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INTEGRATION OF AN FDTD ANALYSIS OF LOSSY MULTICONDUCTOR TRANSMISSION LINES WITHIN A GENERAL-PURPOSE CIRCUIT SIMULATOR

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Abstract: In this paper, a multiport circuit model for a finite-difference time-domain (FDTD) analysis of the multiconductor transmission line equations is derived. The multiport model enables easy integration of a lossy transmission-line network within a general-purpose nonlinear circuit simulator. The model can be viewed as a link between a distributed-system analysis method, FDTD, and a lumped-element network solver, the circuit simulator. The multiport circuit model consists of resistors and dependent current sources which are updated at every time step by the FDTD algorithm. The approach is capable of dealing with distributed sources, as well as lumped sources. The analysis of a two-conductor transmission line with nonlinear terminations is presented as an application of this approach.

INTRODUCTION

Integration of multiconductor transmission line (MTL) analysis techniques, which arise in applications ranging from tiny interconnects in VLSI technology and electronic chip packaging to very long over-head lines and cables in power transmission, within a circuit simulator is becoming a requirement for all state-of-the-art simulators [1]. Accurate and efficient simulation is important for all these applications. A fundamental difficulty encountered in integrating transmission-line simulation into a transient circuit simulator arises because network nonlinearities and/or time dependent components require a time-domain analysis whereas transmission lines characteristics such as conductor loss and dispersion are best described in the frequency domain. The issue of mixed time-frequency modeling of lossy coupled multiconductor transmission lines has been studied in both the electronics and the power systems communities for many years [2].

FDTD methods are a common way of approximating the time-domain response of transmission lines [3], [4]. The MTL equations are discretized both in time and space and the resulting difference equations are usually solved using the leap-frog scheme [5]. The main problem with interfacing circuit-simulator nodes to the solution of the MTL equations is that in the standard leap-frog FDTD implementation, the discretized line voltages and currents are not collocated in space and time, whereas the circuit-simulator nodal voltages and currents are collocated in both space and time.

In this paper we present an approach, which is an improvement to the technique addressed in [6], for the inclusion of an FDTD formulation capable of modeling lossy transmission lines within a circuit simulator. In this method, each conductor of the transmission line is represented as a two-port stamp, which includes dependent current sources. The current sources are updated at each time step by the FDTD algorithm. The two-port stamp contains only resistive elements, which makes the proposed model independent from the scheme used by the simulator for approximating derivatives.

FORMULATION OF THE METHOD

FDTD methods are commonly used for the numerical solution of partial differential equations in electro-magnetics [7], as well as MTL equations [5]. The derivatives in the MTL equations can be discretized and approximated with various finite differences. We chose an explicit time-space second-order central finite-difference scheme, similar to that described by Paul [5]. In this section, the discretization of the MTL equations is presented first, and then the generation of a link between the variables in the FDTD model of the line and the terminal variables is discussed.

Discretization of MTL Equations

Consider an (M+1)-conductor uniform transmission line. The general MTL equations in the Laplace domain are

$$\frac{\partial}{\partial x}\mathbf{V}(x,s) + \mathbf{Z}(s)\mathbf{I}(x,s) = \mathbf{0} \tag{1a}$$

$$\frac{\partial}{\partial x}\mathbf{I}(x,s) + \mathbf{Y}(s)\mathbf{V}(x,s) = \mathbf{0} \tag{1b}$$

where $\mathbf{V}(x,s)$ and $\mathbf{I}(x,s)$ are M-vectors of the line voltage and current, respectively. Matrices $\mathbf{Z}(s)$ and $\mathbf{Y}(s)$ represent the line series impedance and shunt admittance matrices, respectively. The space variable is x and s is the Laplace variable.

