basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

SENIOR CERTIFICATE EXAMINATIONS/ NATIONAL SENIOR CERTIFICATE EXAMINATIONS

MATHEMATICS P2

2022

MARKS: 150

TIME: 3 hours

This question paper consists of 14 pages and 1 information sheet.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 10 questions.

2. Answer ALL the questions in the SPECIAL ANSWER BOOK provided.

3. Clearly show ALL calculations, diagrams, graphs, etc. that you have used in determining your answers.

4. Answers only will NOT necessarily be awarded full marks.

5. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.

6. If necessary, round off answers correct to TWO decimal places, unless stated otherwise.

7. Diagrams are NOT necessarily drawn to scale.

8. An information sheet with formulae is included at the end of the question paper.

9. Write neatly and legibly.

QUESTION 1

The table below shows the mass (in kg) of the school bags of 80 learners.

MASS (in kg)	FREQUENCY
$5 < m \leq 7$	6
$7 < m \leq 9$	18
$9 < m \leq 11$	21
$11 < m \leq 13$	19
$13 < m \leq 15$	11
$15 < m \leq 17$	4
$17 < m \leq 19$	1

1.1 Write down the modal class of the data. (1)

1.2 Complete the cumulative frequency column in the table in the ANSWER BOOK. (2)

1.3 Draw a cumulative frequency graph (ogive) for the given data on the grid provided in the ANSWER BOOK. (3)

1.4 Use the graph to determine the median mass for this data. (2)

1.5 The international guideline for the mass of a school bag is that it should not exceed 10% of a learner's body mass.

1.5.1 Calculate the estimated mean mass of the school bags. (2)

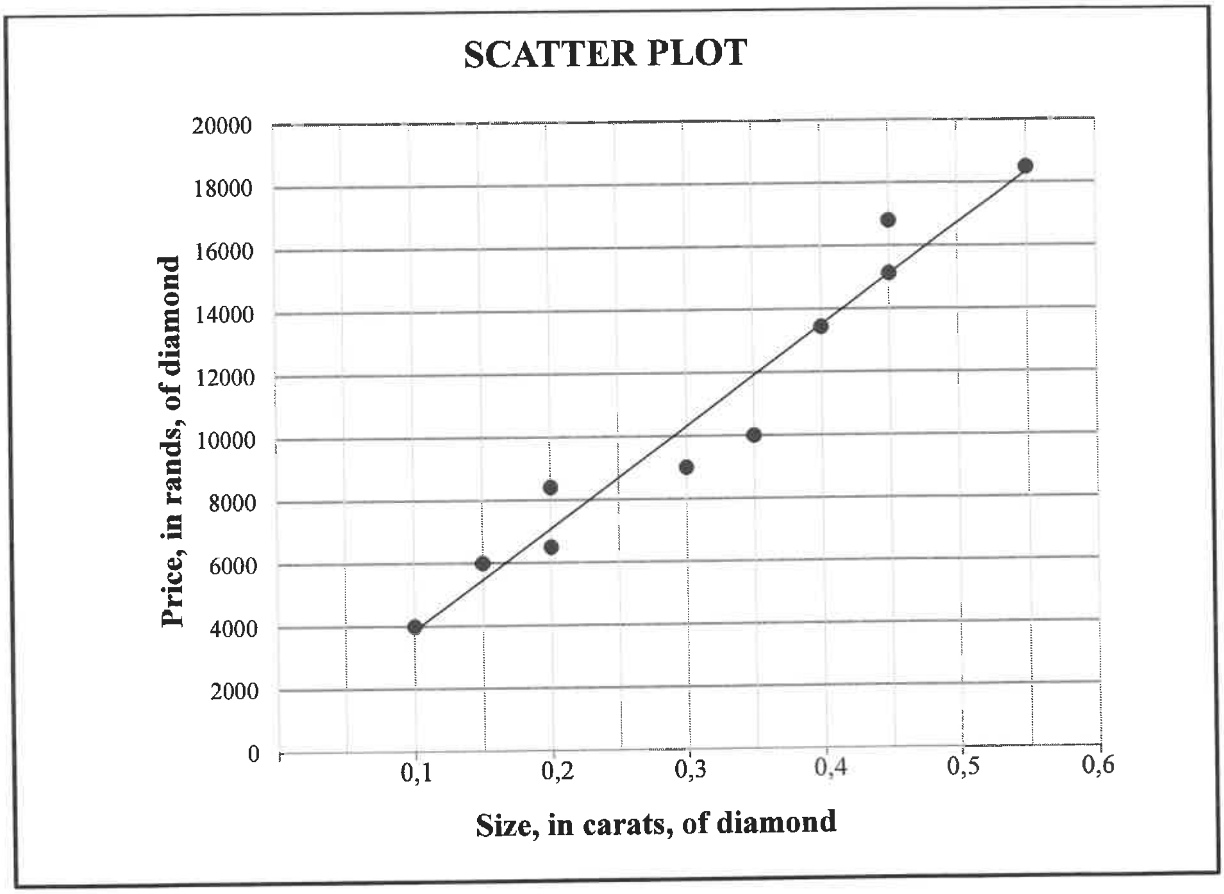
1.5.2 The mean mass of this group of learners was found to be 80 kg. On average, are these school bags satisfying the international guideline with regard to mass? Motivate your answer. (2)

[12]

QUESTION 2

The table below shows the size (in carats) and the price (in rands) of 10 diamonds that were sold by a diamond trader. This information is also presented in the scatter plot below. The least squares regression line for the data is drawn.

Size, in carats, of diamond (x)	0,1	0,15	0,2	0,2	0,3	0,35	0,4	0,45	0,45	0,55
Price, in rands, of diamond (y)	4 000	6 000	6 500	8 400	9 000	10 000	13 440	15 120	16 800	18 480



2.1 Determine the equation of the least squares regression line for the data. (3)

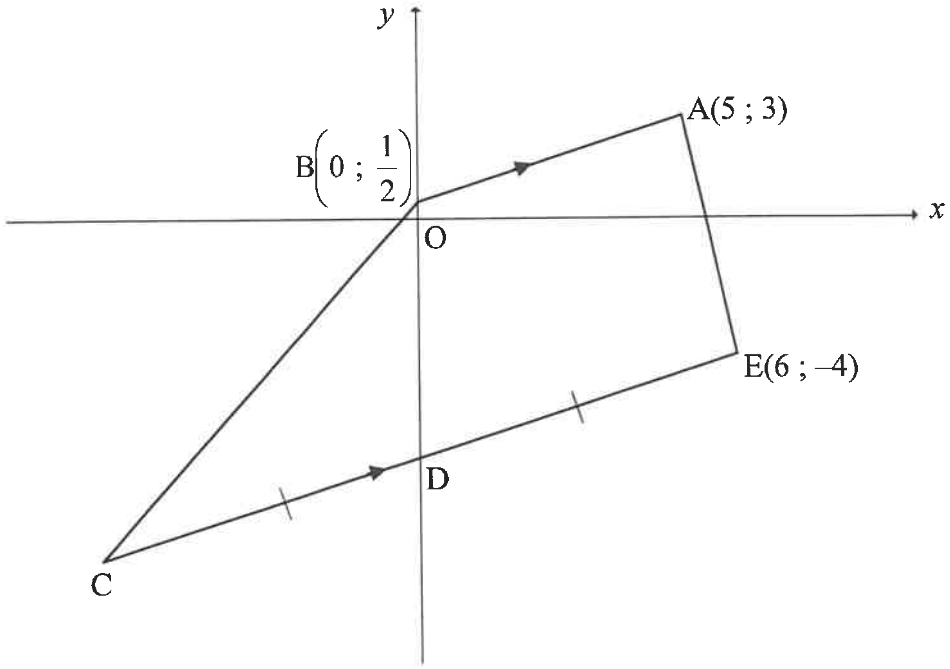
2.2 If the trader sold a diamond that was 0,25 carats in size, predict the selling price of this diamond in rands. (2)

2.3 Calculate the average price increase per 0,05 carat of the diamonds. (2)

2.4 It was later found that the selling price of the 0,35 carat diamond was recorded incorrectly. The correct price is R11 500. When this correction is made to the data set, the correlation between the size and price of these diamonds gets stronger. Explain the reason for this by referring to the given scatter plot. (1)
[8]

QUESTION 3

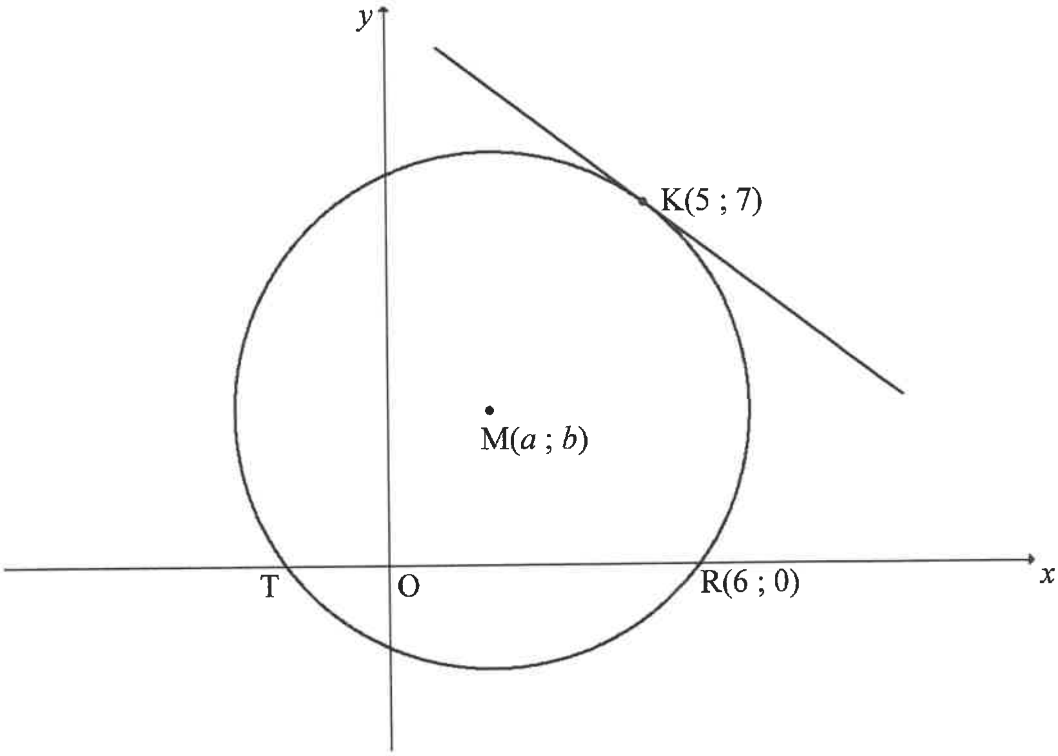
In the diagram, A(5 ; 3), $B\left(0\ ;\ \frac{1}{2}\right)$, C and E(6 ; –4) are the vertices of a trapezium having BA || CE. D is the y-intercept of CE and CD = DE.



