basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

SENIOR CERTIFICATE EXAMINATIONS/
NATIONAL SENIOR CERTIFICATE EXAMINATIONS

MATHEMATICS P2

MAY/JUNE 2023

MARKS: 150

TIME: 3 hours

This question paper consists of 13 pages and 1 information sheet.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 10 questions.
2. Answer ALL the questions in the SPECIAL ANSWER BOOK provided.
3. Clearly show ALL calculations, diagrams, graphs, etc. which you have used in determining your answers.
4. Answers only will NOT necessarily be awarded full marks.
5. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
6. If necessary, round off answers correct to TWO decimal places, unless stated otherwise.
7. Diagrams are NOT necessarily drawn to scale.
8. An information sheet with formulae is included at the end of the question paper.
9. Write neatly and legibly.

QUESTION 1

1.1 The owner of a small company wishes to establish whether advertising in a regional newspaper is effective. The table below shows the amount spent on advertising and the corresponding sales figures for the last 9 years.

Amount spent on advertising (in rands) (x)	21 300	23 700	24 800	30 540	24 100	40 680	22 400	35 250	29 110
Sales (in rands) (y)	311 500	326 700	349 200	470 000	316 100	564 200	314 000	487 300	392 900

1.1.1 Determine the equation of the least squares regression line for the data. (3)

1.1.2 Predict the sales for a year in which the company will spend R28 500 on advertising. (2)

1.1.3 Write down the correlation coefficient of the data. (1)

1.1.4 Describe the association between the amount spent on advertising in the regional newspaper and the sales of this company. (1)

1.2 The profit that the small company made over the same 9 years is given in the table below.

Profit (in rands)	110 750	107 376	152 338	244 480	144 021	275 994	121 900	207 636	187 700

1.2.1 Calculate the mean profit made over the 9 years. (2)

1.2.2 Write down the standard deviation for the data. (1)

1.2.3 Determine the number of years in which the company made a profit that was greater than one standard deviation above the mean. (2)
[12]

QUESTION 2

The ages of the people who attended a music concert was summarised in the table below.

AGE	NUMBER OF PEOPLE
$5 < x \leq 15$	20
$15 < x \leq 25$	25
$25 < x \leq 35$	60
$35 < x \leq 45$	90
$45 < x \leq 55$	55
$55 < x \leq 65$	40
$65 < x \leq 75$	30

2.1 Write down the modal class of the data. (1)

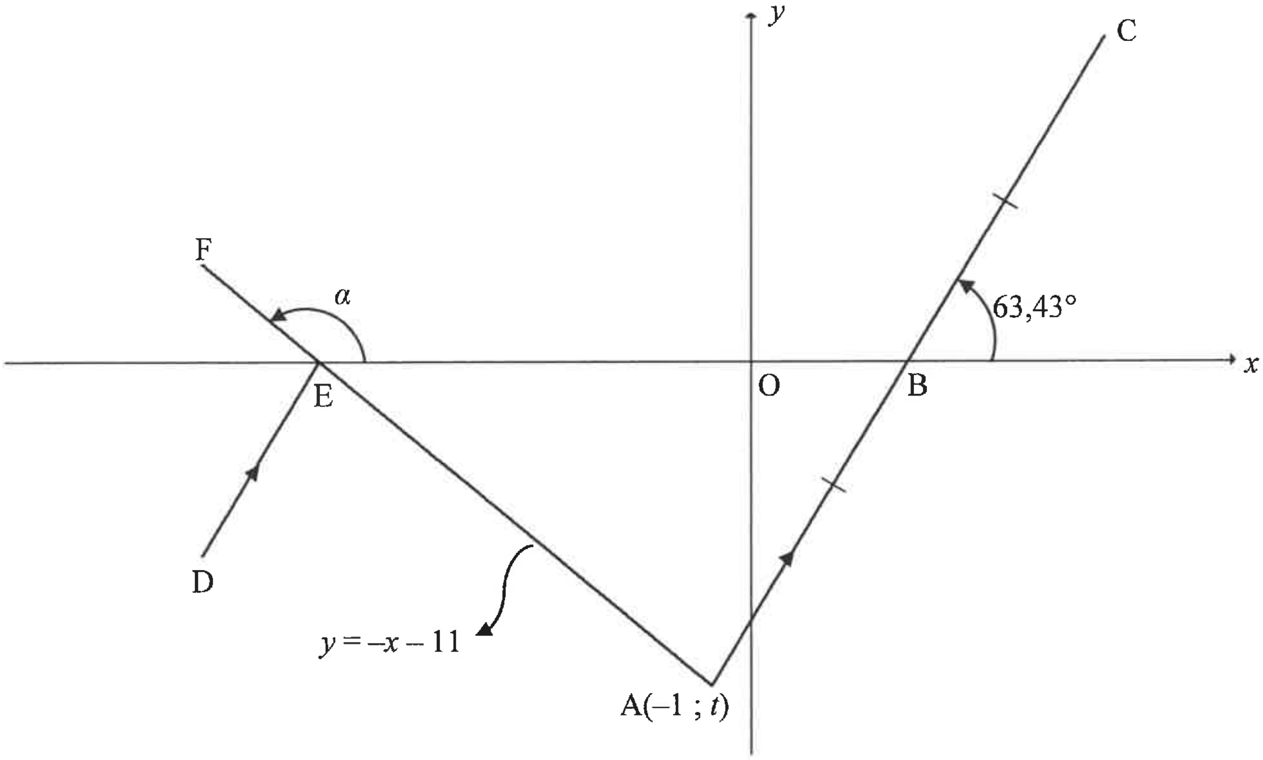
2.2 How many people attended the music concert? (1)

2.3 On the grid provided in the ANSWER BOOK, draw a cumulative frequency graph (ogive) to represent the above data. (4)

2.4 Use the cumulative frequency graph to determine the median age of the people who attended the music concert. (2)
[8]

QUESTION 3

In the diagram, the equation of line AF is $y = -x - 11$. B, a point on the x-axis, is the midpoint of the straight line joining A$(-1\ ;\ t)$ and C. The angles of inclination of AF and AC are α and $63{,}43°$ respectively. AF cuts the x-axis in E. D is a point such that DE || AC.



3.1 Calculate the:

3.1.1 Value of t (2)

3.1.2 Size of α (2)

3.1.3 Gradient of AC, to the nearest whole number (2)

3.2 Determine the equation of AC in the form $y = mx + k$. (2)

3.3 Calculate the:

3.3.1 Coordinates of C (3)

3.3.2 Size of $\hat{FED}$ (3)

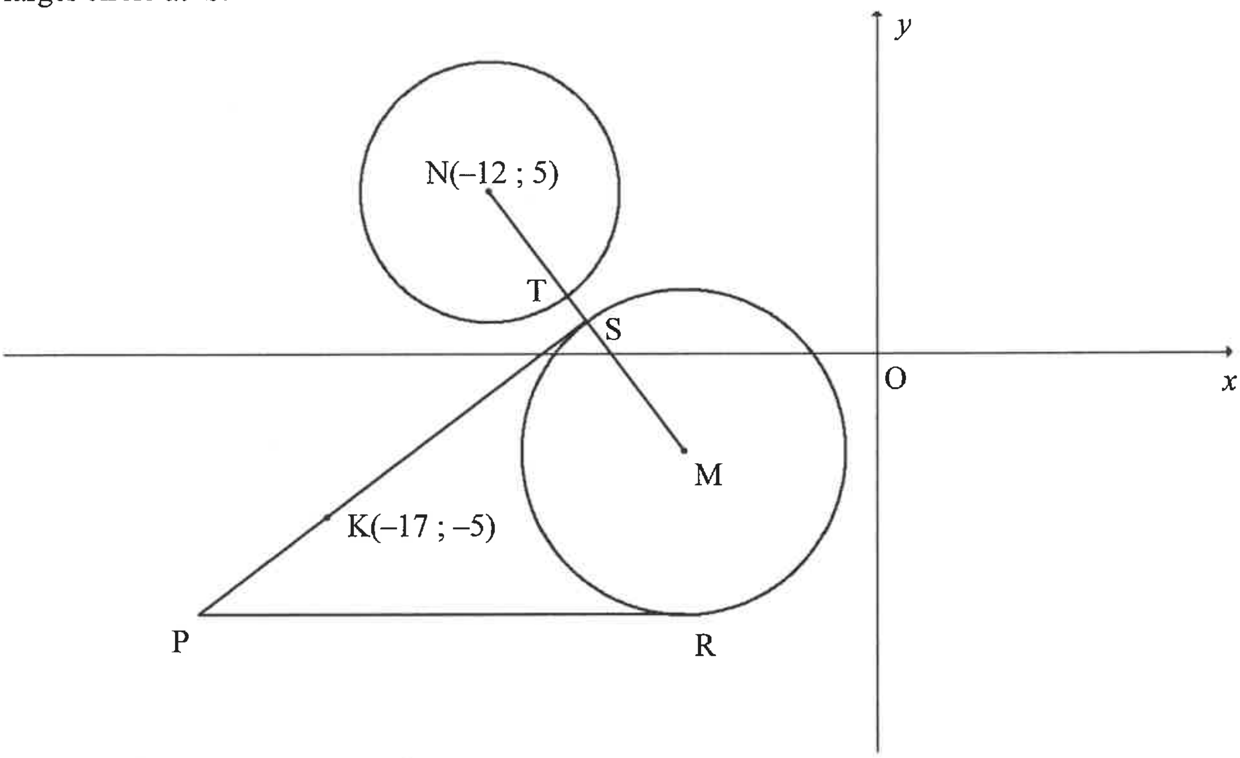
3.4 G is a point such that EAGC, in that order, is a parallelogram.

