The normal (or Gaussian) distribution is a symmetric, unimodal distribution with tails growing thinner from its center, where mean, median and mode are located. The distance from the center is often expressed in standard deviations, notated $\sigma$; this is a compact method for expressing the likelihood of – assumedly – normally-distributed data.

\*\*Mathematical Definition:\*\*

\newline

Probability density function:

$$f\_X\left(x;\mu,\sigma^2\right)=\frac{1}{\sqrt{2\pi\sigma^2}}\mathrm {e}^{-\frac{\left(x-\mu\right)^2}{2\sigma^2}}$$

Cumulative distribution function:

$$F\_X\left(x;\mu,\sigma^2\right)=\frac{1}{\sqrt{2\pi\sigma^2}}\int\_{-\infty}^{x} \mathrm{e}^{-\frac{\left(x-\mu\right)^2}{2\sigma^2}}\mathrm{d}x$$

Expected value:

$$\mathrm{E}\left(X\right)=\mu$$

Variance:

$$\mathrm{Var}\left(X\right)=\sigma^2$$

Standard (or z-) score:

$$z=\frac{x-\mu}{\sigma}$$

The portion of data within one, two and three $\sigma$ from the population mean $\mu$ in both the left and right tail is summarized below:

\begin{align\*}P(\mu-\sigma \leq x \leq \mu+ \sigma) &\approx 68.27\% \\ P(\mu-2\sigma \leq x \leq \mu+2 \sigma) &\approx 95.45\% \\ P(\mu-3\sigma \leq x \leq \mu+3 \sigma) &\approx 99.73\% \end{align\*}

Where $x$ is a realization of the random variable $X$.

From the formula for the probability density function it can be gleaned that the tails of the Gaussian distribution decline like $O(e^{-\lambda|x|^2})$, i.e. they are extremely thin, making this distribution unsuitable for modeling phenomena with heavy tails.