3.1	Calculate the gradient of AB.	(2)
3.2	Determine the equation of CE in the form $y = mx + c$.	(3)
3.3	Calculate the:	
	3.3.1 Coordinates of C	(3)
	3.3.2 Area of quadrilateral ABCD	(4)
3.4	If point K is the reflection of E in the y-axis:	
	3.4.1 Write down the coordinates of K	(2)
	3.4.2 Calculate the:	
	(a) Perimeter of ΔKEC	(4)
	(b) Size of $K\hat{C}E$	(3) **[21]**

QUESTION 4

In the diagram, the circle centred at M(a ; b) is drawn. T and R(6 ; 0) are the x-intercepts of the circle. A tangent is drawn to the circle at K(5 ; 7).



4.1 M is a point on the line $y = x + 1$.

4.1.1 Write b in terms of a. (1)

4.1.2 Calculate the coordinates of M. (5)

4.2 If the coordinates of M are (2 ; 3), calculate the length of:

4.2.1 The radius of the circle (2)

4.2.2 TR (2)

4.3 Determine the equation of the tangent to the circle at K. Write your answer in the form $y = mx + c$. (5)

4.4 A horizontal line is drawn as a tangent to the circle M at the point N(c ; d), where $d < 0$.

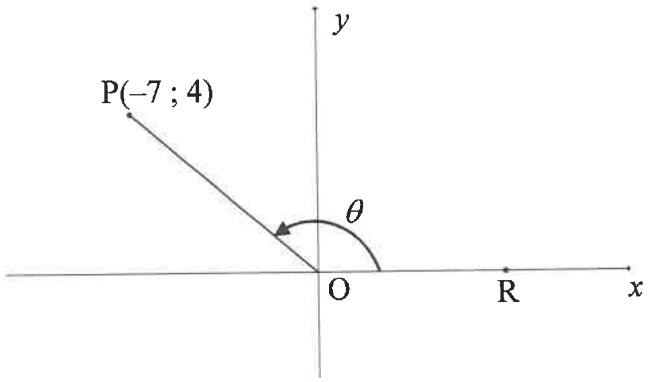
4.4.1 Write down the coordinates of N. (2)

4.4.2 Determine the equation of the circle centred at N and passing through T. Write your answer in the form $(x-a)^2 + (y-b)^2 = r^2$. (3)

[20]

QUESTION 5

5.1 In the diagram below, P(–7 ; 4) is a point in the Cartesian plane. R is a point on the positive x-axis such that obtuse $\hat{POR} = \theta$.



Calculate, **without using a calculator**, the:

5.1.1 Length OP (2)

5.1.2 Value of:

(a) $\tan\theta$ (1)

(b) $\cos(\theta - 180°)$ (2)

5.2 Determine the general solution of: $\sin x\cos x + \sin x = 3\cos^2 x + 3\cos x$ (7)

5.3 Given the identity: $\dfrac{\sin 3x}{1-\cos 3x} = \dfrac{1+\cos 3x}{\sin 3x}$

5.3.1 Prove the identity given above. (3)

5.3.2 Determine the values of x, in the interval $x \in [0° ; 60°]$, for which the identity will be undefined. (3)

[18]

QUESTION 6

6.1 **Without using a calculator**, simplify the following expression to a single trigonometric term:

$$\frac{\sin 10°}{\cos 440°} + \tan(360° - \theta).\sin 2\theta$$ (6)

6.2 Given: $\sin(60° + 2x) + \sin(60° - 2x)$

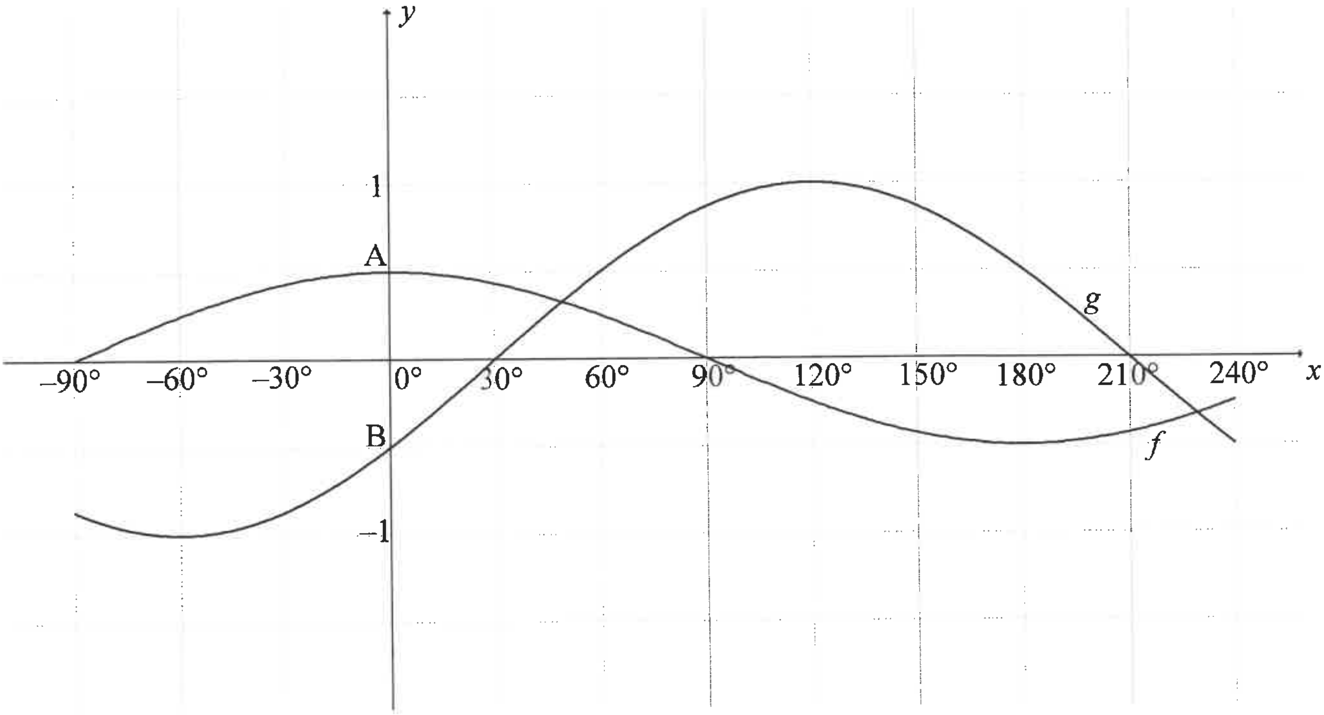
6.2.1 Calculate the value of k if $\sin(60° + 2x) + \sin(60° - 2x) = k \cos 2x$. (3)

6.2.2 If $\cos x = \sqrt{t}$, **without using a calculator**, determine the value of $\tan 60°[\sin(60° + 2x) + \sin(60° - 2x)]$ in terms of t. (3)

[12]

QUESTION 7

In the diagram below, the graphs of $f(x)=\frac{1}{2}\cos x$ and $g(x)=\sin(x-30°)$ are drawn for the interval $x \in [-90°\ ;\ 240°]$. A and B are the y-intercepts of f and g respectively.



7.1 Determine the length of AB. (2)

7.2 Write down the range of $3f(x)+2$. (2)

7.3 Read off from the graphs a value of x for which $g(x)-f(x)=\frac{\sqrt{3}}{2}$. (2)

7.4 For which values of x, in the interval $x \in [-90°\ ;\ 240°]$, will:

7.4.1 $f(x).g(x)>0$ (2)

7.4.2 $g'(x-5°)>0$ (2)

[10]

QUESTION 8

FIGURE I shows a ramp leading to the entrance of a building. B, C and D lie on the same horizontal plane. The perpendicular height (AC) of the ramp is 0,5 m and the angle of elevation from B to A is 15°. The entrance of the building (AE) is 0,915 m wide.