Determine the equation of a circle centred at G and passing through the point B. Write your answer in the form $(x-a)^2 + (y-b)^2 = r^2$. (4)

[18]

QUESTION 4

In the diagram, the equation of the circle centred at N(–12 ; 5) is $x^2 + y^2 + 24x - 10y + 153 = 0$. The equation of the circle centred at M is $(x+6)^2 + (y+3)^2 = 25$. PS and PR are tangents to the circle centred at M at S and R respectively. PR is parallel to the x-axis. K(–17 ; –5) is a point on PS. The straight line joining N and M cuts the smaller circle at T and the larger circle at S.



4.1 Write down the coordinates of M. (2)

4.2 Calculate the:

4.2.1 Length of the radius of the smaller circle (2)

4.2.2 Length of TS (4)

4.3 Determine the equation of the tangent:

4.3.1 PR (2)

4.3.2 PS, in the form $y = mx + c$ (5)

4.4 Quadrilateral PSMR is drawn. Calculate the:

4.4.1 Perimeter of PSMR (5)

4.4.2 Ratio of $\dfrac{\text{area of } \Delta\text{NPS}}{\text{area of quadrilateral PSMR}}$ (2)

[22]

QUESTION 5

5.1 **Without using a calculator**, simplify the following expression to a single trigonometry ratio:

$$\frac{1-\sin(-\theta)\cos(90°+\theta)}{\cos(\theta-360°)}$$ (5)

5.2 Given that $\cos 20° = p$

Without using a calculator, write EACH of the following in terms p:

5.2.1 $\cos 200°$ (2)

5.2.2 $\sin(-70°)$ (2)

5.2.3 $\sin 10°$ (3)

5.3 Determine, **without using a calculator**, the value of:

$$\cos(A+55°)\cos(A+10°)+\sin(A+55°)\sin(A+10°)$$ (3)

5.4 Consider: $\dfrac{\cos 2x+\sin 2x-\cos^2 x}{\sin x-2\cos x}=-\sin x$

5.4.1 Prove the above identity. (3)

5.4.2 Determine the value of $\dfrac{\cos 2x+\sin 2x-\cos^2 x}{-3\sin^2 x+6\sin x\cos x}$ (3)

5.5 Given: $3\tan 4x=-2\cos 4x$

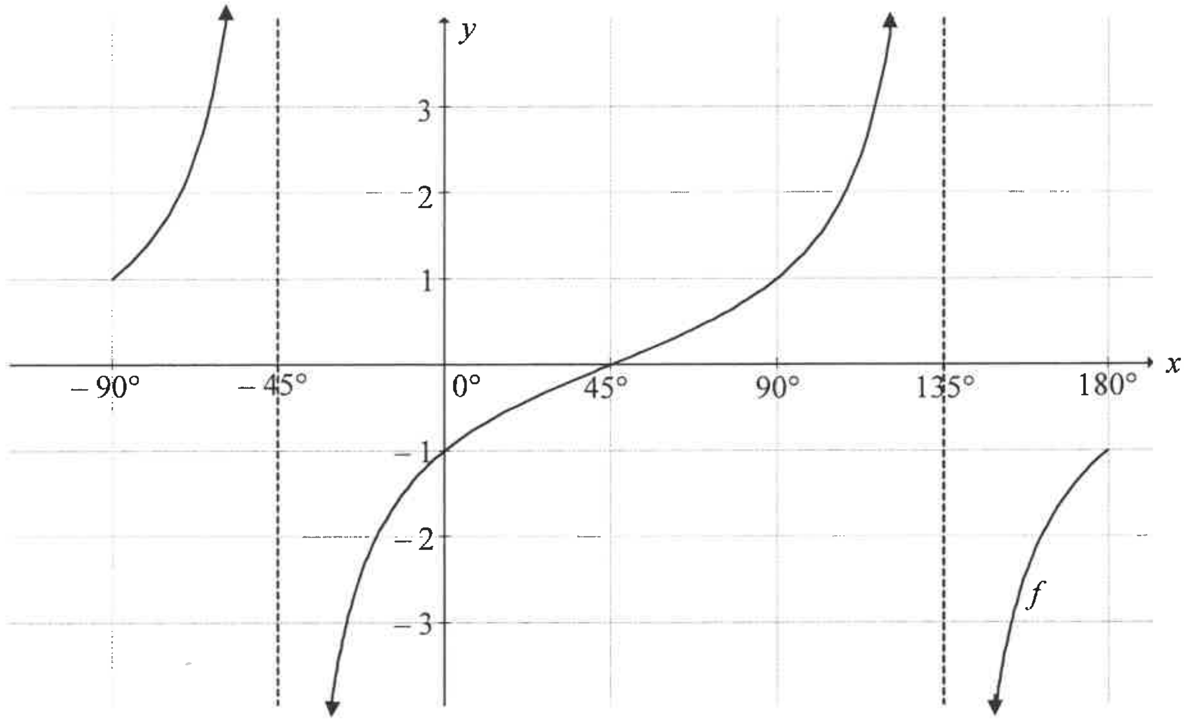
5.5.1 **Without using a calculator**, show that $\sin 4x=-0{,}5$ is the only solution to the above equation. (4)

5.5.2 Hence, determine the general solution of x in the equation $3\tan 4x=-2\cos 4x$ (3)

[28]

QUESTION 6

In the diagram below, the graph of $f(x) = \tan(x - 45°)$ is drawn for $x \in [-90° ; 180°]$.



6.1 Write down the period of f. (1)

6.2 Draw the graph of $g(x) = -\cos 2x$ for the interval $x \in [-90° ; 180°]$ on the grid given in the ANSWER BOOK. Show ALL intercepts with the axes, as well as the minimum and maximum points of the graph. (3)

6.3 Write down the range of g. (1)

6.4 The graph of g is shifted $45°$ to the left to form the graph of h. Determine the equation of h in its simplest form. (2)

6.5 Use the graph(s) to determine the values of x in the interval $x \in [-90° ; 90°]$ for which:

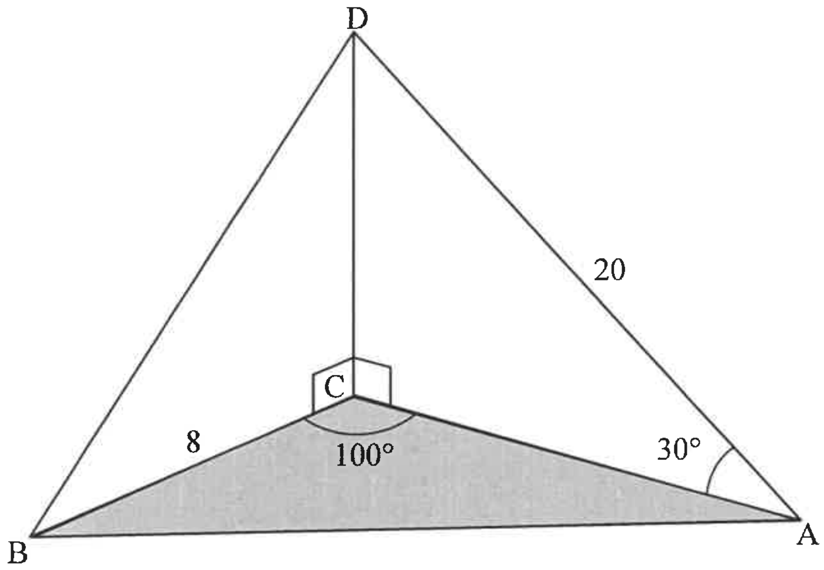
6.5.1 $f(x) > 1$ (2)

6.5.2 $2\cos 2x - 1 > 0$ (4)

[13]

QUESTION 7

In the diagram, A, B and C are points in the same horizontal plane. D is a point directly above C, that is $DC \perp AC$ and $DC \perp BC$. It is given that $A\hat{C}B = 100°$, $C\hat{A}D = 30°$, $AD = 20$ units and $BC = 8$ units.



