basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P2

NOVEMBER 2022

MARKS: 150

TIME: 3 hours

This question paper consists of 13 pages and 1 information sheet.

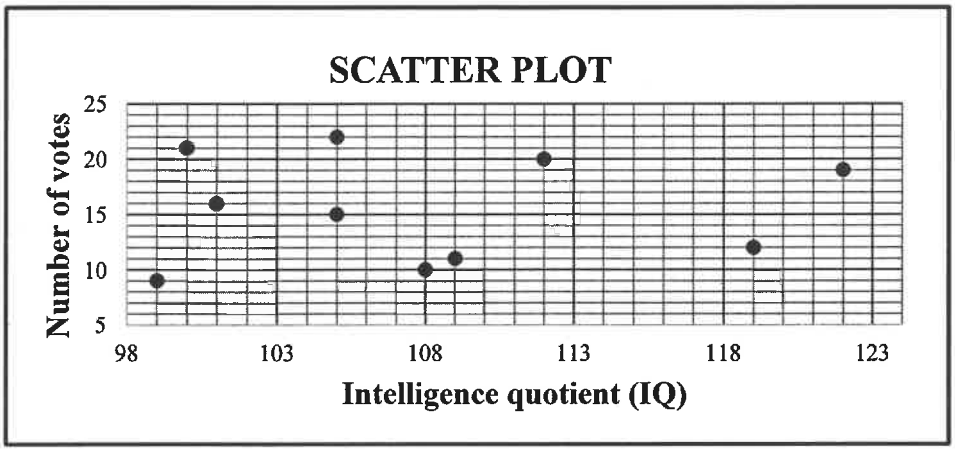
INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 10 questions.
2. Answer ALL the questions in the SPECIAL ANSWER BOOK provided.
3. Clearly show ALL calculations, diagrams, graphs, etc. which you have used in determining your answers.
4. Answers only will NOT necessarily be awarded full marks.
5. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
6. If necessary, round off answers correct to TWO decimal places, unless stated otherwise.
7. Diagrams are NOT necessarily drawn to scale.
8. An information sheet with formulae is included at the end of the question paper.
9. Write neatly and legibly.

QUESTION 1

The matric class of a certain high school had to vote for the chairperson of the RCL (representative council of learners). The scatter plot below shows the IQ (intelligence quotient) of the 10 learners who received the most votes and the number of votes that they received.



Before the election, the popularity of each of these ten learners was established and a popularity score (out of a 100) was assigned to each. The popularity scores and the number of votes of the same 10 learners who received the most votes are shown in the table below.

Popularity score (x)	32	89	35	82	50	59	81	40	79	65
Number of votes (y)	9	22	10	21	11	15	20	12	19	16

1.1 Calculate the:

1.1.1 Mean number of votes that these 10 learners received (2)

1.1.2 Standard deviation of the number of votes that these 10 learners received (1)

1.2 The learners who received fewer votes than one standard deviation below the mean were not invited for an interview. How many learners were invited? (2)

1.3 Determine the equation of the least squares regression line for the data given in the table. (3)

1.4 Predict the number of votes that a learner with a popularity score of 72 will receive. (2)

1.5 Using the scatter plot and table above, provide a reason why:

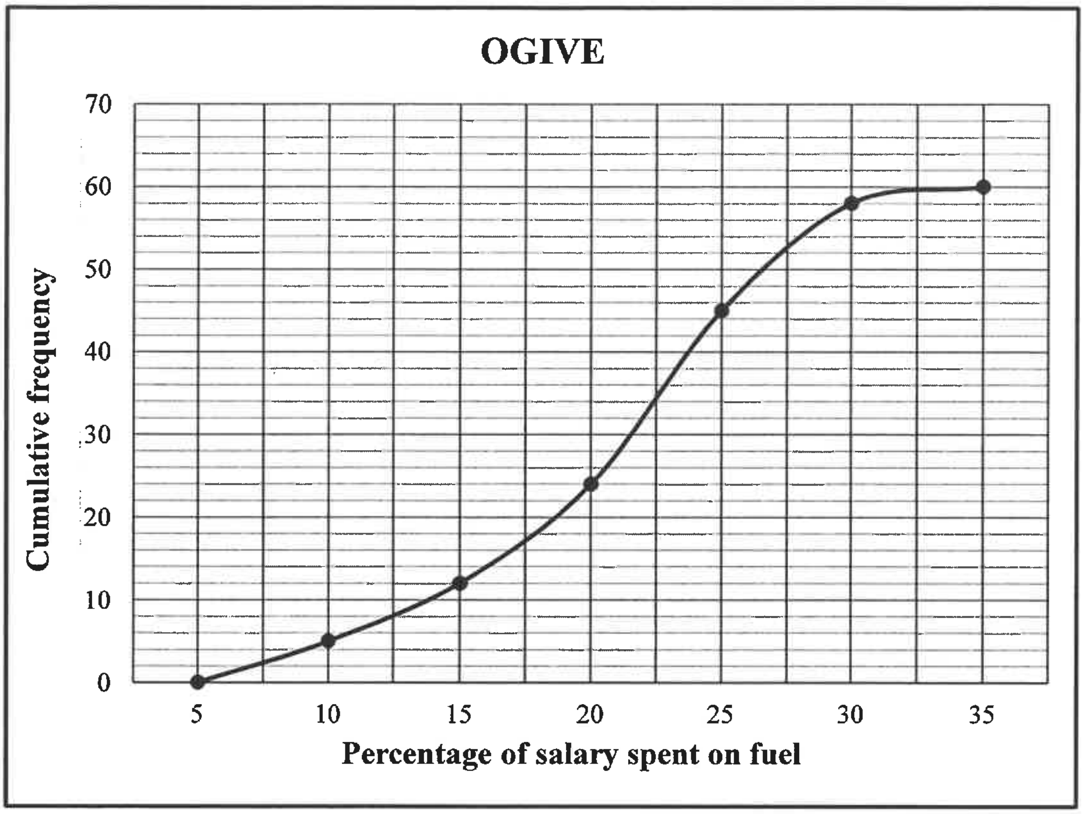
1.5.1 IQ is not a good indicator of the number of votes that a learner could receive (1)

1.5.2 The prediction in QUESTION 1.4 is reliable (1)

[12]

QUESTION 2

A company conducted research among all its employees on what percentage of their monthly salary was spent on fuel in a particular month. The data is represented in the ogive (cumulative frequency graph) below.



2.1 How many people are employed at this company? (1)

2.2 Write down the modal class of the data. (1)

2.3 How many employees spent more than 22,5% of their monthly salary on fuel? (2)

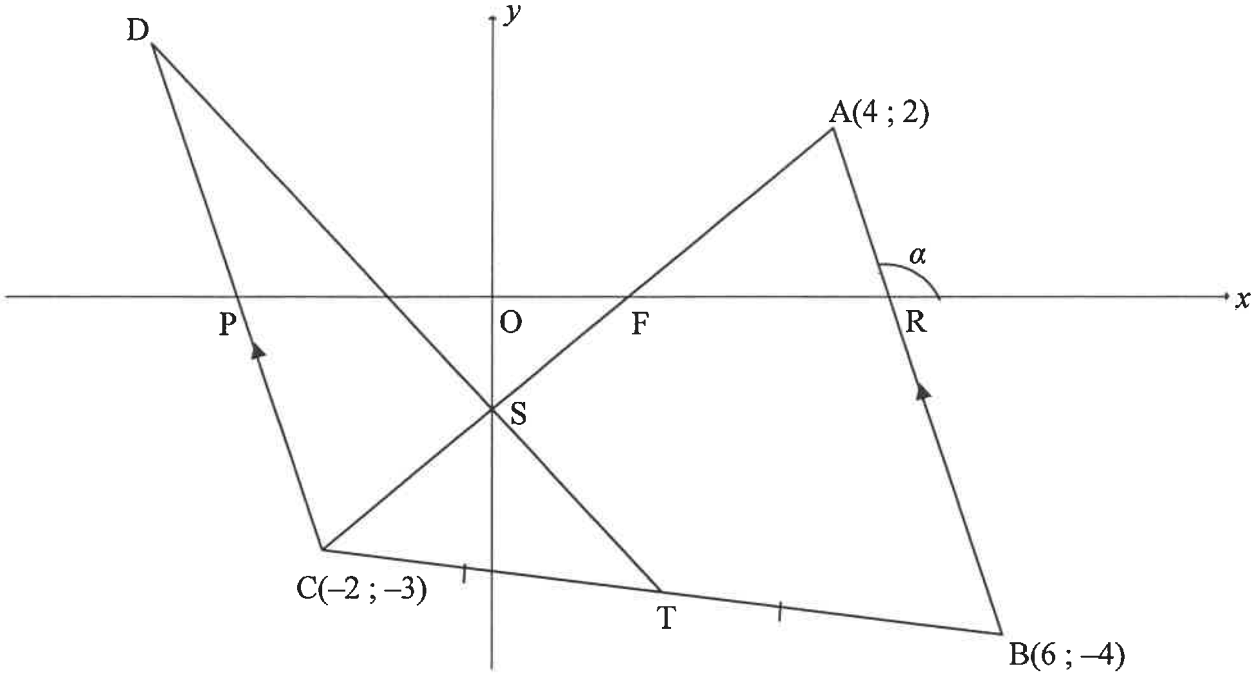
2.4 An employee spent R2 400 of his salary on fuel in that particular month. Determine the monthly salary of this employee if he spends 7% of his salary on fuel. (2)

2.5 The monthly salaries of these employees remains constant and the number of litres of fuel used in each month also remains constant. If the fuel price increases from R21,43 per litre to R22,79 per litre at the beginning of the next month, how will the above ogive change? (2)

[8]

QUESTION 3

In the diagram, A(4 ; 2), B(6 ; –4) and C(–2 ; –3) are vertices of ΔABC. T is the midpoint of CB. The equation of line AC is $5x-6y=8$. The angle of inclination of AB is α. ΔDCT is drawn such that CD || BA. The lines AC and DT intersect at S, the y-intercept of AC. P, F and R are the x-intercepts of DC, AC and AB respectively.