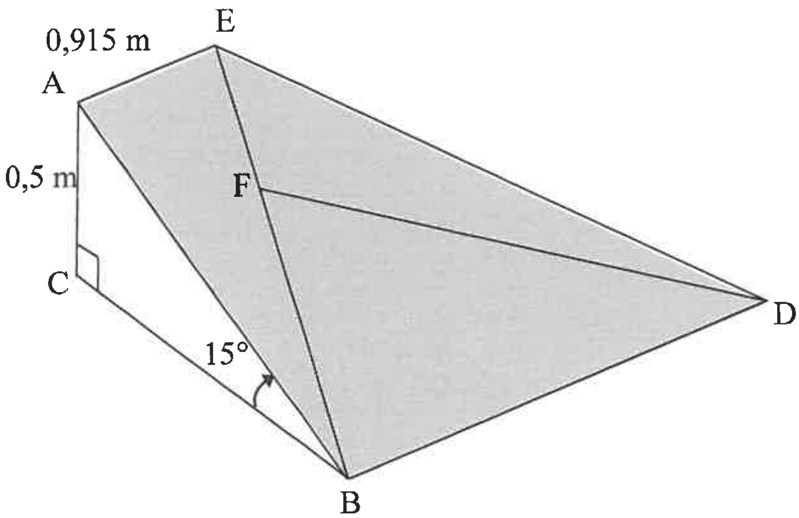


FIGURE I

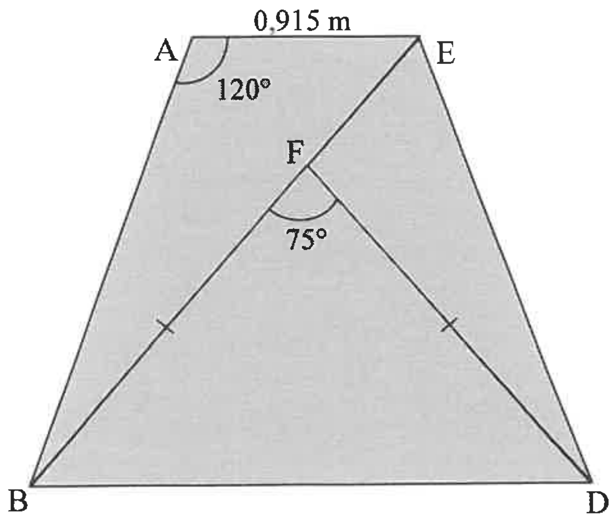


FIGURE II (top view)

8.1 Calculate the length of AB. (2)

8.2 Figure II shows the top view of the ramp. The area of the top of the ramp is divided into three triangles, as shown in the diagram.

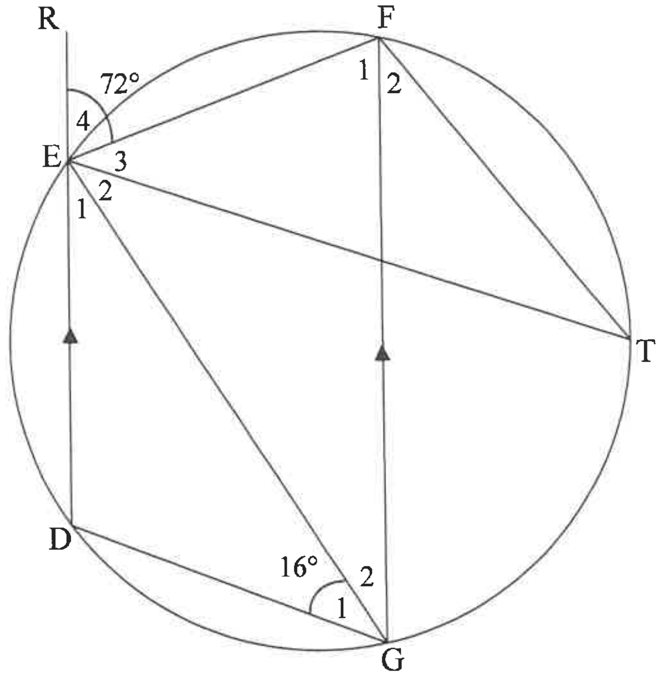
If $B\hat{A}E = 120°$, calculate the length of BE. (3)

8.3 Calculate the area of ΔBFD if $B\hat{F}D = 75°$, $BF = FD$ and $BF = \frac{5}{7}BE$. (3)

[8]

QUESTION 9

9.1 In the diagram, DEFG is a cyclic quadrilateral with DE || GF. DE is produced to R. T is another point on the circle. EG, FT and ET are drawn. $\hat{E}_4 = 72°$ and $\hat{G}_1 = 16°$.



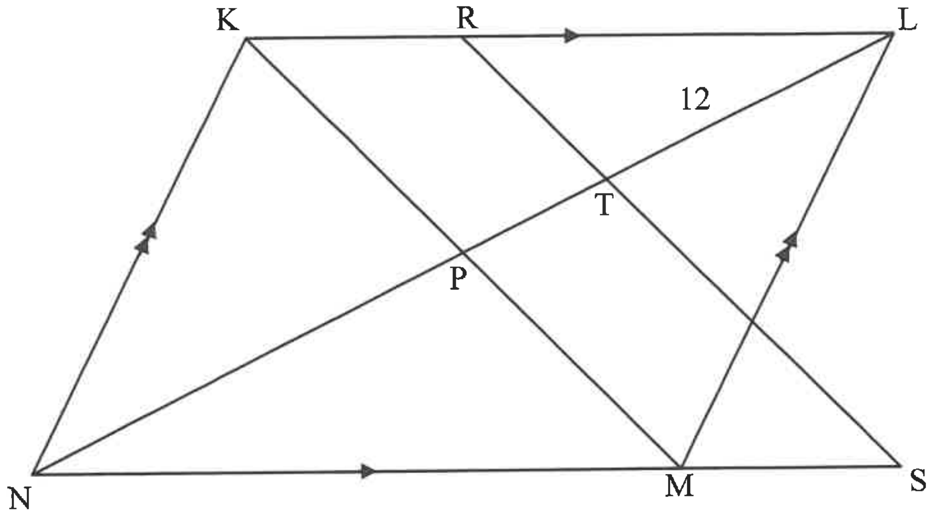
Determine, with reasons, the size of the following angles:

9.1.1 $D\hat{G}F$ (2)

9.1.2 $\hat{T}$ (2)

9.1.3 $G\hat{E}F$ (2)

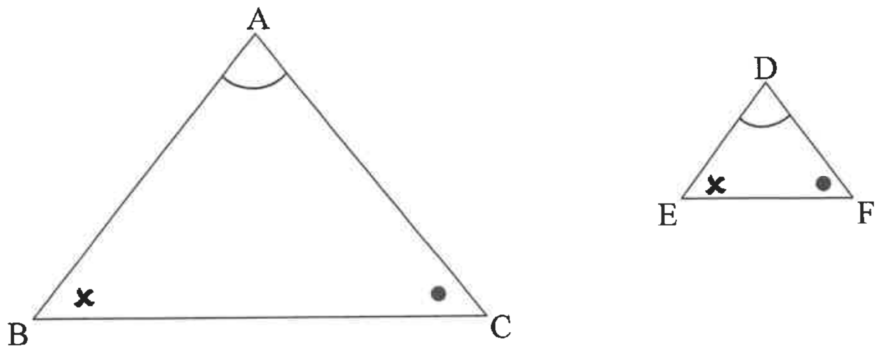
9.2 In the diagram, the diagonals of parallelogram KLMN intersect at P. NM is produced to S. R is a point on KL and RS cuts PL at T.
NM : MS = 4 : 1, NL = 32 units and TL = 12 units.



9.2.1	Determine, with reasons, the value of the ratio NP : PT in simplest form.	(4)
9.2.2	Prove, with reasons, that KM $\parallel$ RS.	(2)
9.2.3	If NM = 21 units, determine, with reasons, the length of RL.	(4)
		[16]

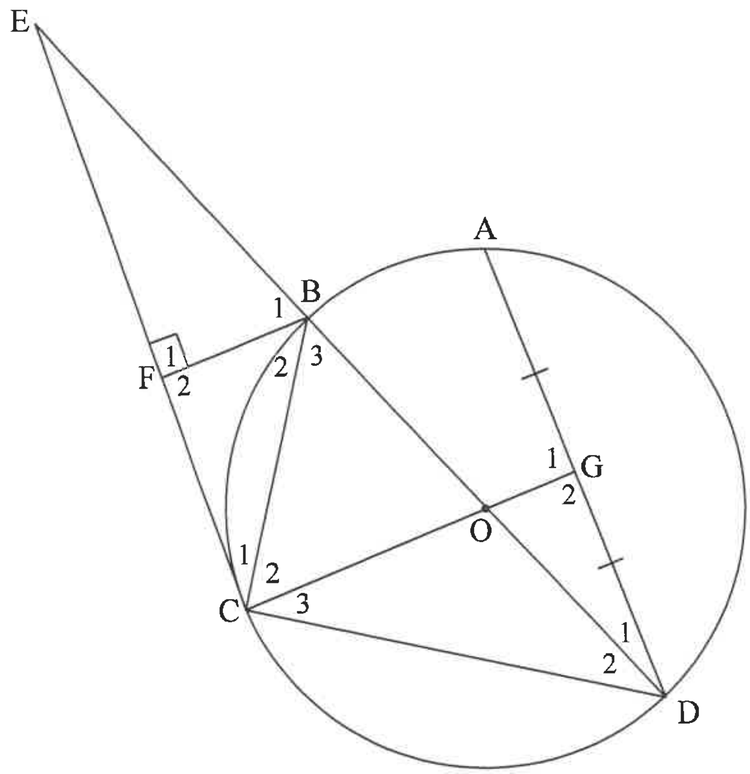
QUESTION 10

10.1 In the diagram, ΔABC and ΔDEF are drawn such that $\hat{A}=\hat{D}$, $\hat{B}=\hat{E}$ and $\hat{C}=\hat{F}$.



Use the diagram in the ANSWER BOOK to prove the theorem which states that if two triangles are equiangular, then the corresponding sides are in proportion, i.e. $\frac{AB}{DE}=\frac{AC}{DF}$. (6)

10.2 In the diagram, O is the centre of a circle passing through A, B, C and D. EC is a tangent to the circle at C. Diameter DB produced meets tangent EC at E. F is a point on EC such that BF ⊥ EC. Radius CO produced bisects AD at G. BC and CD are drawn.



