The first step of our research project is to quantify the:

\* heavy tails (kurtosis)

\* asymmetry (skewness)

\* tail correlation

\* volatility clustering

of the distributions of log-returns of the following financial assets:

In figure 3 it is shown that the standardized S&P500 log-returns are not normally distributed as the observations in the tails, especially the lower tail, are outside the confidence band of the standard normal distribution. Additionally, it is shown that the observations in the lower tail are more pronounced than in the upper tail, which contrasts with the characteristic of asymmetry as under the normality assumption the observations would be evenly distributed. In contrast to the S&P500 index, the JPY/USD exchange rate shows an inverted asymmetry, which can be explained by considering what happens in the minds of investors during an economic collapse such as the 2008 financial crisis. When markets crash investors tend to panic sell risky assets to minimize their losses and invest in more secure ones. This behavior leads to even further losses in the riskier assets, yet higher than usual gains in the more secure ones. These assets are considered “safe haven” instruments as they promise higher returns during market turmoil e.g. the Japanese Yen, the Swiss Franc and government bonds. In table 2 correlations between assets are illustrated to highlight the concept of safe haven instruments. It is shown that stock indices have a negative relationship with the Japanese Yen, which coincides with the concept of safe haven instruments during market turmoil. Further a t-test for the correlation was carried out to evaluate which correlations are statistically significant (@Plackett). In table 3 the significant t-values relating to the values in table 2 have been highlighted.

As demonstrated in figure 4, the absolute S&P500 log-returns are heavily auto-correlated, which indicates that there is volatility clustering. Volatility clusters imply that large changes tend to be followed by large changes, of either sign, and small changes tend to be followed by small changes @Mandelbrot*,* thus the variance itself is not constant and time-dependent. The normal distribution on the other hand assumes constant variance, thus the observations would disperse evenly around its mean. In figure 4 it is also highlighted that the volatility clustering is the most prominent during times of market collapse e.g. 2008 financial crisis. The time-dependent volatility can be estimated by using a GARCH model that assumes volatility to be a latent variable instead of a constant. In figure 5 it is shown that the estimated volatility for the normally distributed sample is dispersing evenly around its true volatility, whereas it varies drastically for the S&P500 log-returns. During market turmoil the estimated volatility is almost six times bigger than what the empirical volatility implies, which could be lead back to the likelihood of tail events. As the occurrence of tail events is usually very unlikely, it seems that they arise more often than the normal distribution assumes and especially during market turbulence.

The scatterplot of SP500 index data and ten-year US bonds futures in figure 6 displays outliers in all four quadrants, as opposed to no outliers in the corners of the scatterplot of two independent normally distributed samples. The key fact is that the relationship between assets cannot be adequately represented by correlation alone as shown in figure 6, since the normal distribution is incapable of replicating the outliers.

Another issue with normally distributed assets is that a portfolio does not deviate for whichever risk measure used to construct it. The normal distribution does not distinguish between different risk profiles as for example the value-at-risk optimized portfolios are the same for all levels of $\alpha$ and equal to the volatility optimized portfolio as illustrated in the following equations:

$$\sigma^2=w\_1^2 \sigma\_1^2+w\_2^2 \sigma\_2^2+w\_1 w\_2 \mathrm{Cov}(X,Y)$$

$$\frac{\partial \sigma^2}{\partial w\_1}=2 w\_1 \sigma\_1^2+w\_2 \mathrm{Cov}(X,Y)$$

$$\frac{\partial \sigma^2}{\partial w\_2}=2 w\_2 \sigma\_2^2+w\_1 \mathrm{Cov}(X,Y)$$

$$w\_1=\frac{w\_2 \sigma\_2^2 - w\_2 \mathrm{Cov}(X, Y)}{\sigma\_1^2 - \mathrm{Cov}(X, Y}$$

$$q\_{\alpha}=\mu\_\mathrm{pf}+\sigma\_\mathrm{pf} q\_{\alpha, \mathrm{norm}}$$

$$q\_{\alpha}=\mu+\sqrt{w\_1^2 \sigma\_1^2 +w\_2^2 \sigma\_2^2+w\_1 w\_2 \mathrm{Cov}(X,Y)} \ q\_{\alpha, norm}$$

$$\frac{\partial q\_{\alpha}}{\partial w\_1}=\frac{\partial q\_{\alpha}}{\partial w\_2}$$

$$w\_1=\frac{w\_2 \sigma\_2^2 - w\_2 \mathrm{Cov}(X, Y)}{\sigma\_1^2 - \mathrm{Cov}(X, Y}$$

Where $q\_{\alpha, \mathrm{norm}}$ is the $\alpha$-quantile of the standard normal distribution.

Although the normal distribution is not suitable for modeling daily log-returns, it still has its purposes when the frequency is increased e.g. yearly log-returns. In figure 7 it is shown that the effect of kurtosis and skewness decrease after aggregating the log-returns and the observations are within the confidence interval. Thus, the normal distribution approximates the characteristics of log-returns better when the horizon is increased. Therefore, the horizon must play an important role for the evaluation of a suitable distribution model. Although, the effect of kurtosis and skewness can be reduced, the effect of tail correlation persists.