7.1 Calculate the length of:

7.1.1 AC (2)

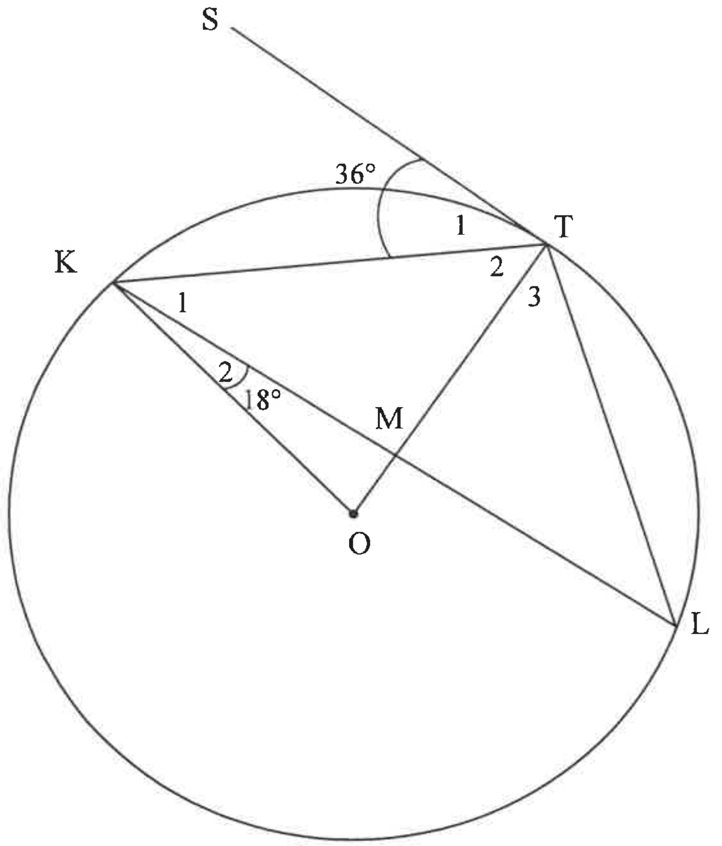
7.1.2 AB (3)

7.2 If it is further given that $A\hat{B}D = 73{,}4°$, calculate the size of $A\hat{D}B$. (3)

[8]

QUESTION 8

8.1 In the diagram, O is the centre of the circle. K, T and L are points on the circle. KT, TL, KL, OK and OT are drawn. OT intersects KL at M. ST is a tangent to the circle at T. $S\hat{T}K = 36°$ and $O\hat{K}L = 18°$.



8.1.1 Determine, giving reasons, the size of:

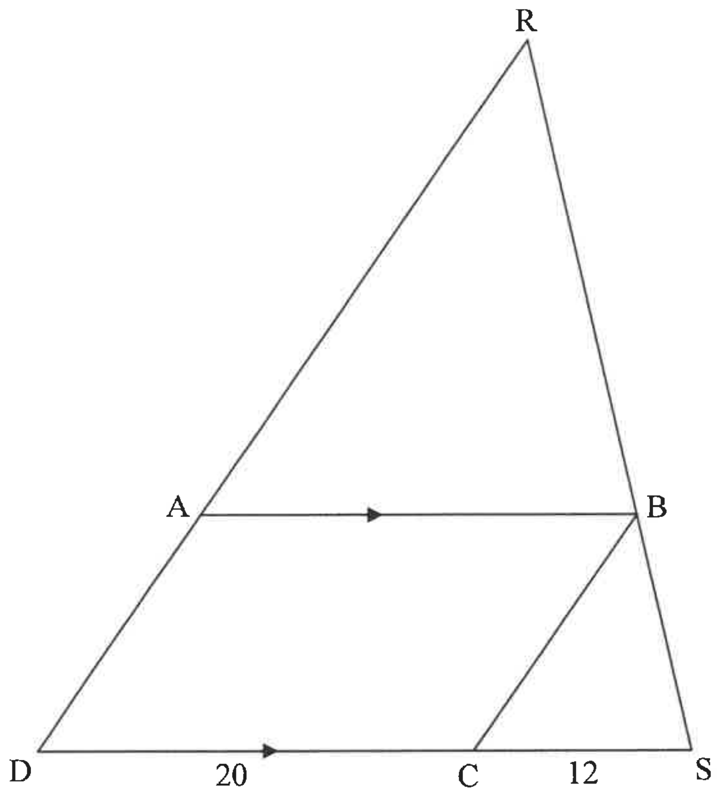
(a) $\hat{T}_2$ (2)

(b) $\hat{L}$ (2)

(c) $K\hat{O}T$ (2)

8.1.2 Prove, giving reasons, that KM = ML. (3)

8.2 In the diagram, ΔRDS is drawn. A, B and C are points on RD, RS and DS respectively such that $AB \parallel DS$ and $RB : BS = 5 : 3$. $DC = 20$ units and $CS = 12$ units.



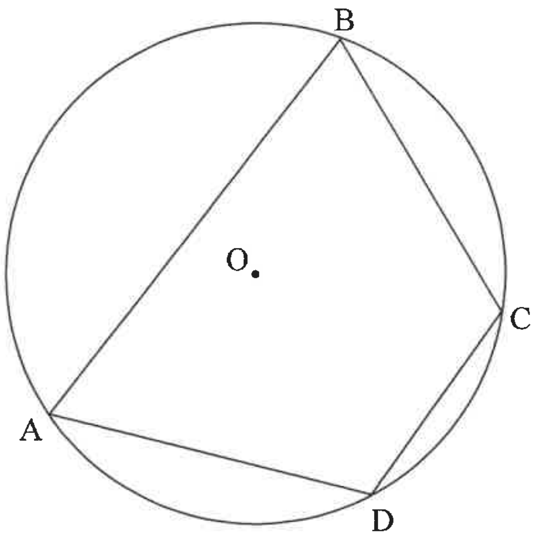
8.2.1 Prove, giving reasons, that $BC \parallel AD$. (3)

8.2.2 If it is further given that $RD = 48$ units, calculate, giving reasons, the value of the ratio $AD : AB$. (3)

[15]

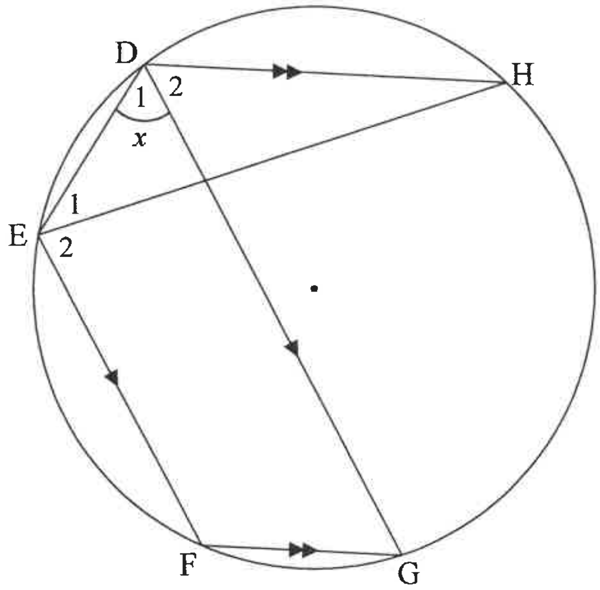
QUESTION 9

9.1 In the diagram, O is the centre of the circle. ABCD is a cyclic quadrilateral.



Use the diagram in the ANSWER BOOK to prove the theorem which states that the opposite angles of a cyclic quadrilateral are supplementary, that is prove that $\hat{B} + \hat{D} = 180°$. (5)

9.2 In the diagram, DEFG is a cyclic quadrilateral such that EF || DG. H is another point on the circle such that DH || FG. Chord EH is drawn. Let $\hat{D}_1 = x$.

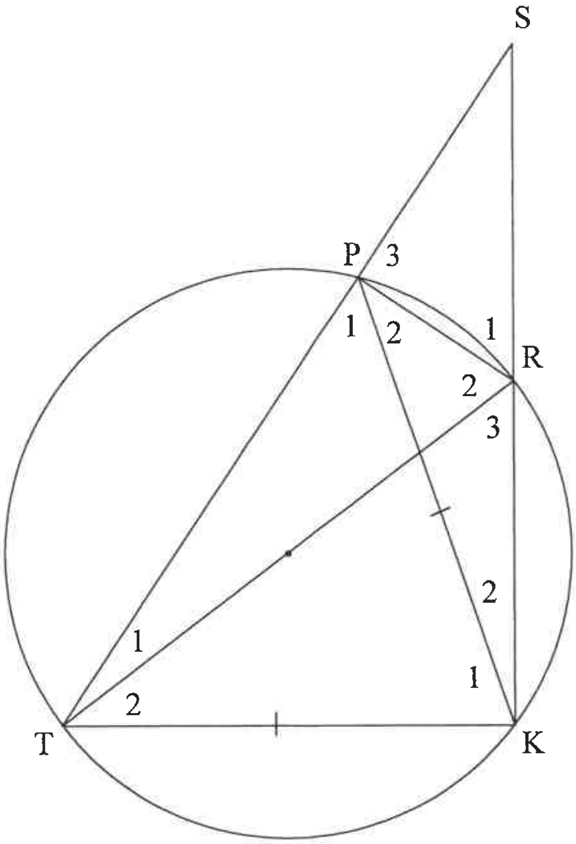


Prove, giving reasons, that $\hat{D}_1 = \hat{D}_2$. (4)

[9]

QUESTION 10

In the diagram, TR is a diameter of the circle. PRKT is a cyclic quadrilateral. Chords TP and KR are produced to intersect at S. Chord PK is drawn such that PK = TK.