Assuming the line per-unit-length parameters to be frequency-independent we can write,

$$\mathbf{Z}(s) = \mathbf{R} + s\mathbf{L} \tag{2a}$$

$$\mathbf{Y}(s) = \mathbf{G} + s\mathbf{C}\,. \tag{2b}$$

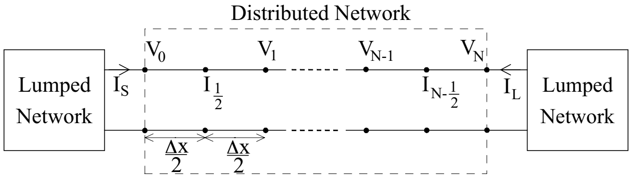

Fig. 1. Interlaced voltages and currents of the FDTD approximation of one of the conductors of an MTL, which is connected to networks with lumped elements. Despite the lines voltages and currents, the terminal voltages and currents are both collocated in time and space.

In (2), **R**, **L**, **G**, and **C** are the per-unit-length resistance, inductance, conductance, and capacitance matrices, respectively.

We can write (1) in the time domain as

$$\frac{\partial}{\partial x}\mathbf{V}(x,t) + \mathbf{L}\frac{\partial}{\partial t}\mathbf{I}(x,t) + \mathbf{R}\mathbf{I}(x,t) = 0 \tag{3a}$$

$$\frac{\partial}{\partial x}\mathbf{I}(x,t) + \mathbf{C}\frac{\partial}{\partial t}\mathbf{V}(x,t) + \mathbf{G}\mathbf{V}(x,t) = 0\,. \tag{3b}$$

To discretize (3), we divide the line into N spatial segments each of length Δx, and consider the time step to be Δt. In order to ensure the stability of the discretization and second-order accuracy of the scheme, the voltages and currents are interlaced as shown in Figs. 1 and 2. As shown, each voltage and adjacent current point are separated by $\Delta x/2$ in space and $\Delta t/2$ in time. This is also true for the terminals but in order to incorporate the terminal constraints we need to collocate the line voltages and currents at the terminations. The procedure for this will be described later. The second-order central-difference approximations to (3) become [5]

$$\frac{\mathbf{V}_{k+1}^{n} - \mathbf{V}_{k}^{n}}{\Delta x} + \mathbf{L}\frac{\mathbf{I}_{k+\frac{1}{2}}^{n+\frac{1}{2}} - \mathbf{I}_{k+\frac{1}{2}}^{n-\frac{1}{2}}}{\Delta t} + \mathbf{R}\frac{\mathbf{I}_{k+\frac{1}{2}}^{n+\frac{1}{2}} + \mathbf{I}_{k+\frac{1}{2}}^{n-\frac{1}{2}}}{2} = 0 \tag{4a}$$

$$\frac{\mathbf{I}_{k+\frac{1}{2}}^{n+\frac{1}{2}} - \mathbf{I}_{k-\frac{1}{2}}^{n+\frac{1}{2}}}{\Delta x} + \mathbf{C}\frac{\mathbf{V}_{k}^{n+1} - \mathbf{V}_{k}^{n}}{\Delta t} + \mathbf{G}\frac{\mathbf{V}_{k}^{n+1} + \mathbf{V}_{k}^{n}}{2} = 0. \tag{4b}$$

In (4), k and n are space and time indices, respectively. Solving (4a) for $I_{k+\frac{1}{2}}^{n+\frac{1}{2}}$ and (4b) for V_k^{n+1}, we obtain the update equations as

$$\mathbf{I}_{k+\frac{1}{2}}^{n+\frac{1}{2}} = \mathbf{Z}_p^{-1}\left(\mathbf{Z}_m\mathbf{I}_{k+\frac{1}{2}}^{n-\frac{1}{2}} + \mathbf{V}_k^n - \mathbf{V}_{k+1}^n\right) \tag{5a}$$

where

$$\mathbf{Z}_p = \Delta x\left(\frac{\mathbf{L}}{\Delta t} + \frac{\mathbf{R}}{2}\right) \tag{5b}$$

$$\mathbf{Z}_m = \Delta x\left(\frac{\mathbf{L}}{\Delta t} - \frac{\mathbf{R}}{2}\right) \tag{5c}$$

