. DCE is a tangent to the circle at C.



Prove the theorem which states that the angle between the tangent to a circle and the chord drawn from the point of contact is equal to the angle in the alternate segment, i.e. $B\hat{C}E = \hat{A}$. (5)

8.2 In the diagram, PQRS is a cyclic quadrilateral with RQ = RS. ST is a tangent to the circle at S. SR is produced to N. $\hat{R}_2 = 68°$.



Determine, with reasons, the size of:

8.2.1 $\hat{P}$ (2)

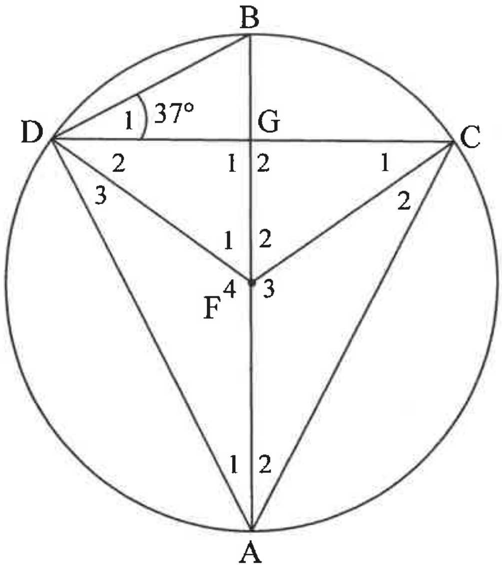
8.2.2 $\hat{Q}_1$ (2)

8.2.3 $\hat{S}_1$ (2)

[11]

QUESTION 9

In the diagram, AB is a diameter of the circle, with centre F. AB and CD intersect at G. FD and FC are drawn. BA bisects $\hat{CAD}$ and $\hat{D}_1 = 37°$.



9.1 Determine, giving reasons, any three other angles equal to $\hat{D}_1$. (4)

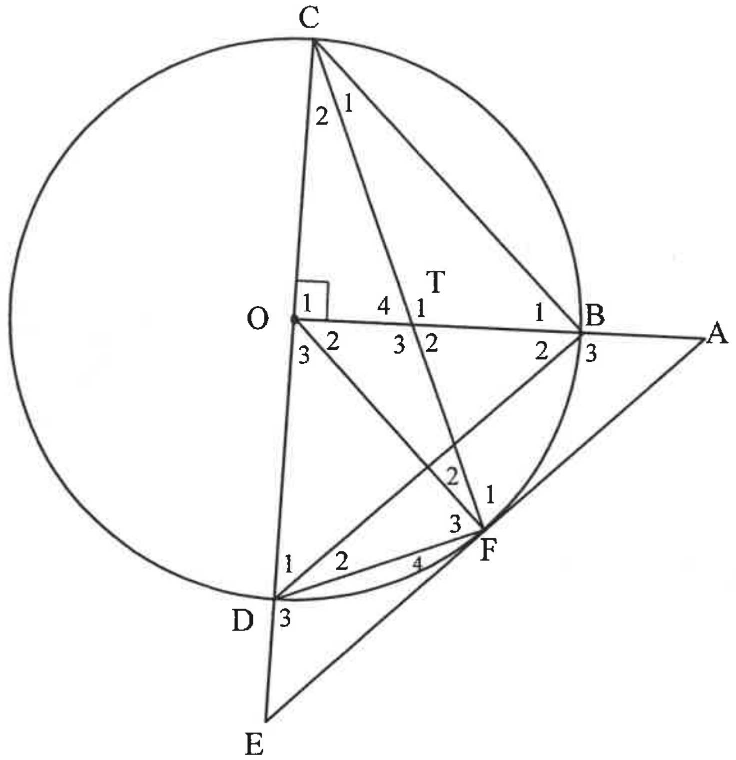
9.2 Show that DG = GC. (4)

9.3 If it is further given that the radius of the circle is 20 units, calculate the length of BG. (4)

[12]

QUESTION 10

In the diagram, COD is the diameter of the circle with centre O. EA is a tangent to the circle at F. AO ⊥ CE. Diameter COD produced intersects the tangent to the circle at E. OB produced intersects the tangent to the circle at A. CF intersects OB in T. CB, BD, OF and FD are drawn.



Prove, with reasons, that:

10.1 TODF is a cyclic quadrilateral (4)

10.2 $\hat{D}_3 = \hat{T}_1$ (3)

10.3 $\Delta TFO \ ||| \ \Delta DFE$ (5)

10.4 If $\hat{B}_2 = \hat{E}$, prove that DB || EA. (2)

10.5 Prove that $DO = \frac{TO.FE}{AB}$ (5)

[19]

TOTAL: 150

INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $\quad A = P(1-ni)$ $\quad A = P(1-i)^n$ $\quad A = P(1+i)^n$

$T_n = a + (n-1)d$ $\quad S_n = \frac{n}{2}[2a + (n-1)d]$

$T_n = ar^{n-1}$ $\quad S_n = \frac{a(r^n - 1)}{r-1}$; $r \neq 1$ $\quad S_\infty = \frac{a}{1-r}$; $-1 < r < 1$

$F = \frac{x[(1+i)^n - 1]}{i}$ $\quad P = \frac{x[1-(1+i)^{-n}]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad M\left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad y - y_1 = m(x - x_1)$ $\quad m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad m = \tan\theta$

$$(x-a)^2 + (y-b)^2 = r^2$$

InΔABC: $\quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$$a^2 = b^2 + c^2 - 2bc.\cos A$$

$$\text{area } \Delta ABC = \frac{1}{2}ab.\sin C$$

$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$ $\quad \sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$ $\quad \cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases} \qquad \sin 2\alpha = 2\sin\alpha.\cos\alpha$$

$\bar{x} = \frac{\sum x}{n}$ $\quad \sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $\quad$ P(A or B) = P(A) + P(B) – P(A and B)

$\hat{y} = a + bx$ $\quad b = \frac{\sum(x-\bar{x})(y-\bar{y})}{\sum(x-\bar{x})^2}$

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NASIONALE SENIORSERTIFIKAAT-EKSAMEN

MATHEMATICS P2/*WISKUNDE V2*

MARKING GUIDELINES/*NASIENRIGLYNE*

MAY/JUNE/*MEI/JUNIE* 2024

MARKS: 150
PUNTE: 150

These marking guidelines consist of 26 pages./
Hierdie nasienriglyne bestaan uit 26 bladsye.

NOTE:

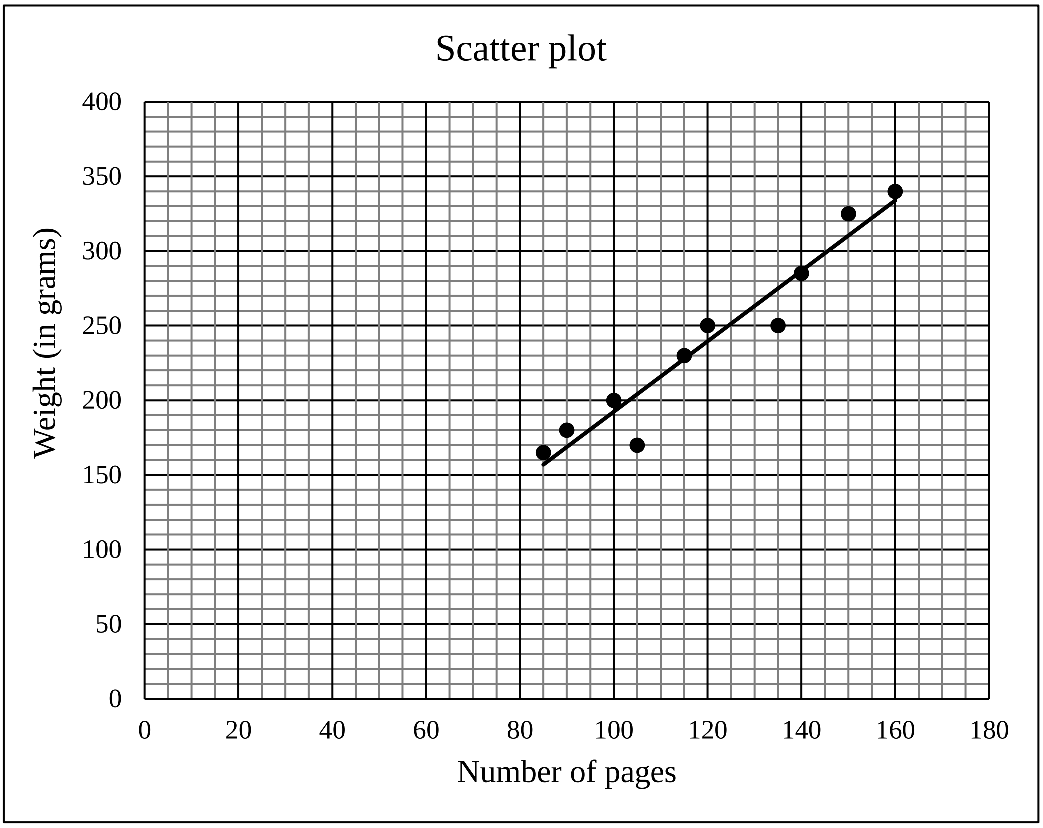
- If a candidate answers a question TWICE, only mark the FIRST attempt.
- If a candidate has crossed out an attempt of a question and did not redo the question, mark the crossed out version.
- Consistent accuracy applies in ALL aspects of the Marking Guidelines. Stop marking at the second calculation error.
- Assuming answers/values in order to solve a problem is NOT acceptable.

LET WEL:

- *As 'n kandidaat 'n vraag TWEE KEER beantwoord, sien slegs die EERSTE poging na.*
- *As 'n kandidaat 'n antwoord op 'n vraag doodtrek en nie oordoen nie, sien die doodgetrekte poging na.*
- *Volgehoue akkuraatheid word in ALLE aspekte van die Nasienriglyne toegepas. Hou op nasien by die tweede berekeningsfout.*
- *Aanvaar van antwoorde/waardes om 'n probleem op te los, word NIE toegelaat nie.*

GEOMETRY	
S	**A mark for a correct statement** **(A statement mark is independent of a reason)**
	'n Punt vir 'n korrekte bewering **('n Punt vir 'n bewering is onafhanklik van die rede)**
R	**A mark for the correct reason** **(A reason mark may only be awarded if the statement is correct)**
	'n Punt vir 'n korrekte rede **('n Punt word slegs vir die rede toegeken as die bewering korrek is)**
S/R	**Award a mark if statement AND reason are both correct**
	Ken 'n punt toe as die bewering EN rede beide korrek is

QUESTION/*VRAAG* 1

1.1	$a = -43,72$ $b = 2,36$ $y = -43,72 + 2,36x$	✓ $a = -43,72$ ✓ $b = 2,36$ ✓ equation (3)
1.2		✓ any correct two points ✓ straight line joining the points for $x \in [85\ ;\ 160]$ (2)
1.3	$y = -43,72 + 2,36(110)$ $y = 215,88$ **OR** y = 215,90 (calculator)	✓ substitution ✓ answer (2) ✓✓ answer (2)
1.4	$y = -43,72 + 2,36(130)$ $y = 263,08$ Percentage increase in weight $= \dfrac{263,08 - 215,88}{215,88} \times 100$ $= 21,86\%$ **OR** $y = 263,08$ Percentage $= \dfrac{263,08}{215,88} \times 100$ $= 121,86\ \%$ Percentage increase in weight $= 121,86 - 100 = 21,86$	✓ y-value ✓ difference between y-values ✓ +ve answer (3) ✓ y-value ✓ difference between % ✓ +ve answer (3)
		[10]

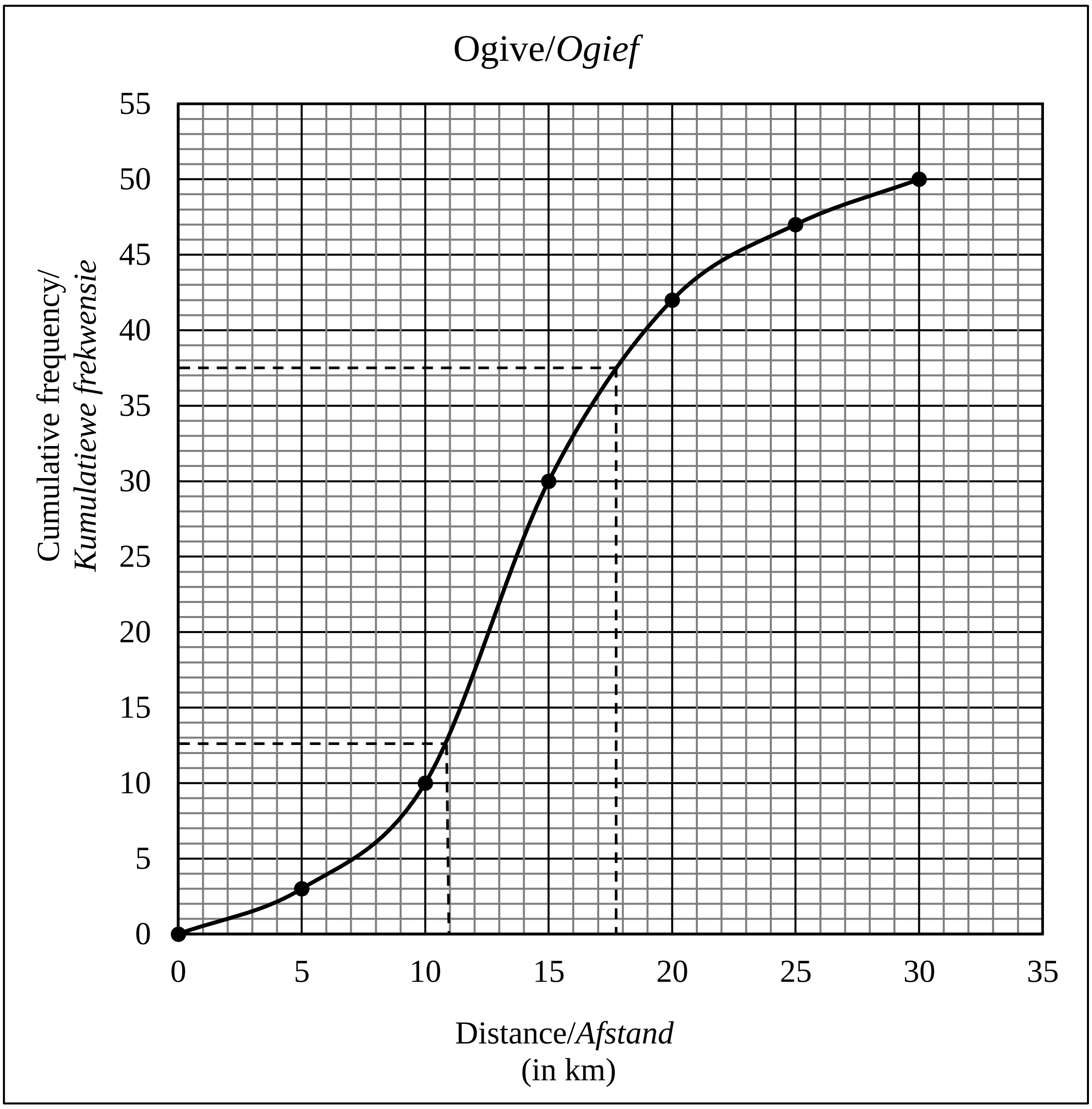
QUESTION/*VRAAG* 2

2.1

Distance (x km)	**Frequency**	**Cumulative frequency**
$0 \le x < 5$	3	3
$5 \le x < 10$	7	10
$10 \le x < 15$	20	30
$15 \le x < 20$	12	42
$20 \le x < 25$	5	47
$25 \le x < 30$	3	50

✓ 10
✓ all values correct
(2)

