basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P2

NOVEMBER 2023

MARKS: 150

TIME: 3 hours

This question paper consists of 13 pages, 1 information sheet and an answer book of 23 pages.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 10 questions.
2. Answer ALL the questions in the SPECIAL ANSWER BOOK provided.
3. Clearly show ALL calculations, diagrams, graphs, etc. which you have used in determining your answers.
4. Answers only will NOT necessarily be awarded full marks.
5. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
6. If necessary, round off answers correct to TWO decimal places, unless stated otherwise.
7. Diagrams are NOT necessarily drawn to scale.
8. An information sheet with formulae is included at the end of the question paper.
9. Write neatly and legibly.

QUESTION 1

Truck drivers travel a certain distance and have a rest before travelling further. A driver kept record of the distance he travelled (in km) on 8 trips and the amount of time he rested (in minutes) before he continued his journey. The information is given in the table below.

Distance travelled (in km) (x)	180	200	400	600	170	350	270	300
Amount of rest time (in minutes) (y)	20	25	55	120	15	50	40	45

1.1 Determine the equation of the least squares regression line for the data. (3)

1.2 If a truck driver travelled 550 km, predict the amount of time (in minutes) that he should rest before continuing his journey. (2)

1.3 Write down the correlation coefficient for the data. (1)

1.4 Interpret your answer to QUESTION 1.3. (1)

1.5 At each stop, the truck driver spent money buying food and other refreshments. The amount spent (in rands) is given in the table below.

100	150	130	200	50	180	200	190

1.5.1 Calculate the mean amount of money he spent at each stop. (2)

1.5.2 Calculate the standard deviation for the data. (1)

1.5.3 At how many stops did the driver spend an amount that was less than one standard deviation below the mean? (2)

[12]

QUESTION 2

At a certain school, the staff committee wanted to determine how many glasses of water the staff members drank during a school day. All teachers present on a specific day were interviewed. The information is shown in the table below.

NUMBER OF GLASSES OF WATER DRANK PER DAY	NUMBER OF STAFF MEMBERS
$0 \leq x < 2$	5
$2 \leq x < 4$	15
$4 \leq x < 6$	13
$6 \leq x < 8$	5
$8 \leq x < 10$	2

2.1 Complete the cumulative frequency column provided in the table in the ANSWER BOOK. (2)

2.2 How many staff members were interviewed? (1)

2.3 How many staff members drank fewer than 6 glasses of water during a school day? (1)

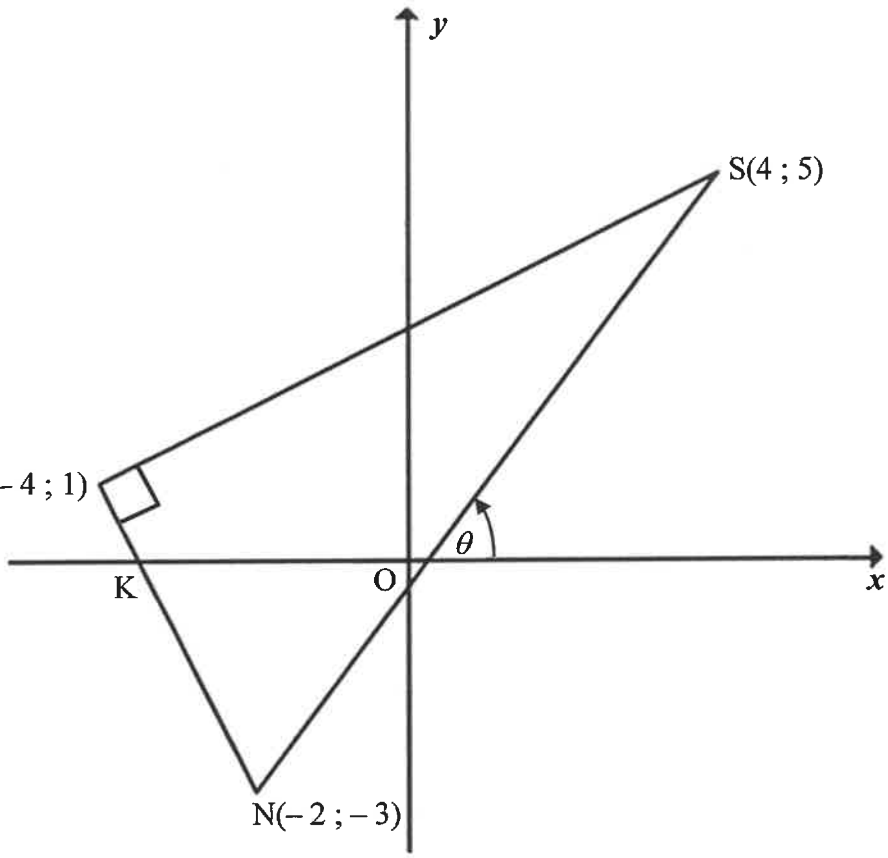
2.4 The staff committee observed that k teachers were absent on the day of the interviews. It was found that half of these k teachers drank from 0 to fewer than 2 (that is $0 \leq x < 2$) glasses of water per day, while the remainder of them drank from 4 to fewer than 6 (that is $4 \leq x < 6$) glasses of water per day. When these k teachers are included in the data, the estimated mean is 4 glasses of water per staff member per day.

How many teachers were absent on the day of the interviews? (4)

[8]

QUESTION 3

In the figure, L$(-4\ ;\ 1)$, S$(4\ ;\ 5)$ and N$(-2\ ;\ -3)$ are the vertices of a triangle having $S\hat{L}N = 90°$. LN intersects the x-axis at K.

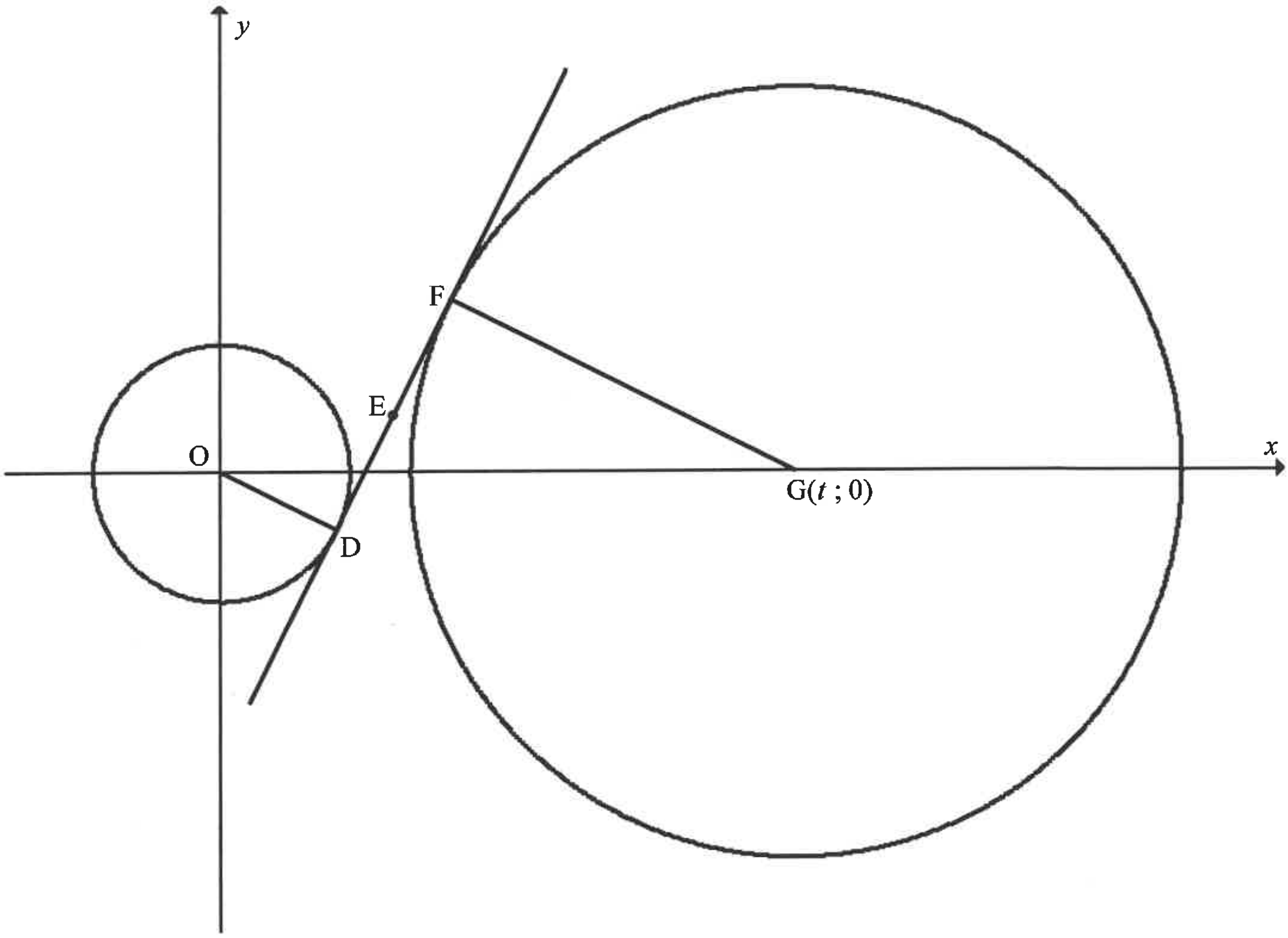


3.1	Calculate the length of SL. Leave your answer in surd form.	(2)
3.2	Calculate the gradient of SN.	(2)
3.3	Calculate the size of θ, the angle of inclination of SN.	(2)
3.4	Calculate the size of $L\hat{N}S$.	(3)
3.5	Determine the equation of the line which passes through L and is parallel to SN. Write your answer in the form $y = mx + c$.	(3)
3.6	Calculate the area of ΔLSN.	(3)
3.7	Calculate the coordinates of point P, which is equidistant from L, S and N.	(3)
3.8	Calculate the size of $L\hat{P}S$.	(2)

[20]

QUESTION 4

In the diagram, the circle with centre O has the equation $x^2 + y^2 = 20$. $G(t\ ;\ 0)$ is the centre of the larger circle. A common tangent touches the circles at D and F respectively, such that $D(p\ ; -2)$ lies in the 4th quadrant.