3.1 Calculate the:

3.1.1 Gradient of AB (2)

3.1.2 Size of α (2)

3.1.3 Coordinates of T (2)

3.1.4 Coordinates of S (2)

3.2 Determine the equation of CD in the form $y=mx+c$. (3)

3.3 Calculate the:

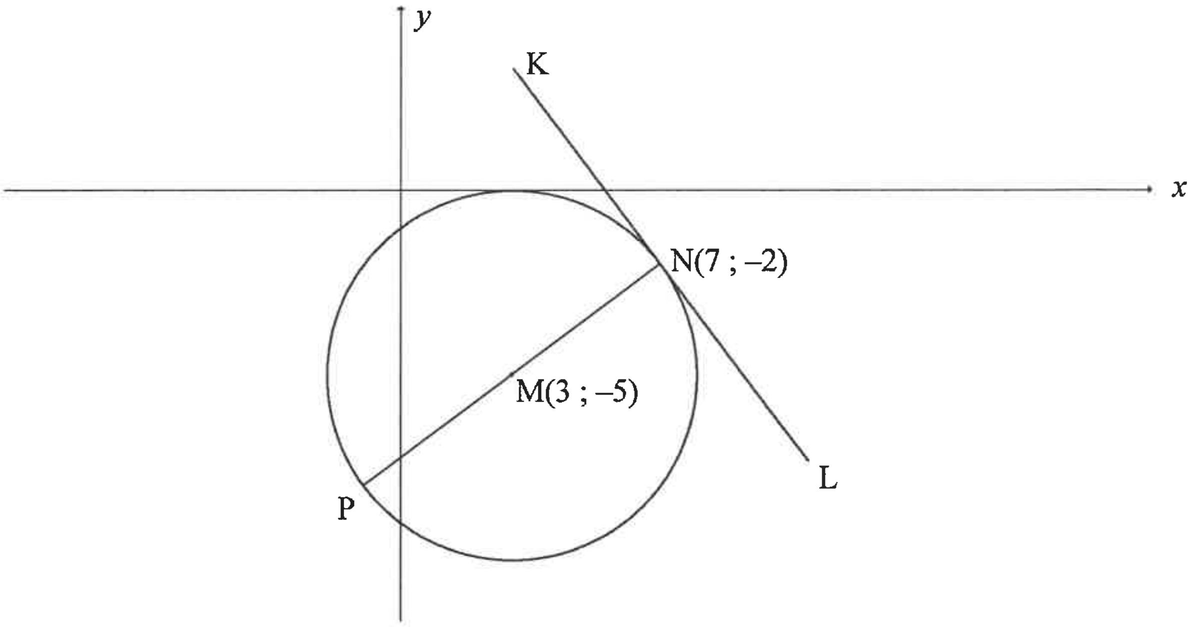
3.3.1 Size of $D\hat{C}A$ (4)

3.3.2 Area of POSC (5)

[20]

QUESTION 4

In the diagram, M(3 ; –5) is the centre of the circle having PN as its diameter. KL is a tangent to the circle at N(7 ; –2).



4.1 Calculate the coordinates of P. (2)

4.2 Determine the equation of:

4.2.1 The circle in the form $(x-a)^2+(y-b)^2=r^2$ (3)

4.2.2 KL in the form $y=mx+c$ (5)

4.3 For which values of k will $y=-\frac{4}{3}x+k$ be a secant to the circle? (4)

4.4 Points A(t ; t) and B are not shown on the diagram.

From point A, another tangent is drawn to touch the circle with centre M at B.

4.4.1 Show that the length of tangent AB is given by $\sqrt{2t^2+4t+9}$. (2)

4.4.2 Determine the minimum length of AB. (4)

[20]

QUESTION 5

5.1 Given that $\sqrt{13}\sin x + 3 = 0$, where $x \in (90°\,;\,270°)$.

Without using a calculator, determine the value of:

5.1.1 $\sin(360° + x)$ (2)

5.1.2 $\tan x$ (3)

5.1.3 $\cos(180° + x)$ (2)

5.2 Determine the value of the following expression, **without using a calculator**:

$$\frac{\cos(90° + \theta)}{\sin(\theta - 180°) + 3\sin(-\theta)}$$ (5)

5.3 Determine the general solution of the following equation:

$(\cos x + 2\sin x)(3\sin 2x - 1) = 0$ (6)

5.4 Given the identity: $\cos(x + y).\cos(x - y) = 1 - \sin^2 x - \sin^2 y$

5.4.1 Prove the identity. (4)

5.4.2 Hence, determine the value of $1 - \sin^2 45° - \sin^2 15°$, **without using a calculator.** (3)

5.5 Consider the trigonometric expression: $16\sin x.\cos^3 x - 8\sin x.\cos x$

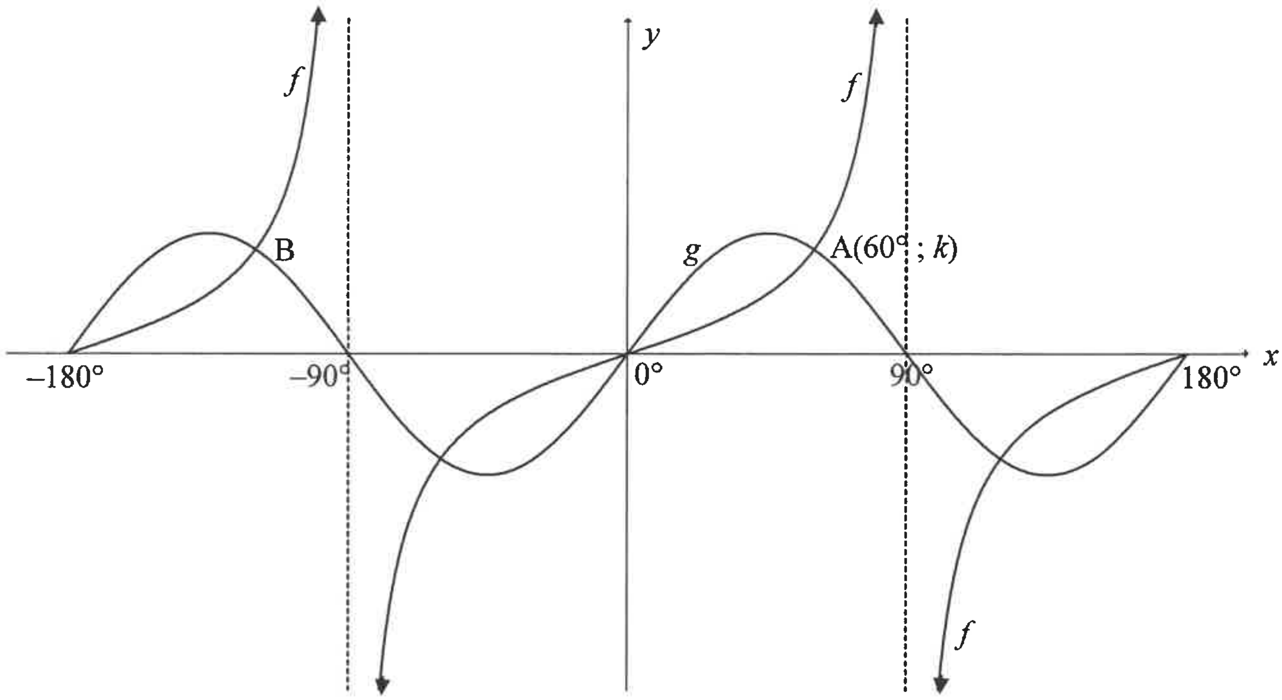
5.5.1 Rewrite the expression as a single trigonometric ratio. (4)

5.5.2 For which value of x in the interval $x \in [0°\,;\,90°]$ will $16\sin x.\cos^3 x - 8\sin x.\cos x$ have its minimum value? (1)

[30]

QUESTION 6

In the diagram below, the graphs of $f(x) = \tan x$ and $g(x) = 2\sin 2x$ are drawn for the interval $x \in [-180° ; 180°]$ A$(60° ; k)$ and B are two points of intersection of f and g.



6.1 Write down the period of g. (1)

6.2 Calculate the:

6.2.1 Value of k (1)

6.2.2 Coordinates of B (1)

6.3 Write down the range of $2g(x)$. (2)

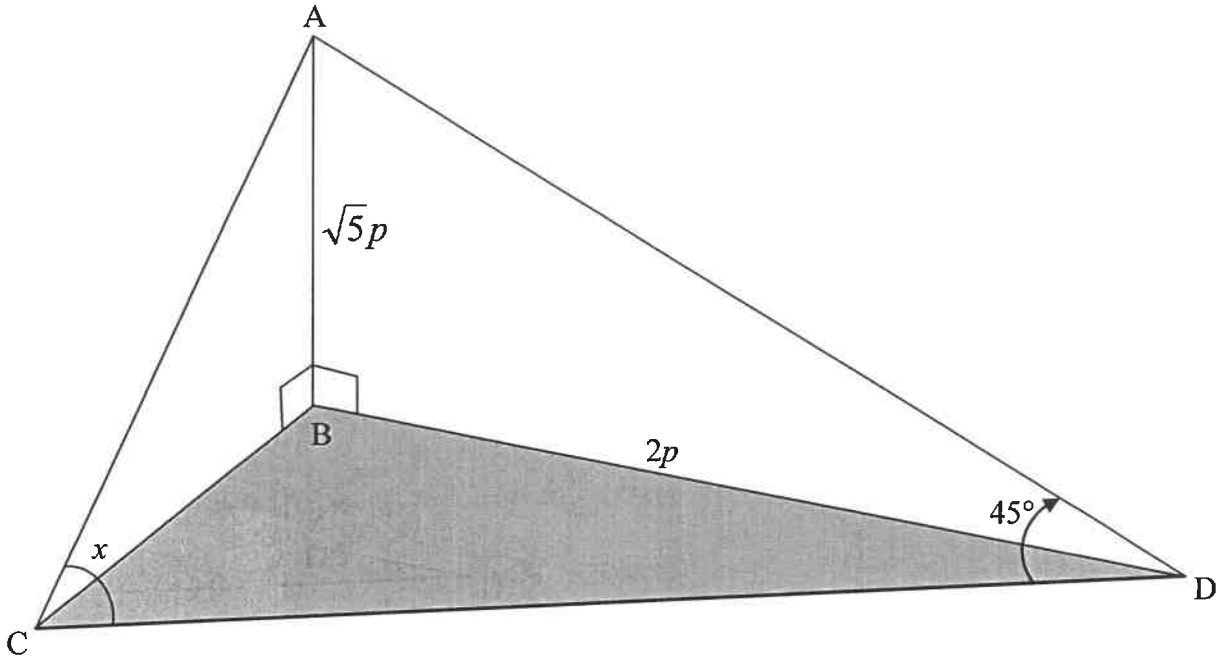
6.4 For which values of x will $g(x+5°) - f(x+5°) \le 0$ in the interval $x \in [-90° ; 0°]$? (2)

6.5 Determine the values of p for which $\sin x.\cos x = p$ will have exactly two real roots in the interval $x \in [-180° ; 180°]$. (3)

[10]

QUESTION 7

AB is a vertical flagpole that is $\sqrt{5}p$ metres long. AC and AD are two cables anchoring the flagpole. B, C and D are in the same horizontal plane.
$BD = 2p$ metres, $A\hat{C}D = x$ and $A\hat{D}C = 45°$.