10.2.1 Prove, with reasons, that:

(a) FB || CG (3)

(b) ΔFCB ||| ΔCDB (5)

10.2.2 Give a reason why $\hat{G}_1 = 90°$. (1)

10.2.3 Prove, with reasons, that $CD^2 = CG.DB$. (5)

10.2.4 Hence, prove that $DB = CG + FB$. (5)
[25]

TOTAL: 150

INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $\quad A = P(1-ni)$ $\quad A = P(1-i)^n$ $\quad A = P(1+i)^n$

$T_n = a + (n-1)d$ $\quad S_n = \frac{n}{2}[2a + (n-1)d]$

$T_n = ar^{n-1}$ $\quad S_n = \frac{a(r^n - 1)}{r-1}$; $r \neq 1$ $\quad S_\infty = \frac{a}{1-r}$; $-1 < r < 1$

$F = \frac{x[(1+i)^n - 1]}{i}$ $\quad P = \frac{x[1-(1+i)^{-n}]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad \mathrm{M}\left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad y - y_1 = m(x - x_1)$ $\quad m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad m = \tan\theta$

$$(x-a)^2 + (y-b)^2 = r^2$$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$a^2 = b^2 + c^2 - 2bc.\cos A$

$area\ \Delta ABC = \frac{1}{2} ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\quad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\quad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$$

$\sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum x}{n}$ $\quad \sigma^2 = \frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $\quad P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$\hat{y} = a + bx$ $\quad b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$

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SENIORSERTIFIKAAT-EKSAMEN
NATIONAL SENIOR CERTIFICATE EXAMINATIONS/
NASIONALE SENIORSERTIFIKAAT-EKSAMEN

MATHEMATICS P2/*WISKUNDE V2*

MARKING GUIDELINES/*NASIENRIGLYNE*

2022

MARKS: 150
PUNTE: 150

These marking guidelines consist of 20 pages./
Hierdie nasienriglyne bestaan uit 20 bladsye.

NOTE:

- If a candidate answers a question TWICE, mark only the FIRST attempt.
- If a candidate has crossed out an attempt at an answer and not redone the question, mark the crossed-out version.
- Consistent accuracy applies in ALL aspects of the marking guidelines. Stop marking at the second calculation error.
- Assuming answers/values in order to solve a problem is NOT acceptable.

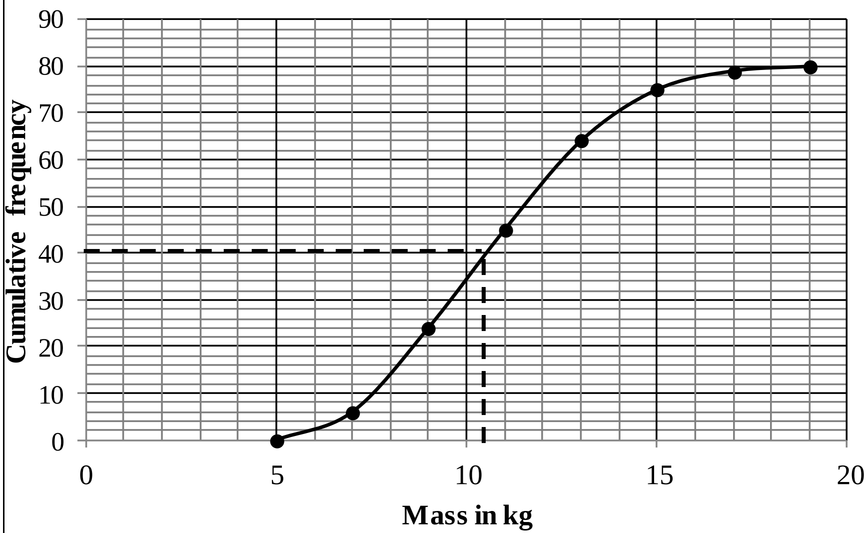
LET WEL:

- *As 'n kandidaat 'n vraag TWEE KEER beantwoord, sien slegs die EERSTE poging na.*
- *As 'n kandidaat 'n antwoord van 'n vraag doodtrek en nie oordoen nie, merk die doodgetrekte poging.*
- *Volgehoue akkuraatheid word in ALLE aspekte van die nasienriglyne toegepas. Hou op nasien by die tweede berekeningsfout.*
- *Aanvaar van antwoorde/waardes om 'n probleem op te los, word NIE toegelaat nie.*

GEOMETRY • *MEETKUNDE*	
S	**A mark for a correct statement** ***(A statement mark is independent of a reason)***
	'n Punt vir 'n korrekte bewering ***('n Punt vir 'n bewering is onafhanklik van die rede)***
R	**A mark for the correct reason** ***(A reason mark may only be awarded if the statement is correct)***
	'n Punt vir 'n korrekte rede ***('n Punt word slegs vir die rede toegeken as die bewering korrek is)***
S/R	**Award a mark if statement AND reason are both correct**
	Ken 'n punt toe as die bewering EN rede beide korrek is

QUESTION/*VRAAG* 1

<table>
<tr><td>1.1</td><td>Modal class: $9 < m \leq 11$</td><td>✓ answer
(1)</td></tr>
<tr><td>1.2</td><td>
<table>
<tr><th>Mass (in kg)</th><th>Frequency</th><th>Cumulative frequency</th></tr>
<tr><td>$5 < m \leq 7$</td><td>6</td><td>6</td></tr>
<tr><td>$7 < m \leq 9$</td><td>18</td><td>24</td></tr>
<tr><td>$9 < m \leq 11$</td><td>21</td><td>45</td></tr>
<tr><td>$11 < m \leq 13$</td><td>19</td><td>64</td></tr>
<tr><td>$13 < m \leq 15$</td><td>11</td><td>75</td></tr>
<tr><td>$15 < m \leq 17$</td><td>4</td><td>79</td></tr>
<tr><td>$17 < m \leq 19$</td><td>1</td><td>80</td></tr>
</table>
</td><td>✓ adding

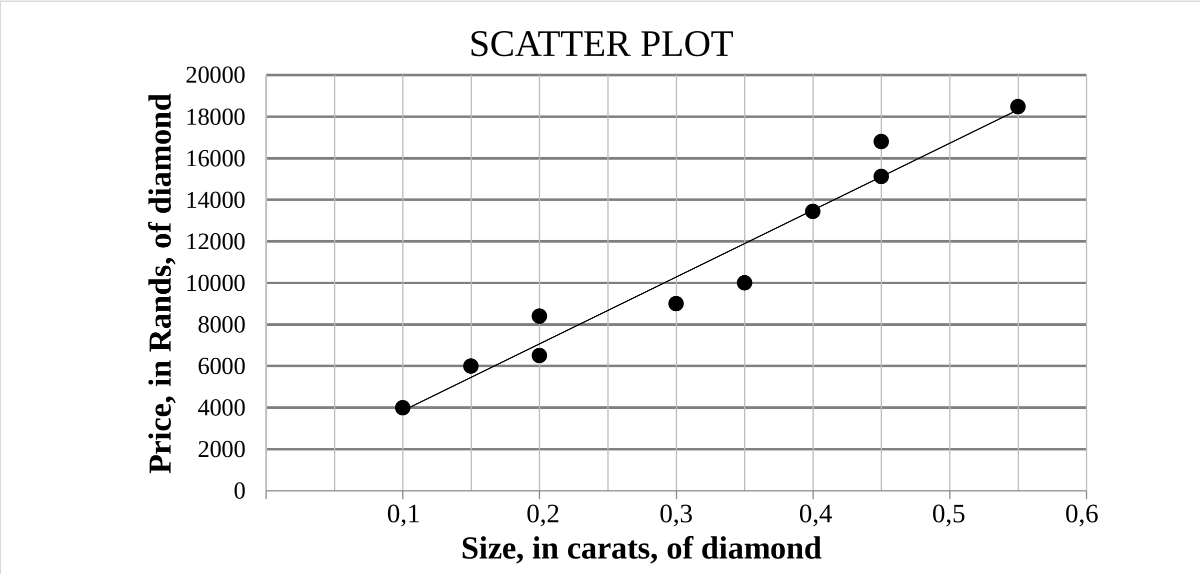
✓ 80
(2)</td></tr>
<tr><td>1.3</td><td>
</td><td>✓ grounding (5 ; 0)
✓ points
✓ shape
(3)</td></tr>
<tr><td>1.4</td><td>Median mass: 10,5 kg</td><td>✓✓ answer
(2)</td></tr>
<tr><td>1.5.1</td><td>$\bar{x} = \frac{(6\times 6 + 18\times 8 + 21\times 10 + 19\times 12 + 11\times 14 + 4\times 16 + 1\times 18)}{80}$
$= \frac{854}{80}$
$= 10,68$
Answer only 2/2</td><td>✓ 854
✓ answer
(2)</td></tr>
<tr><td>1.5.2</td><td>Learners' bags are heavier than the stipulated international guideline.
Estimated mean = 10,68 kg
10% of 80 kg
= 8 kg
10,68 kg > 8 kg</td><td>✓ answer

