# Bachelor's Thesis Presentation

## The Influence of the Investment Horizon on the Asset Allocation

Joël Meili & Fabio Bührer

Zurich University of Applied Sciences Winterthur

July 1st 2019

# Contents

# Goals of the Research

- Understand the role of the investment horizon in portfolio allocation
- Establishing a risk measure
- Building a model for the evolution of assets' value
- Modeling fat tails of financial assets

Section 1

# Aspects of Portfolio Optimization

# Risk Measures

## Standard deviation

- Risk quantified by movements from the mean (volatility), no level of confidence applicable
- Does not capture skewness or kurtosis (fat tails), but is coherent

## Value-at-risk

- Intuitive indication of potential losses, level of confidence applicable
- Does capture skewness and kurtosis (fat tails), but is not coherent

## Expected shortfall

- Intuitive indication of potential losses, level of confidence applicable
- Does capture skewness and kurtosis (fat tails) and is coherent

## Our solution in this thesis

- Use value-at-risk or expected shortfall as risk measures

# Mean-Variance Analysis

## What is mean-variance analysis

- Optimization of weights to find efficient frontier (lowest portfolio standard deviation for given portfolio return)

## Advantages

- Easy computation
- Explicit results (weights)

## Disadvantages

- Drawbacks of standard deviation as a risk measure
- No consideration of desired investment horizon

# Fat Tails in Financial Returns: Basics

- In fat-tailed data, extreme movements are relatively frequent (compared to thin-tailed distribution)
- Thin-tailed distributions decay like $O(e^{-\lambda|x|}), \lambda \in \mathbb{R}_{>0}$
- Fat-tailed distributions decay like $O(|x|^{-\alpha}), \alpha \in \mathbb{R}_{>0}$
- Thin-tailed or fat-tailed model: choice can make huge difference in portfolio optimization

### Heavy-tailed

- Distributions are called "heavy-tailed" or "fat-tailed" when the tails decay like a power law $O(|x|^{-\alpha}), \alpha \in \mathbb{R}_{>0}$

### Our solution in this thesis

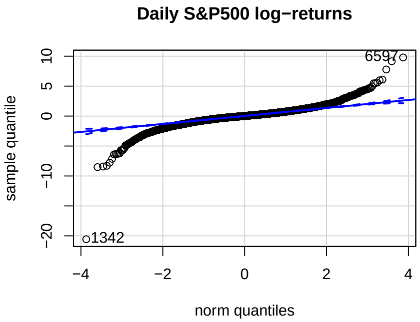
- Use of the Student's t-distribution and generalized hyperbolic distributions

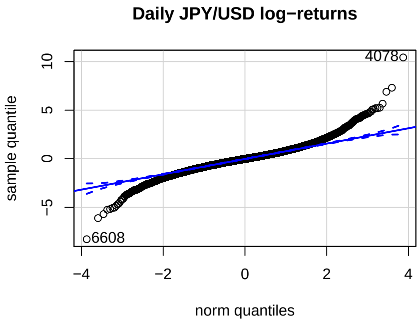
Section 2

# Problems with the Normality Assumption

# Kurtosis and Skewness of Log-Returns

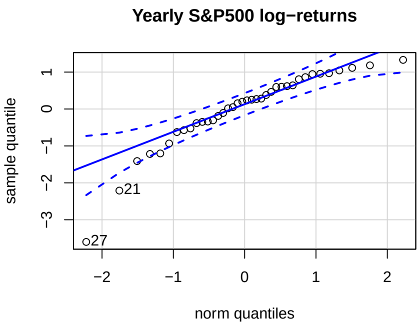
- Log-returns are often heavy tailed (kurtosis) especially for short-term horizons
- Stocks are usually negatively skewed and safe haven instruments e.g. JPY/USD are usually positively skewed

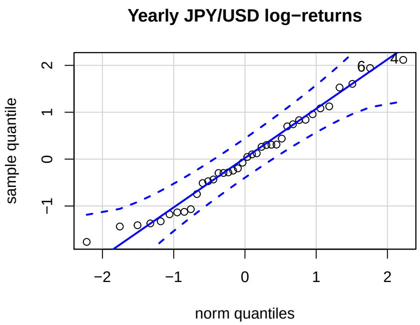


# Kurtosis and Skewness of Log-Returns

- Effect of kurtosis is reduced significantly when the horizon is increased (aggregated log-returns)
- Assumption of normally distributed log-returns seems appropriate



# Tail Correlation (Gaussian vs. Student's t Copula)

- Multivariate log-return modelling implies tail correlation
- Tail correlation incorporates nonlinear dependencies that cannot be modeled solely with the covariance matrix

## Bivariate Gaussian Probability Density

$$f(\vec{x}) = \frac{1}{2\pi} \det \Sigma^{-\frac{1}{2}} e^{-\frac{1}{2}(\vec{x}-\vec{\mu})'\Sigma^{-1}(\vec{x}-\vec{\mu})}$$

Where $\mu$ is the location vector and $\Sigma$ is the covariance matrix.