4.1 Given that $D(p\ ; -2)$ lies on the smaller circle, show that $p = 4$. (2)

4.2 $E(6\ ;\ 2)$ is the midpoint of DF. Determine the coordinates of F. (3)

4.3 Determine the equation of the common tangent, DF, in the form $y = mx + c$. (4)

4.4 Calculate the value of t. Show ALL working. (3)

4.5 Determine the equation of the larger circle in the form $ax^2 + by^2 + cx + dy + e = 0$. (4)

4.6 The smaller circle must be translated by k units along the x-axis to touch the larger circle internally. Calculate the possible values of k. (4)

[20]

QUESTION 5

5.1 Given: $\sin\beta = \frac{1}{3}$, where $\beta \in (90°\ ;\ 270°)$

Without using a calculator, determine each of the following:

5.1.1 $\cos\beta$ (3)

5.1.2 $\sin 2\beta$ (3)

5.1.3 $\cos(450° - \beta)$ (3)

5.2 Given: $\dfrac{\cos^4 x + \sin^2 x.\cos^2 x}{1+\sin x}$

5.2.1 Prove that $\dfrac{\cos^4 x + \sin^2 x.\cos^2 x}{1+\sin x} = 1 - \sin x$ (4)

5.2.2 For what value(s) of x in the interval $x \in [0°\ ;\ 360°]$ is $\dfrac{\cos^4 x + \sin^2 x.\cos^2 x}{1+\sin x}$ undefined? (2)

5.2.3 Write down the minimum value of the function defined by $y = \dfrac{\cos^4 x + \sin^2 x.\cos^2 x}{1+\sin x}$ (2)

5.3 Given: cos(A – B) = cosAcosB + sinAsinB

5.3.1 Use the above identity to deduce that sin(A – B) = sinAcosB – cosAsinB (3)

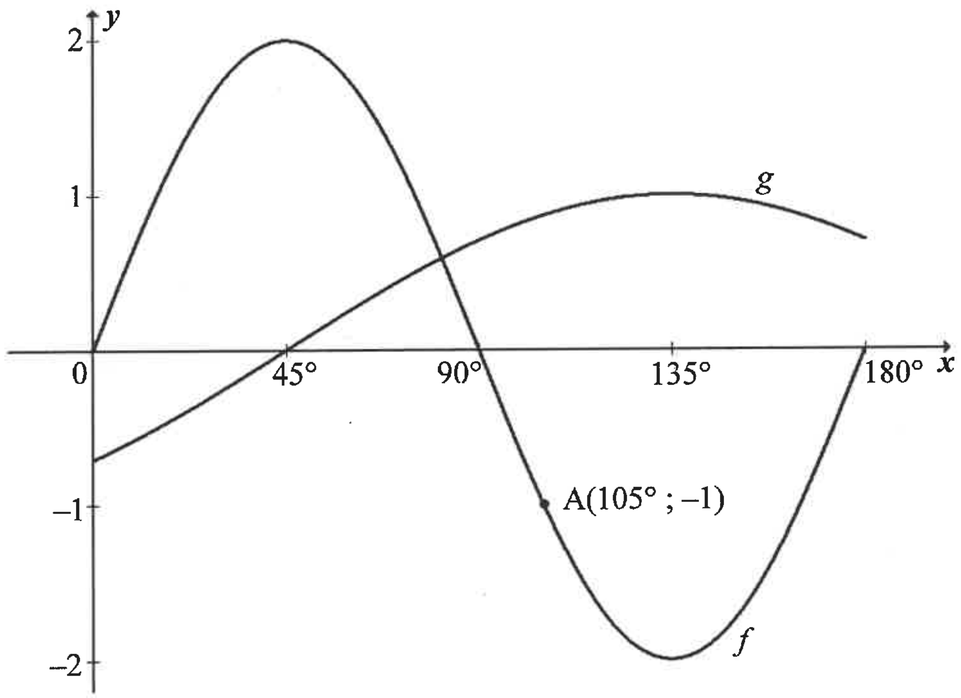
5.3.2 Hence, or otherwise, determine the general solution of the equation $\sin 48°\cos x - \cos 48°\sin x = \cos 2x$ (5)

5.4 Simplify $\dfrac{\sin 3x + \sin x}{\cos 2x + 1}$ to a single trigonometric ratio. (6)

[31]

QUESTION 6

In the diagram, the graphs of $f(x)=2\sin 2x$ and $g(x)=-\cos(x+45°)$ are drawn for the interval $x\in[0°\,;180°]$. A$(105°\,;-1)$ lies on f.

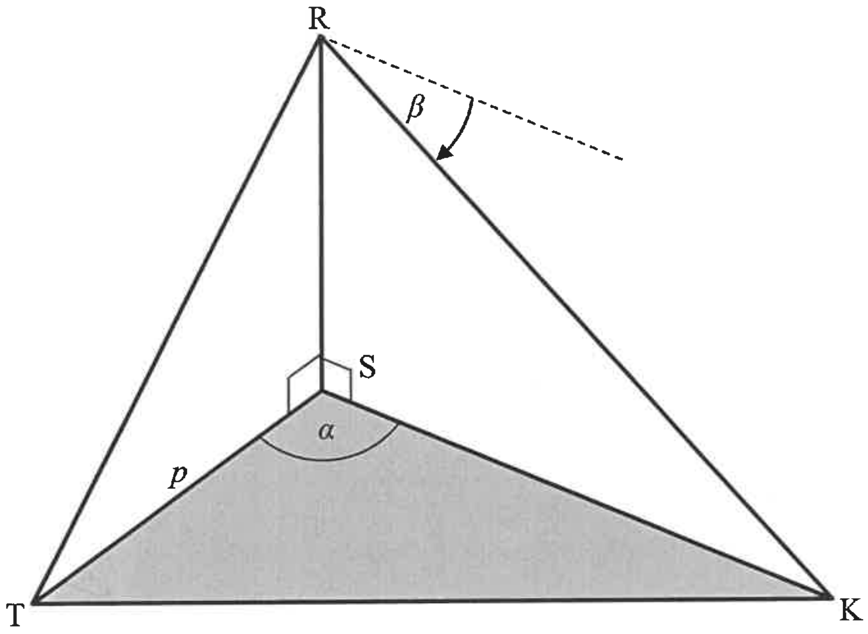


6.1	Write down the period of f.	(1)
6.2	Determine the range of g in the interval $x \in [0°\,;180°]$.	(2)
6.3	Determine the values of x, in the interval $x \in [0°\,;180°]$, for which:	
	6.3.1 $f(x).g(x)>0$	(2)
	6.3.2 $f(x)+1\leq 0$	(2)
6.4	Another graph p is defined as $p(x)=-f(x)$. D$(k\,;-1)$ lies on p. Determine the value(s) of k in the interval $x \in [0°\,;180°]$.	(3)
6.5	Graph h is obtained when g is translated $45°$ to the left. Determine the equation of h. Write your answer in its simplest form.	(2)

[12]

QUESTION 7

In the diagram, S, T and K lie in the same horizontal plane. RS is a vertical tower. The angle of depression from R to K is β. $T\hat{S}K = \alpha$, TS $= p$ metres and the area of ΔSTK is q m^2.



7.1 Determine the length of SK in terms of p, q and α. (2)

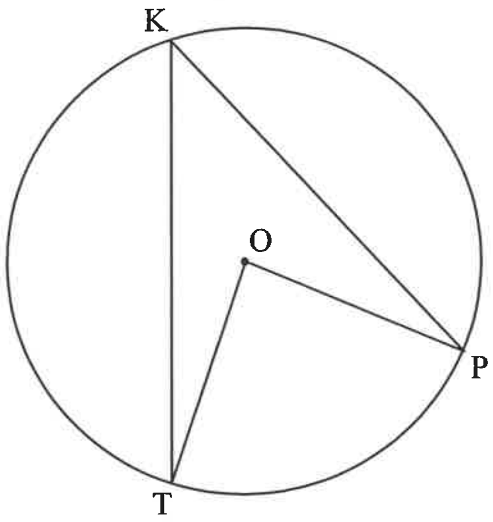
7.2 Show that $\text{RS} = \dfrac{2q\tan\beta}{p\sin\alpha}$ (2)

7.3 Calculate the size of α if $\alpha < 90°$ and RS = 70 m, $p = 80$ m, $q = 2\ 500$ m^2 and $\beta = 42°$. (3)

[7]