✓ 8 kg
(2)</td></tr>
</table>

	OR/ *OF* Learners' bags are heavier than the stipulated international guideline. Estimated mean $= \frac{10,68}{80} \times 100$ $= 13,35\%$ $13,35\% > 10\%$	✓ answer ✓ 13,35% (2)
		[12]

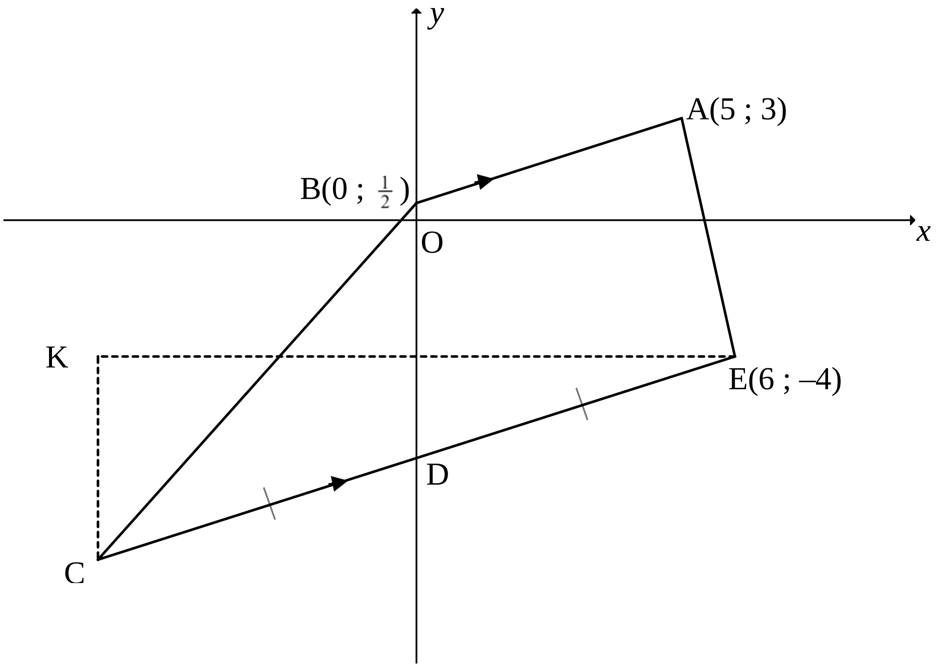
QUESTION/*VRAAG* 2

Size, in carats, of diamond (x)	0,1	0,15	0,2	0,2	0,3	0,35	0,4	0,45	0,45	0,55
Price, in rands, of diamond (y)	4 000	6 000	6 500	8 400	9 000	10 000	13 440	15 120	16 800	18 480



2.1	$a = 634{,}382\ldots$ $b = 32\ 189{,}263\ldots$ $\hat{y} = 634{,}38 + 32189{,}26x$ Answer only 3/3	✓ a ✓ b ✓ equation (3)
2.2	$\hat{y} = 634{,}38 + 32189{,}26(0{,}25)$ = R8 681,70 **OR/*OF*** $\hat{y} = \text{R}8\,681{,}70$ (if using calculator)	✓ substitution ✓ answer (2) ✓✓ answer (2)
2.3	Average price increase $= \text{R}\dfrac{32189{,}26}{20}$ per 0,05 carat = R1 609,46 per 0,05 carat **OR/*OF*** Average price increase $= 0{,}05 \times 32\,189{,}26$ = R1 609,46 per 0,05 carat **OR/*OF*** at 0,3: $\hat{y} = \text{R}10\,291{,}16$ ∴ Average price increase = 10 291,16 – 8 681,70 = R1 609,46 per 0,05 carat Answer only 2/2	✓ divide gradient by 20 ✓ answer (2) ✓ multiply gradient by 0,05 ✓ answer (2) ✓ Estimated price of a 0,3 carat diamond ✓ answer (2)
2.4	The point (0,35 ; 11500) is closer to the least squares regression line.	✓ reason (1)
		[8]

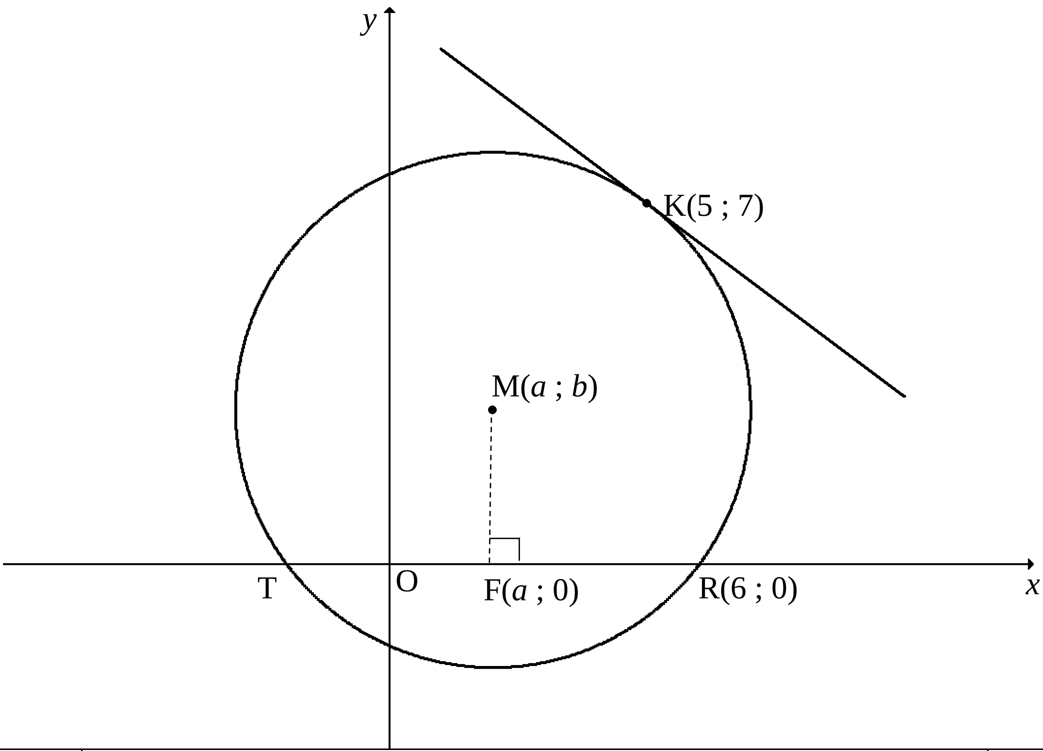
QUESTION/*VRAAG* 3



3.1	$m_{AB}=\dfrac{3-\frac{1}{2}}{5-0}$ $m_{AB}=\dfrac{1}{2}$ Answer only 2/2	✓ substitution ✓ answer (2)
3.2	$m_{CE}=m_{BA}=\dfrac{1}{2}$ $-4=\dfrac{1}{2}(6)+c$ **OR/*OF*** $y-(-4)=\dfrac{1}{2}(x-6)$ $c=-7$ $y=\dfrac{1}{2}x-7$	✓ gradient ✓ substitution of E ✓ answer (3)
3.3.1	D(0 ; –7) $\dfrac{x_C+6}{2}=0$ $\dfrac{y_C+(-4)}{2}=-7$ $x_C=-6$ $y_C=-10$ C(–6 ; –10) Answer only 3/3	✓ D(0 ; –7) ✓ $x_C=-6$ ✓ $y_C=-10$ (3)
3.3.2	Area Δ BCD $=\dfrac{1}{2}(7,5)(6)$ $= 22,5$ Area Δ ABD $=\dfrac{1}{2}(7,5)(5)$ $= 18,75$ Area ABCD = 22,5 + 18,75 = 41,25 units^2	✓ subst of correct base and height into the area formula ✓ area ΔBCD = 22,5 ✓ area ΔABD = 18,75 ✓ answer (4)

3.4.1	K(–6 ; –4)	✓ $x_K = -6$ ✓ $y_K = -4$ (2)
3.4.2a	KC = 6 units; KE = 12 units; $CE = \sqrt{(6)^2 + (12)^2}$ [Pythagoras] $CE = \sqrt{180} = 6\sqrt{5} = 13,42$ Perimeter ΔKEC = 6 + 12 + $\sqrt{180}$ = 31,42 units	✓ KC = 6 units ✓ KE = 12 units ✓ CE ✓ answer (4)
3.4.2b	$\tan K\hat{C}E = \frac{KE}{KC} = \frac{12}{6} = 2$ $K\hat{C}E = 63{,}43°$	✓ trig ratio ✓ $\tan K\hat{C}E = 2$ ✓ answer (3)
	OR/*OF*	
	$\sin K\hat{C}E = \frac{KE}{CE} = \frac{12}{\sqrt{180}} = \frac{2\sqrt{5}}{5}$ $K\hat{C}E = 63{,}43°$	✓ trig ratio ✓ $\sin K\hat{C}E = \frac{12}{\sqrt{180}}$ ✓ answer (3)
	OR/*OF*	
	$m_{CE} = \frac{1}{2}$ $\tan\theta = \frac{1}{2}$ $\theta = 26{,}57°$ $K\hat{C}E = 90° - 26{,}57°$ $K\hat{C}E = 63{,}43°$	✓ $\tan\theta = \frac{1}{2}$ ✓ $\theta = 26{,}57°$ ✓ answer (3)
	OR/*OF*	
	$KE^2 = KC^2 + CE^2 - 2(KC)(CE)\cos K\hat{C}E$ $(12)^2 = (6)^2 + (\sqrt{180})^2 - 2(6)(\sqrt{180})(\cos K\hat{C}E)$ $\cos K\hat{C}E = \frac{\sqrt{5}}{5}$ $K\hat{C}E = 63{,}43°$	 ✓ substitution into cosine rule ✓ trig ratio ✓ answer (3)
		[21]

