Portfolio optimization usually involves minimizing the risk of a portfolio for a given level of expected return. This requires a definition of risk. In Markovitz portfolio optimization, risk is defined as the standard deviation of returns. As the standard deviation makes no reference to a client’s investment horizon, this yields the same optimal portfolio for long-term investors as for short-term investors, which seems counter-intuitive.

To address this issue, we use Value at risk and expected shortfall over a given investment horizon as risk measures, instead of the standard deviation of returns. We include a review and comparison of these different risk measures, their advantages and their shortcomings.

In addition, we account for the fact that the distributions of market returns are not exactly normal, but exhibit different degrees of fat tails. Based on analyzing 35 years of daily market returns, we model them by univariate and multivariate skewed Student’s t-distributions, whose numbers of degrees of freedom range from 3 to 10. Fewer degrees of freedom, i.e., fatter tails, correspond to shorter-term returns. We include a review of normal distributions, symmetric Student’s t-Distributions, skewed Student’s t-distributions and their generalization, the generalized hyperbolic distributions.

We then first use numerical techniques to minimize Value at Risk over a given investment horizon for a portfolio consisting of assets that are described by symmetric Student’s t distributions with different degrees of freedom. We find that the optimal portfolio now depends on the investment horizon. In particular, the appetite for heavy-tailed assets grows with longer investment horizons, while for shorter-term horizons the assets with more normally-distributed returns are overweighted.

In a second step, we analytically minimize Expected Shortfall over various levels, corresponding to different investment horizons, for a portfolio consisting of assets that are described by the multivariate skewed Student’s t distribution. The results confirm again that longer-term investors should take more tail risk, while short-term investors should avoid it.

Implications for institutional portfolio management are briefly discussed.