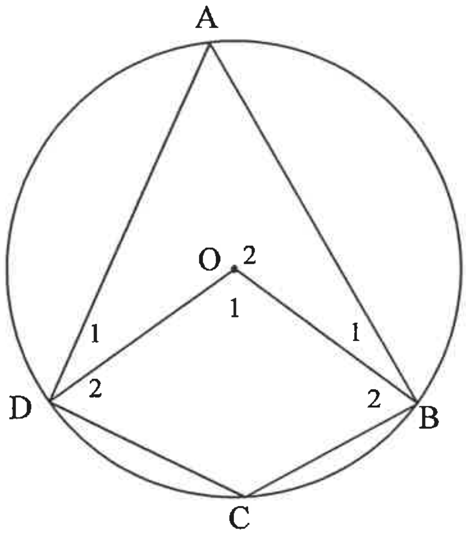
QUESTION 8

8.1 In the diagram, O is the centre of the circle.



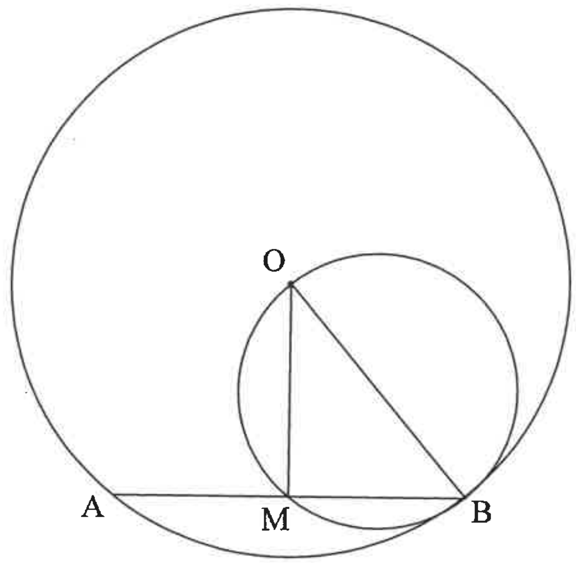
Use the diagram above to prove the theorem which states that the angle subtended by a chord at the centre of the circle is equal to twice the angle subtended by the same chord at the circumference, that is, prove that $T\hat{O}P = 2T\hat{K}P$. (5)

8.2 In the diagram, O is the centre of the circle and ABCD is a cyclic quadrilateral. OB and OD are drawn.



If $\hat{O}_1 = 4x + 100°$ and $\hat{C} = x + 34°$, calculate, giving reasons, the size of x. (5)

8.3 In the diagram, O is the centre of the larger circle. OB is a diameter of the smaller circle. Chord AB of the larger circle intersects the smaller circle at M and B.



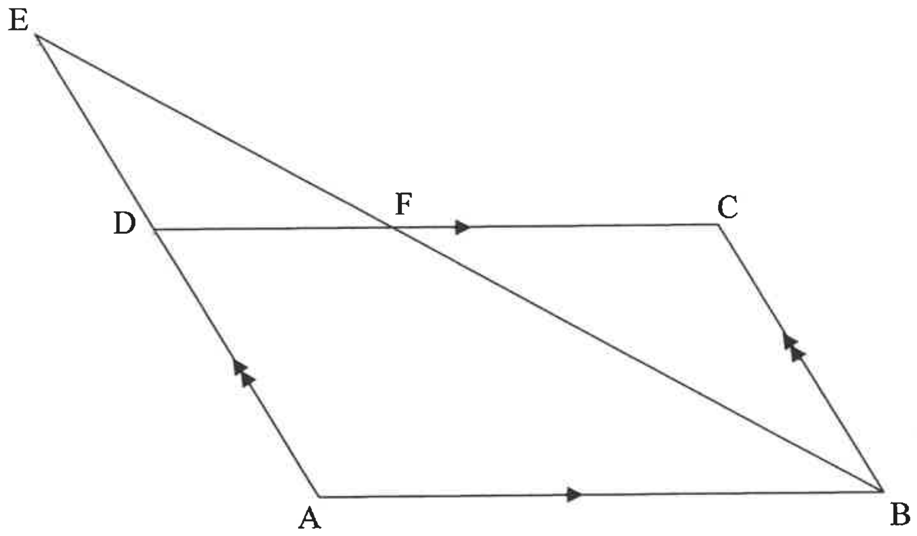
8.3.1 Write down the size of $O\hat{M}B$. Provide a reason. (2)

8.3.2 If $AB = \sqrt{300}$ units and $OM = 5$ units, calculate, giving reasons, the length of OB. (4)

[16]

QUESTION 9

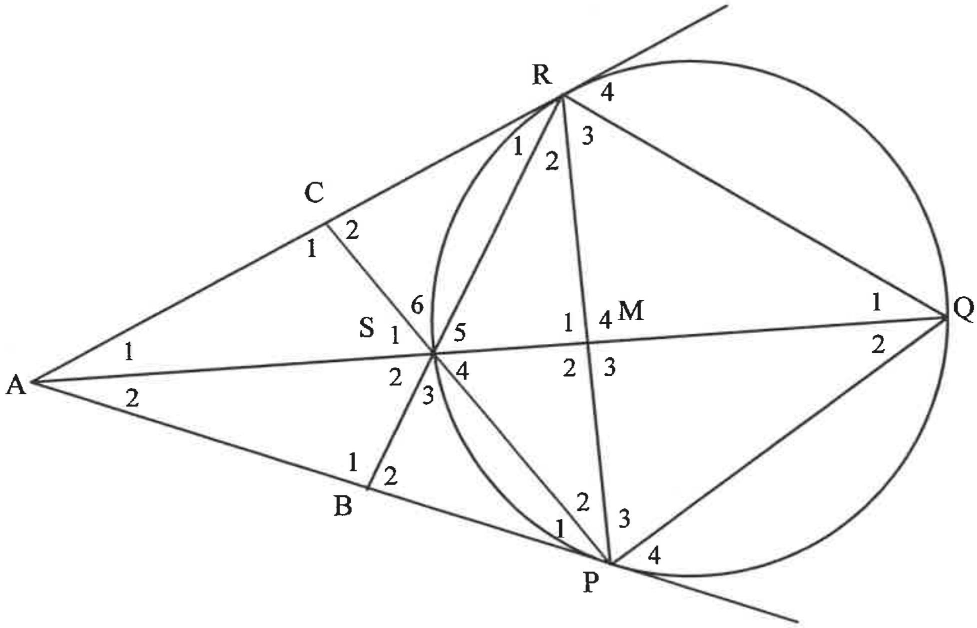
In the diagram, ABCD is a parallelogram with $AB = 14$ units. AD is produced to E such that $AD : DE = 4 : 3$. EB intersects DC in F. $EB = 21$ units.



9.1	Calculate, with reasons, the length of FB.	(3)
9.2	Prove, with reasons, that $\Delta EDF \mathbin{\|\|\|} \Delta EAB$.	(3)
9.3	Calculate, with reasons, the length of FC.	(3)
		[9]

QUESTION 10

In the diagram, PQRS is a cyclic quadrilateral such that PQ = PR. The tangents to the circle through P and R meet QS produced at A. RS is produced to meet tangent AP at B. PS is produced to meet tangent AR at C. PR and QS intersect at M.



Prove, giving reasons, that:

10.1 $\hat{S}_3 = \hat{S}_4$ (5)

10.2 SMRC is a cyclic quadrilateral (4)

10.3 RP is a tangent to the circle passing through P, S and A at P (6)

[15]

TOTAL: 150

INFORMATION SHEET

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$A=P(1+ni)$ $\quad A=P(1-ni)$ $\quad A=P(1-i)^n$ $\quad A=P(1+i)^n$

$T_n=a+(n-1)d$ $\quad S_n=\frac{n}{2}\left[2a+(n-1)d\right]$

$T_n=ar^{n-1}$ $\quad S_n=\frac{a\left(r^n-1\right)}{r-1}\ ;r\neq 1$ $\quad S_\infty=\frac{a}{1-r};\ -1<r<1$

$F=\frac{x\left[(1+i)^n-1\right]}{i}$ $\quad P=\frac{x\left[1-(1+i)^{-n}\right]}{i}$

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ $\quad M\left(\frac{x_1+x_2}{2};\frac{y_1+y_2}{2}\right)$

$y=mx+c$ $\quad y-y_1=m\left(x-x_1\right)$ $\quad m=\frac{y_2-y_1}{x_2-x_1}$ $\quad m=\tan\theta$

$$(x-a)^2+(y-b)^2=r^2$$

In ΔABC: $\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$

$a^2=b^2+c^2-2bc.\cos A$

$area\ \Delta ABC=\frac{1}{2}ab.\sin C$

$\sin(\alpha+\beta)=\sin\alpha.\cos\beta+\cos\alpha.\sin\beta$ $\quad \sin(\alpha-\beta)=\sin\alpha.\cos\beta-\cos\alpha.\sin\beta$

$\cos(\alpha+\beta)=\cos\alpha.\cos\beta-\sin\alpha.\sin\beta$ $\quad \cos(\alpha-\beta)=\cos\alpha.\cos\beta+\sin\alpha.\sin\beta$

$$\cos 2\alpha=\begin{cases}\cos^2\alpha-\sin^2\alpha\\1-2\sin^2\alpha\\2\cos^2\alpha-1\end{cases}$$

$\sin 2\alpha=2\sin\alpha.\cos\alpha$

$\bar{x}=\frac{\sum_{i=1}^{n}x_i}{n}$ $\quad \sigma^2=\frac{\sum_{i=1}^{n}(x_i-\bar{x})^2}{n}$

$P(A)=\frac{n(A)}{n(S)}$ $\quad P(A \text{ or } B)=P(A)+P(B)-P(A \text{ and } B)$

$\hat{y}=a+bx$ $\quad b=\frac{\sum(x-\bar{x})(y-\bar{y})}{\sum(x-\bar{x})^2}$

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NATIONAL
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GRADE 12/*GRAAD 12*

MATHEMATICS P2/*WISKUNDE V2*

NOVEMBER 2023

MARKING GUIDELINES/*NASIENRIGLYNE*

MARKS/PUNTE: 150

These marking guidelines consist of 23 pages./
Hierdie nasienriglyne bestaan uit 23 bladsye.

NOTE:

- If a candidate answers a question TWICE, only mark the FIRST attempt.
- If a candidate has crossed out an attempt of a question and not redone the question, mark the crossed out version.
- Consistent accuracy applies in ALL aspects of the Marking Guidelines. Stop marking at the second calculation error.
- Assuming answers/values in order to solve a problem is NOT acceptable.

