The skewed Student’s t-distribution (SST) is a limiting case of the GH distribution that occurs when $\lambda = -\frac{\nu}{2}$, $\chi = \nu$, and $\psi = 0$, $\nu$ being the degrees of freedom of one tail. By calibrating the parameters, the tails of the SST distribution can be made to have different weights, so that it may be used to model phenomena which display asymmetric behavior and exhibit extreme movements in one direction more often than in the other, e.g. financial instruments with large losses more frequent than windfall profits, a characteristic of many equities. This means that in the multivariate case, for $\gamma \neq 0$ the tails behave as follows (@Haff):

\begin{align\*} \gamma < 0 &: \begin{cases} \text{The left tail decays like} & O\left(|x|^{-\left(\frac{\nu}{2} + 1\right)}\right) \\ \text{The right tail decays like} & O\left(|x|^{-\left(\frac{\nu}{2} + 1\right)} e^{-2 |\gamma| |x|} \right) \end{cases} \\ \gamma > 0 &: \begin{cases} \text{The left tail decays like} & O\left(|x|^{-\left(\frac{\nu}{2} + 1\right)} e^{-2 |\gamma| |x|} \right) \\ \text{The right tail decays like} & O\left(|x|^{-\left(\frac{\nu}{2} + 1\right)}\right) \end{cases} \\ \ \end{align\*}

\*\*Mathematical Definition:\*\*

\newline

Probability density function:

$$f\_X(\boldsymbol{x}; \nu, \boldsymbol{\mu}, \Sigma, \boldsymbol{\gamma}) = c \frac{K\_\frac{\nu + d}{2} \left( \sqrt{\left( \nu + (\boldsymbol{x} - \boldsymbol{\mu})^{\top} \Sigma^{-1} (\boldsymbol{x} - \boldsymbol{\mu}) \right) \boldsymbol{\gamma}^\top \Sigma^{-1} \boldsymbol{\gamma}} \right) e^{(\boldsymbol{x} - \boldsymbol{\mu})^{\top} \Sigma^{-1} \boldsymbol{\gamma}}}{\left( \left( \nu + \left(\boldsymbol{x} - \boldsymbol{\mu}\right)^{\top} \Sigma^{-1} \left(\boldsymbol{x} - \boldsymbol{\mu}\right) \right) \boldsymbol{\gamma}^\top \Sigma^{-1} \boldsymbol{\gamma} \right)^{- \frac{\nu + 2}{4}} \left(1 + \frac{(\boldsymbol{x} - \boldsymbol{\mu})^{\top} \Sigma^{-1} (\boldsymbol{x} - \boldsymbol{\mu})}{\nu} \right)^{\frac{\nu + d}{2}}}$$

Where $c$ is the normalizing constant

$$c = \frac{2^{\left(1 - \frac{\nu + d}{2}\right)}}{\Gamma \left(\frac{\nu}{2} \right) \left(\pi \nu \right)^{\frac{d}{2}} |\Sigma|^{\frac{1}{2}}}$$

Expected value:

$$\mathrm{E}(\boldsymbol{X}) = \boldsymbol{\mu} + \frac{ \nu}{\nu - 2} \Sigma^{-1} \boldsymbol{\gamma}$$

Covariance:

$$\mathrm{Cov}(\boldsymbol{X}) = \begin{cases}\infty & \text{for } \nu \leq 4 \\ \frac{2 \nu^{2}}{(\nu - 2)^{2} (\nu - 4)}\boldsymbol{\gamma}^\top \Sigma^{-1} \boldsymbol{\gamma} + \frac{\nu}{\nu - 2} I & \text{for } \nu > 4 \end{cases}$$

Because of the covariance formula, the estimated $\tilde{\nu}$ for an asymmetric model delivers a distribution with finite covariance only if $\tilde{\nu} > 4$. Being a special case of the GH distribution, the SST distribution becomes a (non-central) symmetric Student’s t-distribution when $\boldsymbol{\gamma} \rightarrow \boldsymbol{0}$ (@QRM, @Haff).