10.1 Prove, giving reasons, that:

10.1.1 SR is a diameter of a circle passing through points S, P and R (4)

10.1.2 $\hat{S} = \hat{P}_2$ (5)

10.1.3 $\Delta SPK \parallel\!| \Delta PRK$ (3)

10.2 If it is further given that SR = RK, prove that $ST = \sqrt{6}RK$. (5)
[17]

TOTAL: 150

INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $\quad$ $A = P(1-ni)$ $\quad$ $A = P(1-i)^n$ $\quad$ $A = P(1+i)^n$

$T_n = a + (n-1)d$ $\quad$ $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

$T_n = ar^{n-1}$ $\quad$ $S_n = \frac{a\left(r^n - 1\right)}{r-1}$; $r \neq 1$ $\quad$ $S_\infty = \frac{a}{1-r}$; $-1 < r < 1$

$F = \frac{x\left[(1+i)^n - 1\right]}{i}$ $\quad$ $P = \frac{x\left[1-(1+i)^{-n}\right]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad$ $M\left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad$ $y - y_1 = m(x - x_1)$ $\quad$ $m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad$ $m = \tan\theta$

$$(x-a)^2 + (y-b)^2 = r^2$$

In ΔABC: $\quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$$a^2 = b^2 + c^2 - 2bc.\cos A$$

$$\text{area}\,\Delta ABC = \frac{1}{2}ab.\sin C$$

$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$ $\quad$ $\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$ $\quad$ $\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$ $\quad$ $\sin 2\alpha = 2\sin\alpha\cos\alpha$

$\bar{x} = \frac{\sum fx}{n}$ $\quad$ $\sigma^2 = \frac{\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $\quad$ $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$\hat{y} = a + bx$ $\quad$ $b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$

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MATHEMATICS P2/*WISKUNDE V2*

MARKING GUIDELINES/*NASIENRIGLYNE*

MAY/JUNE/*MEI/JUNIE* 2023

MARKS: 150
PUNTE: 150

These marking guidelines consist of 21 pages./
Hierdie nasienriglyne bestaan uit 21 bladsye.

NOTE:

- If a candidate answers a question TWICE, mark only the FIRST attempt.
- If a candidate has crossed out an attempt at an answer and not redone the question, mark the crossed-out version.
- Consistent accuracy applies in ALL aspects of the marking guidelines. Stop marking at the second calculation error.
- Assuming answers/values in order to solve a problem is NOT acceptable.

LET WEL:

- *As 'n kandidaat 'n vraag TWEE KEER beantwoord, sien slegs die EERSTE poging na.*
- *As 'n kandidaat 'n antwoord van 'n vraag doodtrek en nie oordoen nie, merk die doodgetrekte poging.*
- *Volgehoue akkuraatheid word in ALLE aspekte van die nasienriglyne toegepas. Hou op nasien by die tweede berekeningsfout.*
- *Aanvaar van antwoorde/waardes om 'n probleem op te los, word NIE toegelaat nie.*

GEOMETRY • *MEETKUNDE*	
S	**A mark for a correct statement** ***(A statement mark is independent of a reason)***
	'n Punt vir 'n korrekte bewering ***('n Punt vir 'n bewering is onafhanklik van die rede)***
R	**A mark for the correct reason** ***(A reason mark may only be awarded if the statement is correct)***
	'n Punt vir 'n korrekte rede ***('n Punt word slegs vir die rede toegeken as die bewering korrek is)***
S/R	**Award a mark if statement AND reason are both correct**
	Ken 'n punt toe as die bewering EN rede beide korrek is

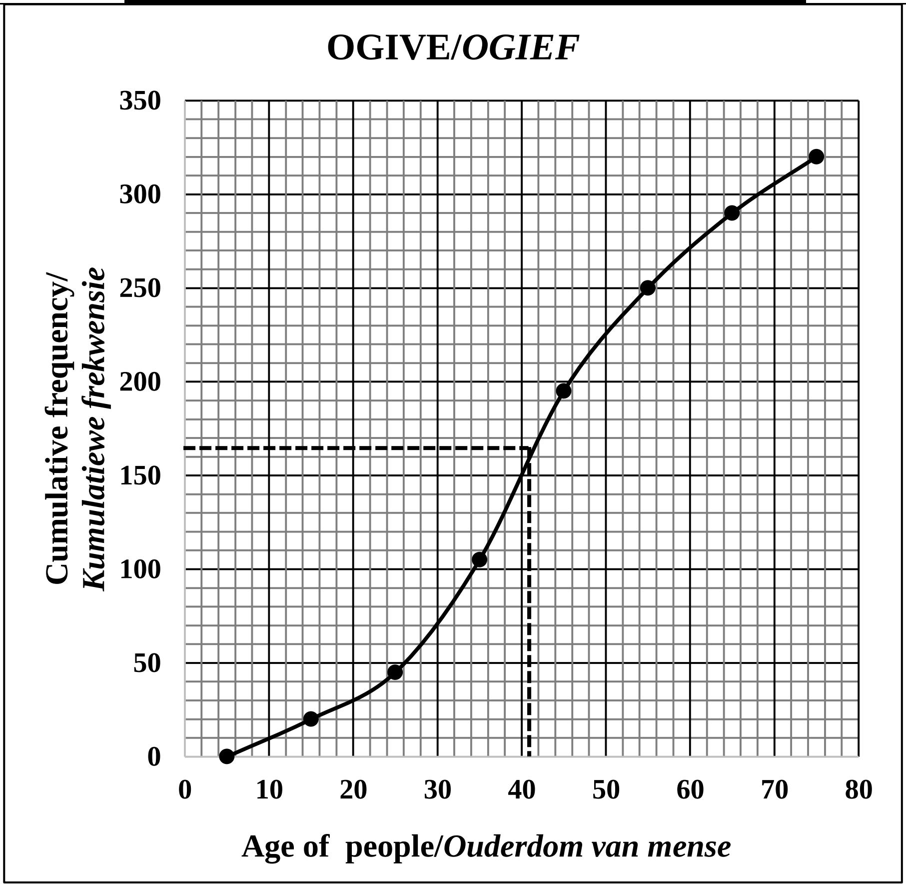
QUESTION/*VRAAG* 1

1.1.1	$a = 1730,22$ $b = 13,96$ $\hat{y} = 1730,22 + 13,96x$	✓ $a = 1730,22$ ✓ $b = 13,96$ ✓ equation (3)
1.1.2	$\hat{y} = 1730,22 + 13,96x$ $\hat{y} = 1730,22 + 13,96(28\,500)$ $\hat{y} = \text{R}399\ 590,22$ **OR/*OF*** $\hat{y} =$ R399 599,64 (calc)	✓ substitution ✓ answer (2) ✓✓ answer (2)
1.1.3	$r = 0,98002\ldots$ $r = 0,98$	✓ answer (1)
1.1.4	There is a very strong positive correlation between the amount spent on advertising and sales. / *Daar is 'n baie sterk positiewe korrelasie tussen die bedrag spandeer op advertensie en die verkope.*	✓ strong positive (1)
1.2.1	$\bar{x} = \dfrac{1\,552\,195}{9}$ $\bar{x} = 172\,466,11$	✓ $\bar{x} = \dfrac{1\,552\,195}{9}$ ✓ answer (2)
1.2.2	$\sigma = 56\,950,09$	✓ answer (1)
1.2.3	$\bar{x} + \sigma$ $= 172\,466,11 + 56\,950,09$ $= 229\,416,20$ 2 years/*jaar*	✓ $\bar{x} + \sigma$ ✓ answer (2)
		[12]