NOTA:

- *As 'n kandidaat 'n vraag TWEE KEER beantwoord, merk slegs die EERSTE poging.*
- *As 'n kandidaat 'n antwoord van 'n vraag doodtrek en nie oordoen nie, merk die doodgetrekte poging.*
- *Volgehoue akkuraatheid word in ALLE aspekte van die Nasienriglyne toegepas. Hou op nasien by die tweede berekeningsfout.*
- *Aanvaar van antwoorde/waardes om 'n probleem op te los, word NIE toegelaat nie.*

GEOMETRY	
S	**A mark for a correct statement (A statement mark is independent of a reason)**
	'n Punt vir 'n korrekte bewering ('n Punt vir 'n bewering is onafhanklik van die rede)
R	**A mark for the correct reason (A reason mark may only be awarded if the statement is correct)**
	'n Punt vir 'n korrekte rede ('n Punt word slegs vir die rede toegeken as die bewering korrek is)
S/R	**Award a mark if statement AND reason are both correct**
	Ken 'n punt toe as die bewering EN rede beide korrek is

QUESTION/*VRAAG* 1

1.1	$a = -23,846\ldots$ $b = 0,227\ldots$ $\hat{y} = -23,85 + 0,23x$	✓ $a = -23,846\ldots$ ✓ $b = 0,227\ldots$ ✓ equation (3)
1.2	$\hat{y} = -23,85 + 0,23(550)$ $y = 102,65$ **OR** $y = 101,02$	✓ substitution of 550 ✓ answer (2) ✓✓ $y = 101,02$ (calculator) (2)
1.3	$r = 0,98$	✓ $r = 0,98$ (1)
1.4	Very strong positive correlation	✓ strong positive (1)

50	100	130	150	180	190	200	200

1.5.1	$\bar{x} = \frac{1200}{8}$ $\bar{x} = 150$ **OR** $\bar{x} = 150$	✓ 1200 ✓ answer (2) ✓✓ $\bar{x} = 150$ (2)
1.5.2	$\sigma = 50,50$	✓ $\sigma = 50,50$ (1)
1.5.3	$\bar{x} - \sigma$ $= 150 - 50,50$ $= 99,50$ $\therefore$ 1 stop	✓ calculation of $\bar{x} - \sigma$ ✓ answer (2)
		[12]

QUESTION/*VRAAG* 2

<table>
<tr><td>2.1</td><td>

Number of glasses of water per day	Number of staff members	Cumulative frequency
$0 \le x < 2$	5	5
$2 \le x < 4$	15	20
$4 \le x < 6$	13	33
$6 \le x < 8$	5	38
$8 \le x < 10$	2	40

</td><td>✓ 5; 20
✓ 40
(2)</td></tr>
<tr><td>2.2</td><td>40 staff members</td><td>✓ answer
(1)</td></tr>
<tr><td>2.3</td><td>33 staff members</td><td>✓ answer
(1)</td></tr>
<tr><td>2.4</td><td>

$\bar{x} = \dfrac{\left(1\times\left(5+\frac{k}{2}\right)\right)+(3\times15)+\left(5\times\left(13+\frac{k}{2}\right)\right)+(7\times5)+(9\times2)}{40+k} = 4$

$5+\frac{k}{2}+45+65+\frac{5k}{2}+35+18=160+4k$
$3k+168=160+4k$
$k=8$

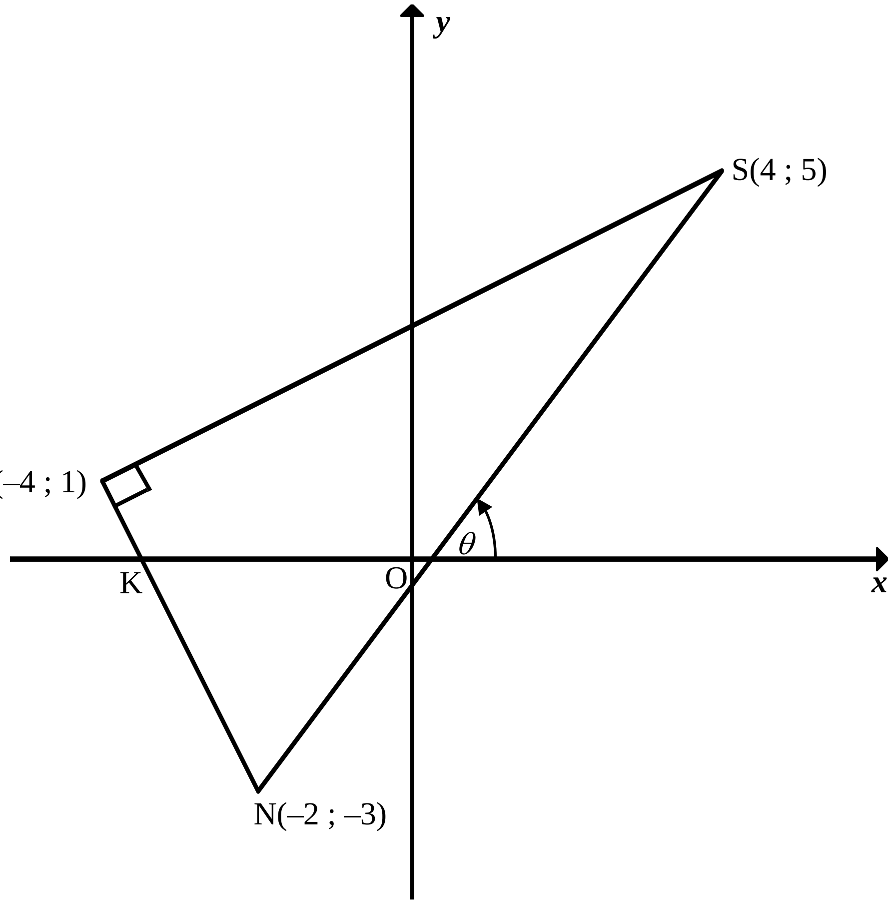
OR

$\bar{x} = \dfrac{(1\times5)+(15\times3)+(13\times5)+(5\times7)+(2\times9)}{40}$
$= 4,2$
$\bar{x}_{\text{old}} - \bar{x}_{\text{current}} = 4,2-4$
$= 0,2$
$\therefore 0,2\times40$
= 8 teachers

</td><td>✓ answer from Q2.2 + k
✓ $\left(1\times\left(5+\frac{k}{2}\right)\right)$
✓ $\left(5\times\left(13+\frac{k}{2}\right)\right)$
✓ answer
(4)

✓ 4,2
✓ $\bar{x}_{\text{old}}-4$
✓ difference
✓ answer
(4)</td></tr>
<tr><td></td><td></td><td>**[8]**</td></tr>
</table>

QUESTION/*VRAAG* 3

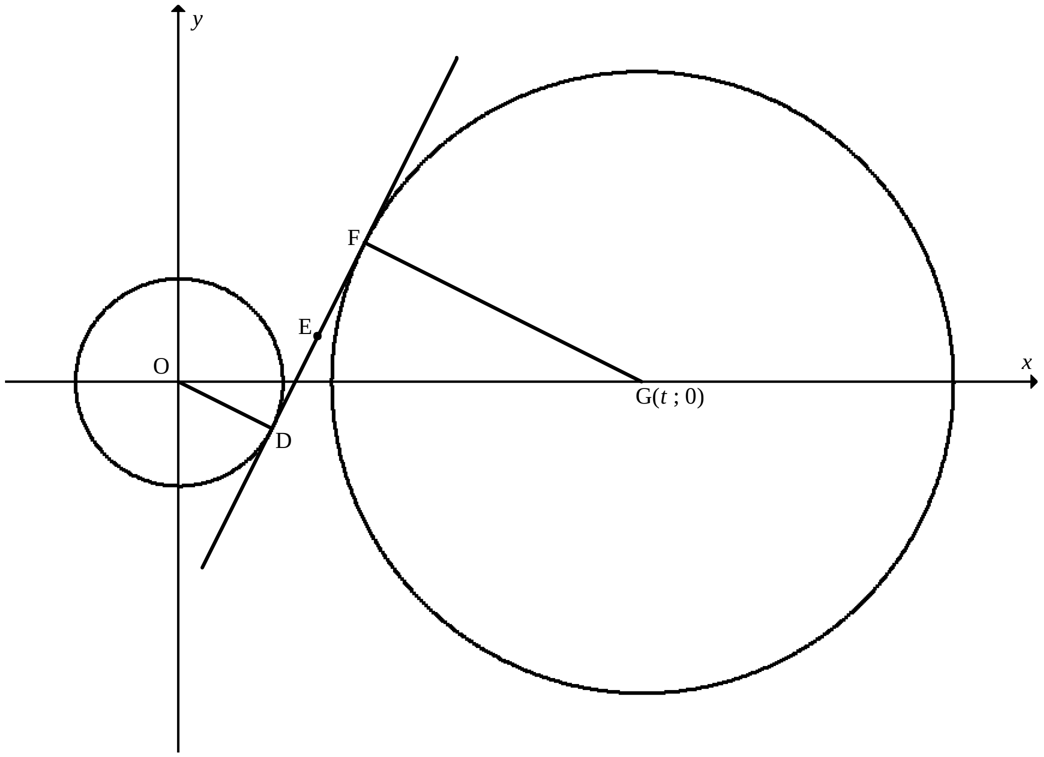


3.1	$SL=\sqrt{(x_S-x_L)^2+(y_S-y_L)^2}$ $SL=\sqrt{(4-(-4))^2+(5-1)^2}$ $SL=\sqrt{80}=4\sqrt{5}=8,94$ units	✓ substitution of S and L into correct formula ✓ answer (2)
3.2	$m_{SN}=\dfrac{5-(-3)}{4-(-2)}$ $m_{SN}=\dfrac{4}{3}$	✓ substitution of S and N into correct formula ✓ answer (2)
3.3	$m=\tan\theta=\dfrac{4}{3}$ $\theta=53,13°$	✓ $\tan\theta=m_{SN}$ ✓ answer (2)
3.4	$m_{LN}=\dfrac{1-(-3)}{-4-(-2)}$ $m_{LN}=-2$ $L\hat{K}O=116,565...°$ $L\hat{N}S=116,565...°-53,13°$ $L\hat{N}S=63,44°$	✓ $m_{LN}=-2$ ✓ size of $L\hat{K}O$ ✓ answer (3)