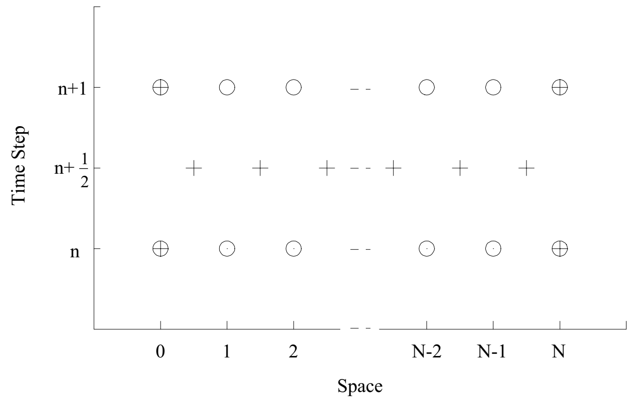

Fig. 2. Time-space distribution of the line voltages (o) and currents (+). At the terminal, the voltage and currents are collocated both in time and space.

and,

$$\mathbf{V}_k^{n+1} = \mathbf{Y}_p^{-1}\left(\mathbf{Y}_m\mathbf{V}_k^n + \mathbf{I}_{k-\frac{1}{2}}^{n+\frac{1}{2}} - \mathbf{I}_{k+\frac{1}{2}}^{n+\frac{1}{2}}\right) \tag{6a}$$

where

$$\mathbf{Y}_p = \Delta x\left(\frac{\mathbf{C}}{\Delta t} + \frac{\mathbf{G}}{2}\right) \tag{6b}$$

$$\mathbf{Y}_m = \Delta x\left(\frac{\mathbf{C}}{\Delta t} - \frac{\mathbf{G}}{2}\right). \tag{6c}$$

This update scheme is illustrated in Fig. 2. The values of the line voltages at time step n and previous value of the line currents are used to update the line currents at time step $n+\frac{1}{2}$, and then the line voltages (except for the terminal voltages) are updated at time step n+1. The updating process of the terminal voltages and currents is taken care of by the circuit simulator which is discussed next.

Integration within the Circuit Simulator

The update equations of the FDTD scheme cannot be directly applied to the terminal voltages since they are not collocated in space or time, whereas the terminal conditions relate the voltage and current at the same position and time (see Fig. 1). Here, the terminal currents, which are denoted as I_S and I_L, are collocated in time and space with the terminal voltages. To resolve this problem, the second MTL equation given in (3b) is discretized at $x = 0$ as [8],

$$\frac{1}{\frac{\Delta x}{2}}\left(\mathbf{I}_{\frac{1}{2}}^{n+\frac{1}{2}} - \frac{\mathbf{I}_S^{n+1} + \mathbf{I}_S^n}{2}\right) + \mathbf{C}\frac{\mathbf{V}_0^{n+1} - \mathbf{V}_0^n}{\Delta t} + \mathbf{G}\frac{\mathbf{V}_0^{n+1} + \mathbf{V}_0^n}{2} = 0. \tag{7}$$

Solving (7) for $\mathbf{I}_S^{n+1}$, yields,

$$\mathbf{I}_S^{n+1} = \mathbf{I}_{SH}^{n,n+\frac{1}{2}} + \mathbf{G}_{eq}\mathbf{V}_0^{n+1} \tag{8a}$$

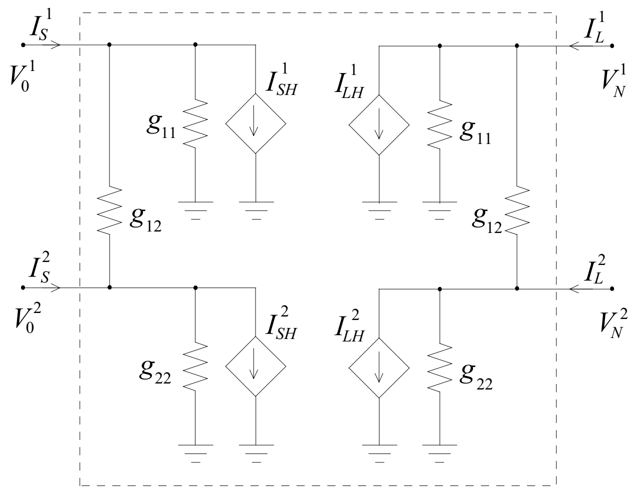

Fig. 3. The multiport stamp for a two-conductor transmission line circuit model. The FDTD algorithm at each time step updates history current sources, $\mathbf{I_{SH}}$ and $\mathbf{I_{SL}}$. $\mathbf{G_{eq}}$ is determined by the line parameters and time and space steps. Superscripts 1 and 2 refer to the conductor number.

