Value-at-risk (VaR) measures the value that the potential loss will not exceed at a specific confidence level over a stated time horizon. For example, a one-day 95%-VaR of 1 million USD means that losses should not exceed 1 million USD with a certainty of 95% (@QRM).

**\*\*Mathematical Definition:\*\***

\newline

Value-at-risk:

Where is the significance level.

It has been established that VaR does not always qualify as a coherent risk measure because, in general, it does not conform to the subadditivity axiom (@Artzner). This means that the sum of the VaR of multiple, single assets can be less than the VaR of the single, diversified portfolio composed of the same instruments. This phenomenon violates the notion of risk reduction through diversification, whereby the risk of a portfolio of assets with imperfectly correlated behaviors is less than the sum of each instrument’s risk. At most, the level of risk of a diversified portfolio should equal the sum of the individual asset risks, and only in the case where all instruments are perfectly correlated.

However, VaR adheres to subadditivity when asset returns follow an elliptical distribution, e.g. any distribution that belongs to the set of normal variance mixture distributions :

Where is the location vector, is a random variable, is a real-valued matrix, and

Compared to variance, value-at-risk at various levels of confidence is a more fine-grained measure of risk, since it states the extent of potential losses. However, VaR can be viewed as an optimistic view of the loss scenario; contrary to the principle of conservatism, the quantile given by VaR is the least bad outcome that satisfies the conditions of a specific timespan and a given level of confidence. Despite it being a more specific risk metric than variance, VaR fails to quantify the real loss an instrument is expected to make when the downside event takes place. Moreover, comparing VaRs for assets with different distributions can be misleading. An assets can be riskier than another for the same VaR if the left tail of the distribution of its returns is comparatively heavier. Still, if accounted for its limitations, VaR can serve as an indicator and warning signal for managing risks related to financial instruments. Notwithstanding its technical and conceptual shortcomings, presently VaR plays a significant role in financial risk management and is included as a measure of risk in regulatory frameworks such as Basel II.