	OR SN = 10 units $\sin \hat{LNS} = \frac{4\sqrt{5}}{10}$ $\hat{LNS} = 63{,}44°$ **OR** $LN = 2\sqrt{5}$ units $\tan \hat{LNS} = \frac{4\sqrt{5}}{2\sqrt{5}}$ $\hat{LNS} = 63{,}44°$ **OR** SN = 10 units $LN = 2\sqrt{5}$ units $\cos \hat{LNS} = \frac{2\sqrt{5}}{10}$ $\hat{LNS} = 63{,}44°$	✓ SN = 10 units ✓ correct trig ratio ✓ answer (3) ✓ $LN = 2\sqrt{5}$ units ✓ correct trig ratio ✓ answer (3) ✓ SN = 10 units and $LN = 2\sqrt{5}$ units ✓ correct trig ratio ✓ answer (3)
3.5	$m = \frac{4}{3}$ $1 = \frac{4}{3}(-4) + c$ **OR** $y - 1 = \frac{4}{3}(x - (-4))$ $c = \frac{19}{3}$ $y - 1 = \frac{4}{3}x + \frac{16}{3}$ $y = \frac{4}{3}x + \frac{19}{3}$ $y = \frac{4}{3}x + \frac{19}{3}$	✓ m_{SN} ✓ substitution of m_{SN} & L ✓ equation (3)
3.6	$SL = 4\sqrt{5}$ $LN = \sqrt{(-4-(-2))^2 + (1-(-3))^2}$ $LN = \sqrt{20} = 2\sqrt{5}$ Area $\Delta LSN = \frac{1}{2}(4\sqrt{5})(2\sqrt{5})$ $= 20 \text{ units}^2$ **OR**	✓ $LN = \sqrt{20} = 2\sqrt{5}$ ✓ substitution into formula ✓ answer (3)

	SN = 10 units LN = $\sqrt{(-4-(-2))^2+(1-(-3))^2}$ LN = $\sqrt{20} = 2\sqrt{5}$ Area ΔLSN = $\frac{1}{2}(10)(2\sqrt{5})\sin 63,44°$ = 20 units^2	✓ LN = $\sqrt{20} = 2\sqrt{5}$ ✓ substitution into formula ✓ answer (3)
3.7	$\hat{L} = 90°$ SN is a diameter of circle S, L, N [chord subtends 90° **OR** converse $\angle$ in semi-circle] Centre of circle = P$\left(\frac{4+(-2)}{2};\frac{5+(-3)}{2}\right)$ = P$(1;1)$ **OR** Let the coordinates of P be (*a* ; *b*). Then, PL = PN: $(-4-a)^2+(1-b)^2=(-2-a)^2+(-3-b)^2$ $a-2b=-1$ …..equation 1 If PS = PN, then: $4a+2b=6$ ………….. equation 2 Solving simultaneously yields: $a=1$ and $b=1$ and P$(1;1)$ **OR** If PL = PN, then: $a-2b=-1$ …..equation 1 If PS = PL, then: $2a+b=3$.....equation 2 Solving simultaneously yields: $a=1$ and $b=1$ and P$(1;1)$	✓ SN is a diameter of circle S, L, N ✓ *x*-value ✓ *y*-value (3) ✓ 2 correct linear equations ✓ *x*-value ✓ *y*-value (3) ✓ 2 correct linear equations ✓ *x*-value ✓ *y*-value (3)
3.8	$L\hat{P}N = \theta = 53,13°$ [alt $\angle$s; LP || x–axis] $\therefore L\hat{P}S = 126,87°$ **OR** $L\hat{N}S = 63,44°$ $\therefore L\hat{P}S = 126,88°$ [$\angle$ at centre = 2 × $\angle$ at circumference] **OR** $L\hat{S}N = 26,56°$ [sum of $\angle$s in Δ] $S\hat{L}P = 26,56°$ [$\angle$s opp equal radii] $\therefore L\hat{P}S = 126,88°$ [sum of $\angle$s in Δ] **OR** $(4\sqrt{5})^2 = 5^2+5^2-2(5)(5)\cos L\hat{P}S$ $\cos L\hat{P}S = -\frac{3}{5}$ $\therefore L\hat{P}S = 126,87°$	✓ $L\hat{P}N$ ✓ answer (2) ✓ $L\hat{N}S$ ✓ answer (2) ✓ $L\hat{S}N$ ✓ answer (2) ✓ correct substitution into cosine formula ✓ answer (2)
		[20]

QUESTION/*VRAAG* 4

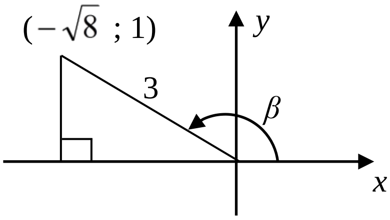


4.1	D(p ; -2) $x^2 + y^2 = 20$ $p^2 + (-2)^2 = 20$ $p^2 = 16$ $p = \pm 4$ $p = 4$	✓ substitution of point $D(p\ ;-2)$ ✓ $p^2 = 16$ (2)
4.2	$\frac{4+x_F}{2} = 6$ $\qquad$ $\frac{-2+y_F}{2} = 2$ $x_F = 8$ $\qquad$ $y_F = 6$ $F(8;6)$ **OR** $x_E - x_D = 6-4$ $\qquad$ $y_E - y_D = 2-(-2)$ $= 2$ $\qquad$ $= 4$ $x_F = 6+2 = 8$ $\qquad$ $y_F = 2+4 = 6$ $F(8;6)$	✓ method ✓ x-value ✓ y-value (3) ✓ method ✓ x-value ✓ y-value (3)

4.3	$m_{DE} = \frac{-2-2}{4-6}$ $m_{DE} = 2$ $-2 = 2(4)+c$ **OR** $y-(-2) = 2(x-4)$ $c = -10$ $y+2 = 2x-8$ $y = 2x-10$ $y = 2x-10$ **OR** $m_{OD} = -\frac{2}{4} = -\frac{1}{2}$ $\therefore m_{DE} = 2$ [tan ⊥ radius] $-2 = 2(4)+c$ **OR** $y-(-2) = 2(x-4)$ $c = -10$ $y+2 = 2x-8$ $y = 2x-10$ $y = 2x-10$	✓ correct substitution ✓ gradient of DE, DF or EF ✓ substitution of point D(4 ; −2) or E(6 ; 2) or F(8 ; 6) ✓ answer (4) ✓ correct gradient of OD ✓ gradient of DE ✓ substitution of point D(4 ; −2) or E(6 ; 2) or F(8 ; 6) ✓ answer (4)
4.4	$m_{DE} = 2$ $\therefore m_{GF} = -\frac{1}{2}$ [tan ⊥ radius] $\frac{0-6}{t-8} = -\frac{1}{2}$ $-(t-8) = 2(-6)$ $t = 20$ **OR** $y = 2x-10$ $0 = 2x-10$ $x = 5$ A(5 ; 0) In ΔAFG: FA ⊥ FG $FA^2 = (6-0)^2 + (8-5)^2 = 45$ $FG^2 = (t-8)^2 + (0-6)^2$ $= t^2 - 16t + 100$ $GA^2 = (t-5)^2$ $= t^2 - 10t + 25$ $\therefore GA^2 = GF^2 + FA^2$ $t^2 - 10t + 25 = t^2 - 16t + 100 + 45$ $6t = 120$ $t = 20$	 ✓ correct gradient of GF ✓ substitution of F ✓ answer (3) ✓ x-intercept of DF ✓ substitution into Pythagoras ✓ answer (3)

4.5	$F(8;6)$ G(20 ; 0) $(8-20)^2+(6-0)^2=r^2$ $r^2=180$ $(x-20)^2+y^2=180$ $x^2+y^2-40x+220=0$	✓ substitution of F and G ✓ value of r^2 ✓ equation of circle ✓ answer (4)
4.6	Smaller circle $r=2\sqrt{5}$ Larger circle $r=6\sqrt{5}$ G(20 ; 0) $k=20-(6\sqrt{5}-2\sqrt{5})$ or $k=20+(6\sqrt{5}-2\sqrt{5})$ $=20-4\sqrt{5}$ $=20+4\sqrt{5}$ = 11,06 units = 28,94 units **OR** Smaller circle $r=2\sqrt{5}$ $k=2\left(2\sqrt{5}\right)+20-8\sqrt{5}$ or $k=2\left(6\sqrt{5}\right)+20-8\sqrt{5}$ $=20-4\sqrt{5}$ $=20+4\sqrt{5}$ = 11,06 units = 28,94 units **OR** $x^2+y^2-40x+220=0$ $y=0$ $\therefore x^2-40x+220=0$ $\therefore x=20+6\sqrt{5}$ or $x=20-6\sqrt{5}$ $\therefore k=20+6\sqrt{5}-\sqrt{20}$ or $k=20-6\sqrt{5}+\sqrt{20}$ $\therefore k=20+4\sqrt{5}$ $\therefore k=20-4\sqrt{5}$ = 11,06 units = 28,94 units	✓ $r=2\sqrt{5}$ ✓ method ✓ answer ✓ answer (4) ✓ $r=2\sqrt{5}$ ✓ method ✓ answer ✓ answer (4) ✓ x-intercepts ✓ method ✓ answer ✓ answer (4)
		[20]