## Bivariate Student's t Probability Density

$$f(\vec{x}) = \frac{\Gamma\frac{\nu+2}{2}}{\Gamma\frac{\nu}{2}\nu\pi|\Sigma|^{\frac{1}{2}}}[1 + \frac{1}{\nu}(\vec{x} - \vec{\mu})^T\Sigma^{-1}(\vec{x} - \vec{\mu})]^{-\frac{(\nu+2)}{2}}$$

Where $\Gamma$ is the gamma function and $\nu$ is the degrees of freedom parameter.

# Tail Correlation (Gaussian vs. Student's t Copula)

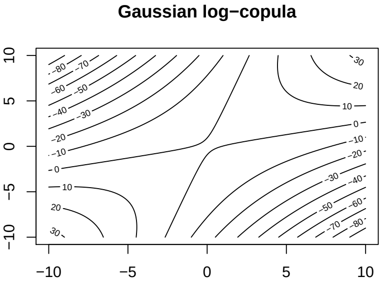
- The probability density function of a copula is defined by the Sklar's theorem
- The Gaussian copula diverges only in two of the corners whereas the Student's t copula diverges in all four corners

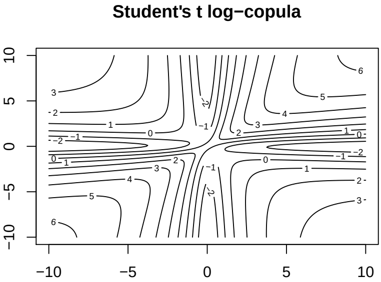
**Sklar's Theorem**

$$c(\vec{x}) = \frac{f(\vec{x})}{\overline{f}(x_1)\overline{f}(x_2)}$$

Where $\overline{f}(x)$ is the univariate probability density function.

# Tail Correlation (Gaussian vs. Student's t Copula)



Section 3

# Parameter Fitting of Financial Log-Returns

# Univariate Symmetric Student's t-Distribution

- Fitted univariate symmetric Student's t-distributions to financial assets using a likelihood maximization algorithm described in the GHYP-package (https://CRAN.R-project.org/package=ghyp)
- Bootstrapped N = 100 samples to evaluate confidence interval for estimates of $\nu$

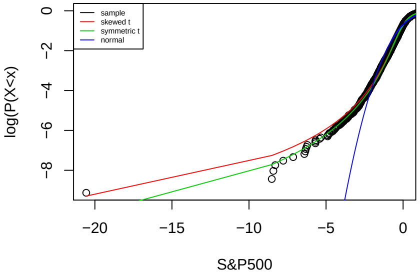
| | $\nu_{DIJA}$ | $\nu_{S\&P500}$ | $\nu_{10YUSBondFutures}$ | $\nu_{JPY/USD}$ | $\nu_{GOLD}$ |
|---|---|---|---|---|---|
| $\overline{\mu} - \overline{\sigma}$ | 2.962 | 2.780 | 3.903 | 3.904 | 2.903 |
| $\overline{\mu}$ | 3.078 | 2.885 | 4.068 | 4.085 | 3.015 |
| $\overline{\mu} + \overline{\sigma}$ | 3.194 | 2.989 | 4.233 | 4.265 | 3.127 |

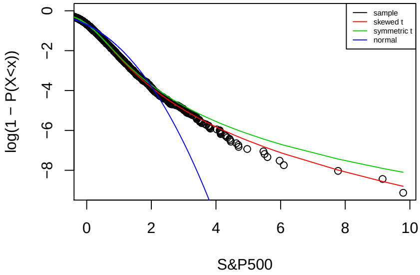
# Symmetric vs. Skewed Student's t-Distribution

- Both variants have fat-tailed distributions
- Symmetric Student's t-distribution does not allow intrinsic skewness of some instruments
- Skewed Student's t-distribution is a generalized version of symmetric distribution
- Skewed Student's t-distribution's skewness parameter dictates degree of asymmetry
- Negative skewness: left tail is fatter (risky assets)
- Positive skewness: right tail is fatter (hedge assets)

# Graphical Assessment of Goodness of Fit for Univariate Skewed Student's t-Distribution

- Skewed Student's t-distribution captures extreme events in the tails better than symmetric Student's t-distribution
- Graphical assessment provides further motivation for the use of the skewed Student's t-distribution