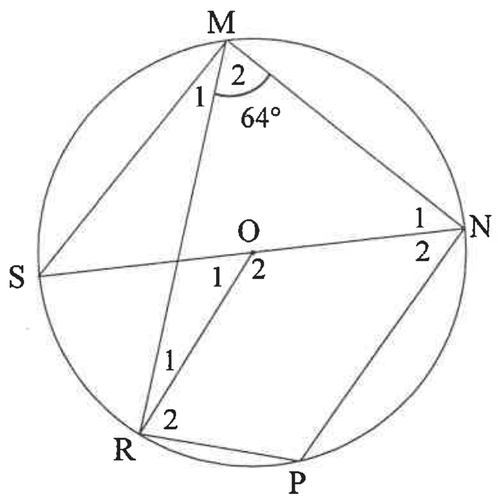
7.1 Determine the length of AD in terms of p. (2)

7.2 Show that the length of $CD = \dfrac{3p(\sin x + \cos x)}{\sqrt{2}\sin x}$. (5)

7.3 If it is further given that $p = 10$ and $x = 110°$, calculate the area of ΔADC. (3)
[10]

QUESTION 8

8.1 In the diagram, O is the centre of the circle. MNPR is a cyclic quadrilateral and SN is a diameter of the circle. Chord MS and radius OR are drawn. $\hat{M}_2 = 64°$.



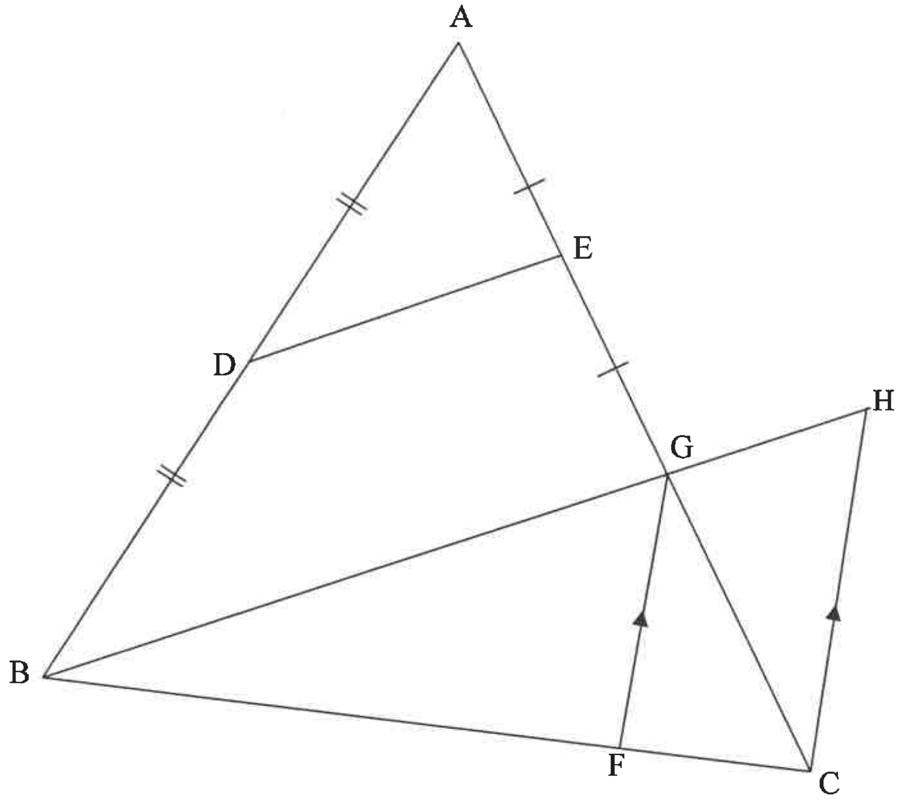
Determine, giving reasons, the size of the following angles:

8.1.1 $\hat{P}$ (2)

8.1.2 $\hat{M}_1$ (2)

8.1.3 $\hat{O}_1$ (2)

8.2 In the diagram, ΔABG is drawn. D and E are midpoints of AB and AG respectively. AG and BG are produced to C and H respectively. F is a point on BC such that $FG \parallel CH$.



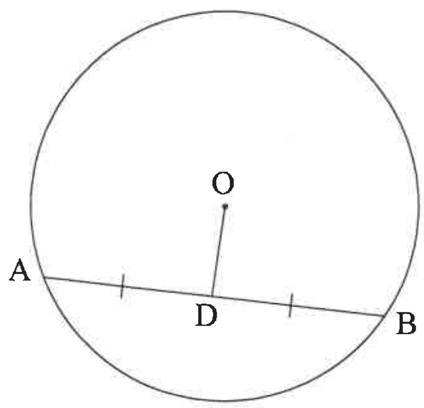
8.2.1 Give a reason why $DE \parallel BH$. (1)

8.2.2 If it is further given that $\frac{FC}{BF} = \frac{1}{4}$, $DE = 3x - 1$ and $GH = x + 1$, calculate, giving reasons, the value of x. (6)

[13]

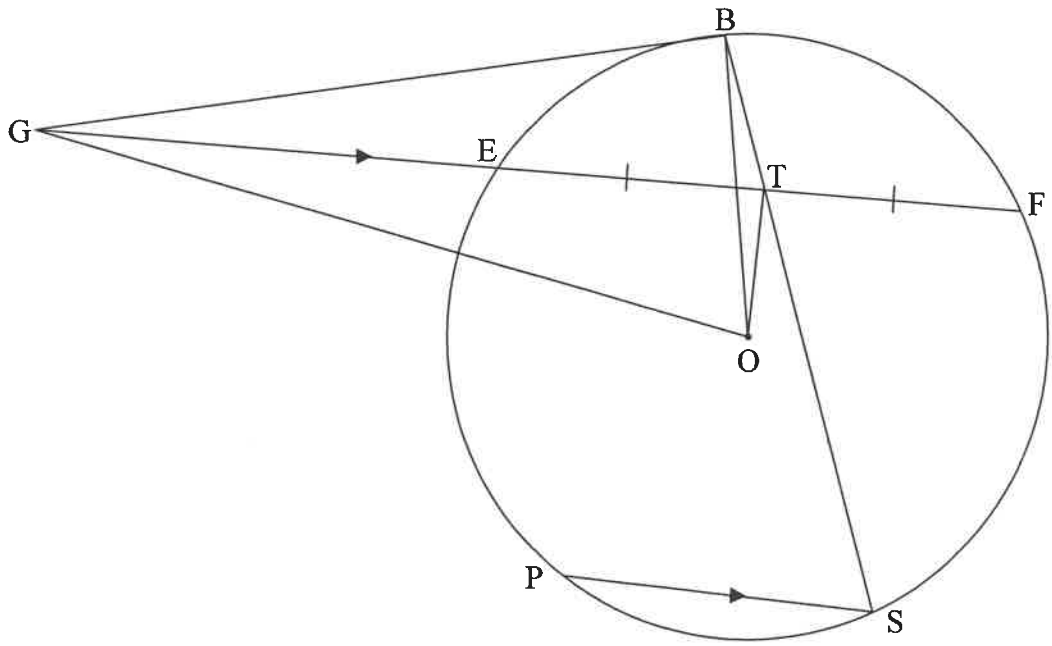
QUESTION 9

9.1 In the diagram, O is the centre of a circle. OD bisects chord AB.



Prove the theorem that states that the line from the centre of a circle that bisects a chord is perpendicular to the chord, i.e. $OD \perp AB$. (5)

9.2 In the diagram, E, B, F, S and P are points on the circle centred at O. GB is a tangent to the circle at B. FE is produced to meet the tangent at G. OT is drawn such that T is the midpoint of EF. GO and BO are drawn. BS is drawn through T. $PS \parallel GF$.



Prove, giving reasons, that:

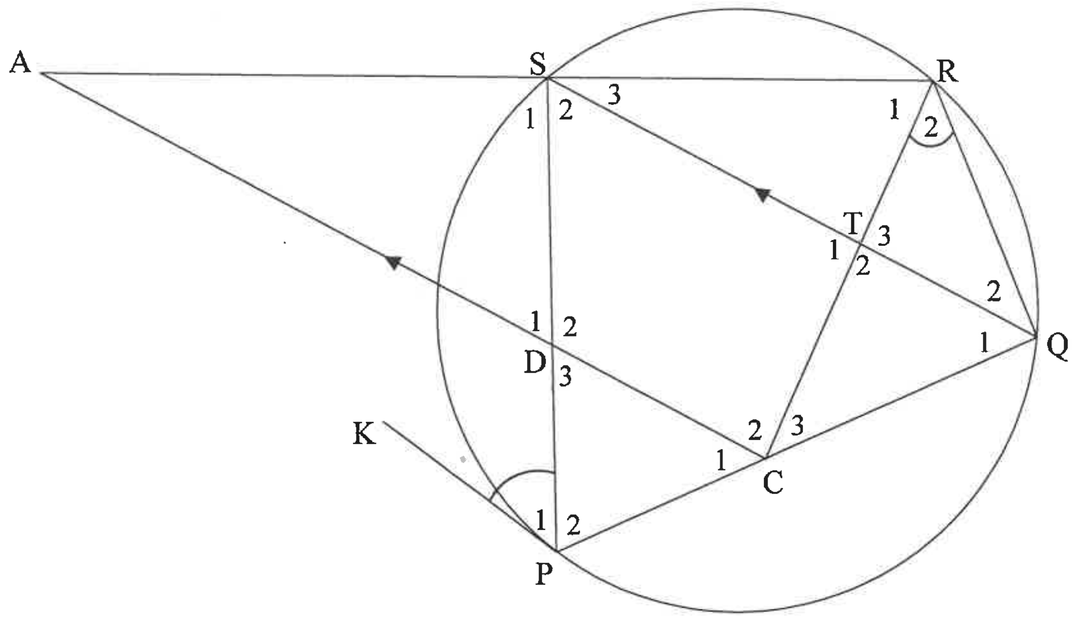
9.2.1 OTBG is a cyclic quadrilateral (5)

9.2.2 $G\hat{O}B = \hat{S}$ (4)

[14]

QUESTION 10

In the diagram, PQRS is a cyclic quadrilateral. KP is a tangent to the circle at P. C and D are points on chords PQ and PS respectively and CD produced meets RS produced at A. CA || QS. RC is drawn. $\hat{P}_1 = \hat{R}_2$.