QUESTION/VRAAG 5

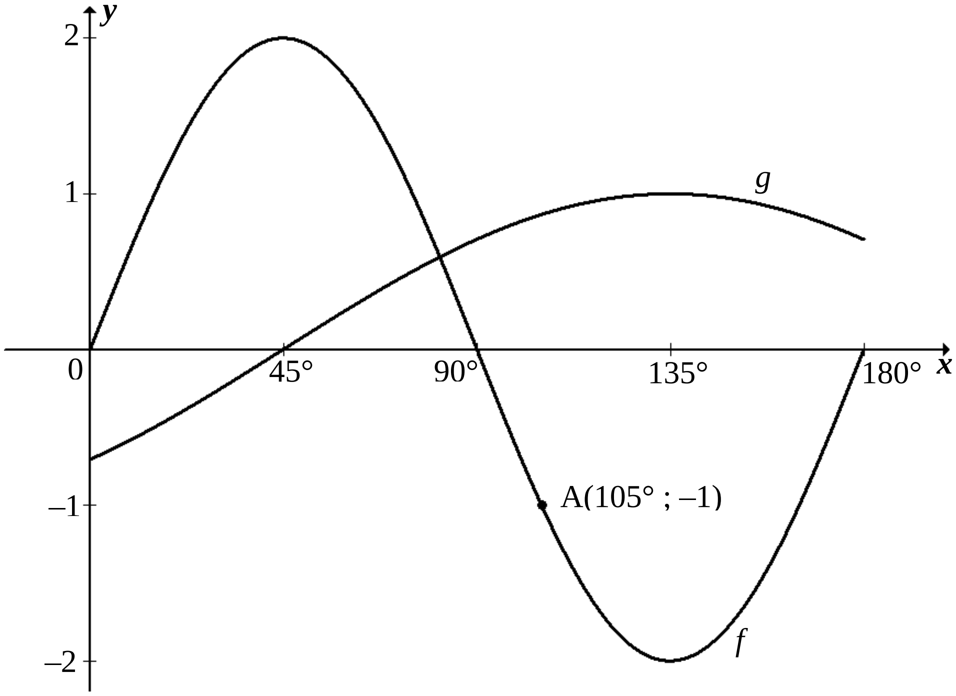
5.1.1	$\sin\beta = \frac{1}{3}$ $\beta \in (90°;270°)$  $x = -\sqrt{8} = -2\sqrt{2}$ $\cos\beta$ $= \frac{-2\sqrt{2}}{3}$ **OR** $\sin\beta = \frac{1}{3}$ $\beta \in (90°;270°)$ $\cos^2\beta = 1 - \sin^2\beta$ $\cos^2\beta = 1 - \left(\frac{1}{3}\right)^2$ $\cos^2\beta = \frac{8}{9}$ $\cos\beta = \frac{-\sqrt{8}}{3}$ $= \frac{-2\sqrt{2}}{3}$	✓ $x^2 + y^2 = r^2$ ✓ $x = -2\sqrt{2}$ ✓ answer (3) ✓ square identity ✓ $\cos^2\beta$ ✓ answer (3)
5.1.2	$\sin 2\beta$ $= 2\sin\beta\cos\beta$ $= 2\left(\frac{1}{3}\right)\left(\frac{-\sqrt{8}}{3}\right)$ $= \frac{-2\sqrt{8}}{9}$ **OR** $2\left(\frac{-2\sqrt{2}}{9}\right)$ $= \frac{-4\sqrt{2}}{9}$	✓ double angle ✓ substitution ✓ answer (3)
5.1.3	$\cos(450° - \beta)$ $= \cos(90° - \beta)$ $= \sin\beta$ $= \frac{1}{3}$ **OR**	✓ $\cos(90° - \beta)$ ✓ co-ratio ✓ answer (3)

	$\cos(450° - \beta)$ $= \cos 450° \cos\beta + \sin 450° \sin\beta$ $= \cos 90° \cos\beta + \sin 90° \sin\beta$ $= \sin\beta$ $= \dfrac{1}{3}$	✓ expansion ✓ reduction ✓ answer (3)
5.2.1	LHS $= \dfrac{\cos^4 x + \sin^2 x.\cos^2 x}{1+\sin x}$ $= \dfrac{\cos^2 x\left(\cos^2 x + \sin^2 x\right)}{1+\sin x}$ $= \dfrac{1-\sin^2 x}{1+\sin x}$ $= \dfrac{(1-\sin x)(1+\sin x)}{1+\sin x}$ $= 1-\sin x$ = RHS	✓ factors ✓ $\sin^2 x + \cos^2 x = 1$ ✓ $\cos^2 x = 1-\sin^2 x$ ✓ factors (4)
	OR	
	LHS $= \dfrac{\cos^4 x + \sin^2 x.\cos^2 x}{1+\sin x}$ $= \dfrac{\cos^4 x + \left(1-\cos^2 x\right)\cos^2 x}{1+\sin x}$ $= \dfrac{\cos^4 x + \cos^2 x - \cos^4 x}{1+\sin x}$ $= \dfrac{1-\sin^2 x}{1+\sin x}$ $= \dfrac{(1-\sin x)(1+\sin x)}{1+\sin x}$ $= 1-\sin x$ = RHS	✓ $\sin^2 x = 1-\cos^2 x$ ✓ expansion ✓ $\cos^2 x = 1-\sin^2 x$ ✓ factors (4)
	OR	
	RHS $= 1-\sin x$ $= (1-\sin x)\times\dfrac{1+\sin x}{1+\sin x}$ $= \dfrac{1-\sin^2 x}{1+\sin x}$ $= \dfrac{\cos^2 x}{1+\sin x}$ $= \dfrac{\cos^2 x\left(\sin^2 x + \cos^2 x\right)}{1+\sin x}$ $= \dfrac{\cos^4 x + \cos^2 x.\sin^2 x}{1+\sin x}$ = LHS	✓ $\times\dfrac{1+\sin x}{1+\sin x}$ ✓ product ✓ $1-\sin^2 x = \cos^2 x$ ✓ $1 = \cos^2 x + \sin^2 x$ (4)

5.2.2	$\sin x+1=0$ $\sin x=-1$ ref. $\angle = 90°$ $x = 270°$	✓ $\sin x+1=0$ ✓ $x = 270°$ (2)
5.2.3	$y=\dfrac{\cos^4 x+\sin^2 x.\cos^2 x}{1+\sin x}$ $y=1-\sin x$ $\therefore$ Minimum = 0	✓✓ Minimum = 0 (2)
5.3.1	sin (A – B) $=\cos\left[90°-\left(\mathrm{A}-\mathrm{B}\right)\right]$ $=\cos\left[(90°-\mathrm{A})-(-\mathrm{B})\right]$ $=\cos\left(90°-\mathrm{A}\right)\cos\left(-\mathrm{B}\right)+\sin\left(90°-\mathrm{A}\right)\sin\left(-\mathrm{B}\right)$ $=\sin\mathrm{A}\cos\mathrm{B}+\cos\mathrm{A}\left(-\sin\mathrm{B}\right)$ $=\sin\mathrm{A}\cos\mathrm{B}\ -\cos\mathrm{A}\sin\mathrm{B}$ **OR** sin (A – B) $=\cos\left[90°-\left(\mathrm{A}-\mathrm{B}\right)\right]$ $=\cos\left[(90°+\mathrm{B})-\mathrm{A}\right]$ $=\cos\left(90°+\mathrm{B}\right)\cos\mathrm{A}+\sin\left(90°+\mathrm{B}\right)\sin\mathrm{A}$ $=-\sin\mathrm{B}\cos\mathrm{A}+\cos\mathrm{B}\sin\mathrm{A}$ $=\sin\mathrm{A}\cos\mathrm{B}-\cos\mathrm{A}\sin\mathrm{B}$	✓ co-ratio ✓ compound angle ✓ reduction (3) ✓ co-ratio ✓ compound angle ✓ reduction (3)
5.3.2	$\sin 48°\cos x-\cos 48°\sin x=\cos 2x$ $\sin\left(48°-x\right)=\cos 2x$ $\sin\left(48°-x\right)=\sin\left(90°-2x\right)$ $48°-x=90°-2x+k.360°$ or $48°-x=180°-\left(90°-2x\right)+k.360°$ $x=42°+k.360°$ $\quad -3x=42°+k.360°$ $x=-14°-k.120°$; $k\in Z$ **OR** $\sin 48°\cos x-\cos 48°\sin x=\cos 2x$ $\sin\left(48°-x\right)=\cos 2x$ $\cos\left(90°-48°+x\right)=\cos 2x$ $\cos\left(42°+x\right)=\cos 2x$ $42°+x=2x+k.360°$ or $42°+x=360°-2x+k.360°$ $-x=-42°+k.360°$ $\quad 3x=318°+k.360°$ $x=42°-k.360°$ $\quad x=106°+k.120°$; $k\in Z$	✓ compound angle ✓ co-ratio ✓ both equations ✓ general solution ✓ general solution; $k\in Z$ (5) ✓ compound angle ✓ co-ratio ✓ both equations ✓ general solution ✓ general solution; $k\in Z$ (5)

