The expectation-maximization (EM) algorithm is a numerical method for the estimation of statistical parameters. The EM procedure is formalized as a problem where both unknown parameters $\boldsymbol{\theta}$ and missing values $\boldsymbol{Z}$ have to be estimated simultaneously, the observed data being $\boldsymbol{X}$. The EM algorithm alternates between the E-step and the M-step to solve this problem.

\*\*Mathematical definition:\*\*

The steps of the EM method at the $t$-th iteration are the following (@Neath):

1. Random initialization of the estimates $\boldsymbol{\tilde{\theta}}\_t$

2. E-step: Define $Q(\boldsymbol{\theta} | \boldsymbol{\tilde{\theta}}\_t) = \mathrm{E}\_{\boldsymbol{Z}|\boldsymbol{X}, \boldsymbol{\tilde{\theta}}\_t} \left[\ln\left(L \left(\boldsymbol {\theta}; \boldsymbol{X}, \boldsymbol{Z} \right) \right) \right]$

Where $Q(\boldsymbol{\theta} | \boldsymbol{\tilde{\theta}}\_t)$ is the expected value of the log-likelihood $\ln\left(L \left(\boldsymbol {\theta}; \boldsymbol{X}, \boldsymbol{Z} \right)\right)$ with complete data, given $\boldsymbol{X}$ and the estimates $\boldsymbol{\tilde{\theta}}\_t$.

3. M-step: Find $\boldsymbol{\tilde{\theta}}\_{t+1} = \underset{\boldsymbol{\theta}}{\mathrm{arg\max}} \ Q(\boldsymbol{\theta} | \boldsymbol{\tilde{\theta}}\_t)$

Where $\boldsymbol{\tilde{\theta}}\_{t+1}$ is the estimate which maximizes $Q(\boldsymbol{\theta} | \boldsymbol{\tilde{\theta}}\_t)$.

4. Iterate the E- and M-steps until a chosen degree of convergence is reached

The multicycle expectation conditional maximization (MCECM) algorithm is a variant of the EM algorithm used in this paper. The MCECM algorithm replaces the M-step with a CM-step, i.e. a sequence of conditional maximizations (CMs) of $\mathrm{E}\_{\boldsymbol{Z}|\boldsymbol{X}, \boldsymbol{\tilde{\theta}}\_t} \left[\ln\left(L \left(\boldsymbol {\theta}; \boldsymbol{X}, \boldsymbol{Z} \right) \right) \right]$, and performs multiple E-steps during the same iteration, e.g. before each conditional maximization (@BDA). In this paper, the estimation of the parameters of the multivariate skewed Student’s t-distribution is carried out by way of the MCECM algorithm implemented in the R ghyp package (@GHYP).