2.2



✓ grounding

✓ plotting a min of 3 points (cf at upper limits)

✓ smooth, increasing curve
(3)

2.3

$Q_3 = 17{,}8$

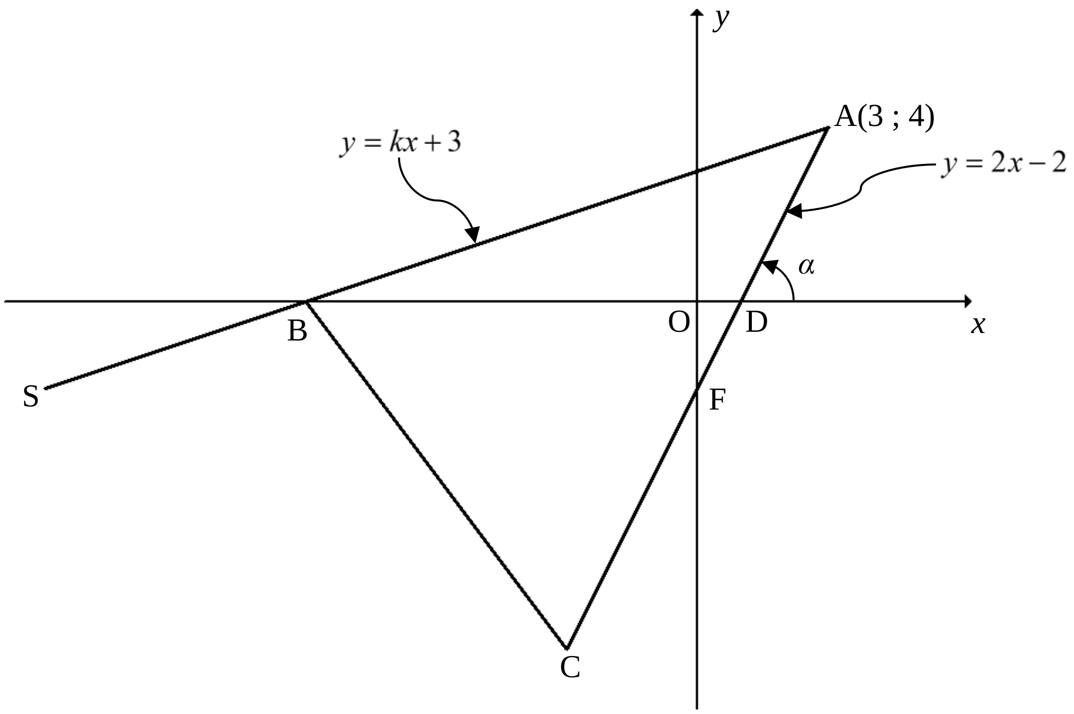
$Q_1 = 11$

IQR = 6,8

✓ Q_3 (accept between 17-19) and Q_1 (accept between 10-12,5)
✓ answer (accept 5-9)
(2)

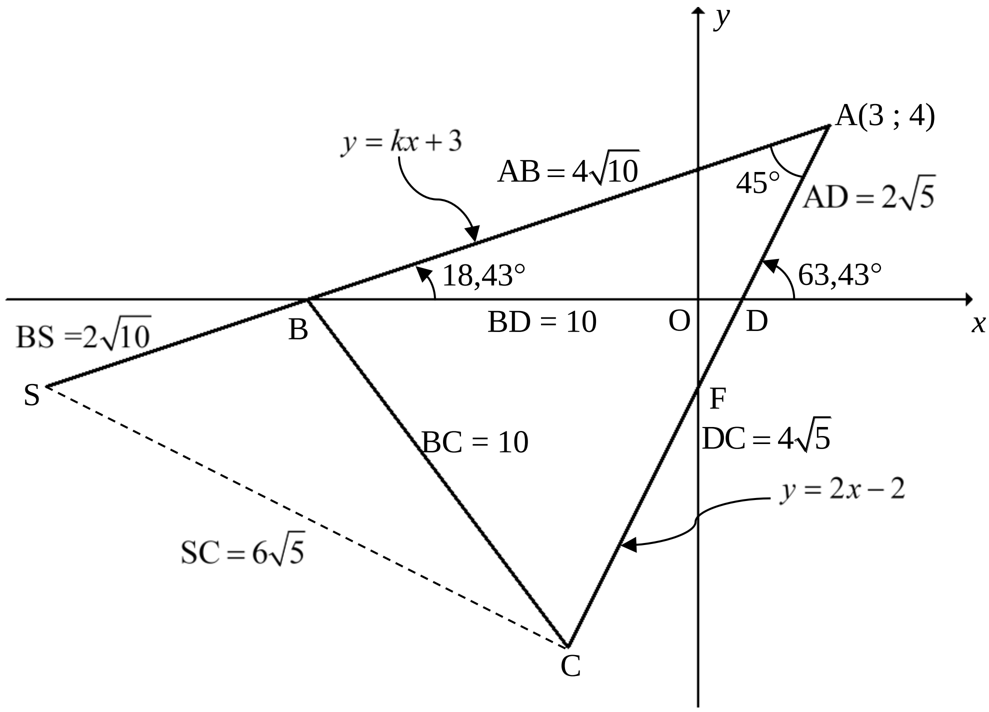
2.4	$5 \leq x < 10$	✓ $5 \leq x < 10$ (1)
2.5	Estimated mean $= \dfrac{2,5(3)+7,5(11)+12,5(20)+17,5(8)+22,5(5)+27,5(3)}{50}$ $= \dfrac{675}{50}$ $= 13,5$ km	✓ new frequencies ✓ $\sum fx$ ✓ answer (3)
		[11]

QUESTION/*VRAAG* 3



3.1	$y = kx + 3$ $4 = k(3) + 3$ $3k = 1$ $\therefore k = \frac{1}{3}$ **OR** y-intercept of AB: (0 ; 3) $m_{AB} = \frac{4-3}{3-0}$ $= \frac{1}{3}$ $\therefore k = \frac{1}{3}$	✓ substitution (3 ; 4) (1) ✓ substitution (0 ; 3) (1)
3.2	$0 = \frac{1}{3}x + 3$ $-3 = \frac{1}{3}x$ $x = -9$ $B(-9 ;\ 0)$	✓ $y = 0$ ✓ answer (2)