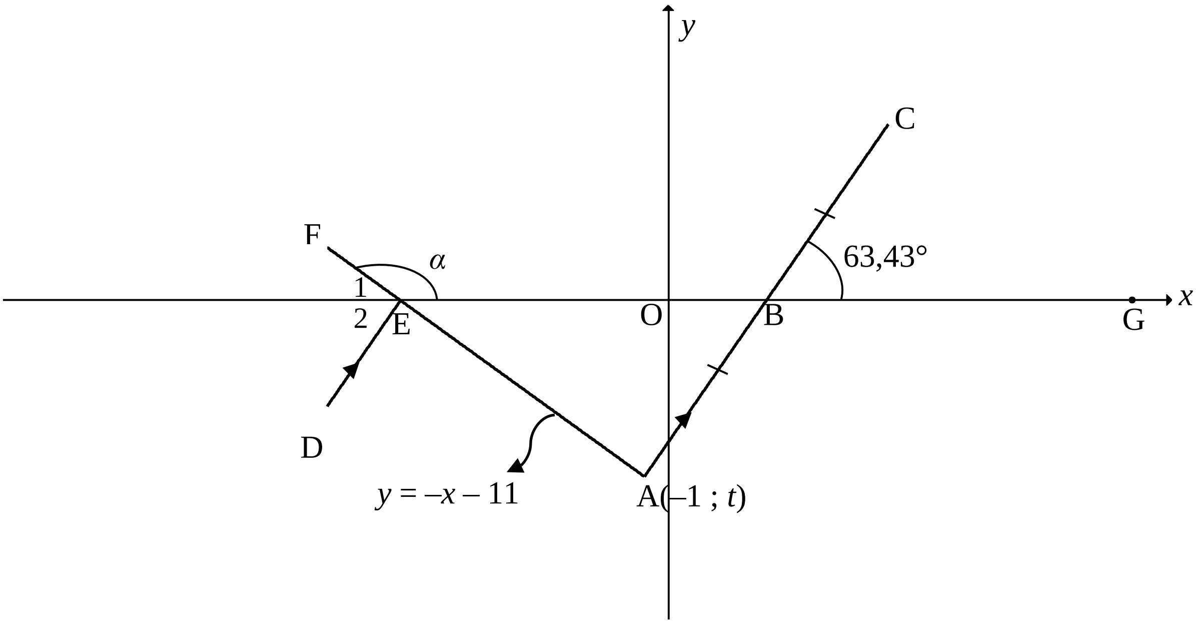
QUESTION/*VRAAG 2*

2.1	$35 < x \leq 45$	✓ answer (1)
2.2	320 people/*mense*	✓ answer (1)
2.3	see table and ogive below	✓ cumulative frequency ✓ grounding ✓ plotting at upper limit ✓ shape (4)
2.4	Median = 41	✓✓ answer (2)
		[8]

AGE	NUMBER OF PEOPLE	CUMULATIVE FREQUENCY
$5 < x \leq 15$	20	20
$15 < x \leq 25$	25	45
$25 < x \leq 35$	60	105
$35 < x \leq 45$	90	195
$45 < x \leq 55$	55	250
$55 < x \leq 65$	40	290
$65 < x \leq 75$	30	320



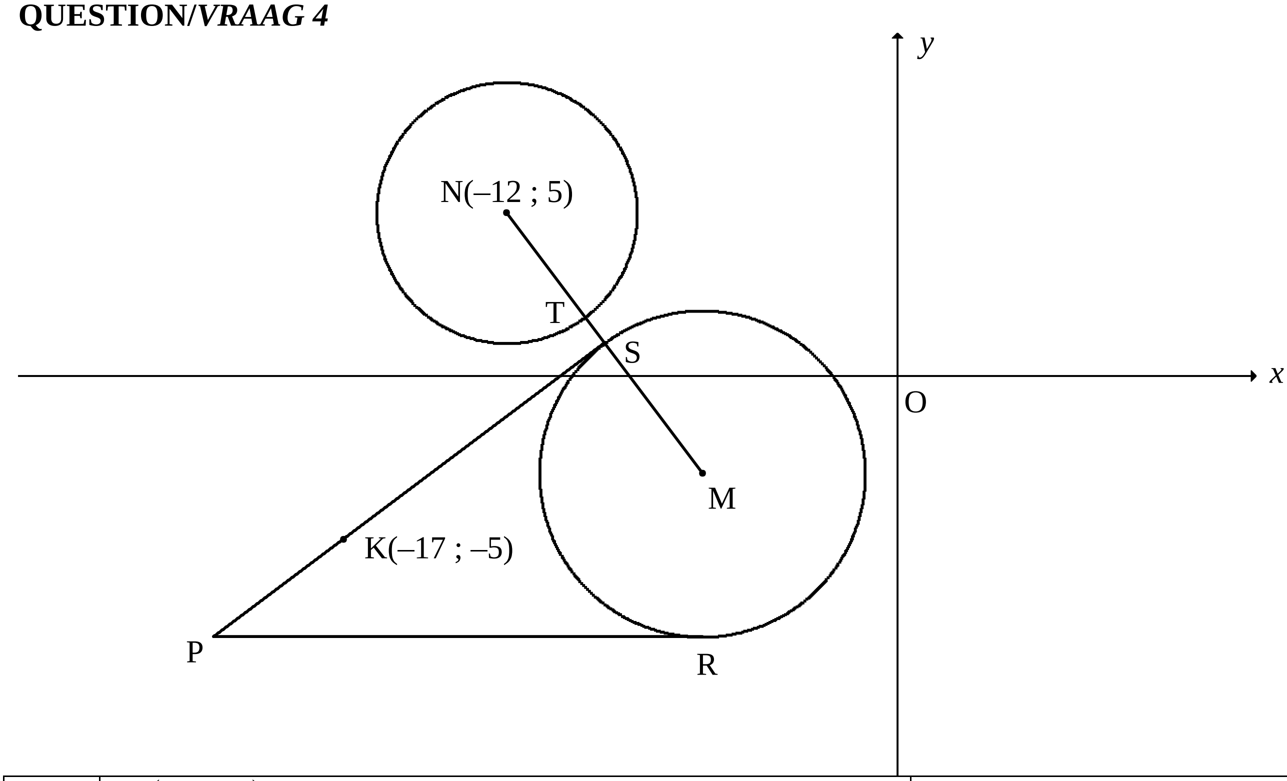
QUESTION/*VRAAG* 3



3.1.1	$y=-x-11$ A(–1 ; t) $t=-(-1)-11$ $t=-10$	✓ substitution ✓ value of t (2)
3.1.2	$\tan\alpha=-1$ $ref.\angle=45°$ $\therefore\alpha=135°$	✓ $\tan\alpha=-1$ ✓ 135° (2)
3.1.3	$\tan 63{,}43°=m_{AC}$ $m_{AC}=2$	✓ $\tan 63{,}43°=m_{AC}$ ✓ answer (2)
3.2	$m_{AC}=2$ A(–1 ; –10) $y=2x+k$ **OR/*OF*** $y-y_1=2(x-x_1)$ $-10=2(-1)+k$ $\quad$ $y-(-10)=2(x-(-1))$ $k=-8$ $\quad$ $y=2x-8$ $y=2x-8$	✓ substitution of m and A ✓ equation (2)

3.3.1	$y=2x-8$ $0=2x-8$ $x_B=4$ $\frac{x_C+(-1)}{2}=4$ $\quad$ $\frac{y_C+(-10)}{2}=0$ $x_C=9$ $\quad$ $y_C=10$ **OR/*OF*** by translation / *met translasie* $A\rightarrow B\ (x;y)\rightarrow(x+5;y+10)$ $B\rightarrow C\ (4;0)\rightarrow(4+5;\ 0+10)=(9\ ;10)$	✓ $x_B=4$ ✓ $x_C=9$ ✓ $y_C=10$ (3) ✓ $(x+5;y+10)$ ✓ $x_C=9$ ✓ $y_C=10$ (3)
3.3.2	$A\hat{B}E=63,43°$ [vert. opp $\angle$'s =] $\hat{E}_2=63,43°$ [corres. $\angle$'s, DE || AB] $\hat{E}_1=45°$ [$\angle$s on a str line] $F\hat{E}D=108,43°$ **OR/*OF*** $E\hat{A}B=135°-63,43°$ $E\hat{A}B=71,57°$ $D\hat{E}A=E\hat{A}B=71,57°$ $F\hat{E}D=108,43°$ **OR/*OF*** $A\hat{B}E=63,43°$ [vert. opp $\angle$'s] $D\hat{E}O=116,57°$ [co-int. $\angle$'s, DE || AB] $F\hat{E}D=360°-(116,57°+135°)$ $=108,43°$	✓ $A\hat{B}E=63,43°$ ✓ $\hat{E}_1=45°$ ✓ $F\hat{E}D=108,43°$ (3) ✓ $E\hat{A}B=71,57°$ ✓ $D\hat{E}A=E\hat{A}B=71,57°$ ✓ $F\hat{E}D=108,43°$ (3) ✓ $A\hat{B}E=63,43°$ ✓ $D\hat{E}O=116,57°$ ✓ $F\hat{E}D=108,43°$ (3)
3.4	$y=0$ $x_E=-11$ $\frac{x_G+(-11)}{2}=4$ $x_G=19$ $(x-19)^2+y^2=15^2$ $(x-19)^2+y^2=225$	✓ $x_E=-11$ ✓ $x_G=19$ ✓ $(x-19)^2+y^2$ ✓ 225 (4)
		[18]

QUESTION/*VRAAG 4*

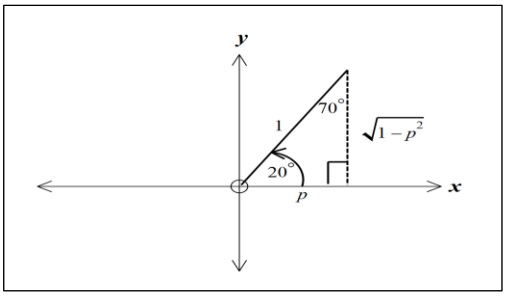


4.1	$M(-6;-3)$	✓ –6 ✓ –3 (2)
4.2.1	$x^2+y^2+24x-10y+153=0$ $(x+12)^2+(y-5)^2=-153+144+25$ $(x+12)^2+(y-5)^2=16$ $r^2 = 16$ $r = 4$ units	✓ $r^2=-153+144+25$ ✓ length of radius (2)
4.2.2	$NM=\sqrt{(-12-(-6))^2+(5-(-3))^2}$ NM = 10 units SM = 5 units $\therefore$TS = 10 – 5 – 4 = 1 unit	✓ substitution into distance formula ✓ NM = 10 units ✓ SM = 5 units ✓ answer (4)
4.3.1	R(–6 ; –8) $y = -8$	✓ $y_R=-8$ ✓ answer (2)