Prove, giving reasons, that:

10.1 $\hat{S}_1 = \hat{T}_2$ (4)

10.2 $\dfrac{AD}{AR} = \dfrac{AS}{AC}$ (5)

10.3 $AC \times SD = AR \times TC$ (4)

[13]

TOTAL: 150

INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1 + ni)$ $A = P(1 - ni)$ $A = P(1 - i)^n$ $A = P(1 + i)^n$

$T_n = a + (n-1)d$ $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

$T_n = ar^{n-1}$ $S_n = \frac{a\left(r^n - 1\right)}{r - 1}$; $r \neq 1$ $S_\infty = \frac{a}{1 - r}$; $-1 < r < 1$

$F = \frac{x\left[(1+i)^n - 1\right]}{i}$ $P = \frac{x\left[1 - (1+i)^{-n}\right]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\mathrm{M}\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $y - y_1 = m(x - x_1)$ $m = \frac{y_2 - y_1}{x_2 - x_1}$ $m = \tan\theta$

$(x - a)^2 + (y - b)^2 = r^2$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$a^2 = b^2 + c^2 - 2bc.\cos A$

area $\Delta ABC = \frac{1}{2} ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$ $\sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum x}{n}$ $\sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$\hat{y} = a + bx$ $b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$

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GRADE 12/*GRAAD 12*

MATHEMATICS P2/*WISKUNDE V2*

NOVEMBER 2022

MARKING GUIDELINES/*NASIENRIGLYNE*

MARKS/*PUNTE*: 150

These marking guidelines consist of 24 pages.
Hierdie nasienriglyne bestaan uit 24 bladsye.

NOTE:

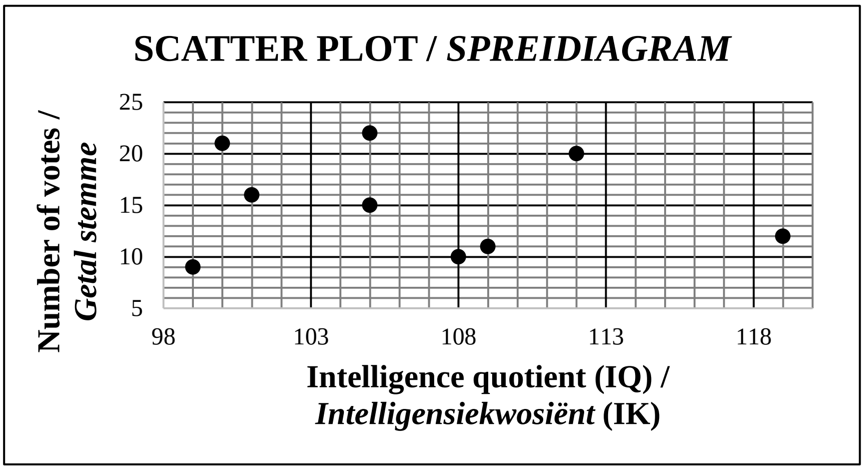
- **If a candidate answers a question TWICE, only mark the FIRST attempt.**
- **If a candidate has crossed out an attempt of a question and not redone the question, mark the crossed out version.**
- **Consistent accuracy applies in ALL aspects of the marking memorandum. Stop marking at the second calculation error.**
- **Assuming answers/values in order to solve a problem is NOT acceptable.**

NOTA:

- ***As 'n kandidaat 'n vraag TWEE KEER beantwoord, merk slegs die EERSTE poging.***
- ***As 'n kandidaat 'n antwoord van 'n vraag doodtrek en nie oordoen nie, merk die doodgetrekte poging.***
- ***Volgehoue akkuraatheid word in ALLE aspekte van die memorandum toegepas. Hou op nasien by die tweede berekeningsfout.***
- ***Aanvaar van antwoorde/waardes om 'n probleem op te los, word NIE toegelaat nie.***

GEOMETRY/*MEETKUNDE*	
S	**A mark for a correct statement** **(A statement mark is independent of a reason)**
	'n Punt vir 'n korrekte bewering ***('n Punt vir 'n bewering is onafhanklik van die rede)***
R	**A mark for the correct reason** **(A reason mark may only be awarded if the statement is correct)**
	'n Punt vir 'n korrekte rede ***('n Punt word slegs vir die rede toegeken as die bewering korrek is)***
S/R	**Award a mark if statement AND reason are both correct**
	Ken 'n punt toe as die bewering EN rede beide korrek is

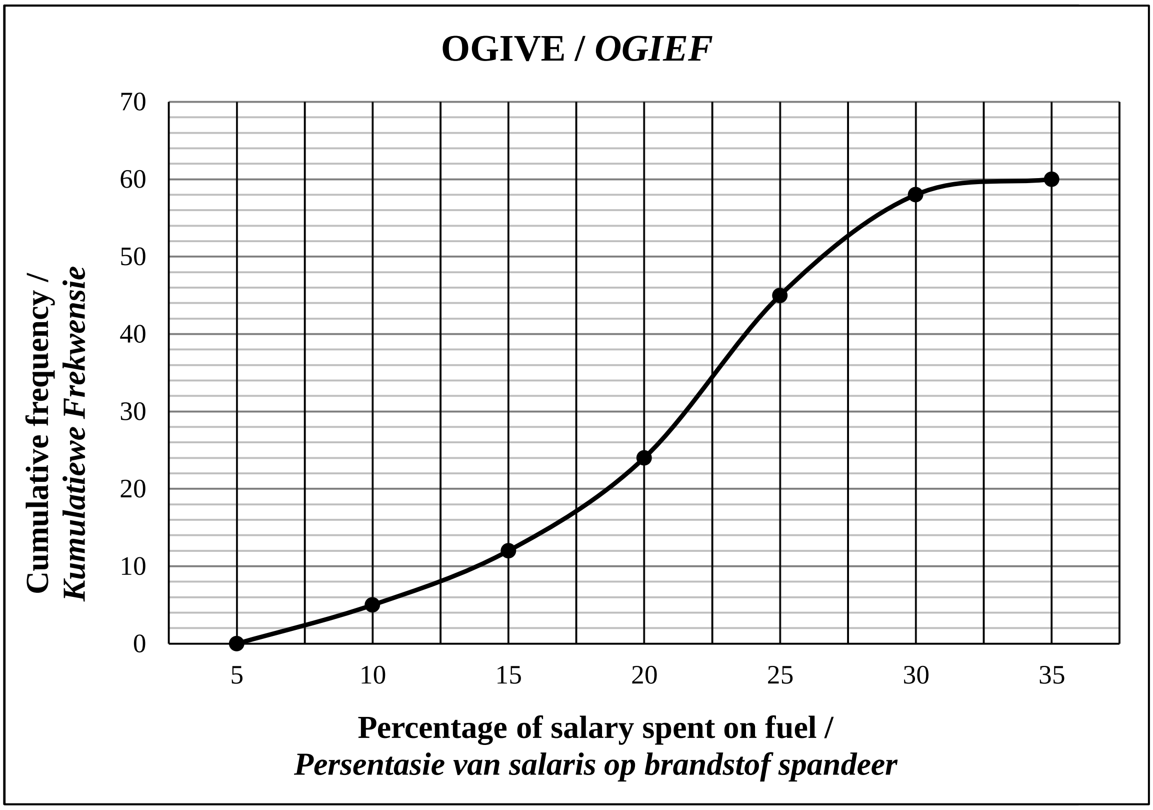
QUESTION/*VRAAG* 1



Popularity score (x) ***Gewildheidspunt* (x)**	32	89	35	82	50	59	81	40	79	65
Number of votes (y) ***Getal stemme* (y)**	9	22	10	21	11	15	20	12	19	16

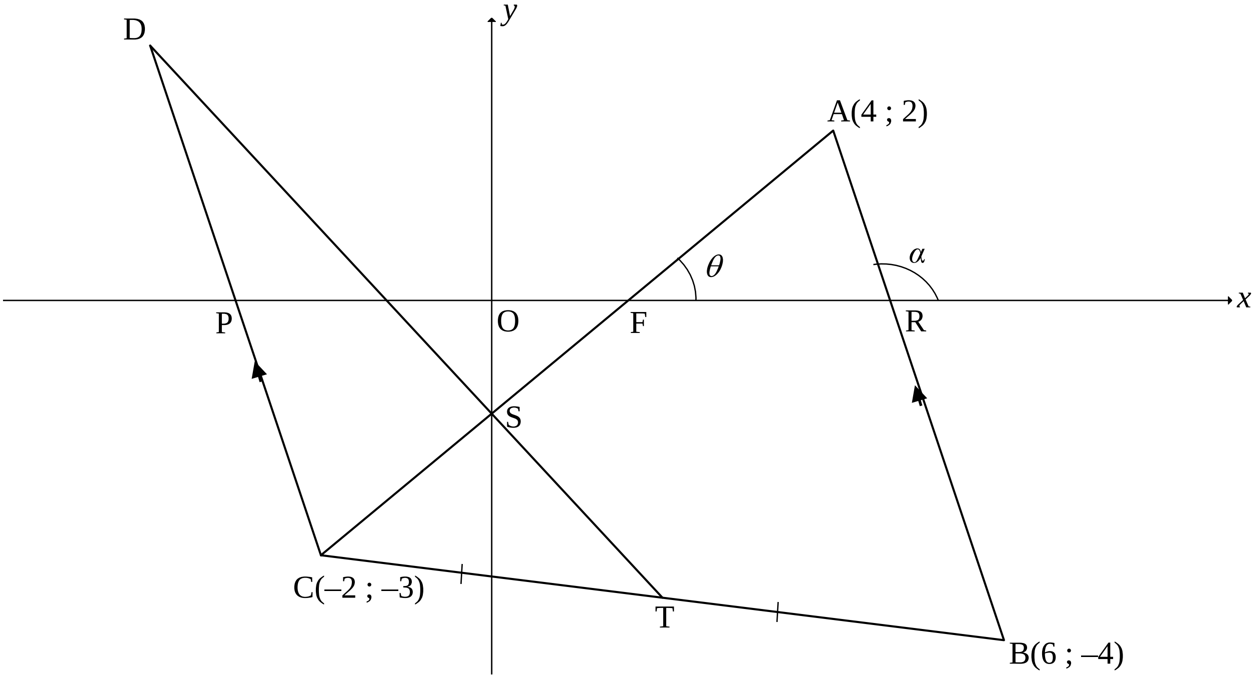
1.1.1	$\bar{y} = \frac{155}{10}$ $= 15,5$ ANSWER ONLY: Full marks	✓155 ✓answer (2)
1.1.2	SD = 4,59	✓ answer (1)
1.2	$\bar{y} - \text{SD}$ = 15,5 – 4,59 = 10,91 ∴ 10 – 2 = 8 learners	✓ value of $\bar{y} - \text{SD}$ ✓ answer (2)
1.3	a = 1,7709… b = 0,2243… $\hat{y} = 1,77 + 0,22x$	✓ a ✓ b ✓ equation (3)
1.4	$\hat{y} = 1,77 + 0,22(72)$ = 17,61 ≈ 18 votes **OR/*OF*** $\hat{y} = 17,92 \approx 18$ votes	✓ substitution ✓ answer (2) ✓✓ answer (2)
1.5.1	Points are all scattered therefore low correlation and unrealistic prediction./*Punte is versprei daarom 'n lae korrelasie en onrealistiese voorspelling.*	✓ R (1)
1.5.2	r = 0,98/correlation very strong/*korrelasie baie sterk* ∴ a reliable prediction/*'n betroubare voorspelling*	✓ S (1)
		[12]