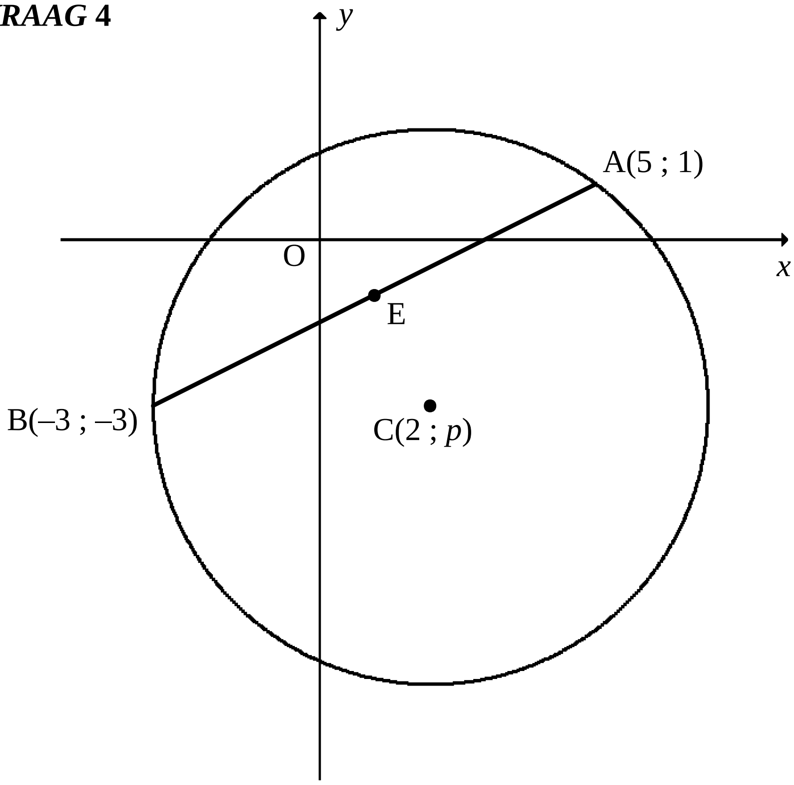
3.3	$F(0\,;-2)$ $F\left(\frac{x+3}{2}\,;\,\frac{y+4}{2}\right)$ $\frac{x+3}{2}=0 \qquad \frac{y+4}{2}=-2$ $x=-3 \qquad y=-8$ $C(-3\,;-8)$	✓ $F(0\,;-2)$ ✓ $\frac{x+3}{2}=0\ ;\ \frac{y+4}{2}=-2$ ✓ x-value ✓ y-value (4)
	OR by translation $F(0;-2)$ $A \to F\,(x;y)\to(x-3;y-6)$ $F\to C(0;-2)\to(0-3;-2-6)=(-3;-8)$	 ✓ $F(0;-2)$ ✓ $(x-3\,;y-6)$ ✓ x-value ✓ y-value (4)
3.4	$m_{BC}=\frac{0-(-8)}{-9-(-3)}$ OR $m_{BC}=\frac{-8-0}{-3-(-9)}$ $m_{BC}=-\frac{4}{3}$ $y=-\frac{4}{3}x+c$ $(-2)=-\frac{4}{3}(-15)+c$ $c=-22$ $y=-\frac{4}{3}x-22$	✓ substitution of B and C into the gradient formula ✓ m_{BC} ✓ $m_{line}=m_{BC}$ ✓ substitution of $S(-15;-2)$ ✓ equation (5)
	OR $m_{BC}=\frac{0-(-8)}{-9-(-3)}$ OR $m_{BC}=\frac{-8-0}{-3-(-9)}$ $m_{BC}=-\frac{4}{3}$ $y-y_1=-\frac{4}{3}(x-x_1)$ $y-(-2)=-\frac{4}{3}(x-(-15))$ $y+2=-\frac{4}{3}x-20$ $y=-\frac{4}{3}x-22$	 ✓ substitution into the gradient formula ✓ m_{BC} ✓ $m_{line}=m_{BC}$ ✓ substitution of $S(-15;-2)$ ✓ equation (5)



3.5	$\tan\alpha = m_{AC} = 2$ $\alpha = 63,43°$ $\tan A\hat{B}D = m_{AS} = \frac{1}{3}$ $A\hat{B}D = 18,43°$ $B\hat{A}C = \alpha - A\hat{B}D$ $B\hat{A}C = 63,43° - 18,43°$ $B\hat{A}C = 45°$	✓ $\tan\alpha = m_{AC} = 2$ ✓ $\alpha = 63,43°$ ✓ $\tan A\hat{B}D = m_{AS} = \frac{1}{3}$ ✓ $A\hat{B}D = 18,43°$ ✓ answer (5)
	OR $AB = \sqrt{(-9-3)^2 + (0-4)^2}$ $AB = 4\sqrt{10}$ BD = 10 $AD = \sqrt{(3-1)^2 + (4-0)^2}$ $AD = 2\sqrt{5}$ $BD^2 = AB^2 + AD^2 - 2AB.AD\cos B\hat{A}C$ $(10)^2 = (4\sqrt{10})^2 + (2\sqrt{5})^2 - 2(4\sqrt{10})(2\sqrt{5})\cos B\hat{A}C$ $\cos B\hat{A}C = \frac{\sqrt{2}}{2}$ $B\hat{A}C = 45°$	✓ length of AB ✓ calculation of remaining 2 lengths ✓ substitution into cosine-rule ✓ rewriting in terms of $\cos B\hat{A}C$ ✓ answer (5)

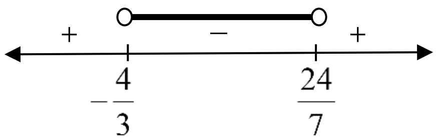
3.6	A(3 ; 4) and S(–15 ; – 2) $AS = \sqrt{(x_A - x_S)^2 + (y_A - y_S)^2}$ $AS = \sqrt{(3-(-15))^2 + (4-(-2))^2}$ $AS = \sqrt{360} = 6\sqrt{10} = 18{,}97$ $\dfrac{\text{Area of } \Delta ABD}{\text{Area of } \Delta ASC} = \dfrac{\frac{1}{2}(BD)(\perp h)}{\frac{1}{2}(AS)(AC)\sin B\hat{A}C}$ $\dfrac{\text{Area of } \Delta ABD}{\text{Area of } \Delta ASC} = \dfrac{\frac{1}{2}(10)(4)}{\frac{1}{2}(6\sqrt{10})(6\sqrt{5})\sin 45°}$ $\dfrac{\text{Area of } \Delta ABD}{\text{Area of } \Delta ASC} = \dfrac{2}{9}$	✓ $AS = \sqrt{(3-(-15))^2 + (4-(-2))^2}$ ✓ length of AS ✓ Area ΔABD ✓ Area ΔASC ✓ answer (5)
	OR $AS = \sqrt{(3-(-15))^2 + (4-(-2))^2}$ $AS = \sqrt{360} = 6\sqrt{10} = 18{,}97$ $AB = \sqrt{(-9-3)^2 + (0-4)^2} = 4\sqrt{10}$ $AD = \sqrt{(3-1)^2 + (4-0)^2} = 2\sqrt{5}$ $\dfrac{\text{Area of } \Delta ABD}{\text{Area of } \Delta ASC} = \dfrac{\frac{1}{2}(AB)(AD)\sin\hat{A}}{\frac{1}{2}(AS)(AC)\sin\hat{A}}$ $= \dfrac{\frac{1}{2}(4\sqrt{10})(2\sqrt{5})\sin\hat{A}}{\frac{1}{2}(6\sqrt{10})(6\sqrt{5})\sin\hat{A}}$ $= \dfrac{2}{9}$	✓ $AS = \sqrt{(3-(-15))^2 + (4-(-2))^2}$ ✓ length of AS ✓ Area ΔABD ✓ Area ΔASC ✓ answer (5)
		[22]

QUESTION/*VRAAG* 4



4.1	$E\left(\frac{5+(-3)}{2};\frac{1+(-3)}{2}\right)$ $\therefore E(1;-1)$	✓ $x=1$ ✓ $y=-1$ (2)
4.2	$AB=\sqrt{(x_A-x_B)^2+(y_A-y_B)^2}$ $AB=\sqrt{(5-(-3))^2+(1-(-3))^2}$ $AB=\sqrt{80}=4\sqrt{5}=8,94$ units	✓ $AB=\sqrt{80}=4\sqrt{5}=8,94$ (1)
4.3	$m_{AB}=\frac{1-(-3)}{5-(-3)}$ $m_{AB}=\frac{1}{2}$ $\therefore m_{CE}=-2$ [CE $\perp$ AB] $E(1;-1)$ $y=-2x+c$ **OR** $y-y_1=-2(x-x_1)$ $(-1)=-2(1)+c$ $y-(-1)=-2(x-1)$ $c=1$ $y=-2x+1$ $y=-2x+1$	✓ $m_{AB}=\frac{1}{2}$ ✓ m_{CE} ✓ substitution of E ✓ equation (4)

4.4	$y=-2x+1$ $p=-2(2)+1$ $p=-3$ **OR** $m_{\text{CE}}=-2$ $\dfrac{p-(-1)}{2-1}=-2$ $p+1=-2$ $p=-3$	✓ substitution of C(2 ; p) into ⊥ bisector of AB (1) ✓ substitution of C and E into the gradient formula (1)
4.5	BC = r = 5 units $\therefore (x-2)^2+(y+3)^2=25$ $x^2-4x+4+y^2+6y+9=25$ $x^2+y^2-4x+6y-12=0$	✓ BC = r = 5 units ✓ $(x-2)^2+(y+3)^2$ ✓ r^2 ✓ $x^2-4x+4+y^2+6y+9=25$ (4)