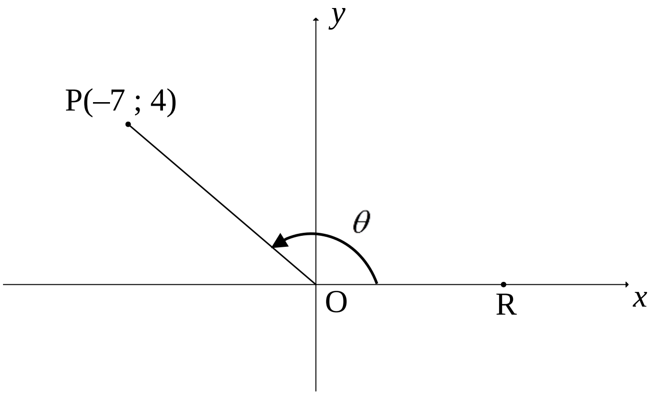
QUESTION/*VRAAG* 4



4.1.1	$y = x + 1$ $b = a + 1$	✓ $b = a + 1$ (1)
4.1.2	$MR^2 = MK^2$ $(a-6)^2 + (b-0)^2 = (a-5)^2 + (b-7)^2$ $(a-6)^2 + (a+1)^2 = (a-5)^2 + (a+1-7)^2$ $a^2 + 2a + 1 = a^2 - 10a + 25$ $12a = 24$ $a = 2$ $b = 3$ $\therefore M(2 ; 3)$	✓ equating radii / solving simultaneously ✓ substitution $b = a + 1$ ✓ $12a = 24$ ✓ $a = 2$ ✓ $b = 3$ (5)
4.2.1	$(6-2)^2 + (0-3)^2 = r^2$ $r = 5$ **OR/*OF*** $(2-5)^2 + (3-7)^2 = r^2$ $r = 5$ Answer only 2/2	✓ substitution R and M ✓ $r = 5$ (2) ✓ substitution K and M ✓ $r = 5$ (2)

4.2.2	T(–2 ; 0) TR = 8 units [line from centre ⊥ to chord] **OR/*OF*** M(2 ; 3) F(a ; 0) FR = 4 units TR = 8 units [line from centre ⊥ to chord] **OR/*OF*** $(x-2)^2+(0-3)^2=25$ $x^2-4x+4+9=25$ $x^2-4x-12=0$ $(x-6)(x+2)=0$ $x=6$ or $x=-2$ TR = 8 units Answer only 2/2	✓ T(–2 ; 0) ✓ answer (2) ✓ 4 units ✓ answer (2) ✓ x values ✓ answer (2)
4.3	$m_{\text{radius}}=\frac{7-3}{5-2}$ $m_{\text{radius}}=\frac{4}{3}$ $m_{\text{tangent}}=-\frac{3}{4}$ $7=-\frac{3}{4}(5)+c$ **OR/*OF*** $y-7=-\frac{3}{4}(x-5)$ $c=\frac{43}{4}$ $y=-\frac{3}{4}x+\frac{43}{4}$ $y=-\frac{3}{4}x+\frac{43}{4}$	✓ substitution ✓ $m_{\text{radius}}=\frac{4}{3}$ ✓ $m_{\text{tangent}}=-\frac{3}{4}$ ✓ substitution ✓ answer (5)
4.4.1	N(2 ; –2)	✓ $x_N=2$ ✓ $y_N=-2$ (2)
4.4.2	$(-2-2)^2+(0+2)^2=r^2$ $r^2=20$ $(x-2)^2+(y+2)^2=20$	✓ substitution ✓ $r^2=20$ ✓ answer (3)
		[20]

QUESTION/*VRAAG* 5



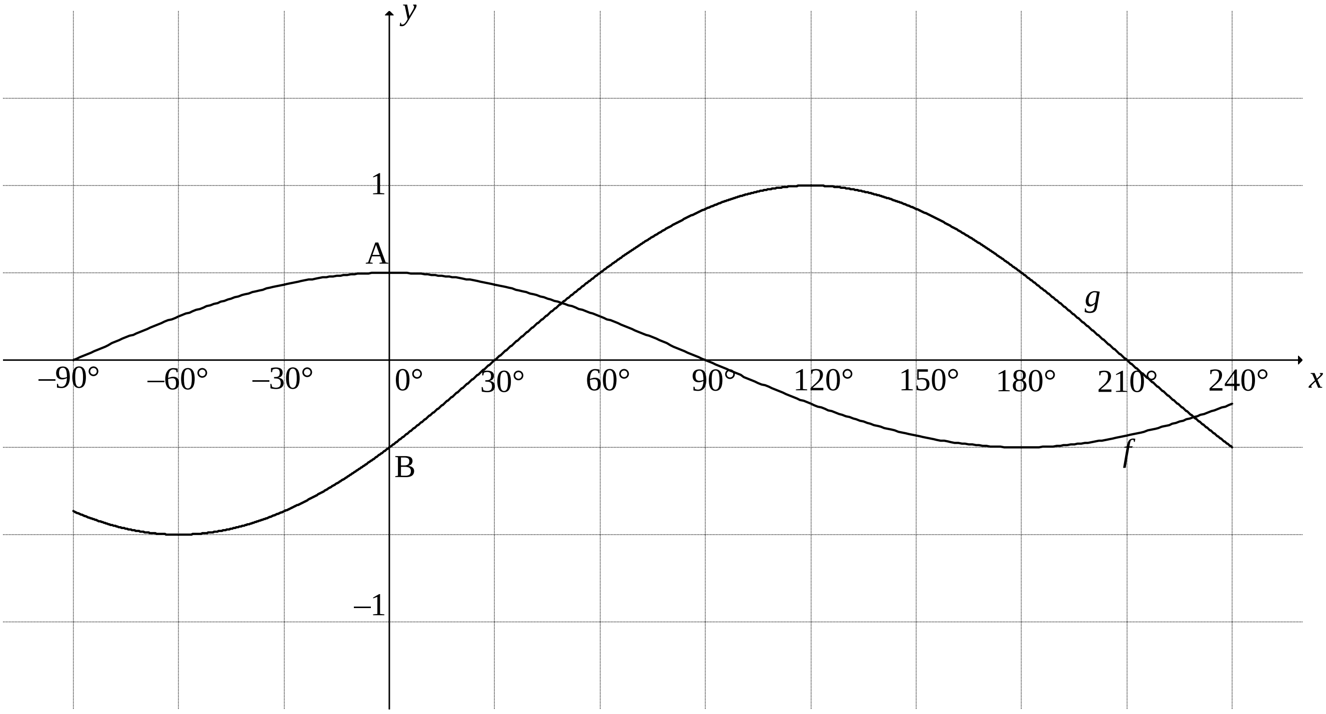
5.1.1	$OP=\sqrt{(-7)^2+(4)^2}$ $=\sqrt{65}$ Answer only 2/2	✓ substitution ✓ answer (2)
5.1.2(a)	$\tan\theta=\dfrac{4}{-7}$	✓ answer (1)
5.1.2(b)	$\cos(\theta-180°)=-\cos\theta$ $=\dfrac{7}{\sqrt{65}}$	✓ reduction ✓ answer (2)
5.2	$\sin x\cos x+\sin x=3\cos^2 x+3\cos x$ $\sin x\cos x+\sin x-3\cos^2 x-3\cos x=0$ $\sin x(\cos x+1)-3\cos x(\cos x+1)=0$ $(\cos x+1)(\sin x-3\cos x)=0$ $\cos x=-1$ or $\sin x=3\cos x$ $\tan x=3$ x = 180° + k.360° or x = 71,57° + k.180° ; $k\in Z$ **OR/*OF*** $\sin x\cos x+\sin x=3\cos^2 x+3\cos x$ $\sin x\cos x+\sin x-3\cos^2 x-3\cos x=0$ $\sin x(\cos x+1)-3\cos x(\cos x+1)=0$ $(\cos x+1)(\sin x-3\cos x)=0$ $\cos x=-1$ or $\sin x=3\cos x$ $\tan x=3$ x = 180° + k.360° or x = 71,57° + k.360° or x = 251,57° + k.360°; $k\in Z$	✓ RHS = 0 ✓ grouping ✓ factors ✓ both equations ✓ x = 180° ✓ x = 71,57° ✓ + k.180°; $k\in Z$ (7) ✓ RHS = 0 ✓ grouping ✓ factors ✓ both equations ✓ x = 180° ✓ x = 71,57° and 251,57° ✓ + k.360°; $k\in Z$ (7)

5.3.1	$\text{LHS} = \frac{\sin 3x}{1-\cos 3x} \times \frac{1+\cos 3x}{1+\cos 3x}$ $= \frac{(\sin 3x)(1+\cos 3x)}{(1-\cos 3x)(1+\cos 3x)}$ $= \frac{(\sin 3x)(1+\cos 3x)}{1-\cos^2 3x}$ $= \frac{(\sin 3x)(1+\cos 3x)}{\sin^2 3x}$ $= \frac{1+\cos 3x}{\sin 3x}$ = RHS **OR/*OF*** $\text{LHS} = \frac{\sin 3x}{1-\cos 3x} \times \frac{\sin 3x}{\sin 3x}$ $= \frac{\sin^2 3x}{\sin 3x(1-\cos 3x)}$ $= \frac{1-\cos^2 3x}{\sin 3x(1-\cos 3x)}$ $= \frac{(1-\cos 3x)(1+\cos 3x)}{\sin 3x(1-\cos 3x)}$ $= \frac{1+\cos 3x}{\sin 3x}$ = RHS	✓ multiply by "1" ✓ $1-\cos^2 3x$ ✓ square identity (3) ✓ multiply by "1" ✓ square identity ✓ factors (3)
5.3.2	undefined when $\sin 3x = 0$ and $1-\cos 3x = 0$ $3x = 0°$ or $3x = 180°$ and $3x = 0°$ or $3x = 360°$ $x = 0°$ or $x = 60°$	✓ $\sin 3x = 0$ and $1-\cos 3x = 0$ ✓ 0° ✓ 60° (3)
		[18]

