The generalized autoregressive conditional heteroskedasticity (GARCH) statistical model is used for modeling time series, in particular to model data where the error terms $\epsilon\_t$, e.g. the difference between the logarithms of the $t$-th observation and the previous one $\ln(x\_t) - \ln(x\_{t-1})$, are affected by alternating periods of lower and higher variance, the latter known in finance as volatility clusters. GARCH models are generalized autoregressive methods because they generate forecasts for the instantaneous error variance $\sigma^{2}\_{t}$ based on an autoregressive moving average (ARMA) process, as opposed to the autoregressive (AR) process used in simpler ARCH models; conditional heteroskedasticity means that $\sigma^{2}\_{t}$ depends on past data, allowing it to change over time.

\*\*Mathematical Definition\*\*:

A GARCH($n$, $m$) model for $\epsilon\_t$ has:

$$\epsilon\_t = \sigma\_t u\_t$$

$$\sigma^{2}\_{t} = c \sigma^{2} + \sum\_{j=1}^{n} \alpha\_j \sigma^{2}\_{t-j} + \sum\_{k=1}^{m} \beta\_k \epsilon^{2}\_{t-k}$$

Where $\epsilon\_t$ is the $t$-th error term, $u\_t$ is a white noise process with independent and identically distributed random variables having $\mathrm{E}(u\_t) = 0$ and $\mathrm{Var}(u\_t) = 1$, $\sigma^{2}\_{t}$ is the conditional variance $\mathrm{Var}(\epsilon\_t |\sigma\_{t-1}, \dots, \sigma\_{t-n}, \epsilon\_{t-1}, \dots, \epsilon\_{t-m})$, $c$ is the fixed weight of the unconditional variance $\sigma^{2} = \mathrm{Var}(\epsilon\_t)$, $n$ and $m$ are the specified lag lengths, and $\alpha\_j$ and $\beta\_k$ are the weights of past data points for each respective given lag $j$ and $k$; the quantities $c$, $\beta\_k$, and $\alpha\_j$ have to be estimated (@OEKO2).

In this paper, the GARCH model is used to confirm the existence of volatility clusters in the data and thus justify methods that do not assume normally distributed logarithmic returns.