4.6	$(x-2)^2+(y+3)^2=25$ **OR** $x^2+y^2-4x+6y-12=0$ $y=tx+8$ $(x-2)^2+(tx+8+3)^2=25$ **OR** $x^2+(tx+8)^2-4x+6(tx+8)-12=0$ $x^2-4x+4+t^2x^2+22tx+121-25=0$ **OR** $x^2+t^2x^2+16tx+64-4x+6tx+48-12=0$ $x^2(t^2+1)+x(22t-4)+100=0$ $\Delta<0$ $(22t-4)^2-4(t^2+1)(100)<0$ $484t^2-176t+16-400t^2-400<0$ $84t^2-176t-384<0$ $21t^2-44t-96<0$ $(7t-24)(3t+4)<0$ CV: $\dfrac{24}{7}$; $-\dfrac{4}{3}$  $\therefore t\in\left(-\dfrac{4}{3}\ ;\ \dfrac{24}{7}\right)$ **OR** $-\dfrac{4}{3}<t<\dfrac{24}{7}$	✓ substitution of $y=tx+8$ ✓ standard form ✓ $\Delta<0$ ✓standard form of Δ ✓ critical values ✓ answer (6)
		[18]

QUESTION/*VRAAG* 5

5.1.1	$\sin 220°$ $= -\sin 40°$ $= -p$	✓ $-\sin 40°$ ✓ answer (2)
5.1.2	$\cos^2 50°$ $= \sin^2 40°$ $= p^2$	✓ $\sin^2 40$ ✓ answer (2)
5.1.3	$\cos(-80°)$ $= \cos 80°$ $= 1 - 2\sin^2 40°$ $= 1 - 2p^2$ **OR** $\cos(-80°)$ $= \cos 80°$ $= \cos(30° + 50°)$ $= \cos 30° \cos 50° - \sin 30° \sin 50°$ $= \frac{\sqrt{3}p}{2} - \frac{\sqrt{1-p^2}}{2}$	✓ $\cos 80°$ ✓ double angle ✓ answer (3) ✓ $\cos 80°$ ✓ expansion ✓ answer (3)
5.2.1	LHS = $\tan x(1-\cos^2 x) + \cos^2 x$ $= \frac{\sin x}{\cos x}(\sin^2 x) + \cos^2 x$ $= \frac{\sin^3 x + \cos^3 x}{\cos x}$ $= \frac{(\sin x + \cos x)(\sin^2 x - \sin x \cos x + \cos^2 x)}{\cos x}$ $= \frac{(\sin x + \cos x)(1 - \sin x \cos x)}{\cos x}$ $=$ RHS **OR**	✓ $\frac{\sin x}{\cos x}$ ✓ $\sin^2 x$ ✓ simplification ✓ factorisation of cubes ✓ $\sin^2 x + \cos^2 x = 1$ (5)

	$\text{RHS} = \dfrac{(\sin x+\cos x)(1-\sin x\cos x)}{\cos x}$ $= \dfrac{\sin x-\sin^2 x\cos x+\cos x-\sin x\cos^2 x}{\cos x}$ $= \tan x-\sin^2 x+1-\sin x\cos x$ $= \tan x+\cos^2 x-\sin x\cos x$ $= \tan x\left(1-\dfrac{\sin x\cos x}{\tan x}\right)+\cos^2 x$ $= \tan x\left(1-\dfrac{\sin x\cos x}{\frac{\sin x}{\cos x}}\right)+\cos^2 x$ $= \tan x\left(1-\cos^2 x\right)+\cos^2 x$ $= \text{LHS}$	✓ multiplication ✓ $\div$ by $\cos x$ ✓ $-\sin^2 x+1=\cos^2 x$ ✓ factorisation ✓ $\tan x=\dfrac{\sin x}{\cos x}$ (5)
5.2.2	$\cos x=0$ or where $\tan x$ is undefined $x=90°+k.360°$ or $x=270°+k.360°$ $x=90°$ or $x=-90°$	✓ $\cos x=0$ or $\tan x$ undefined ✓ $x=90°$ ✓ $x=-90°$ (3)
5.3.1	$\dfrac{\sin 150°+\cos^2 x-1}{2}$ $=\dfrac{\sin 30°+\cos^2 x-1}{2}$ $=\dfrac{\frac{1}{2}-\left(1-\cos^2 x\right)}{2}$ $=\left(\dfrac{1}{2}-\sin^2 x\right)\times\dfrac{1}{2}$ $=\dfrac{1-2\sin^2 x}{4}$ $=\dfrac{\cos 2x}{4}$	✓ $\sin 30°$ ✓ $\sin 30°=\dfrac{1}{2}$ ✓ factor ✓ $1-\cos^2 x=\sin^2 x$ ✓ simplification ✓ answer in terms of cos $2x$ (6)
5.3.2	$\dfrac{\sin 150°+\cos^2 x-1}{2}=\dfrac{1}{25}$ $\dfrac{\cos 2x}{4}=\dfrac{1}{25}$ $\cos 2x=\dfrac{4}{25}$ $\text{ref}\angle=80,79...°$ $2x=80,79...°+k.360°$ or $2x=279,20...°+k.360°$ $x=40,40°+k.180°$ or $x=139,60°+k.180°$; $k\in Z$	✓ answer $5.3.1=\dfrac{1}{25}$ ✓ $2x=80,79°$ ✓ $2x=279,20...°$ ✓ $x=40,40°$ and $x=139,60°$ ✓ $+k.180°$; $k\in Z$ (5)

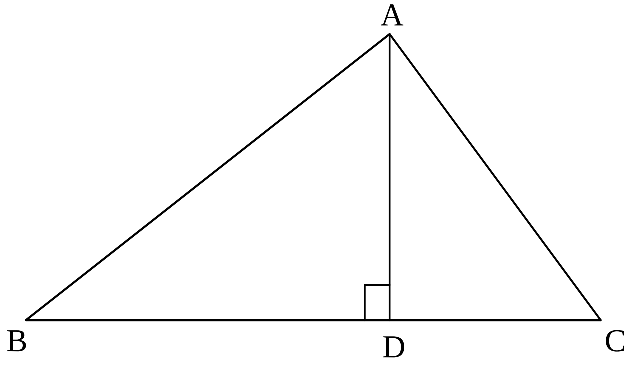
	OR $\frac{\sin 150° + \cos^2 x - 1}{2} = \frac{1}{25}$ $\sin 150° + \cos^2 x - 1 = \frac{2}{25}$ $\sin 30° + \cos^2 x - 1 = \frac{2}{25}$ $\cos^2 x = \frac{29}{50}$ $\cos x = \pm\sqrt{\frac{29}{50}}$ $x = 40,40° + k.360°$ or $x = 319,60° + k.360°$; $k \in Z$ or $x = 139,60° + k.360°$ or $x = 220,40° + k.360°$; $k \in Z$	✓ $\cos^2 x = \frac{29}{50}$ ✓ $x = 40,40°$ ✓ $x = 139,60°$ ✓ x = 220,40° and $x = 319,60°$ ✓ + k.360° ; $k \in Z$ (5)
		[26]