QUESTION/*VRAAG* 2



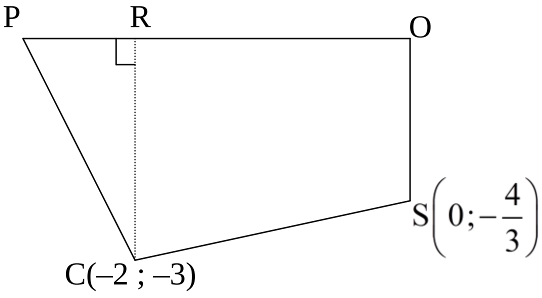
2.1	60 employees	✓ answer (A) (1)
2.2	$20 < x \le 25$	✓ answer (1)
2.3	60 – 34 = 26 employees ANSWER ONLY: Full marks	✓ 34 ✓ answer (2)
2.4	Salary $= \frac{100}{7} \times 2400$ Salary = R34 285,71 ANSWER ONLY: Full marks	✓ method ✓ answer (2)
2.5	∴ Ogive/Cumulative frequency graph will shift to the right/will become steeper. ∴ *Ogief/Kumulatiewe frekwensie grafiek sal na regs skuif/sal steiler wees.*	✓✓ answer (2)
		[8]

QUESTION/*VRAAG* 3



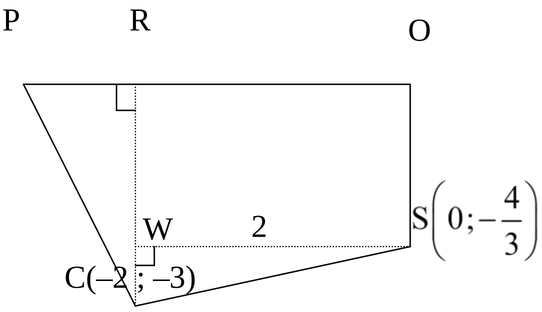
3.1.1	$m_{AB}=\frac{2-(-4)}{4-6}$ **OR** $m_{AB}=\frac{-4-2}{6-4}$ $m_{AB}=-3$ ANSWER ONLY: Full marks	✓ substitution ✓ answer (2)
3.1.2	$\tan\alpha=m_{AB}=-3$ $\alpha=108{,}43°$ ANSWER ONLY: Full marks	✓ $\tan\alpha=m_{AB}=-3$ ✓ answer (2)
3.1.3	$T\left(\frac{x_1+x_2}{2};\frac{y_1+y_2}{2}\right)$ $T\left(\frac{-2+6}{2};\frac{-3-4}{2}\right)$ $T\left(2;\frac{-7}{2}\right)$	✓ $x_T=2$ ✓ $y_T=\frac{-7}{2}$ (2)
3.1.4	$5(0)-6y=8$ $y=-\frac{4}{3}$ $S\left(0;\frac{-4}{3}\right)$	✓ $x_S=0$ ✓ $y_S=\frac{-4}{3}$ (2)
3.2	$m_{CD}=m_{AB}=-3$ $-3=-3(-2)+c$ OR $y-(-3)=-3(x-(-2))$ $c=-9$ $y=-3x-9$ $y=-3x-9$	✓ gradient ✓ substitution of C(–2 ; –3) ✓ equation (3)

3.3.1	$5x-6y=8$ $y=\frac{5}{6}x-\frac{8}{6}$ $\tan\theta=m_{AC}=\frac{5}{6}$ $\theta=39{,}81°$ $\hat{A}=108{,}43°-39{,}81°$ $=68{,}62°$ $D\hat{C}A=68{,}62°$ [alt $\angle$s ; DC$\parallel$AB]	✓ $\tan\theta=m_{AC}=\frac{5}{6}$ ✓ $\theta=39{,}81°$ ✓ $\hat{A}=68{,}62°$ ✓ answer (4)
3.3.2	$P(-3;0)$ and F(1,6 ; 0) Area POSC = Area ΔFPC – Area ΔOFS $=\frac{1}{2}(4,6)(3)-\frac{1}{2}(1,6)\left(\frac{4}{3}\right)$ = 6,9 – 1,07 = 5,83 units^2 **OR/*OF*** $P(-3;0)$ $FC=\sqrt{\left(-2-\frac{8}{5}\right)^2+(-3-0)^2}=\frac{3\sqrt{61}}{5}$ Area $\Delta PFC=\frac{1}{2}(PF)(FC)\sin O\hat{F}S$ $=\frac{1}{2}\left(\frac{23}{5}\right)\left(\frac{3\sqrt{61}}{5}\right)\sin 39{,}81°$ $=6{,}90$ Area $\Delta OFS=\frac{1}{2}\left(\frac{8}{5}\right)\left(\frac{4}{3}\right)$ = 1,07 Area POSC = 6,90 – 1,07 = 5,83 units^2 **OR/*OF***	✓ $P(-3;0)$ ✓ method ✓ $\frac{1}{2}(4,6)(3)$ ✓ $\frac{1}{2}(1,6)\left(\frac{4}{3}\right)$ ✓ answer (5) ✓ $P(-3;0)$ ✓ $\frac{1}{2}\left(\frac{23}{5}\right)\left(\frac{3\sqrt{61}}{5}\right)\sin 39{,}81°$ ✓ $\frac{1}{2}\left(\frac{8}{5}\right)\left(\frac{4}{3}\right)$ ✓ method ✓ answer (5)



$P(-3;0)$	✓ $P(-3;0)$
Area of POSC = Area of OSCR + Area of ΔPRC	✓ method
$=\frac{1}{2}\left(\frac{4}{3}+3\right)\times 2+\frac{1}{2}(1\times 3)$	✓ $\frac{1}{2}\left(\frac{4}{3}+3\right)\times 2$ ✓ $\frac{1}{2}(1\times 3)$
$=\frac{35}{6}$	
$=5{,}83\text{units}^2$	✓ answer (5)

OR/
OF

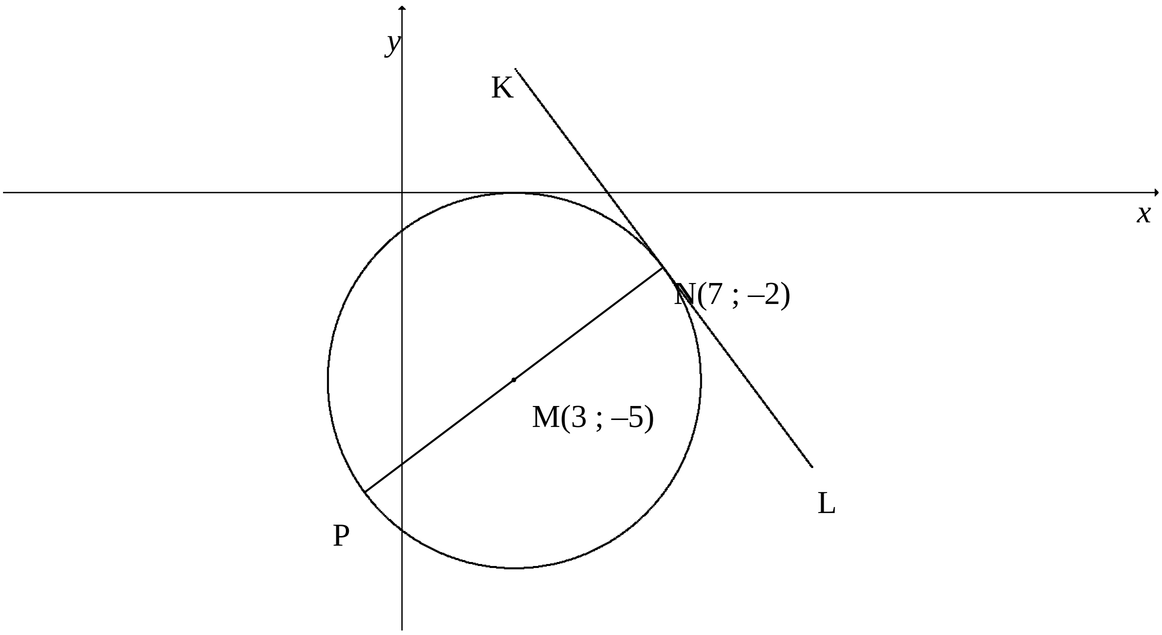


$P(-3;0)$	✓ $P(-3;0)$
Area POSC = Area ROSW + Area ΔPRC + Area ΔWSC	✓ method
$=\left(\frac{4}{3}\right)(2)+\frac{1}{2}(1)(3)+\frac{1}{2}(2)\left(\frac{5}{3}\right)$	✓ $\frac{1}{2}(1)(3)$ ✓ $\left(\frac{4}{3}\right)(2)+\frac{1}{2}(2)\left(\frac{5}{3}\right)$
$=\frac{35}{6}$	
$=5{,}83\text{units}^2$	✓ answer (5)

OR/*OF*

	$P(-3;0)$ Area of ΔPSC = $\frac{1}{2}(PC)(CS)\sin D\hat{C}A$ $=\frac{1}{2}\left(\sqrt{10}\right)\left(\frac{\sqrt{61}}{3}\right)\sin 68,62°$ = 3,833.. Area of ΔPOS = $\frac{1}{2}(PO)(OS)$ $=\frac{1}{2}(3)\left(\frac{4}{3}\right)$ = 2 Area POSC = 3,833… + 2 $=5,83\text{units}^2$	✓ $P(-3;0)$ ✓ $\frac{1}{2}\left(\sqrt{10}\right)\left(\frac{\sqrt{61}}{3}\right)\sin 68,62°$ ✓ $\frac{1}{2}(3)\left(\frac{4}{3}\right)$ ✓ method ✓ answer (5)
		[20]

QUESTION/*VRAAG* 4



4.1	$P(x;y);\ N(7;-2)\ ;\ M(3;-5)$ $\frac{x+7}{2}=3 \qquad \frac{y-2}{2}=-5$ $x=-1 \qquad y=-8$ $P(-1;-8)$	✓ $x_P=-1$ ✓ $y_P=-8$ (2)
4.2.1	$r^2=(7-3)^2+(-2-(-5))^2$ **OR/*OF*** $r^2=(-1-3)^2+(-8-(-5))^2$ $r^2=25$ $(x-3)^2+(y+5)^2=25$	✓ substitution into distance formula ✓ $(x-3)^2+(y+5)^2$ ✓ $r^2=25$ (3)
4.2.2	$m_{\text{radius}}=\frac{-5-(-2)}{3-7}=\frac{3}{4}$ $m_{\text{tangent}}=-\frac{4}{3}$ [radius ⊥ tangent/*raaklyn* ⊥ *radius*] $-2=-\frac{4}{3}(7)+c$ **OR** $y-(-2)=-\frac{4}{3}(x-7)$ $c=\frac{22}{3}$ $\qquad y=-\frac{4}{3}x+\frac{22}{3}$ $y=-\frac{4}{3}x+\frac{22}{3}$	✓ substitution ✓ $m_{\text{radius}}=\frac{-3}{-4}=\frac{3}{4}$ ✓ $m_{\text{tangent}}=-\frac{4}{3}$ ✓ substitution of *m* and N(7 ; –2) ✓ equation (5)
4.3	$-8=-\frac{4}{3}(-1)+c$ $\therefore\ c=-\frac{28}{3}$ $-\frac{28}{3}<k<\frac{22}{3}$	✓ subst *m* and P ✓ value of *c* ✓✓ answer (4)