4.3.2	$m_{NM}=\frac{5-(-3)}{-12-(-6)}$ $m_{NM}=-\frac{4}{3}$ $m_{tangent}=\frac{3}{4}$ $-5=\frac{3}{4}(-17)+c$ **OR/*OF*** $y-y_1=\frac{3}{4}(x-x_1)$ $c=\frac{31}{4}$ $y-(-5)=\frac{3}{4}(x-(-17))$ $y=\frac{3}{4}x+\frac{31}{4}$ $y=\frac{3}{4}x+\frac{31}{4}$	✓ substitution ✓ $m_{NM}=-\frac{4}{3}$ ✓ $m_{tangent}=\frac{3}{4}$ ✓ substitution of *m* and N ✓ equation (5)
	OR/OF NS = SM = 5 $S\left(\frac{-12-6}{2}\ ;\ \frac{5-3}{2}\right)$ S (– 9 ; 1) $m_{SK}=\frac{1-(-5)}{-9+17}$ $=\frac{6}{8}=\frac{3}{4}$ $y+5=\frac{3}{4}(x+17)$ $y=\frac{3}{4}x+\frac{31}{4}$ *or* $y=\frac{3}{4}x+7\frac{3}{4}$	✓ S midpoint ✓ coordinates of S ✓ $m_{tangent}=\frac{3}{4}$ ✓ substitution of *m* and K(–17 ; –5) or S ✓ equation (5)
4.4.1	$-8=\frac{3}{4}x+\frac{31}{4}$ $-32=3x+31$ $3x=-63$ $x=-21$ P(–21 ; –8) R(–6 ; –8) PR = PS = 15 units [tangents from same point] MS = MR = 5 units Perimeter PSMR = 15 + 15 + 5 + 5 = 40 units	✓ $-8=\frac{3}{4}x+\frac{31}{4}$ ✓ $x=-21$ ✓ PR = PS = 15 units ✓ MS = MR = 5 units ✓ answer (5)

4.4.2	$\dfrac{\text{area of } \Delta\text{NPS}}{\text{area of quadrilateral PSMR}}$ $\dfrac{\frac{1}{2}\text{NS.SP}}{\frac{1}{2}\text{SP.MS}+\frac{1}{2}\text{MR.PR}}$ $=\dfrac{\frac{1}{2}(5)(15)}{2\left(\frac{1}{2}\right)(5)(15)}$ $=\dfrac{1}{2}$ **OR** $\Delta\text{NPS} \equiv \Delta\text{SPM} \equiv \Delta\text{MPR}$ $\dfrac{\text{area of } \Delta\text{NPS}}{\text{area of quadrilateral PSMR}}$ $=\dfrac{1}{2}$	✓ substitution ✓ answer (2) ✓ congruent ✓ answer (2)
		[22]

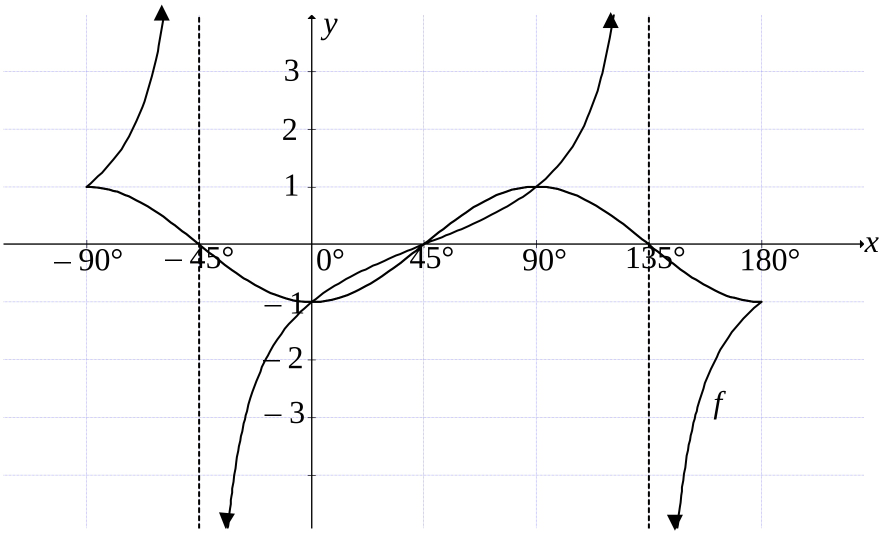
QUESTION/*VRAAG* 5

5.1	$\frac{1-\sin(-\theta)\cos(90°+\theta)}{\cos(\theta-360°)}$ $=\frac{1-(-\sin\theta)(-\sin\theta)}{\cos\theta}$ $=\frac{1-\sin^2\theta}{\cos\theta}$ $=\frac{\cos^2\theta}{\cos\theta}$ $=\cos\theta$	✓ $-\sin\theta$ ✓ $-\sin\theta$ ✓ $\cos\theta$ ✓ $\cos^2\theta$ ✓ answer (5)
5.2.1	$\cos 200°$ $=-\cos 20°$ $=-p$	✓ reduction ✓ answer (2)
5.2.2	$\sin(-70°)$ $=-\sin 70°$ $=-\cos 20°$ $=-p$ **OR/*OF*** $\sin(-70°)$ $=-\sin 70°$ $=-p$  	✓ reduction ✓ answer (2) ✓ reduction ✓ answer (2)
5.2.3	$\sin 10°$ $\cos(2(10°))=1-2\sin^2 10°$ $2\sin^2 10°=1-\cos 20°$ $\sin 10°=\sqrt{\frac{1-\cos 20°}{2}}$ $\sin 10°=\sqrt{\frac{1-p}{2}}$ **OR/*OF*** $\sin 10°$ $\sin(30°-20°)$ $=\sin 30°\cos 20°-\cos 30°\sin 20°$ $=\frac{1}{2}p-\frac{\sqrt{3}}{2}\sqrt{1-p^2}=\frac{p-\sqrt{3}\sqrt{1-p^2}}{2}$ **OR/*OF***	✓ double angle ✓ $\sin 10°$ as subject ✓ answer (3) ✓ using special angle ✓ expanding ✓ answer (3)

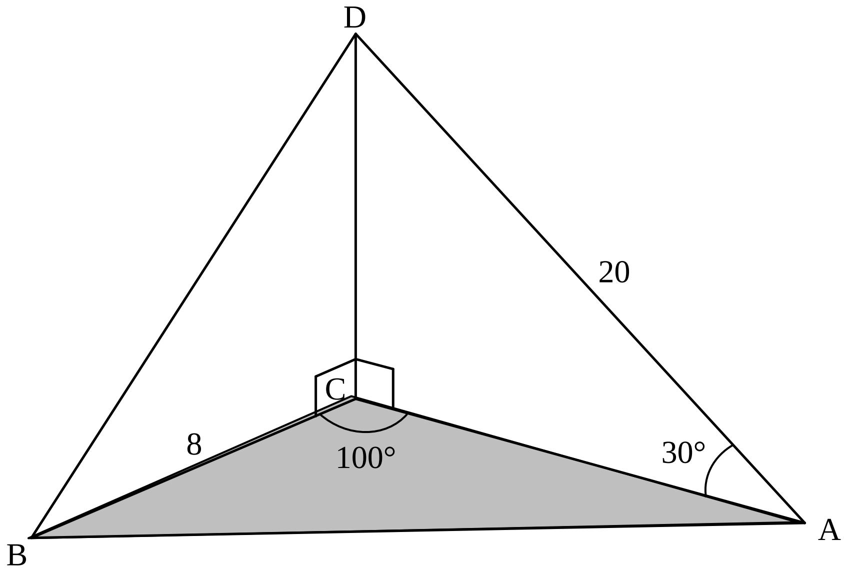
	$\sin 10°$ $\sin(70° - 60°)$ $= \sin 70° \cos 60° - \cos 70° \sin 60°$ $= p.\frac{1}{2} - \sqrt{1-p^2} \times \frac{\sqrt{3}}{2} = \frac{p - \sqrt{3}\sqrt{1-p^2}}{2}$	✓ using special angle ✓ expanding ✓ answer (3)
	OR/*OF*	
	$\sin 10°$ $= \cos 80°$ $\cos(60° + 20°)$ $= \cos 60° \cos 20° - \sin 60° \sin 20°$ $= \frac{1}{2}p - \frac{\sqrt{3}}{2}.\sqrt{1-p^2}$	✓ using special angle ✓ expanding ✓ answer (3)
5.3	$\cos(A + 55°)\cos(A + 10°) + \sin(A + 55°)\sin(A + 10°)$ $= \cos[A + 55° - (A + 10°)]$ $= \cos 45°$ $= \frac{1}{\sqrt{2}}$ or $\frac{\sqrt{2}}{2}$	✓✓ compound identity ✓ answer (3)
5.4.1	LHS $= \frac{\cos 2x + \sin 2x - \cos^2 x}{\sin x - 2\cos x}$ RHS = $-\sin x$ $= \frac{\cos^2 x - \sin^2 x + 2\sin x \cos x - \cos^2 x}{\sin x - 2\cos x}$ $= \frac{-\sin^2 x + 2\sin x \cos x}{\sin x - 2\cos x}$ $= \frac{-\sin x(\sin x - 2\cos x)}{\sin x - 2\cos x}$ $= -\sin x$ $\therefore$ LHS = RHS	✓ $\cos^2 x - \sin^2 x$ ✓ $2\sin x \cos x$ ✓ common factor of $-\sin x$ (3)
5.4.2	$\frac{\cos 2x + \sin 2x - \cos^2 x}{-3\sin^2 x + 6\sin x \cos x}$ $= \frac{\cos 2x + \sin 2x - \cos^2 x}{-3\sin x(\sin x - 2\cos x)}$ $= \frac{\cos 2x + \sin 2x - \cos^2 x}{(\sin x - 2\cos x)} \times \frac{1}{-3\sin x}$ $= (-\sin x) \times \frac{1}{-3\sin x}$ $= \frac{1}{3}$	✓ common factor of $-3\sin x$ ✓ substitution ✓ answer (3)