QUESTION/*VRAAG* 6

6.1	Period = $360°$	✓ $360°$ (1)
6.2	Amplitude $=1$	✓ 1 (1)
6.3	$a=-45°$	✓ $a=-45°$ (1)
6.4	$\sin 2x = k$ $k=\sin(2\times 165°)$ **OR** $k=\sin\left(2\times\left(-75°\right)\right)$ $k=\sin 330°$ $k=\sin\left(-150°\right)$ $k=-\sin 30°$ $k=-\frac{1}{2}$ **OR** $k=\cos\left(165°-45°\right)$ **OR** $k=\cos\left(-75°-45°\right)$ $k=\cos 120°$ $k=\cos\left(-120°\right)$ $k=-\cos 60°$ $k=-\frac{1}{2}$	✓ $-\sin 30°$ ✓ $-\frac{1}{2}$ (2) ✓ $-\cos 60°$ ✓ $-\frac{1}{2}$ (2)
6.5	Points of intersection are translated $60°$ to the left $x=-15°$	✓ $x=-15°$ (1)
6.6	$\sqrt{2}\sin 2x=\sin x+\cos x$ $\sin 2x=\frac{1}{\sqrt{2}}\sin x+\frac{1}{\sqrt{2}}\cos x$ $\sin 2x=\sin 45°\sin x+\cos 45°\cos x$ $\sin 2x=\cos\left(45°-x\right)$ **OR** $\sin 2x=\cos\left(x-45°\right)$ $\therefore 2$ roots in the interval $x\in[-90°;90°]$	✓ division by $\sqrt{2}$ ✓ special angles ✓ $\cos\left(45°-x\right)$ or $\cos\left(x-45°\right)$ ✓ answer (4)
		[10]

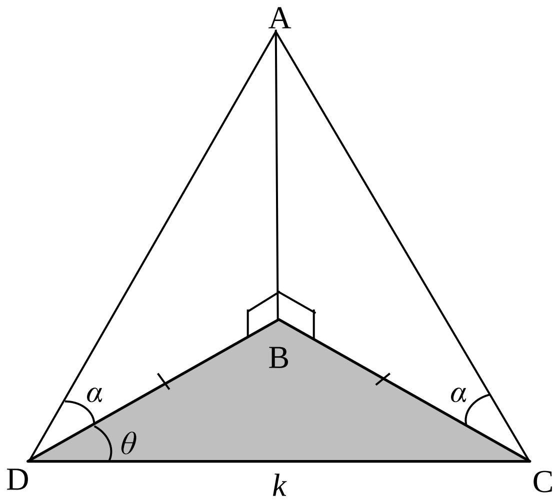
QUESTION/*VRAAG* 7

7.1



7.1.1	$\sin \hat{B} = \frac{AD}{AB}$ $AD = AB \sin \hat{B}$	✓ $\sin \hat{B} = \frac{AD}{AB}$ ✓ answer (2)
7.1.2	Area of $\Delta ABC = \frac{1}{2}(BC)(AD)$ $\therefore$ Area of $\Delta ABC = \frac{1}{2}(BC)(AB)\sin \hat{B}$	✓ $\frac{1}{2}(BC)(AD)$ (1)

7.2



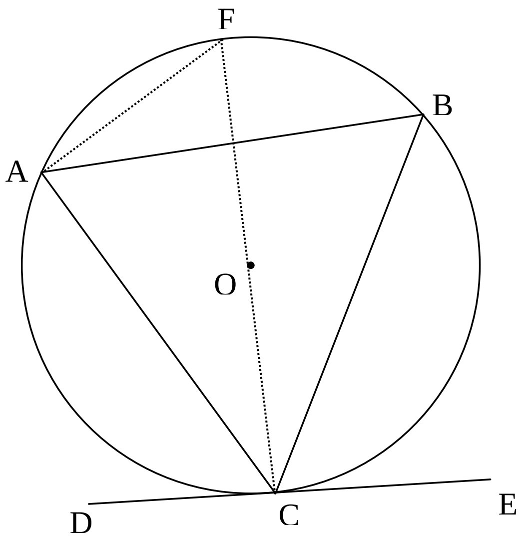
7.2.1	In ΔADB $\sin \alpha = \frac{AB}{AD}$ $AD = \frac{AB}{\sin \alpha}$ In ΔABC $\sin \alpha = \frac{AB}{AC}$ $AC = \frac{AB}{\sin \alpha}$ $AD = AC$ **OR** In ΔADB and ΔACB	✓ $\sin \alpha = \frac{AB}{AD}$ ✓ $\sin \alpha = \frac{AB}{AC}$ (2)

	AB = AB [common side] $\hat{ABD} = \hat{ABC} = 90°$ [given] BD = BC [given] $\Delta ADB \equiv \Delta ACB$ [S∠S] $\therefore$ AD = AC	✓ $\Delta ADB \equiv \Delta ACB$ ✓ R (2)
	OR In ΔADB and ΔACB $A\hat{D}B = A\hat{C}B = \alpha$ [given] $A\hat{B}D = A\hat{B}C = 90°$ [given] AB = AB **OR** BD = BC [common side **OR** given] $\therefore$ $\Delta ADB \equiv \Delta ACB$ [∠∠S] $\therefore$ AD = AC	✓ $\Delta ADB \equiv \Delta ACB$ ✓ R (2)
	OR $AD^2 = AB^2 + DB^2$ [Pythagoras] $AC^2 = AB^2 + BC^2$ [Pythagoras] But DB = BC [given] $\therefore AD^2 = AC^2$ $\therefore$ AD = AC	✓ both Pythagoras statements ✓ DB = BC (2)
7.2.2	$\frac{BD}{\sin\theta} = \frac{k}{\sin(180° - 2\theta)}$ $BD = \frac{k\sin\theta}{\sin 2\theta}$ $BD = \frac{k\sin\theta}{2\sin\theta\cos\theta}$ $BD = \frac{k}{2\cos\theta}$	✓ substitution of $(180° - 2\theta)$ into sine rule ✓ reduction ✓ double angle (3)
	OR $BC^2 = k^2 + BD^2 - 2k(BD)\cos\theta$ $BD^2 = k^2 + BD^2 - 2k(BD)\cos\theta$ $k^2 - 2k(BD)\cos\theta = 0$ $2k(BD)\cos\theta = k^2$ $\therefore BD = \frac{k}{2\cos\theta}$	✓ substitution into cosine-rule ✓ substitution BC with BD into cosine-rule ✓ simplification in terms of BD (3)

7.2.3	Area of $\Delta BCD = \frac{1}{2}(DC)(BD)\left(\sin C\hat{D}B\right)$ $= \frac{1}{2}k\left(\frac{k}{2\cos\theta}\right)\sin\theta$ $= \frac{1}{4}k^2\tan\theta$	✓ substitution into area rule ✓ $\frac{\sin\theta}{\cos\theta} = \tan\theta$ ✓ $\frac{1}{4}k^2\tan\theta$ (3)
	OR	
	Area of $\Delta BCD = \frac{1}{2}(BD)(BC)\left(\sin(180° - 2\theta)\right)$ $= \frac{1}{2}\left(\frac{k}{2\cos\theta}\right)\left(\frac{k}{2\cos\theta}\right)(\sin 2\theta)$ $= \frac{2k^2\sin\theta\cos\theta}{8\cos\theta\cos\theta}$ $= \frac{1}{4}k^2\tan\theta$	✓ substitution into area rule ✓ $\frac{\sin\theta}{\cos\theta} = \tan\theta$ ✓ $\frac{1}{4}k^2\tan\theta$ (3)
		[11]

QUESTION/*VRAAG* 8

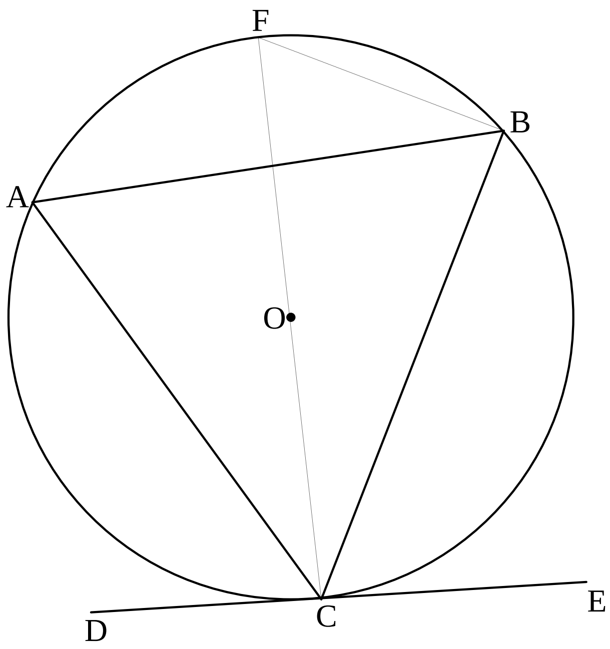
8.1



	Construction: Draw diameter CF and draw AF *Konstruksie: Trek middellyn* CF *en verbind* AF $\hat{FCE} = 90°$ [tan ⊥ radius/*raaklyn ⊥ radius*] $\hat{FAC} = 90°$ [∠ in semi circle/∠ *in halwe sirkel*] $\hat{FAB} = \hat{FCB}$ [∠s same segment/∠*e dieselfde segm*] $\therefore \hat{BAC} = \hat{BCE}$ $\therefore \hat{BCE} = \hat{A}$	✓Constr ✓ S ✓ R ✓ S/R ✓ S/R (5)