where

$$\mathbf{G}_{eq} = \Delta x\left(\frac{\mathbf{G}}{2} + \frac{\mathbf{C}}{\Delta t}\right) \tag{8b}$$

$$\mathbf{I}_{SH}^{n,n+\frac{1}{2}} = 2\mathbf{I}_{\frac{1}{2}}^{n+\frac{1}{2}} - \mathbf{I}_S^n + \Delta x\left(\frac{\mathbf{G}}{2} - \frac{\mathbf{C}}{\Delta t}\right)\mathbf{V}_0^n . \tag{8c}$$

Similarly, (3b) can be discretized at the other terminal as

$$\mathbf{I}_L^{n+1} = \mathbf{I}_{LH}^{n,n+\frac{1}{2}} + \mathbf{G}_{eq}\mathbf{V}_N^{n+1} \tag{9a}$$

where

$$\mathbf{I}_{LH}^{n,n+\frac{1}{2}} = -2\mathbf{I}_{N-\frac{1}{2}}^{n+\frac{1}{2}} - \mathbf{I}_L^n + \Delta x\left(\mathbf{G} - \frac{\mathbf{C}}{\Delta t}\right)\mathbf{V}_N^n . \tag{9b}$$

Observe that the terminal currents at this point are directed into the line in order to provide symmetry of the equivalent circuit. This has been taken into account by the negative sign in the first term at the right hand side of (9b).

The advantage of (8) and (9) is their corresponding circuit representation, which makes the integration of the method within a circuit simulator possible. The circuit representation of (8a) and (9a) for a two-conductor MTL is demonstrated in Fig. 3. This circuit is composed of resistive elements and dependent current sources that are updated by (8c) and (9b). The resistive elements in Fig. 3 are functions of the elements of $\mathbf{G}_{eq}$ and are given by,

$$g_{ij} = \begin{cases} \sum_{j=1}^{N} G_{eq}^{ij} & i = j \\ -G_{eq}^{ij} & i \neq j \end{cases} \tag{10}$$

As illustrated in Fig. 2, in each time step, the FDTD algorithm is called by the circuit simulator and the line's internal currents and voltages, $I_{k+\frac{1}{2}}^{n+\frac{1}{2}}$ $k \in \{0,1,...,N-1\}$ and V_k^{n+1} $k \in \{1,2,...,N-1\}$, are updated based on their past values, $I_{k+\frac{1}{2}}^{n-\frac{1}{2}}$ and V_k^n, and the terminals' voltages past values, V_0^n and V_N^n. Next, the history current sources, I_{SH} and I_{LH}, are updated and their values are passed to the circuit simulator, which then solves the network and updates the terminals' currents and voltages.