QUESTION/*VRAAG* 6

6.1	$\dfrac{\sin 10°}{\cos 440°} + \tan(360° - \theta).\sin 2\theta$ $= \dfrac{\cos 80°}{\cos 80°} - \tan\theta(2\sin\theta\cos\theta)$ $= 1 - \dfrac{\sin\theta}{\cos\theta}(2\sin\theta\cos\theta)$ $= 1 - 2\sin^2\theta$ $= \cos 2\theta$	✓ $-\tan\theta$ ✓ cos 80° ✓ co-ratio ✓ double angle ✓ quotient identity ✓ answer (6)
6.2.1	$\sin(60° + 2x) + \sin(60° - 2x) = k\cos 2x$ $(\sin 60°\cos 2x + \cos 60°\sin 2x) + (\sin 60°\cos 2x - \cos 60°\sin 2x) = k\cos 2x$ $2\sin 60°\cos 2x = k\cos 2x$ $2\left(\dfrac{\sqrt{3}}{2}\right)\cos 2x = k\cos 2x$ $\therefore k = \sqrt{3}$	✓ both expansions correct ✓ special ∠s ✓ answer (3)
6.2.2	$\tan 60°[\sin(60° + 2x) + \sin(60° - 2x)]$ $= \tan 60°[k\cos 2x]$ $= \sqrt{3}(\sqrt{3}\cos 2x)$ $= 3(2\cos^2 x - 1)$ $= 3\left(2\left(\sqrt{t}\right)^2 - 1\right)$ $= 6\left(\sqrt{t}\right)^2 - 3$ $= 6t - 3$	✓ special ∠ ✓ double ∠s ✓ answer i.t.o t (3)
		[12]

QUESTION/*VRAAG* 7



7.1	A$\left(0;\frac{1}{2}\right)$ B$\left(0;-\frac{1}{2}\right)$ $AB=\frac{1}{2}-\left(-\frac{1}{2}\right)$ = 1 unit Answer only 2/2	✓ y-values ✓ answer (2)
7.2	Range of $f: y\in\left[-\frac{1}{2};\frac{1}{2}\right]$ Range of $3f(x)+2: \; y\in\left[\frac{1}{2};3\frac{1}{2}\right]$ **OR/*OF*** $\frac{1}{2}\le y\le 3\frac{1}{2}$	✓ critical values ✓ answer (2)
7.3	$x = 90°$	✓✓ $x = 90°$ (2)
7.4.1	$x\in(30°;\ 90°)\cup(210°;240°]$ **OR/*OF*** $30°<x<90°$ or $210°<x\le 240°$	✓ $x\in(30°;\ 90°)$ ✓ $(210°;240°]$ (2) ✓ $30°<x<90°$ ✓ $210°<x\le 240°$ (2)
7.4.2	$x\in(-55°;\ 125°)$ **OR/*OF*** $-55°<x<125°$	✓ critical values ✓ answer (2) ✓ critical values ✓ answer (2)
		[10]

QUESTION/*VRAAG* 8

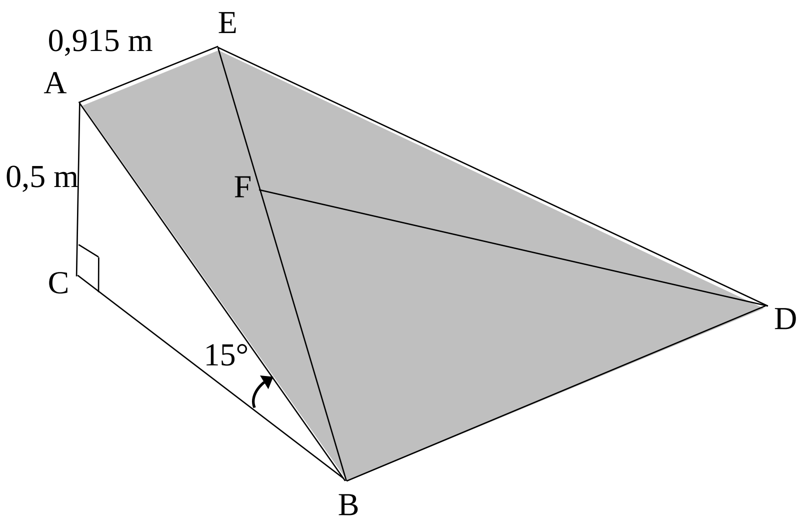


FIGURE I

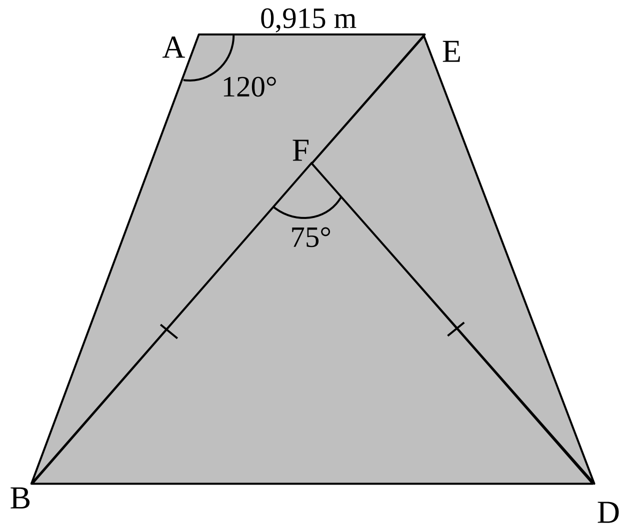
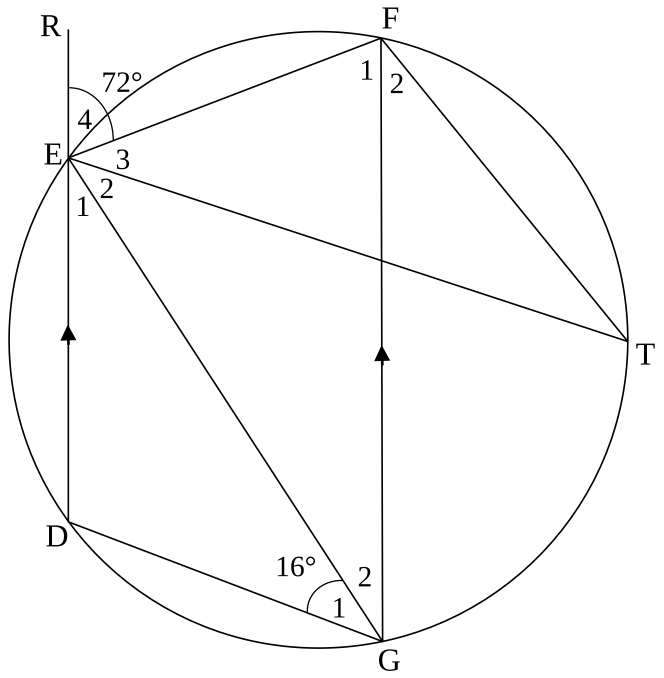


FIGURE II (top view)

8.1	$\frac{0,5}{AB} = \sin 15°$ $AB = \frac{0,5}{\sin 15°}$ $AB = 1,93 \text{ m}$ Answer only 2/2	✓ trig ratio ✓ answer (2)
8.2	$BE^2 = AB^2 + AE^2 - 2(AB)(AE)\cos B\hat{A}E$ $BE^2 = (1,93)^2 + (0,915)^2 - 2(1,93)(0,915)(\cos 120°)$ $BE = 2,52 \text{ m}$	✓ correct use of cosine rule ✓ substitution ✓ answer (3)
8.3	$BF = FD = \frac{5}{7}(2,52) = 1,80 \text{ m}$ $\text{Area } \Delta BFD = \frac{1}{2}(BF)(FD)\sin B\hat{F}D$ $= \frac{1}{2}(1,8)(1,8)(\sin 75°)$ $= 1,56 \text{ m}^2$	✓ BF ✓ correct substitution into the area rule ✓ answer (3)
		[8]