OR

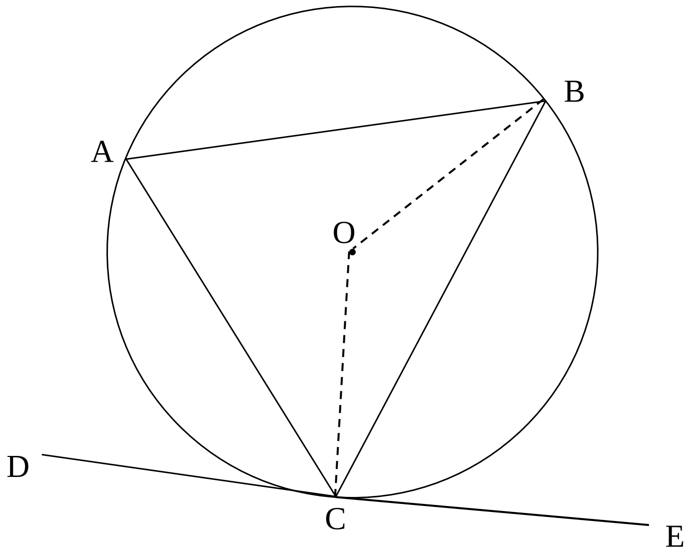
8.1



	Construction: Draw diameter CF and draw FB *Konstruksie: Trek middellyn* CF *en verbind* FB $F\hat{B}C = 90°$ [∠ in semi circle/∠ *in halwe sirkel*] $B\hat{F}C + F\hat{C}B = 90°$ [sum of ∠s in Δ/*binne ∠e v Δ*] $O\hat{C}E = 90°$ [tan ⊥ radius/ *raaklyn ⊥ radius*] $\therefore B\hat{C}E = \hat{F}$ but $\hat{A} = \hat{F}$ [∠s in same seg/∠ *in dies. segment*] $\therefore B\hat{C}E = \hat{A}$	✓ construction ✓ S / R ✓ S ✓ R ✓ S / R (5)

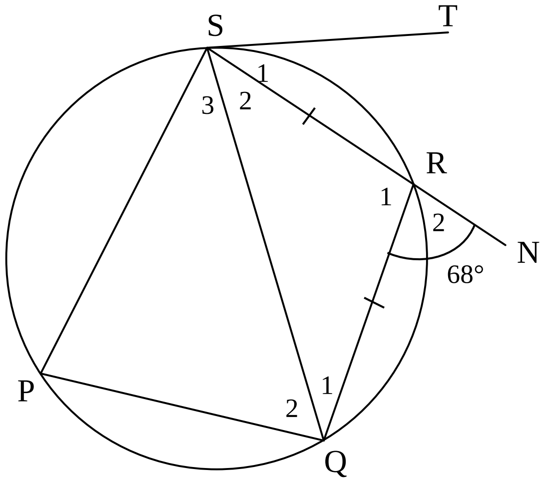
OR

8.1



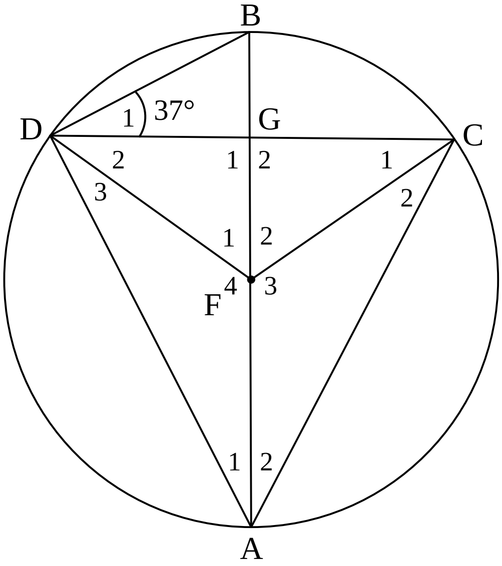
	Construction: Draw radii BO and OC *Konstruksie: Trek radiusse BO en OC* $O\hat{C}E = 90°$ or $B\hat{C}E = 90° - O\hat{C}B$ [tan ⊥ radius / *raaklyn ⊥ radius*] $O\hat{C}B = O\hat{B}C$ [∠s opp equal sides/ *∠e teenoor gelyke sye*] $\therefore\ C\hat{O}B = 180° - 2O\hat{C}B$ [∠s of Δ/*∠e van Δ*] $C\hat{A}B = 90° - O\hat{C}B$ [∠ at centre = 2×∠ circumf/ *midpts∠ = 2× omtreks∠*] $\therefore B\hat{C}E = C\hat{A}B$	✓ construction ✓ S ✓R ✓ S ✓ S/R (5)

8.2



8.2.1	$\hat{P} = \hat{R}_2 = 68°$ [ext ∠ of cyclic quad /*buite ∠ van kvh*]	✓ S ✓ R (2)
8.2.2	$\hat{Q}_1 = \hat{S}_2$ [∠s opp equal sides /∠*e teenoor gelyke sye*] $\hat{Q}_1 + \hat{S}_2 = 68°$ [ext ∠ of Δ/ *buite ∠ van* Δ] $\therefore \hat{Q}_1 = 34°$	✓ S ✓ S (2)
8.2.3	$\hat{S}_1 = \hat{Q}_1 = 34°$ [tan-chord theorem/∠ *tussen rkl en koord*]	✓ S ✓ R (2)
		[11]