4.4.1	$AB^2 = AM^2 - MB^2$ $AB^2 = \left[(t-3)^2 + (t+5)^2\right] - 5^2$ $= t^2 - 6t + 9 + t^2 + 10t + 25 - 25$ $AB = \sqrt{2t^2 + 4t + 9}$	✓ substitution into Pythagoras ✓ simplification (A) (2)
4.4.2	$t = \frac{-4}{2(2)}$ $= -1$ Minimum at $t = -1$ $AB = \sqrt{2(-1)^2 + 4(-1) + 9}$ $AB = \sqrt{7}$ **OR/*OF*** $4t + 4 = 0$ $t = -1$ Minimum at $t = -1$ $AB = \sqrt{2(-1)^2 + 4(-1) + 9}$ $AB = \sqrt{7}$ **OR/*OF*** Length of $AB = \sqrt{2t^2 + 4t + 9}$ $= \sqrt{2\left(t^2 + 2t + \frac{9}{2}\right)}$ $= \sqrt{2\left[(t+1)^2 + \frac{7}{2}\right]}$ $= \sqrt{2(t+1)^2 + 7}$ Minimum at $t = -1$ $AB = \sqrt{2(-1)^2 + 4(-1) + 9}$ $AB = \sqrt{7}$	✓ substitution into correct formula ✓ $t = -1$ ✓ substitution ✓ answer (4) ✓ derivative = 0 ✓ $t = -1$ ✓ substitution ✓ answer (4) ✓ completing of the square ✓ $t = -1$ ✓substitution ✓answer (4)
		[20]

QUESTION/*VRAAG* 5

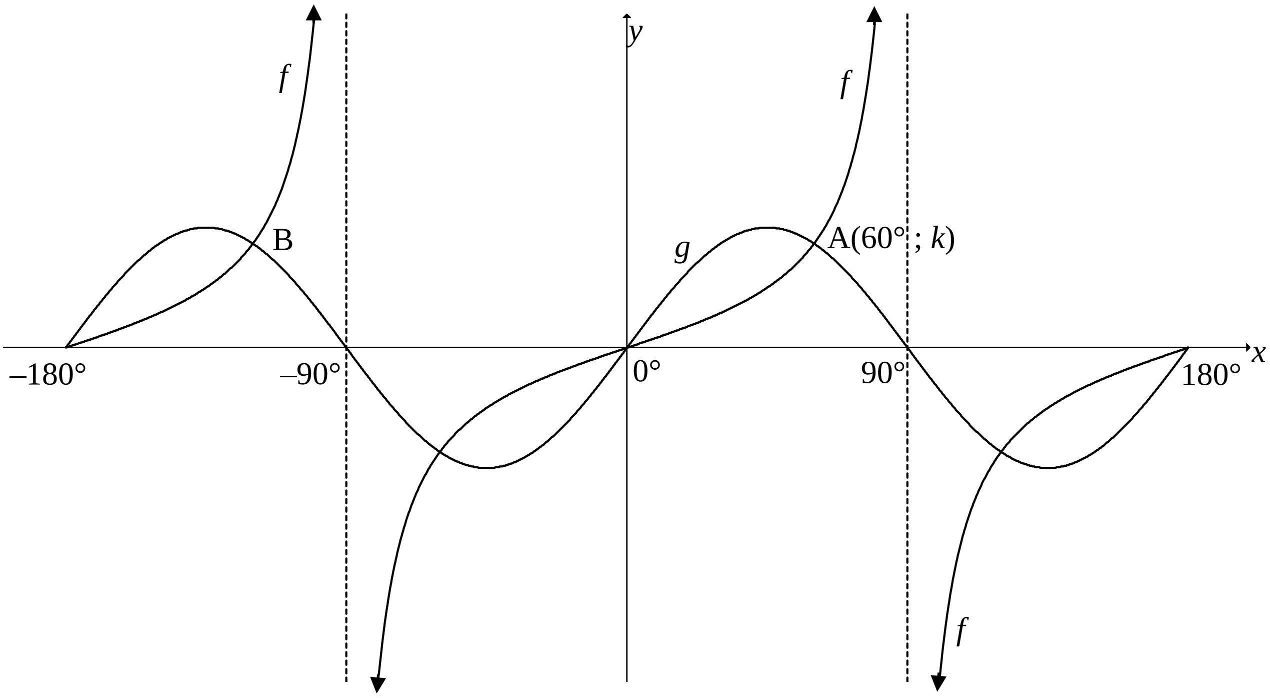
5.1.1	$\sin(360°+x)$ $=\sin x$	✓ + ✓ $\sin x$ (2)
5.1.2	$x-\text{coordinate}=\sqrt{\left(\sqrt{13}\right)^2-(-3)^2}$ $=-2$ $\tan x=\frac{-3}{-2}$ $=\frac{3}{2}$ **OR/*OF*** $x-\text{coordinate}=\sqrt{\left(\sqrt{13}\right)^2-(3)^2}$ $=2$ $\tan x=\frac{3}{2}$	✓✓ substitution ✓ method (3) ✓✓ substitution ✓ method (3)
5.1.3	$\cos(180°+x)$ $=-\cos x$	✓ – ✓ $\cos x$ (2)
5.2	$\frac{\cos(90°+\theta)}{\sin(\theta-180°)+3\sin(-\theta)}$ $=\frac{-\sin\theta}{\sin\left(-\left(180°-\theta\right)\right)-3\sin\theta}$ $=\frac{-\sin\theta}{-\sin\theta-3\sin\theta}$ $=\frac{-\sin\theta}{-4\sin\theta}$ $=\frac{1}{4}$	✓ $-\sin\theta$ ✓ $-3\sin\theta$ ✓ $-\sin\theta$ ✓ simplification ✓ answer (5)

5.3	$(\cos x + 2\sin x)(3\sin 2x - 1) = 0$ $\cos x + 2\sin x = 0$ or $3\sin 2x - 1 = 0$ $\tan x = -\frac{1}{2}$ $\sin 2x = \frac{1}{3}$ ref $\angle$=26,565...° ref $\angle$=19,471...° x = 153,43° + k.180° ; $k \in Z$ x = 9,74° + k.180°; $k \in Z$ **OR/*OF*** or x = 153,43° + k.360° ; $k \in Z$ x = 80,26° + k.180°; $k \in Z$ or x = 333,43° + k.360° ; $k \in Z$	✓ both equations ✓ $\tan x = -\frac{1}{2}$ ✓ $\sin 2x = \frac{1}{3}$ ✓ x = 153,43° **OR** x = 153,43° &333,43° ✓ x = 9,74° & 80,26° ✓ + k.180° ; $k \in$ Z (6)
5.4.1	LHS $= \cos(x+y).\cos(x-y)$ $= [\cos x.\cos y - \sin x.\sin y][\cos x.\cos y + \sin x.\sin y]$ $= \cos^2 x.\cos^2 y - \sin^2 x.\sin^2 y$ $= (1-\sin^2 x)(1-\sin^2 y) - \sin^2 x.\sin^2 y$ $= 1 + \sin^2 x.\sin^2 y - \sin^2 x - \sin^2 y - \sin^2 x.\sin^2 y$ $= 1 - \sin^2 x - \sin^2 y$ = RHS	✓ expansion ✓ simplification ✓ square identity ✓ product (4)
5.4.2	$1 - \sin^2 45° - \sin^2 15°$ $= \cos(45° + 15°).\cos(45° - 15°)$ $= \cos 60°.\cos 30°$ $= \left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$ $= \frac{\sqrt{3}}{4}$ **OR/*OF***	✓ identifying x and y ✓ substitution ✓ answer (3)

	$1-\sin^2 45°-\sin^2 15°$ $=\sin^2 15°+\cos^2 15°-\sin^2 45°-\sin^2 15°$ $=\cos^2 15°-\left(\frac{\sqrt{2}}{2}\right)^2$ $=\cos^2 15°-\frac{1}{2}$ $=\frac{2\cos^2 15°-1}{2}$ $=\frac{\cos 30°}{2}$ $=\frac{\sqrt{3}}{2}\times\frac{1}{2}$ $=\frac{\sqrt{3}}{4}$	✓ identity ✓ substitution ✓ answer (3)
	OR $1-\sin^2 45°-\sin^2 15°$ $=\cos^2 45°-\sin^2(45°-30°)$ $=\left(\frac{1}{\sqrt{2}}\right)^2-(\sin 45°\cos 30°-\cos 45°\sin 30°)^2$ $=\frac{1}{2}-\left(\frac{1}{\sqrt{2}}\times\frac{\sqrt{3}}{2}-\frac{1}{\sqrt{2}}\times\frac{1}{2}\right)^2$ $=\frac{1}{2}-\left(\frac{\sqrt{3}}{2\sqrt{2}}-\frac{1}{2\sqrt{2}}\right)^2$ $=\frac{1}{2}-\left(\frac{\sqrt{3}}{2\sqrt{2}}-\frac{1}{2\sqrt{2}}\right)^2$ $=\frac{1}{2}-\left(\frac{3}{8}-\frac{\sqrt{3}}{4}+\frac{1}{8}\right)$ $=\frac{\sqrt{3}}{4}$	✓ expansion ✓ substitution ✓ answer (3)