5.5.1	$3\tan 4x = -2\cos 4x$ $3\left(\dfrac{\sin 4x}{\cos 4x}\right) = -2\cos 4x$ $3\sin 4x + 2\cos^2 4x = 0$ $3\sin 4x + 2(1-\sin^2 4x) = 0$ $-2\sin^2 4x + 3\sin 4x + 2 = 0$ $2\sin^2 4x - 3\sin 4x - 2 = 0$ $(2\sin 4x + 1)(\sin 4x - 2) = 0$ $\sin 4x = -\dfrac{1}{2}$ or $\sin 4x \neq 2$	✓ identity ✓ $1-\sin^2 4x$ ✓ standard form ✓ factors (4)
5.5.2	$\sin 4x = -\dfrac{1}{2}$ *ref.* $\angle = 30°$ $4x = 210° + k.360°$ or $4x = 330° + k.360°$ $x = 52{,}5° + k.90°$; $k \in Z$ $x = 82{,}5° + k.90°$; $k \in Z$	✓ 210° ; 330° ✓ 52,5° ; 82,5° ✓ $k.90°$; $k \in Z$ (3)
		[28]

QUESTION/VRAAG 6

6.1	Period = 180°	✓ answer (1)
6.2		✓ x-intercepts ✓ turning points ✓ end points (3)
6.3	$y \in [-1;1]$ **OR/*OF*** $-1 \le y \le 1$	✓ answer (1)
6.4	$g(x) = -\cos 2x$ $g(x+45°) = -\cos 2(x+45°)$ $= -\cos(2x+90°)$ $= \sin 2x$	✓ $-\cos 2(x+45°)$ ✓ answer (2)
6.5.1	$x \in (-90°;-45°)$ **OR/*OF*** $-90° < x < -45°$	✓✓ $x \in (-90°;-45°)$ (2)
6.5.2	$2\cos 2x - 1 > 0$ $\cos 2x > \frac{1}{2}$ $-\cos 2x < -\frac{1}{2}$ $x \in (-30°;30°)$ **OR/*OF*** $-30° < x < 30°$	✓ $\cos 2x > \frac{1}{2}$ ✓ $-\cos 2x < -\frac{1}{2}$ ✓ $x = \pm 30°$ ✓ interval (4)
		[13]

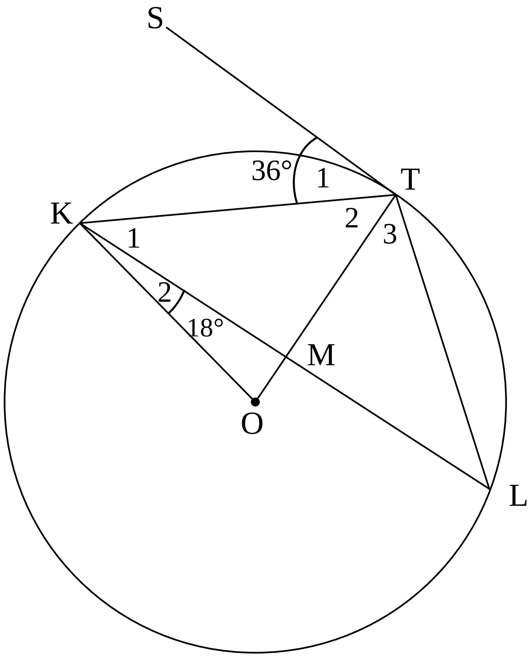
QUESTION/*VRAAG* 7



7.1.1	$\frac{AC}{20} = \cos 30°$ $AC = 20\cos 30°$ $AC = 10\sqrt{3} = 17{,}32$ units **OR/*OF*** $\frac{AC}{\sin 60°} = \frac{20}{\sin 90°}$ $\therefore AC = 20\sin 60 = 17{,}32$	✓ trig ratio ✓ answer (2) ✓ trig ratio ✓ answer (2)
7.1.2	$AB^2 = AC^2 + BC^2 - 2AC.BC\cos A\hat{C}B$ $AB^2 = \left(10\sqrt{3}\right)^2 + 8^2 - 2\left(10\sqrt{3}\right)(8)\cos 100°$ $AB = 20{,}30$ units	✓ cosine formula ✓ substitution into cosine formula ✓ answer (3)
7.2	$\frac{\sin A\hat{D}B}{AB} = \frac{\sin A\hat{B}D}{AD}$ $\frac{\sin A\hat{D}B}{20{,}3} = \frac{\sin 73{,}4°}{20}$ $\sin A\hat{D}B = \frac{20{,}3\sin 73{,}4°}{20}$ $A\hat{D}B = 76{,}58°$	✓ sine formula in ΔABD ✓ substitution into sine formula ✓ answer (3)
		[8]

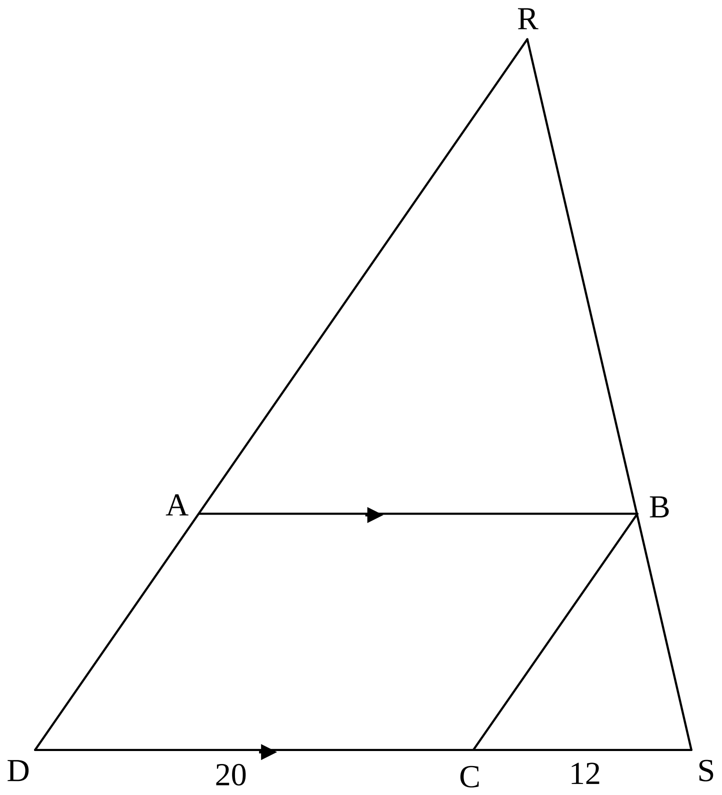
QUESTION/*VRAAG* 8

8.1



8.1.1(a)	$\hat{T}_2 = 54°$ [tan ⊥ rad]	✓ S ✓R (2)
8.1.1(b)	$\hat{L} = 36°$ [tan - chord theorem]	✓ S ✓R (2)
8.1.1(c)	$K\hat{O}T = 72°$ [∠ at centre = 2×∠ at circumference] **OR/*OF*** $O\hat{K}T = \hat{T}_2 = 54°$ [∠s opposite = radii] $K\hat{O}T = 180° - (54° + 54°)$ [sum of int ∠'s of Δ] $= 72°$	✓ S ✓R (2) ✓ S/R ✓ S (2)
8.1.2	$K\hat{M}O = 180° - (18° + 72°)$ $= 90°$ [sum of int ∠'s of Δ] $\therefore$ KM = ML [line from centre ⊥ to chord] **OR/*OF*** $O\hat{K}T = 54°$ [∠s opposite = radii] $\hat{K}_1 = 54° - 18° = 36°$ $T\hat{M}K = 90°$ [sum of int ∠'s of Δ] $\therefore$ KM = ML [line from centre ⊥ to chord]	✓ S ✓ S ✓ R (3) ✓ S ✓ S ✓ R (3)