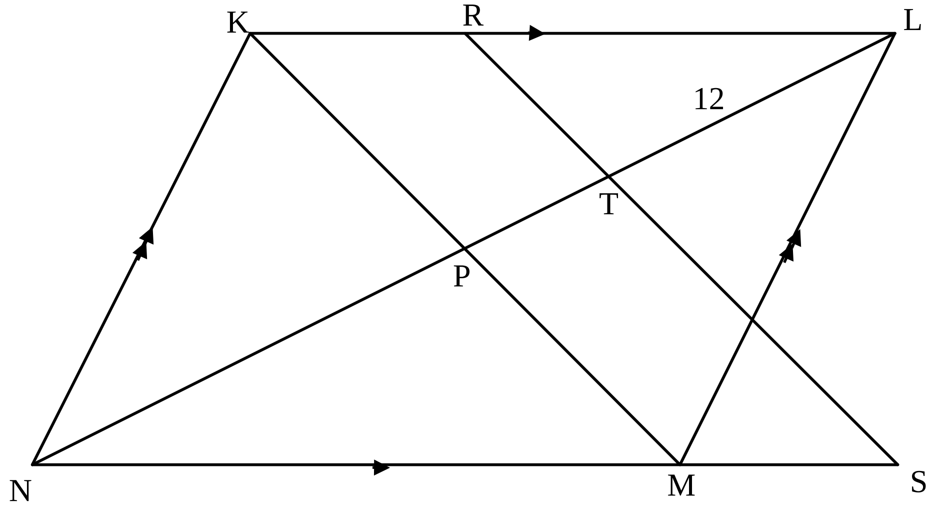
QUESTION/*VRAAG* 9

9.1



9.1.1	$D\hat{G}F = \hat{E}_4 = 72°$ [ext ∠ of cyclic quad/ *buite ∠ v kvh*]	✓ S ✓ R (2)
9.1.2	$\hat{G}_2 = 72° - 16° = 56°$ $\hat{T} = \hat{G}_2 = 56°$ [∠s in the same seg/ *∠e in dies. ⊙ segment*]	✓ S ✓ S / R (2)
9.1.3	$\hat{F}_1 = \hat{E}_4 = 72°$ [alt ∠s; DE ‖ GF / *verw. ∠e*; DE ‖ GF] $\therefore G\hat{E}F = 52°$ [sum of ∠s in Δ / *∠e van* Δ] **OR/*OF*** $\hat{E}_1 = 56°$ [alt ∠s; DE ‖ GF / *verw. ∠e*; DE ‖ GF] $\therefore G\hat{E}F = 52°$ [∠s on a str. line/ *∠e op 'n reguitlyn*]	✓ S / R ✓ S (2) ✓ S / R ✓ S (2)

9.2

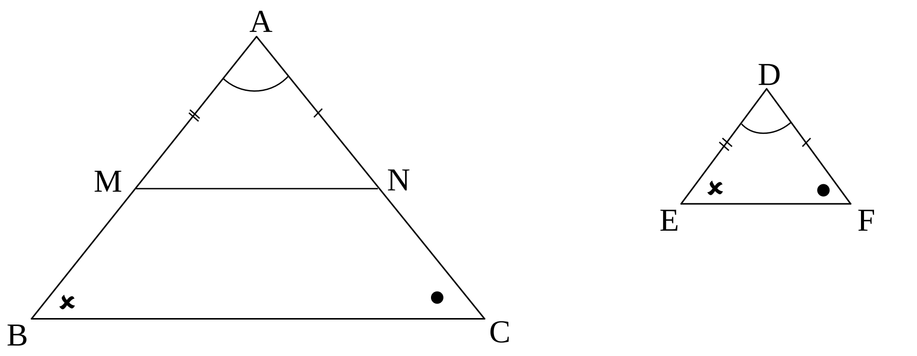


9.2.1	NP = PL = 16 [diag of ||m / *hoeklyne van ||m*] PT = 4 NP : PT = 16 : 4 = 4 : 1	✓ S ✓ R ✓ S ✓ answer (4)
9.2.2	NM : MS = 4 : 1 NP : PT = NM : MS KM || RS [line divides two sides of Δ in prop / *Lyn verdeel 2 sye v Δ eweredig*] **OR/*OF*** [converse prop theorem / *omgekeerde lyn || een sy v Δ*]	✓ S ✓ R (2)
9.2.3	$\frac{RL}{KL} = \frac{TL}{LP}$ [prop theorem; KM || RS **OR** line || one side of Δ / *Lyn || een sy v Δ*] $RL = \frac{12 \times 21}{16}$ $= 15,75$	✓ S ✓ R ✓ S ✓ answer (4)

	OR / *OF* NM : MS = 4 : 1 KR = MS = 5,25 [opp side of $\|\|^m$ / *teenoorst. sye van* $\|\|^m$] KL = NM = 21 RL + 5,25 = 21 RL = 15,75	 ✓ S ✓ R ✓ S ✓ answer (4)
		[16]

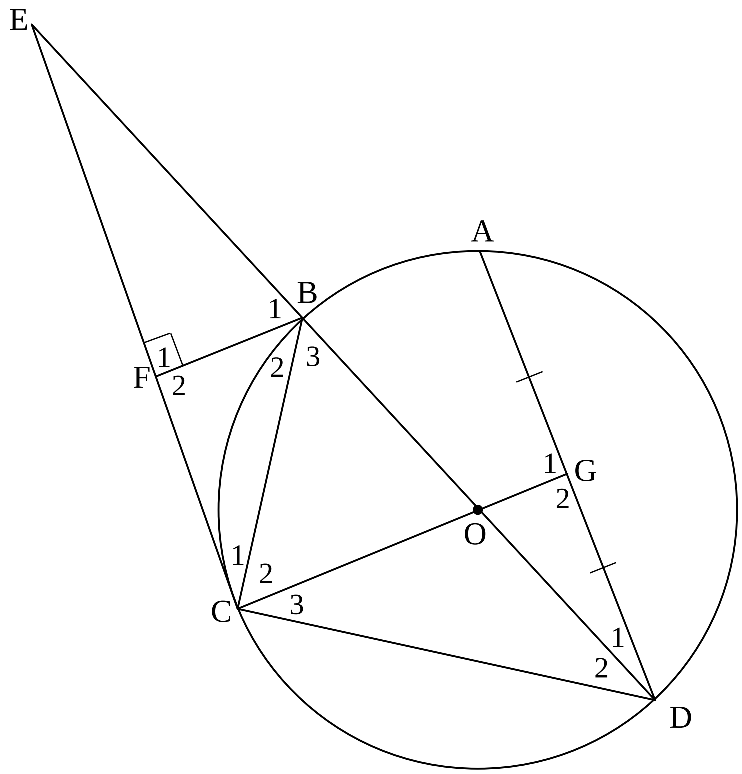
QUESTION/*VRAAG* 10

10.1



10.1	Constr: Let M and N lie on AB and AC respectively such that AM = DE and AN = DF. Draw MN. Proof: In Δ AMN and Δ DEF AM = DE [Constr / *Konstruksie*] AN = DF [Constr / *Konstruksie*] $\hat{A} = \hat{D}$ [Given /*Gegee*] $\therefore\ \Delta AMN \equiv \Delta DEF$ $[s, \angle, s]$ $\therefore A\hat{M}N = \hat{E} = \hat{B}$ MN || BC [corresp $\angle$'s are equal/ *ooreenk.* $\angle$ *e gelyk*] $\frac{AB}{AM} = \frac{AC}{AN}$ [line || one side of Δ **OR/*OF*** prop theorem; MN ||BC / *Lyn* || *een sy v* Δ] $\therefore\ \frac{AB}{DE} = \frac{AC}{DF}$ [AM = DE and AN = DF]	✓Constr ✓S ✓R ✓S /R ✓S ✓R (6)

10.2



10.2.1(a)	$F\hat{C}O = 90°$ [tan ⊥ radius / *raaklyn ⊥ radius*] $\hat{F}_1 = 90°$ [BF ⊥ EC] $\therefore F\hat{C}O = \hat{F}_1 = 90°$ FB ‖ CG [corresp ∠s = / *ooreenk. ∠ gelyk*]	✓ S / R ✓ S ✓ R (3)
10.2.1(b)	In ΔFCB and ΔCDB $B\hat{C}D = 90°$ [∠ in semi-circle / ∠ $\frac{1}{2}$ ⊙] $\hat{F}_2 = 90°$ [BF ⊥ EC] $\therefore \hat{F}_2 = B\hat{C}D = 90°$ $\hat{C}_1 = \hat{D}_2$ [tan chord theorem / ∠ *tussen rkl en koord*] $\hat{B}_2 = \hat{B}_3$ [sum of ∠s in Δ / ∠*e van* Δ] ∴ ΔFCB ||| ΔCDB **OR/*OF*** In ΔFCB and ΔCDB $B\hat{C}D = 90°$ [∠ in semi-circle / ∠ $\frac{1}{2}$ ⊙] $\hat{F}_2 = 90°$ [BF ⊥ EC] $\therefore \hat{F}_2 = B\hat{C}D = 90°$ $\hat{C}_1 = \hat{D}_2$ [tan chord theorem / ∠ *tussen rkl en koord*] ∴ ΔFCB ||| ΔCDB [∠,∠,∠]	✓ S /R ✓ S ✓ S ✓ R ✓ S ✓ S / R ✓ S ✓ S ✓ R ✓ R (5)

10.2.2	$\hat{G}_1 = 90°$ [line from centre to midpt of chord / *midpt.* ⊙; *midpt. koord*]	✓ R (1)
10.2.3	In ΔGCD and ΔCDB $\hat{G}_2 = B\hat{C}D = 90°$ $\hat{C}_3 = \hat{D}_2$ [∠s opp equal sides /∠*e teenoor gelyke sye*] $G\hat{D}C = \hat{B}_3$ [sum of ∠s in Δ /∠*e van* Δ] $\therefore \Delta GCD \mid\mid\mid \Delta CDB$ [∠,∠,∠] $\therefore \frac{CD}{DB} = \frac{CG}{CD}$ [||| Δs] $\therefore CD^2 = CG.DB$	✓ identifying Δs ✓ S ✓ S / R ✓ S **OR** ✓ R ✓ S (5)
10.2.4	$\frac{BC}{DB} = \frac{FB}{BC}$ [ΔFCB ||| ΔCDB] $\therefore BC^2 = DB.FB$ $CD^2 + BC^2 = CG.DB + DB.FB$ $DB^2 = DB(CG + FB)$ $DB = CG + FB$	✓ S ✓ R ✓ S ✓ sum ✓ $DB^2 = CD^2 + BC^2$ (5)
		[25]

TOTAL/*TOTAAL*: 150