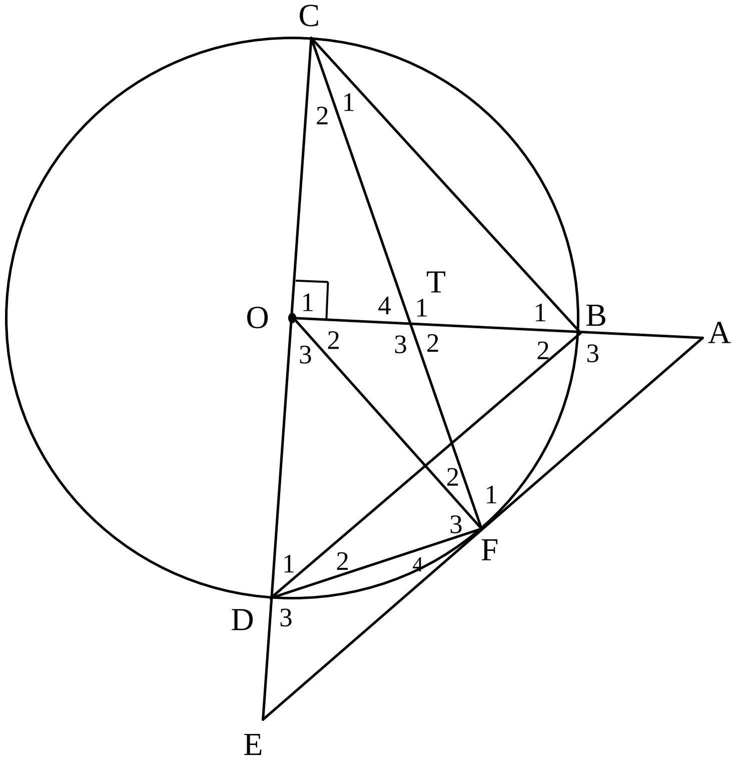
QUESTION/*VRAAG* 9



9.1	$\hat{A}_2 = \hat{D}_1 = 37°$ [$\angle$s in the same seg/$\angle$*e in dies segment*] $\hat{A}_1 = \hat{A}_2 = 37°$ [BA bisects $C\hat{A}D$/BA *halveer* $C\hat{A}D$] $\hat{D}_3 = \hat{A}_1 = 37°$ [$\angle$s opp equal sides/$\angle$*e teenoor gelyke sye*] $\hat{C}_2 = \hat{A}_2 = 37°$ [$\angle$s opp equal sides/$\angle$*e teenoor gelyke sye*]	✓ S ✓ R ✓✓ any other two statements (4)
9.2	$A\hat{D}G = 53°$ [$\angle$ in semi circle/$\angle$ *in halwe sirkel*] $\hat{A}_1 = 37°$ [proved in 9.1/*reeds bewys in 9.1*] $\therefore \hat{G}_1 = 90°$ [sum of $\angle$s in Δ/*binne* $\angle$*e van* Δ] $\therefore$ CG = DG [line from centre $\perp$ to chord/ *lyn uit midpt.* $\perp$ *op koord*] **OR** $\hat{F}_2 = 2\hat{D}_1 = 74°$ [$\angle$ at centre = 2 ×$\angle$ at circumference/ *midpt.* $\angle$*s = 2×omtreks*$\angle$] $\hat{D}_3 = 37°$ [proved in 9.1/*reeds bewys in 9.1*] $\therefore \hat{D}_2 = 16°$ [$\angle$ in semi circle/$\angle$ *in halwe sirkel*] $\hat{C}_1 = \hat{D}_2 = 16°$ [$\angle$s opp equal sides/$\angle$*e teenoor gelyke sye*] $\therefore \hat{G}_2 = 90°$ [sum of $\angle$s in Δ/*binne* $\angle$*e van* Δ] $\therefore$ CG = DG [line from centre $\perp$ to chord/ *lyn uit midpt.* $\perp$ op *koord*]	✓ S ✓ R ✓ S ✓ R (4) ✓ S ✓ R ✓ S ✓ R (4)

9.3	$\hat{F}_2 = 2\hat{D}_1 = 74°$ **OR** $\hat{F}_2 = 2\hat{A}_2 = 74°$ [∠ at centre = 2 ×∠ at circum./ *midpt. ∠s = 2×omtreks∠*] $\frac{FG}{20} = \cos 74°$ $FG = 5,51$ $\therefore BG = 14,49$ units	✓ S ✓ trig ratio ✓ FG ✓ answer (4)
	OR	
	$\hat{F}_2 = 2\hat{D}_1 = 74°$ [∠ at centre = 2 ×∠ at circumference *midpt. ∠ = 2×omtreks∠*] $\frac{FG}{20} = \sin 16°$ $FG = 5,51$ $\therefore BG = 14,49$ units	✓ S ✓ trig ratio ✓ FG ✓ answer (4)
	OR	
	$\frac{DG}{20} = \cos 16°$ DG = 19,23 $\frac{BG}{19,23} = \tan 37°$ BG = 14,49 units	✓ trig ratio ✓ length of DG ✓ trig ratio ✓ answer (4)
	OR	
	$\frac{DG}{20} = \cos 16°$ DG = 19,23 $FG^2 = FD^2 - DG^2$ [Pythagoras] $FG^2 = 20^2 - (19,23)^2$ FG = 5,51 BG = $20 - 5,51$ = 14,49 units	✓ trig ratio ✓ length of DG ✓ correct use of Pythagoras ✓ answer (4)
		[12]

QUESTION/*VRAAG* 10



10.1	$\hat{O}_1 = 90°$ [given/*gegee*] $\hat{F}_2 + \hat{F}_3 = 90°$ [$\angle$ in semi circle/$\angle$ *in halwe sirkel*] $\hat{O}_1 = \hat{F}_2 + \hat{F}_3 = 90°$ $\therefore$ TODF is a cyclic quad [ext $\angle$ = int opp $\angle$/ *buite $\angle$ = teenoorst. binne $\angle$*] **OR** [converse ext $\angle$ of cyclic quad/ *omgekeerde buite $\angle$ v kvh*]	✓ S ✓ R ✓ S ✓ R (4)
10.2	$\hat{T}_1 = \hat{T}_3$ [vert opp $\angle$s =/ *regoorstaande $\angle$e*] But $\hat{D}_3 = \hat{T}_3$ [ext $\angle$ of cyclic quad/ *buite $\angle$ v kvh*] $\therefore \hat{T}_1 = \hat{D}_3$	✓ S / R ✓ S ✓ R (3)
10.3	In ΔDFE and ΔTFO 1) $\hat{D}_3 = \hat{T}_3$ [ext $\angle$ of cyclic quad/ *buite $\angle$ v kvh*] 2) $\hat{F}_4 = \hat{C}_2$ [tan-chord theorem/ *$\angle$ tussen rkl en koord*] but $\hat{C}_2 = \hat{F}_2$ [$\angle$s opp equal sides/ *$\angle$e teenoor gelyke sye*] $\therefore \hat{F}_4 = \hat{F}_2$ 3) $\hat{E} = \hat{O}_2$ [3rd $\angle$ of Δ/*$\angle$e van Δ*] ΔTFO ||| ΔDFE [$\angle\angle\angle$]	✓ S ✓ S / R ✓ S ✓ S ✓ S **OR** R (5)

	OR In ΔDFE and ΔTFO 1) $\hat{D}_3 = \hat{T}_3$ [ext ∠ of cyclic quad/*buite ∠ van Δ*] 2) $\hat{F}_4 = \hat{C}_2$ [tan-chord theorem/*∠ tussen rkl en koord*] $\hat{F}_2 + \hat{F}_3 = 90°$ [∠ in semi circle/*∠ in halwe sirkel*] $\hat{D}_1 + \hat{D}_2 = 90° - \hat{C}_2$ [sum of ∠s in Δ/ *binne ∠e van Δ*] $\hat{E} = 90° - 2\hat{F}_4$ [ext ∠ of Δ/ *buite ∠ van Δ*] $\hat{O}_3 = 2\hat{C}_2$ [∠ at centre = 2 ×∠ at circumference/ *midpt. ∠s = 2×omtreks∠*] $\hat{O}_2 = 90° - 2\hat{F}_4$ [∠s on a str line/*∠e op 'n reguitlyn*] $\hat{O}_2 = \hat{E}$ 3) $\therefore \hat{F}_4 = \hat{F}_2$ [3rd ∠ of Δ/*∠e van Δ*] ΔTFO ||| ΔDFE [∠∠∠]	✓ S ✓ S / R ✓ S ✓ S ✓ S **OR** R (5)
10.4	$\hat{B}_2 = \hat{D}_1$ [∠s opp equal sides/*∠e teenoor gelyke sye*] $\hat{B}_2 = \hat{E}$ [given/*gegee*] $\therefore \hat{D}_1 = \hat{E}$ $\therefore$ DB || EA [corresp ∠s =/*ooreenkomstige ∠e gelyk*]	✓ S / R ✓ R (2)
10.5	In ΔOEA DB || EA [proven/*reeds bewys*] $\frac{OD}{DE} = \frac{OB}{BA}$ [line || one side of Δ/*lyn || een sy van Δ*] **OR** [prop theorem; DB || EA/ *eweredigheid stelling; DB || EA*] $\therefore DE = \frac{DO.AB}{OB}$ $\frac{FO}{FE} = \frac{TO}{DE}$ [ΔTFO ||| ΔDFE] $DE = \frac{TO.FE}{FO}$ $\therefore \frac{DO.AB}{OB} = \frac{TO.FE}{FO}$ $\therefore \frac{DO.AB}{DO} = \frac{TO.FE}{DO}$ [DO = OB = FO] $\therefore DO = \frac{TO.FE}{AB}$	✓ R ✓ S ✓ S / R ✓ S ✓ S (5)
		[19]

TOTAL/*TOTAAL*: 150