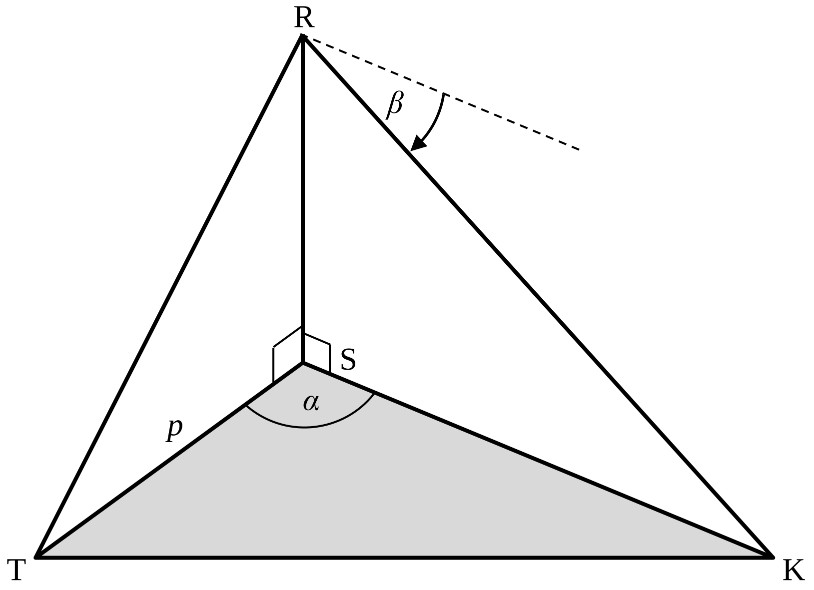
5.4	$\frac{\sin 3x+\sin x}{\cos 2x+1}$ $=\frac{\sin(2x+x)+\sin(2x-x)}{\cos 2x+1}$ $=\frac{\sin 2x\cos x+\cos 2x\sin x+\sin 2x\cos x-\cos 2x\sin x}{2\cos^2 x-1+1}$ $=\frac{2\sin 2x\cos x}{2\cos^2 x}$ $=\frac{2(2\sin x\cos x)\cos x}{2\cos^2 x}$ $=\frac{4\sin x\cos^2 x}{2\cos^2 x}$ $=2\sin x$ **OR** $\frac{\sin 3x+\sin x}{\cos 2x+1}$ $=\frac{\sin(2x+x)+\sin x}{2\cos^2 x-1+1}$ $=\frac{\sin 2x\cos x+\cos 2x\sin x+\sin x}{2\cos^2 x}$ $=\frac{2\sin x\cos x\cos x+\cos 2x\sin x+\sin x}{2\cos^2 x}$ $=\frac{\sin x(2\cos^2 x+\cos 2x+1)}{2\cos^2 x}$ $=\frac{\sin x(2\cos^2 x+2\cos^2 x-1+1)}{2\cos^2 x}$ $=2\sin x$	✓ $3x=(2x+x)$ ✓ expansion ✓ double angle of cos2*x* ✓ simplification ✓ $\sin 2x=2\sin x\cos x$ ✓ answer (6) ✓ $3x=(2x+x)$ ✓ double angle of cos2*x* ✓ expansion ✓ $\sin 2x=2\sin x\cos x$ ✓ common factor ✓ answer (6)
		[31]

QUESTION/*VRAAG* 6



6.1	Period = 180°	✓ 180° (1)
6.2	$y\in\left[-\frac{\sqrt{2}}{2}\ ;1\right]$ **OR** $y\in[-0{,}71;1]$ **OR** $-\frac{\sqrt{2}}{2}\le y\le 1$	✓ $-\frac{\sqrt{2}}{2}$ ✓ $y\in\left[-\frac{\sqrt{2}}{2}\ ;1\right]$ (2)
6.3.1	$x\in(45°\ ;\ 90°)$ **OR** $45°<x<90°$	✓✓ $x\in(45°\ ;\ 90°)$ (2)
6.3.2	$f(x)+1\le 0$ $f(x)\le -1$ $x\in[105°\ ;\ 165°]$ **OR** $105°\le x\le 165°$	✓✓ $x\in[105°\ ;\ 165°]$ (2)
6.4	$p(x)=-2\sin 2x$ $-2\sin 2x=-1$ **OR** $2\sin 2x=1$ k = 15° or k = 75°	✓ reading off $f(x)$ = 1 or $-f(x)$ = –1 ✓ 15° ✓ 75° (3)
6.5	$g(x)=-\cos(x+45°)$ $h(x)=-\cos(x+90°)$ $h(x)=\sin x$	✓ $-\cos(x+90°)$ ✓ answer (2)
		[12]

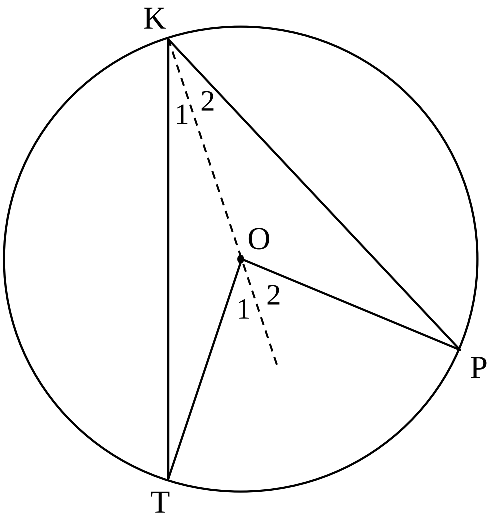
QUESTION/*VRAAG* 7



7.1	Area $\Delta STK = \frac{1}{2}p(SK)\sin\alpha$ $q = \frac{1}{2}p(SK)\sin\alpha$ $SK = \frac{q}{\frac{1}{2}p\sin\alpha}$ $= \frac{2q}{p\sin\alpha}$	✓ substitution into the correct formula ✓ answer (2)
7.2	$R\hat{K}S = \beta$ $\frac{RS}{SK} = \tan\beta$ $RS = \frac{2q\tan\beta}{p\sin\alpha}$ **OR** $\frac{RS}{\sin\beta} = \frac{SK}{\sin(90° - \beta)}$ $RS\cos\beta = SK\sin\beta$ $RS = SK\tan\beta$ $RS = \frac{2q\tan\beta}{p\sin\alpha}$	✓ $R\hat{K}S = \beta$ ✓ correct trig ratio (2) ✓ $R\hat{K}S = \beta$ ✓ $\tan\beta = \frac{\sin\beta}{\cos\beta}$ (2)
7.3	$70 = \frac{2(2500)\tan 42°}{80\sin\alpha}$ $\sin\alpha = \frac{25}{28}\tan 42°$ **OR** $\sin\alpha = 0,80...$ $\alpha = 53,51°$	✓ correct substitution of values into RS ✓ value of $\sin\alpha$ ✓ answer (3)
		[7]

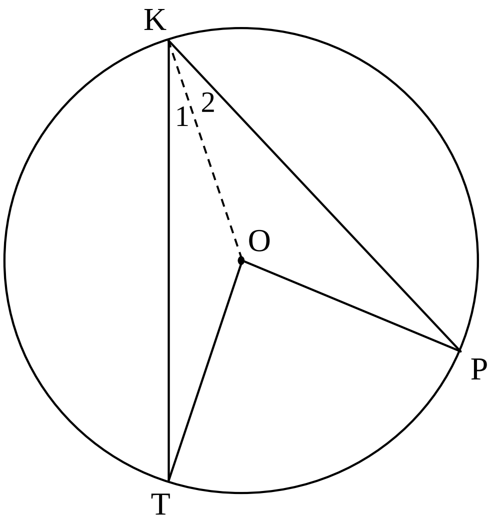
QUESTION/*VRAAG* 8

8.1



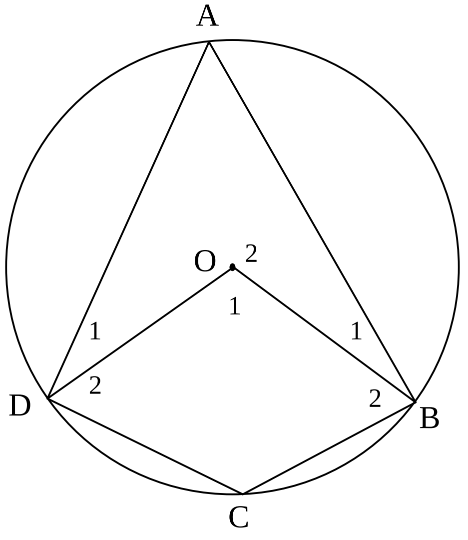
8.1	Construction: Draw KO produced $\hat{O}_1 = \hat{K}_1 + \hat{T}$ [ext $\angle$ of Δ] But $\hat{K}_1 = \hat{T}$ [$\angle$s opp equal sides] $\therefore \hat{O}_1 = 2\hat{K}_1$ $\hat{O}_2 = \hat{K}_2 + P$ [ext $\angle$ of Δ] But $\hat{K}_2 = P$ [$\angle$s opp equal sides] $\therefore \hat{O}_2 = 2\hat{K}_2$ $\therefore \hat{O}_1 + \hat{O}_2 = 2\hat{K}_1 + 2\hat{K}_2$ $= 2\left(\hat{K}_1 + \hat{K}_2\right)$ $\therefore T\hat{O}P = 2T\hat{K}P$ **OR**	✓ construction ✓ S / R ✓ S ✓ S ✓ S (5)

8.1



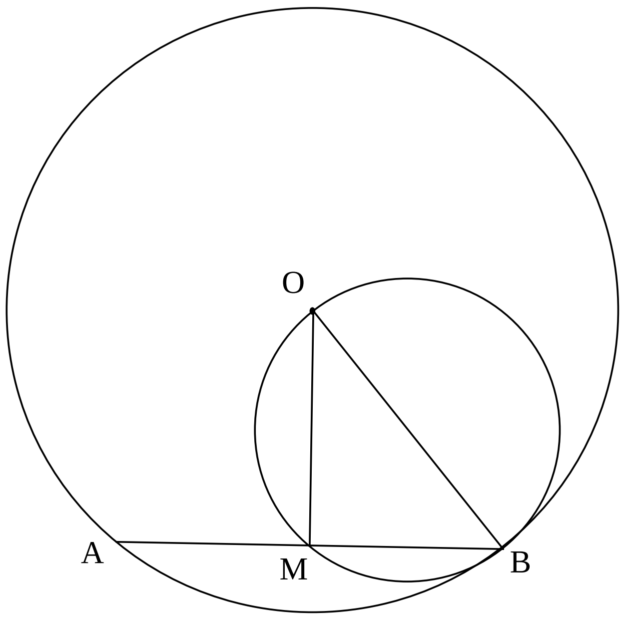
8.1	Construction: Draw KO $\hat{T} = \hat{K}_1$ [$\angle$s opp. equal sides] $\therefore K\hat{O}T = 180° - 2\hat{K}_1$ [sum of $\angle$s of ΔKOT] $\hat{P} = \hat{K}_2$ [$\angle$s opp. equal sides] $\therefore K\hat{O}P = 180° - 2\hat{K}_2$ [sum of $\angle$s of ΔKOP] $T\hat{O}P = 360° - \left(K\hat{O}T + K\hat{O}P\right)$ [$\angle$s around a point] $= 360° - \left(180° - 2\hat{K}_1 + 180° - 2\hat{K}_2\right)$ $= 2\hat{K}_1 + 2\hat{K}_2$ $= 2\left(\hat{K}_1 + \hat{K}_2\right)$ $\therefore T\hat{O}P = 2T\hat{K}P$	✓ construction ✓ S / R ✓ S ✓ S ✓ S (5)