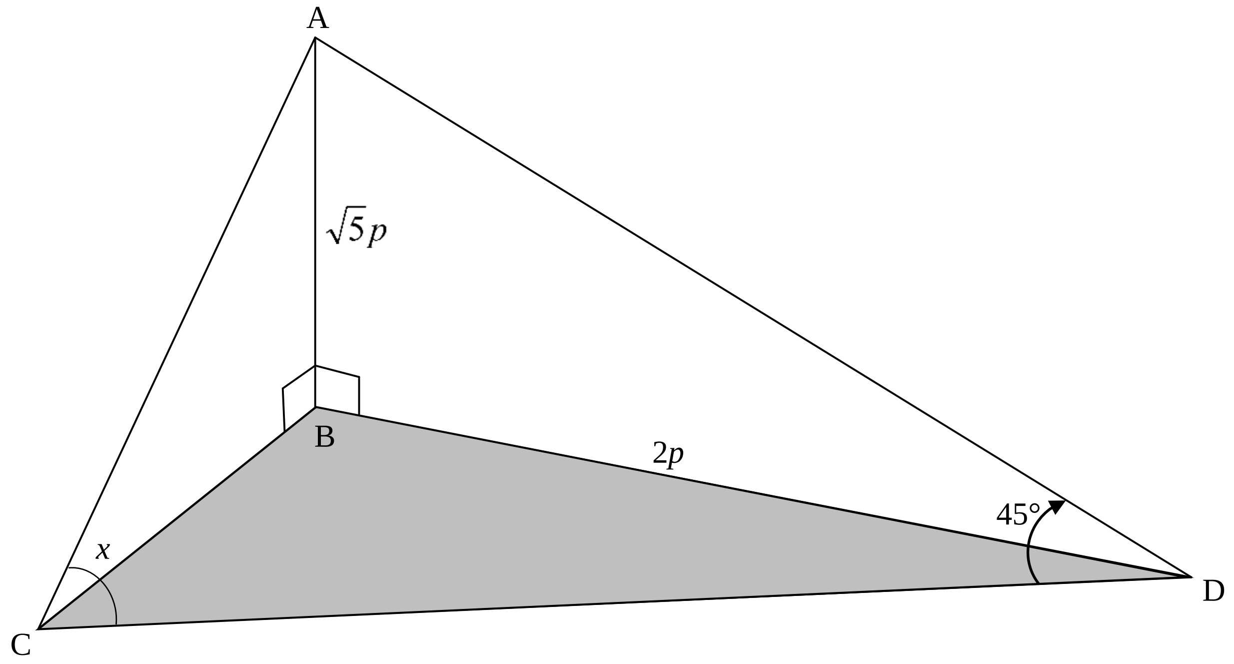
5.5.1	$16\sin x.\cos^3 x - 8\sin x.\cos x$ $= 8\sin x.\cos x(2\cos^2 x - 1)$ $= 4\sin 2x(\cos 2x)$ $= 2\sin 4x$ **OR/*OF*** $16\sin x.\cos^3 x - 8\sin x.\cos x$ $= 16\cos^2 x\left(\frac{1}{2}\sin 2x\right) - 8\left(\frac{1}{2}\sin 2x\right)$ $= 8\left(2\cos^2 x - 1\right)\left(\frac{1}{2}\sin 2x\right)$ $= 4\sin 2x.\cos 2x$ $= 2\sin 4x$	✓ factorisation ✓ 4sin2x ✓ cos2x ✓ double angle (4) ✓ factorisation ✓ $4\sin 2x$ ✓ cos2x ✓ double angle (4)
5.5.2	$16\sin x.\cos^3 x - 8\sin x.\cos x = 2\sin 4x$ Minimum at $x = 67,5°$	✓ answer (1)
		[30]

QUESTION/*VRAAG* 6



6.1	180°	✓ answer (1)
6.2.1	$k=\sqrt{3}=1,73$	✓ answer (1)
6.2.2	$B\left(-120°;\sqrt{3}\right)$	✓ $x=-120°$ (1)
6.3	Range of g: $y\in[-2;\ 2]$ Range of $2g(x)$: $y\in[-4;\ 4]$ **OR/*OF*** ANSWER ONLY: Full marks Range of g: $-2\le y\le 2$ Range of $2g(x)$: $-4\le y\le 4$	✓ $y\in[-2;\ 2]$ ✓ answer (2) ✓ $-2\le y\le 2$ ✓ answer (2)
6.4	$x\in[-65°;\ -5°]$ **OR/*OF*** $-65°\le x\le -5°$	✓✓ $x\in[-65°;\ -5°]$ (2) ✓✓ $-65°\le x\le -5°$ (2)
6.5	$\sin x.\cos x=p$ $4\sin x.\cos x=4p$ $2\sin 2x=4p$ $4p=\pm 2$ $\therefore\ p=-\frac{1}{2}$ or $\frac{1}{2}$ ANSWER ONLY: Full marks	✓ $2\sin 2x=4p$ ✓ $4p=\pm 2$ ✓ answers (3)
		[10]

QUESTION/*VRAAG* 7

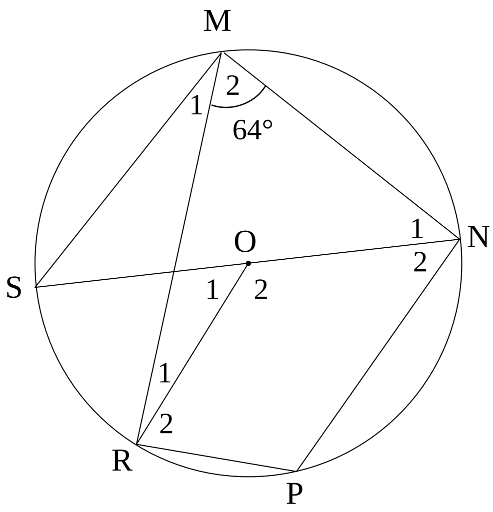


7.1	$AD^2 = AB^2 + BD^2$ $AD^2 = \left(\sqrt{5}p\right)^2 + (2p)^2$ $AD^2 = 9p^2$ $AD = 3p$	 ✓ substitution in Pythagoras ✓ answer (2)
7.2	$\dfrac{CD}{\sin(135° - x)} = \dfrac{3p}{\sin x}$ $CD = \dfrac{3p\sin(135° - x)}{\sin x}$ $CD = \dfrac{3p(\sin 135° \cos x - \cos 135° \sin x)}{\sin x}$ $CD = \dfrac{3p(\sin 45° \cos x + \cos 45° \sin x)}{\sin x}$ $CD = \dfrac{3p\left(\dfrac{\sqrt{2}}{2}\cos x + \dfrac{\sqrt{2}}{2}\sin x\right)}{\sin x}$ $CD = \dfrac{3p\left(\dfrac{\sqrt{2}}{2}\right)(\cos x + \sin x)}{\sin x}$ $CD = \dfrac{3p(\sin x + \cos x)}{\sqrt{2}\sin x}$	✓ correct use of sine rule ✓ $135° - x$ ✓ compound angle ✓ special values ✓ factorisation (5)

7.3	$\text{Area } \Delta\text{ADC} = \frac{1}{2}(\text{AD})(\text{CD})\sin\text{A}\hat{\text{D}}\text{C}$ $= \frac{1}{2}(3p)\left(\frac{3p(\sin x + \cos x)}{\sqrt{2}\sin x}\right)(\sin 45°)$ $= \frac{1}{2}(30)\left(\frac{30(\sin 110° + \cos 110°)}{\sqrt{2}\sin 110°}\right)\sin 45°$ $= 143{,}11 m^2$	✓ correct use of area rule ✓ substitution in area rule ✓ answer (3)
		[10]

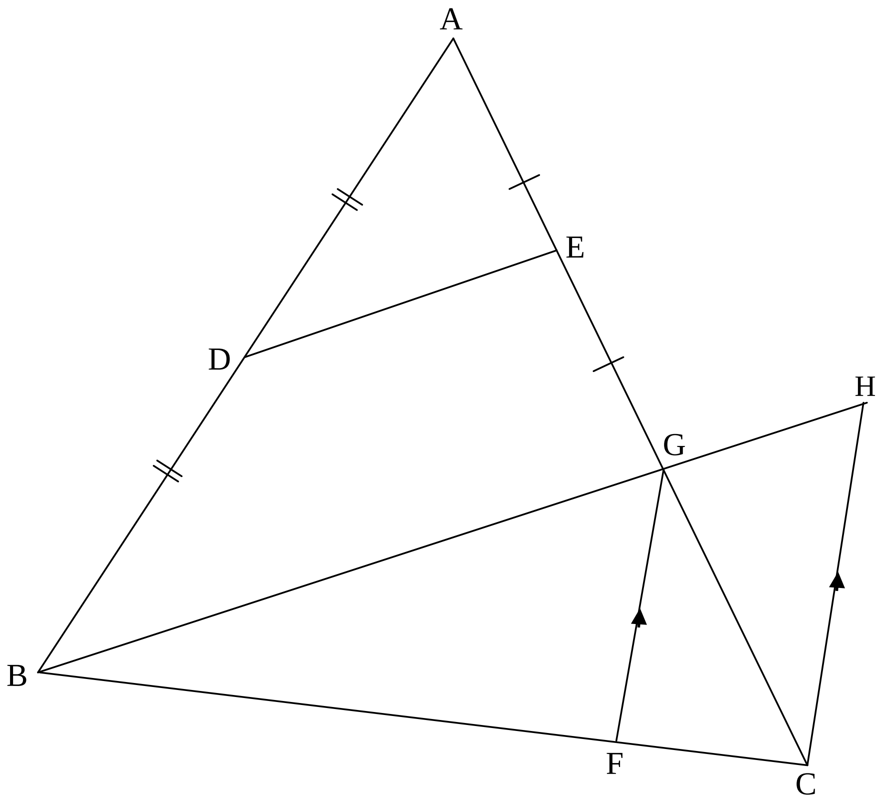
QUESTION/*VRAAG* 8

8.1



8.1.1	$\hat{P} = 116°$ [opp ∠s of cyclic quad/*teenoorst. ∠e van kvh*]	✓ S ✓ R (2)
8.1.2	$\hat{M}_1 + 64° = 90°$ [∠ in semi-circle/∠ *in halwe sirkel*] $\hat{M}_1 = 26°$	✓ R ✓ S (2)
8.1.3	$\hat{O}_1 = 52°$ [∠ at centre = 2 x ∠ at circumference/*midpts.* ∠ = 2 x *omtreks.* ∠]	✓ S ✓R (2)

8.2

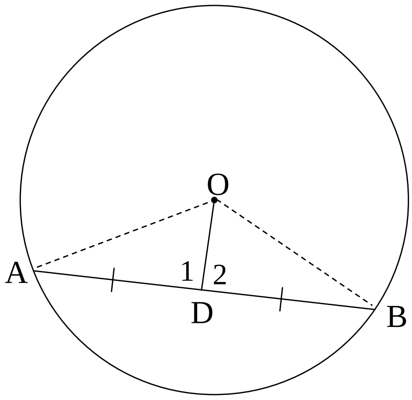


8.2.1	Midpt theorem/*Midpt. Stelling* **OR/*OF*** Converse prop intercept theorem	✓ R (1) ✓ R (1)
8.2.2	BG = 2DE or 6x – 2 [Midpt theorem/*Midpt. stelling*] BG = 6x – 2 $\frac{GH}{BG}=\frac{FC}{BF}$ [line ‖ one side of Δ **OR** prop theorem; FG ‖ CH / *lyn* ‖ *een sy v.* Δ] $\frac{x+1}{6x-2}=\frac{1}{4}$ 4x + 4 = 6x – 2 2x = 6 x = 3 **OR/*OF***	✓ S ✓ R ✓ S ✓R ✓ equation into x ✓ answer (6)