8.2

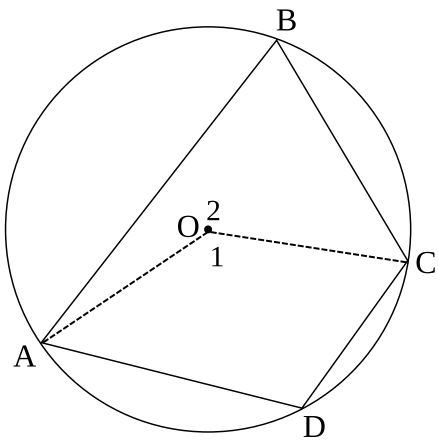


8.2.1	$\frac{DC}{CS}=\frac{20}{12}=\frac{5}{3}$ $\therefore \frac{DC}{CS}=\frac{RB}{BS}$ $\therefore$ BC || DR [converse line || one side of Δ OR sides in the $\therefore$ BC || AD same proportion]	✓ S ✓ S ✓ R (3)
8.2.2	$\frac{AR}{AD}=\frac{RB}{BS}$ [line || one side of Δ] **OR**[Prop Theorem AB || DS] $\frac{AR}{AD}=\frac{5}{3}$ $\frac{48-AD}{AD}=\frac{5}{3}$ $\therefore$ 5AD = 144 − 3AD AD = 18 AB = 20 [opp sides of parm] $\therefore$ AD : AB = 18 : 20 = 9 : 10	✓ $\frac{AR}{AD}=\frac{5}{3}$ ✓ AD = 18 ✓ ratio (3)

	OR/OF $\frac{AR}{RD}=\frac{5}{8}$.............prop thm $AB \parallel DS$ $\frac{AR}{48}=\frac{5}{8}$ $\therefore AR=30$ and $AD=18$ $\therefore \frac{AR}{RD}=\frac{AB}{DS}$........$\vert\vert\vert\ \Delta's$ $\therefore AB=20$ $\therefore AB:AD=18:20=9:10$	✓ $\frac{AR}{RD}=\frac{5}{8}$ ✓ AD = 18 ✓ ratio (3)
		[15]

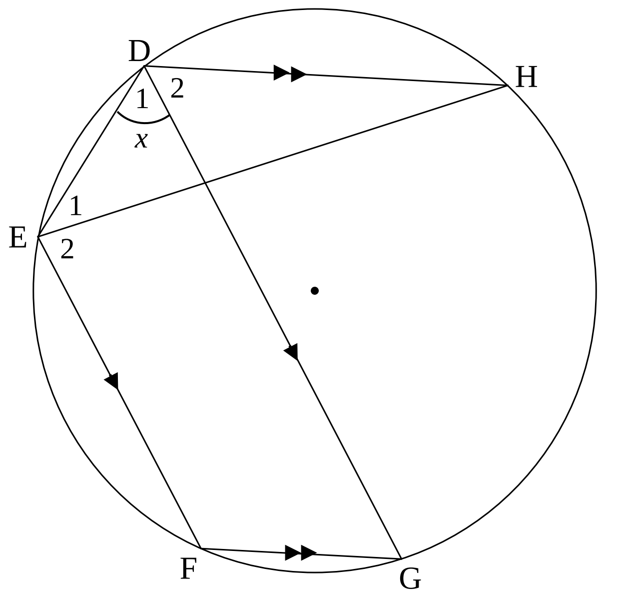
QUESTION/*VRAAG 9*

9.1



9.1	Constr: Draw radii OA and OC. Proof: $\hat{O}_1 = 2\hat{B}$ [$\angle$ at centre $= 2\times\angle$ at circumference] $\hat{O}_2 = 2\hat{D}$ [$\angle$ at centre $= 2\times\angle$ at circumference] $\hat{O}_1 + \hat{O}_2 = 360°$ [revolution] $2\hat{B} + 2\hat{D} = 360°$ [revolution] $\therefore \hat{B} + \hat{D} = 180°$	✓ Construction ✓ S ✓ R ✓ S/R ✓ S (5)

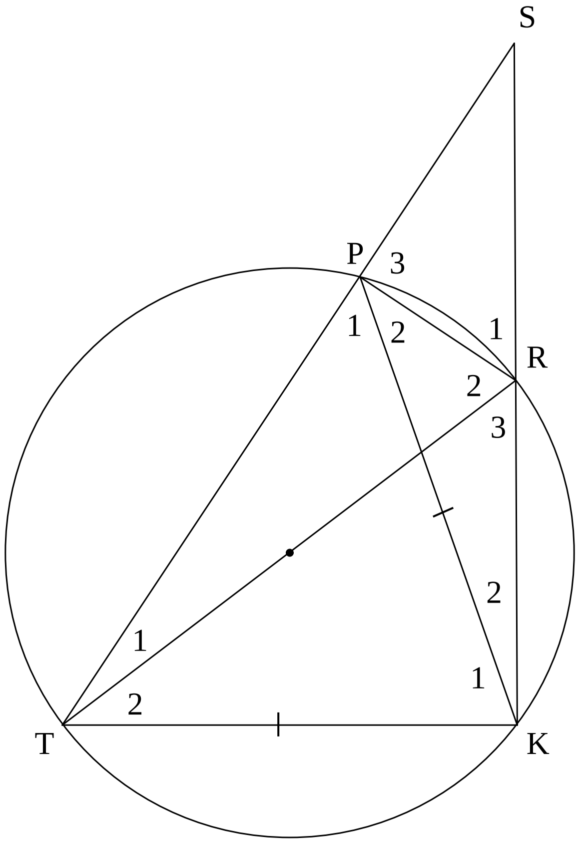
9.2



9.2	$E\hat{F}G = 180° - \hat{D}_1$ [opp ∠'s of cyclic quad] $\therefore E\hat{F}G = 180° - x$ $E\hat{F}G = 180° - \hat{G}$ [co-int ∠'s; EF ∥ DG] $\hat{G} = x$ But $\hat{G} = \hat{D}_2$ [alt ∠'s; DH ∥ FG] $\therefore \hat{D}_1 = \hat{D}_2 = x$	✓S ✓ R ✓S / R ✓ S / R (4)
		[9]

QUESTION/VRAAG 10

10.1



10.1.1	$T\hat{P}R = 90°$ [$\angle$ in semi-circle] $S\hat{P}R = 90°$ [$\angle$'s on a straight line] $\therefore$ SR is a diameter [converse $\angle$ in semi-circle] **OR** $T\hat{K}R = 90°$ [$\angle$ in semi-circle] $S\hat{P}R = 90°$ [ext $\angle$ of cyclic quad] $\therefore$ SR is a diameter [converse $\angle$ in semi-circle] **OR** [chord subtends a right angle]	✓S ✓R ✓S ✓R (4) ✓S ✓R ✓S ✓R (4)

10.1.2	$\hat{R}_1 = P\hat{T}K$ [ext $\angle$ of cyclic quad] $\hat{P}_1 = P\hat{T}K = \hat{R}_1$ [$\angle$s opp equal sides] $\hat{S} + \hat{R}_1 = \hat{P}_1 + P_2$ [ext $\angle$ of Δ] $\therefore\ \hat{S} = \hat{P}_2$ [$\hat{R}_1 = \hat{P}_1$]	✓S ✓R ✓S /R ✓S ✓R (5)
10.1.3	In ΔSPK and ΔPRK $\hat{S} = \hat{P}_2$ [proved] $\hat{K}_2 = \hat{K}_2$ [common] ΔSPK ||| ΔPRK [$\angle, \angle, \angle$] **OR/*OF*** In ΔSPK and ΔPRK $\hat{S} = \hat{P}_2$ [proved] $\hat{K}_2 = \hat{K}_2$ [common] $S\hat{P}K = P\hat{R}K$ [sum of $\angle$s in Δ] ΔSPK ||| ΔPRK	✓S ✓S ✓S/R (3) ✓S ✓S ✓S/R (3)
10.2	$\frac{PK}{RK} = \frac{SK}{PK}$ [ΔSPK ||| ΔPRK] $PK^2 = SK.RK$ $ST^2 = SK^2 + TK^2$ [Pythagoras] TK = PK [Given] $ST^2 = SK^2 + PK^2$ $ST^2 = SK^2 + SK.RK$ $ST^2 = (2RK)^2 + 2RK.RK$ $ST^2 = 6RK^2$ $ST = \sqrt{6}RK$	✓S ✓S ✓ $PK^2 = SK.RK$ ✓SK = 2RK ✓ $ST^2 = 6RK^2$ (5)
		[17]

TOTAL/TOTAAL: 150