8.2



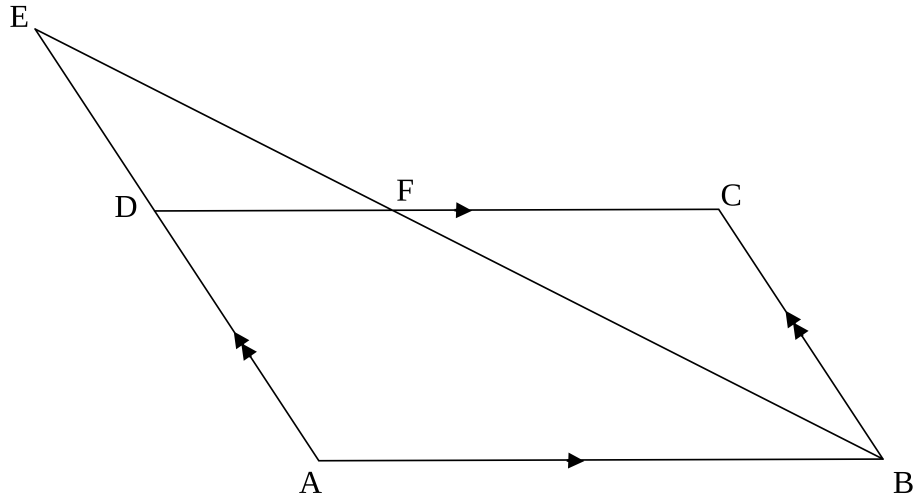
8.2	$\hat{O}_1 = 4x + 100°$ [given] $\therefore \hat{A} = 2x + 50°$ [$\angle$ at centre = 2 × $\angle$ at circumference] $x + 34° + 2x + 50° = 180°$ [opp $\angle$s of cyclic quad] $3x = 96°$ $x = 32°$ **OR** $\hat{O}_2 = 2x + 68°$ [$\angle$ at centre = 2 × $\angle$ at circumference] $4x + 100° + 2x + 68° = 360°$ [$\angle$s round a pt] $6x = 192°$ $x = 32°$ **OR** $\hat{O}_2 = -4x + 260°$ [$\angle$s round a pt] $2\hat{C} = -4x + 260°$ [$\angle$ at centre = 2 × $\angle$ at circumference] $\hat{C} = -2x + 130°$ $x + 34° = -2x + 130°$ $3x = 96°$ $x = 32°$	 ✓ S ✓ R ✓ S ✓ R ✓ answer (5) ✓ S ✓ R ✓ S ✓ R ✓ answer (5) ✓ S ✓ R ✓ S ✓ R ✓ answer (5)

8.3



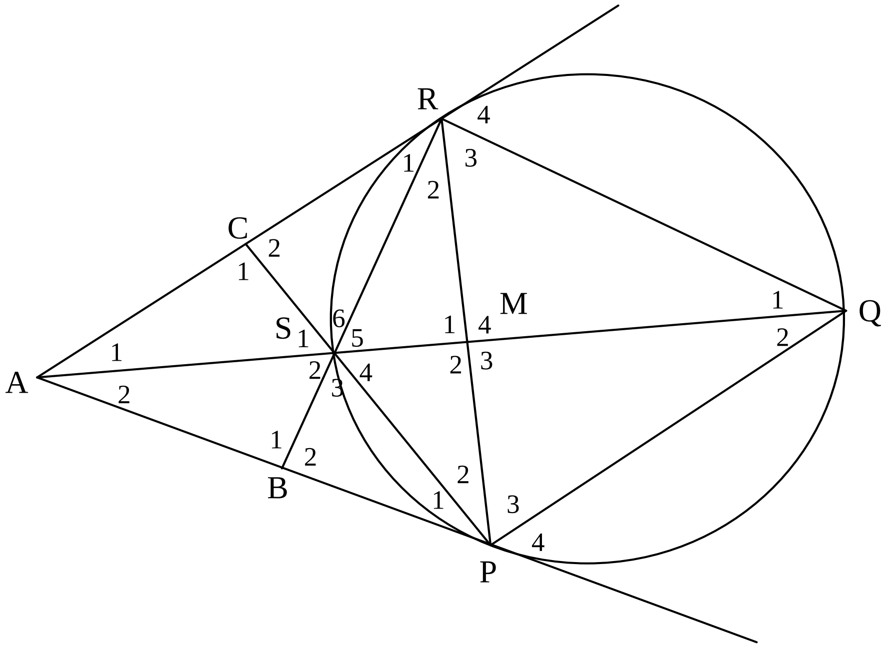
8.3.1	$O\hat{M}B = 90°$ [$\angle$ in semi circle]	✓ S ✓ R (2)
8.3.2	$AB = \sqrt{300} = 10\sqrt{3}$ $\therefore MB = 5\sqrt{3}$ [line from centre $\perp$ to chord] $OB^2 = OM^2 + MB^2$ [Pythagoras] $OB^2 = 5^2 + \left(5\sqrt{3}\right)^2$ OB = 10 units	✓ S ✓ R ✓ S ✓ answer (4)
		[16]

QUESTION/*VRAAG* 9



9.1	$\frac{FB}{EB}=\frac{DA}{EA}$ [prop theorem; DC ∥ AB] **OR** [line ∥ one side of Δ] $FB=\frac{4p\times 21}{7p}$ FB = 12 units	✓ S ✓ R ✓ answer (3)
9.2	In ΔEDF and ΔEAB: $\hat{E}$ is common $E\hat{D}F=\hat{A}$ [corresp ∠s; EA ∥ CB] $E\hat{F}D=E\hat{B}A$ [corresp ∠s; DC ∥ AB] ΔEDF ∣∣∣ ΔEAB [∠;∠;∠]	✓ S ✓ S/R ✓ S **OR** R (3)
9.3	$\frac{DF}{AB}=\frac{ED}{EA}$ [∣∣∣ Δs] $DF=\frac{3p\times 14}{7p}$ DF = 6 units FC = 8 units [DC = AB = 14 units; opp sides of ∥^{m}] **OR** ΔEDF ∣∣∣ ΔBCF [∠;∠;∠] $\frac{ED}{BC}=\frac{DF}{CF}$ [∣∣∣ Δs] $\frac{3}{4}=\frac{14-FC}{FC}$ [BC = AD; opp sides of ∥^{m}] $3FC=56-4FC$ $FC=8$	✓ S ✓ DF = 6 ✓ FC = 14 − DF (3) ✓ ΔEDF ∣∣∣ ΔBCF ✓ $\frac{3}{4}=\frac{14-FC}{FC}$ ✓ answer (3)
		[9]

QUESTION/*VRAAG* 10



10.1	$\hat{S}_3 = P\hat{Q}R$ [ext $\angle$ of cyclic quad] $\hat{R}_3 = P\hat{Q}R$ [$\angle$s opp equal sides] $\therefore \hat{S}_3 = \hat{R}_3$ But $\hat{S}_4 = \hat{R}_3$ [$\angle$s in the same seg] $\therefore \hat{S}_3 = \hat{S}_4$	✓ S ✓ R ✓ S / R ✓ S ✓ R (5)
10.2	$\hat{R}_1 + \hat{R}_2 = P\hat{Q}R$ [tan chord theorem] $\hat{S}_4 = P\hat{Q}R$ [proved in 10.1] $\therefore \hat{S}_4 = \hat{R}_1 + \hat{R}_2$ SMRC is a cyclic quad [converse ext $\angle$ of cyclic quad]	✓ S ✓ R ✓ S ✓ R (4)
10.3	$\hat{S}_3 = \hat{R}_2 + \hat{P}_2$ [ext $\angle$ of Δ] $\hat{S}_4 = \hat{P}_1 + \hat{A}_2$ [ext $\angle$ of Δ] $\therefore \hat{R}_2 + \hat{P}_2 = \hat{A}_2 + \hat{P}_1$ But $\hat{P}_1 = \hat{R}_2$ [tan chord theorem] $\therefore \hat{P}_2 = \hat{A}_2$ RP is a tangent to the circle [converse tan chord theorem] **OR** [$\angle$ between line and chord] **OR** [converse alt seg theorem] **OR**	✓ S ✓ R ✓ S ✓ S ✓ R ✓ R (6)

	In ΔMSP and ΔMPA $\hat{M}_2$ is common AR = AP [tans from same point] $\hat{R}_1 + \hat{R}_2 = \hat{P}_1 + \hat{P}_2$ [$\angle$s opp equal sides] $\hat{S}_4 = \hat{R}_1 + \hat{R}_2$ [proved in 10.2] $\therefore \hat{S}_4 = \hat{P}_1 + \hat{P}_2$ $\therefore \hat{P}_2 = \hat{A}_2$ [sum of $\angle$s in Δ] RP is a tangent to the circle [converse tan chord theorem]	✓ S ✓ S / R ✓ S ✓ S ✓ S ✓ R (6)
		[15]

TOTAL/TOTAAL: 150