The generalized inverse Gaussian (GIG) distribution is a continuous probability distribution family which includes, among others, the Gamma distribution. Its relevance in this paper derives from it being the distribution of the mixing variable $W$ in the variance-mean mixture generating the generalized hyperbolic distribution (@QRM).

**\*\*Mathematical Definition:\*\***

\newline

Probability density function:

$$f\_W \left( w;\lambda,\chi,\psi \right)= \frac{\chi^{-\lambda} \left(\chi \psi \right)^{\frac{\lambda}{2}}}{2K\_\lambda\left(\sqrt{\chi\psi}\right)} w^{\lambda-1} e^{-\frac{1}{2} \left(\frac{\chi}{w}+\psi w \right)}$$

Where $K\_\lambda(\cdot)$ is the modified Bessel function of the third kind with index $\lambda$. The parameters $\chi$ and $\psi$ are subject to:

$$\begin{cases} \chi > 0, \psi \geq 0 & \text{if } \lambda < 0 \\ \chi > 0, \psi > 0 & \text{if } \lambda = 0 \\ \chi \geq 0, \psi > 0 & \text{if } \lambda > 0 \end{cases}$$

Expected value:

$$\mathrm{E}(W) = \sqrt{\frac{\chi}{\psi}} \frac{K\_{\lambda + 1} \left(\sqrt{\chi \psi} \right)}{ K\_{\lambda} \left(\sqrt{\chi \psi} \right)}$$

Variance:

$$\mathrm{Var}(W) = \frac{\chi}{\psi} \left[\frac{K\_{\lambda + 2} \left(\sqrt{\chi \psi} \right)}{ K\_{\lambda} \left(\sqrt{\chi \psi} \right)} - \left( \frac{K\_{\lambda + 1} \left(\sqrt{\chi \psi} \right)}{ K\_{\lambda} \left(\sqrt{\chi \psi} \right)}\right)^{2}\right]$$

The formulas for all the moments are valid only if $\chi$ and $\psi$ are strictly positive.