	$\frac{BF}{FC} = \frac{BG}{GH}$	[line ∥ one side of Δ **OR** prop theorem; FG ∥ CH / *lyn* ∥ *een sy v.* Δ]	✓ S ✓ R
	$\frac{AE}{AG} = \frac{DE}{BG}$	[ΔADE ∥∥ ΔABG]	✓ S ✓R
	BG = 4x + 4		
	$\frac{1}{2} = \frac{3x-1}{4x+4}$		✓ equation into x
	$\therefore 4x + 4 = 6x - 2$		
	$\therefore x = 3$		✓ answer (6)
			[13]

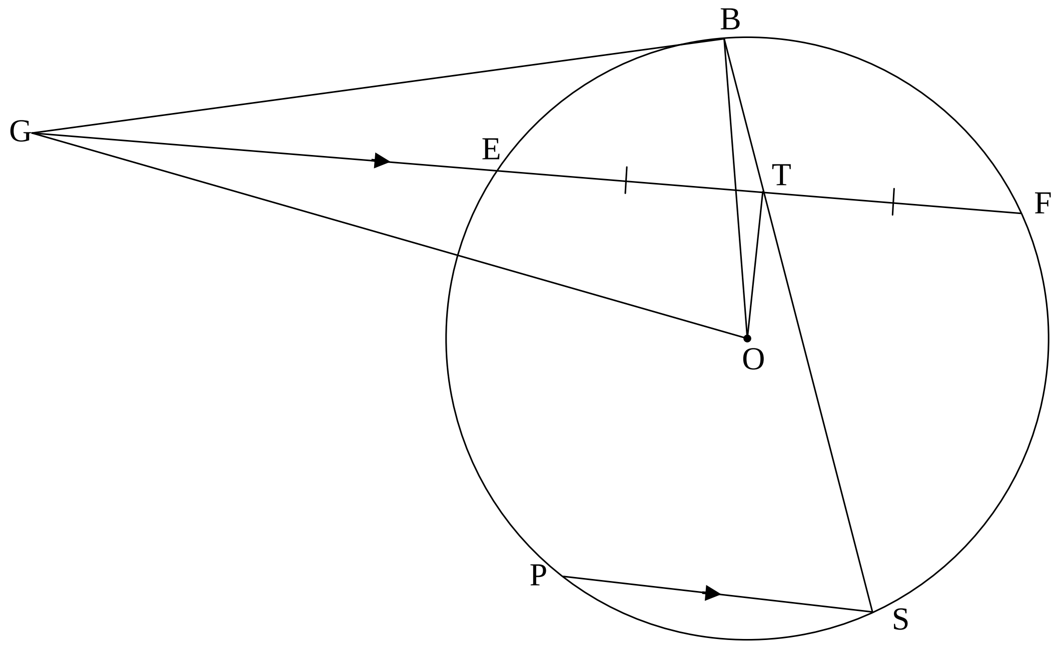
QUESTION/*VRAAG* 9

9.1



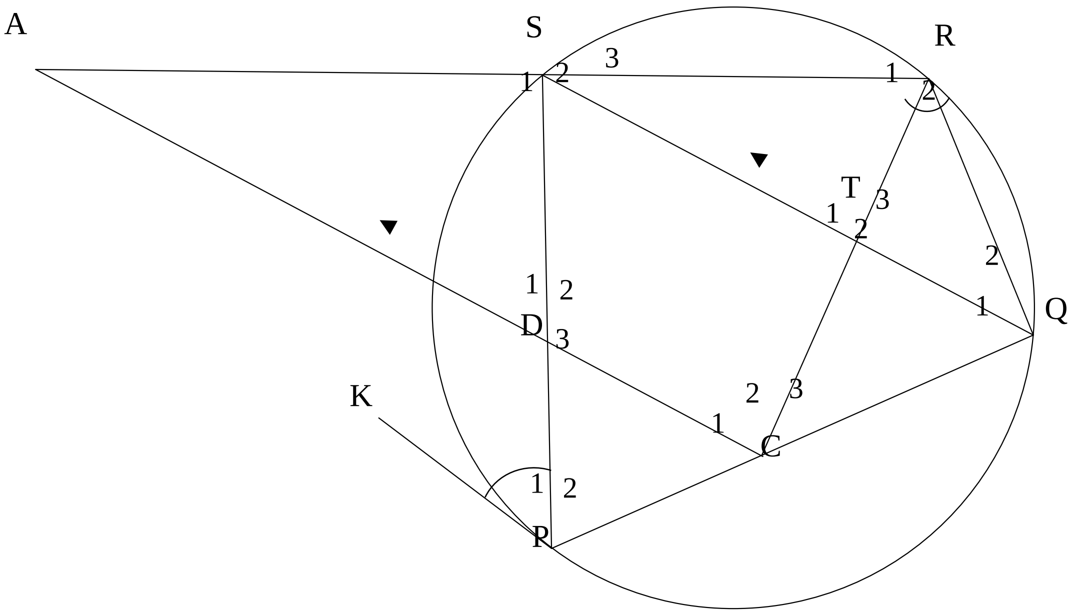
9.1.1	Construction: Draw OA and OB In ΔADO and ΔBDO OA = OB OD = OD AD = DB $\therefore \Delta ADO \equiv \Delta BDO$ ADB is a straight line $\therefore \hat{D}_1 = \hat{D}_2$ $\therefore OD \perp AB$ **OR/*OF*** Construction: Draw OA and OB In ΔADO and ΔBDO AD = DB $\hat{A} = \hat{B}$ OA = OB $\therefore \Delta ADO \equiv \Delta BDO$ ADB is a straight line $\therefore \hat{D}_1 = \hat{D}_2$ $\therefore OD \perp AB$	 [radii/*radiusse*] [common side/*gemeenskaplike sy*] [given/*gegee*] [S;S;S] $\Delta ADO \equiv \Delta BDO$ [$\angle$s on a str line/$\angle$*e op 'n reguitlyn*] [given/*gegee*] [$\angle$s opp; $\angle$s sides /$\angle$*e teenoor gelyke sye*] [radii/*radiusse*] [S;$\angle$;S] $\Delta ADO \equiv \Delta BDO$ [$\angle$s on a str line/$\angle$*e op 'n reguitlyn*]	✓ construction ✓ first pair of sides ✓ other 2 pairs ✓ R ✓ R (5) ✓ construction ✓ first pair of sides ✓ other 2 pairs ✓ R ✓ R (5)

9.2



9.2.1	$O\hat{T}G = 90°$ [line from centre to midpt of chord/ *midpt. sirkel; midpt. koord*] $O\hat{B}G = 90°$ [tan ⊥ radius/*raaklyn* ⊥ *radius*] $\therefore O\hat{T}G = O\hat{B}G = 90°$ ∴ OTBG is a cyclic quadrilateral [line subtends equal ∠s **OR** converse ∠s in the same segment/ *lyn onderspan gelyke ∠e*]	✓S ✓R ✓S ✓R ✓R (5)
9.2.2	$\hat{S} = B\hat{T}G$ [corresp ∠s; GF ‖ PS / *ooreenk.* ∠s; GF ‖ PS] But $B\hat{T}G = G\hat{O}B$ [∠s in the same segment/∠*e in dies. sirkelsegment*] $G\hat{O}B = \hat{S}$	✓S ✓R ✓S ✓R (4)
		[14]

QUESTION/*VRAAG* 10



10.1	$\hat{P}_1 = \hat{Q}_1$ [tan-chord theorem/$\angle$ *tussen raaklyn en koord*] $\hat{S}_1 = \hat{Q}_1 + \hat{Q}_2$ [ext $\angle$ of cyclic quad/*buite* $\angle$ *v. kvh*] $\therefore \hat{S}_1 = \hat{P}_1 + \hat{Q}_2$ $\hat{T}_2 = \hat{R}_2 + \hat{Q}_2$ [ext $\angle$ of Δ/*buite* $\angle$ *v.* Δ] but $\hat{P}_1 = \hat{R}_2$ [given/*gegee*] $\hat{T}_2 = \hat{P}_1 + \hat{Q}_2$ $\therefore \hat{S}_1 = \hat{T}_2 = \hat{P}_1 + \hat{Q}_2$	✓ S ✓ S / R ✓ S ✓ S (4)
10.2	In Δ ASD and ΔACR $\hat{A} = \hat{A}$ [common $\angle$/*gemeenskaplike* $\angle$] $\hat{S}_1 = \hat{T}_2$ [proven/*reeds bewys*] $\hat{T}_2 = \hat{C}_2$ [alt $\angle$s; QS || CA/verw. $\angle$*e*; QS || CA] $\therefore \hat{S}_1 = \hat{C}_2$ $\hat{D}_1 = \hat{R}_1$ [sum of $\angle$s in Δ/$\angle$*e v.* Δ] ΔASD ||| ΔACR $\therefore \frac{AD}{AR} = \frac{AS}{AC}$ [corresponding sides in proportion/ *ooreenstemmende sy in dies. verhouding*] **OR/*OF***	✓ identifying Δ's ✓ S ✓ S / R ✓ S ✓ S (5)

	In Δ ASD and ΔACR $\hat{A} = \hat{A}$ [common $\angle$/*gemeenskaplike* $\angle$] $\hat{S}_1 = \hat{T}_2$ [proven/*gegee*] $\hat{T}_2 = \hat{C}_2$ [alt $\angle$s; QS ‖ CA/verw. $\angle$*e*; QS ‖ CA] $\therefore \hat{S}_1 = \hat{C}_2$ $\Delta ASD \parallel\!\mid \Delta ACR$ [$\angle;\angle;\angle$] $\therefore \frac{AD}{AR} = \frac{AS}{AC}$ [corresponding sides in proportion/ *ooreenstemmende sy in dies. verhouding*]	✓ identifying Δ's ✓ S ✓ S/R ✓ S ✓ R (5)
10.3	$\frac{AS}{AC} = \frac{SD}{CR}$ [$\Delta ASD \parallel\!\mid \Delta ACR$] $\therefore AS = \frac{AC \times SD}{CR}$ $\frac{AS}{AR} = \frac{CT}{CR}$ [line ‖ one side of Δ OR prop theorem; TS ‖ CA/*lyn* ‖ *een sy v.* Δ] $\therefore AS = \frac{AR \times CT}{CR}$ $\therefore \frac{AC \times SD}{CR} = \frac{AR \times CT}{CR}$ $\therefore AC \times SD = AR \times CT$	✓ S ✓ S ✓ R ✓ equating (4)
		[13]

TOTAL/*TOTAAL*: 150