# Multivariate Skewed Student's t-Distribution

- Fitted a multivariate skewed Student's t-distribution to financial assets using an expectation maximization algorithm described in the GHYP-package (https://CRAN.R-project.org/package=ghyp)
- Again bootstrapped N = 100 samples to evaluate confidence interval for the parameter estimates

| | $\nu$ | $\gamma_{DIJA}$ | $\gamma_{S\&P500}$ | $\gamma_{10YUSBondFutures}$ | $\gamma_{JPY/USD}$ | $\gamma_{GOLD}$ |
|---|---|---|---|---|---|---|
| $\overline{\mu} - \overline{\sigma}$ | 4.073 | -0.082 | -0.094 | -0.038 | 0.051 | -0.042 |
| $\overline{\mu}$ | 4.180 | -0.065 | -0.077 | -0.021 | 0.070 | -0.025 |
| $\overline{\mu} + \overline{\sigma}$ | 4.287 | -0.048 | -0.059 | -0.004 | 0.090 | -0.008 |

Section 4

# Results

# Portfolio Optimiztion Using Normal and Non-Normal Distributions

- Optimal portfolio weights are the same for normal and non-normal distributions as long as standard deviation is used as risk measure (same variance for distributions), independent of horizon
- Optimal portfolio weights are the same for standard deviation, value-at-risk or expected shortfall optimized portfolio when normal distributions are used, independent of horizon
- Different portfolio weights when non-normal distributions and value-at-risk or expected shortfall are used as risk measure, dependent of horizon

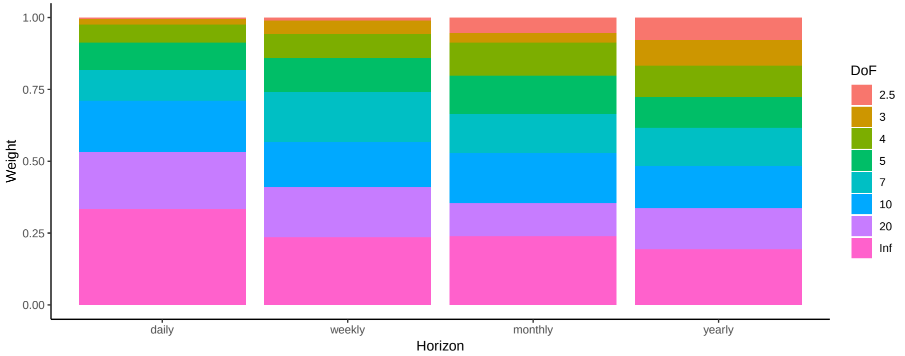
# Monte Carlo Simulation of Univariate Symmetric Student's t-Distributed Log-Returns

- Value-at-risk optimization for simulated univariate symmetric Student's t-distributed log-returns
- Used aggregated log-returns and adjusted value-at-risk level $(\alpha = 1 - \frac{\tau}{26000})$

**Relationship between $\tau$ and $\alpha$**

- $\alpha$ is chosen to cover losses that occur once in a century for each $\tau$, such that results are comparable

# Monte Carlo Simulation of Univariate Symmetric Student's t-distributed Log-Returns



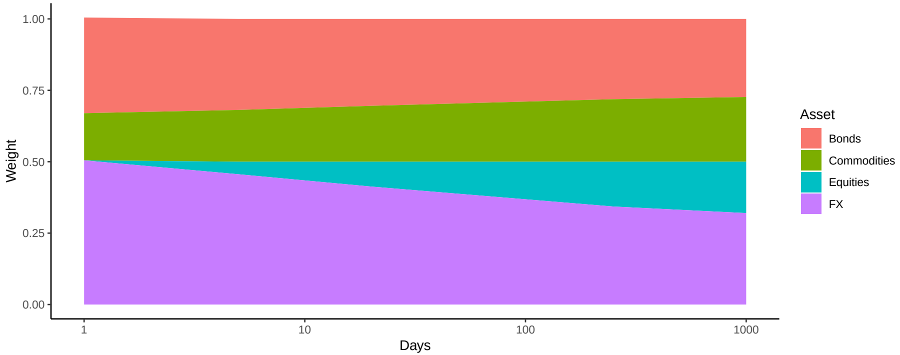
# Portfolio Optimization Using the Multivariate Skewed Student's t-Distribution

- The assets are described by a multivariate skewed Student's t-distribution with 3.5 degrees of freedom
- Expected shortfall of the portfolio is optimized at various confidence levels using the concept of linearity of the marginal distributions, the same relationship between $\tau$ and $\alpha$ persists

### Conservative assumption

- Number of degrees of freedom does not increase as horizon is increased

# Portfolio Optimization Using the Multivariate Skewed Student's t-Distribution



# Conclusion

- Assumption of normally distributed log-returns is invalid, especially for short-term investments
- Kurtosis and skewness are relevant quantities in portfolio optimization
- Optimal portfolio weights are dependent of the horizon when non-normal distributions and value-at-risk or expected shortfall are used for optimization
- The skewed Student's t-distribution is a good proxy for modelling log-returns
- Long-term investors can and should take more tail risks than short-term investors

# Discussion

- Q&A
- Suggestions
- Feedback