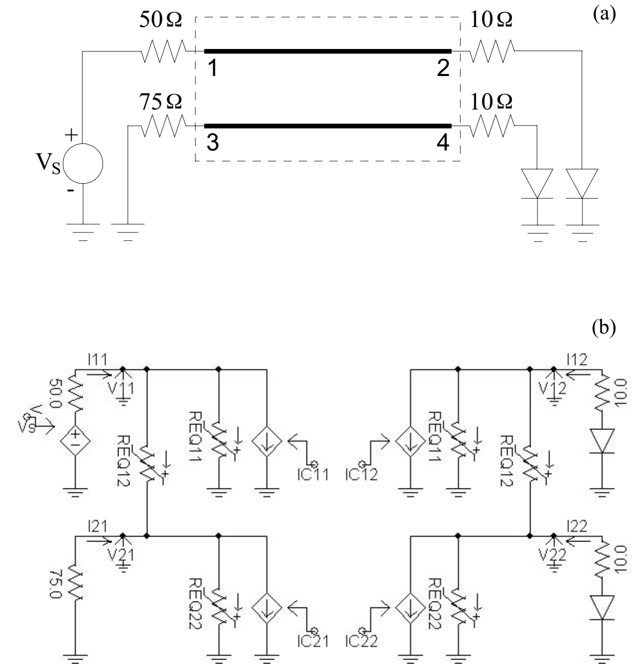

Fig. 4. a) A two-conductor transmission line with nonlinear terminations, adopted from [9], b) The equivalent circuit model for solving the problem using PSCAD. Variables V11, V12, V21, V22, I11, I12, I21, and I22 are the line terminals' variables that are passed to the FDTD algorithm at each time step, and IC11, IC12, IC21, and IC22 are updated by the FDTD algorithm. REQs are determined by (8b) and (10).

SIMULATION RESULTS

We have used PSCAD[1] as a circuit simulator platform to test the algorithm. The platform allows user-defined components that are controlled by a custom FORTRAN code. Note the proposed algorithm, which is independent from the scheme that is used to solve the ordinary differential equations of the circuit simulator, can be implemented with any general circuit simulator.

The FDTD method described in the previous section has the capability of providing access to variables along the whole length of the line, which for example, can be used to simulate the effect of distributed external-field excitation.

[1] Power Systems Computer Aided Design, Manitoba HVDC Research Center.

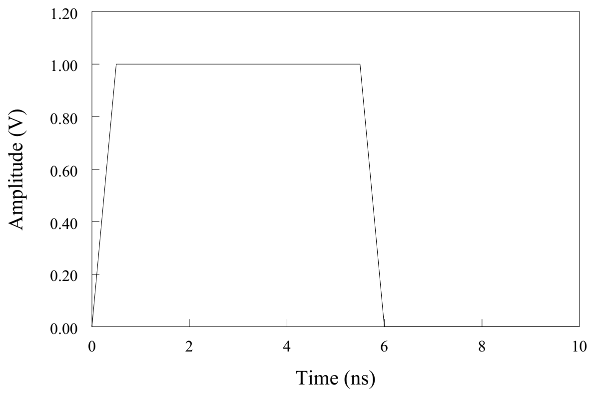

Fig. 5. The excitation waveform, V_S, of Fig. 4a.

The source of such an external field can be a nearby lightning stroke or a plane wave. This method is also able to simulate nonuniform lines for which the per-unit-length parameters are functions of the line's spatial parameter. In this section, the application of this approach to the analysis of lossy multiconductor transmission lines with nonlinear terminations is presented. Consider the two-conductor lossy transmission line shown in Fig. 4a. This configuration was solved by Djordjevic *et al.* [9] using a Green's function method. We use a line length of 0.5 m, and the transmission line parameter matrices are given as,

$$\mathbf{L} = \begin{bmatrix} 309 & 21.7 \\ 21.7 & 309 \end{bmatrix} \text{(nH/M)}$$

$$\mathbf{C} = \begin{bmatrix} 144 & -6.4 \\ -6.4 & 144 \end{bmatrix} \text{(pF/m)}$$

$$\mathbf{R} = \begin{bmatrix} 524 & 33.9 \\ 33.9 & 524 \end{bmatrix} (\text{m}\Omega/\text{m})$$

$$\mathbf{G} = \begin{bmatrix} 905 & -11.8 \\ -11.8 & 905 \end{bmatrix} \text{(nS/m)}\,.$$

The active line is driven by a 1-V trapezoidal voltage source with a rise and fall time of 0.5 ns, and a total duration of 6 ns, as shown in Fig. 5. The resulting terminal voltages at nodes 1-4 of Fig. 4 are shown in Fig. 6. The simulated results match those presented in [9].

CONCLUSION

A method for integrating a finite-difference time-domain approach to the solution of MTL network problems within a general-purpose circuit simulator was presented in this paper. In this method, the line is modeled as a network of resistive elements and dependent current sources, which are updated by the FDTD algorithm at each time step. This approach has the capability of dealing with distributed-source excitation of the line and nonuniform lines. The skin-effect dependence of the line parameters can also be included in this approach.

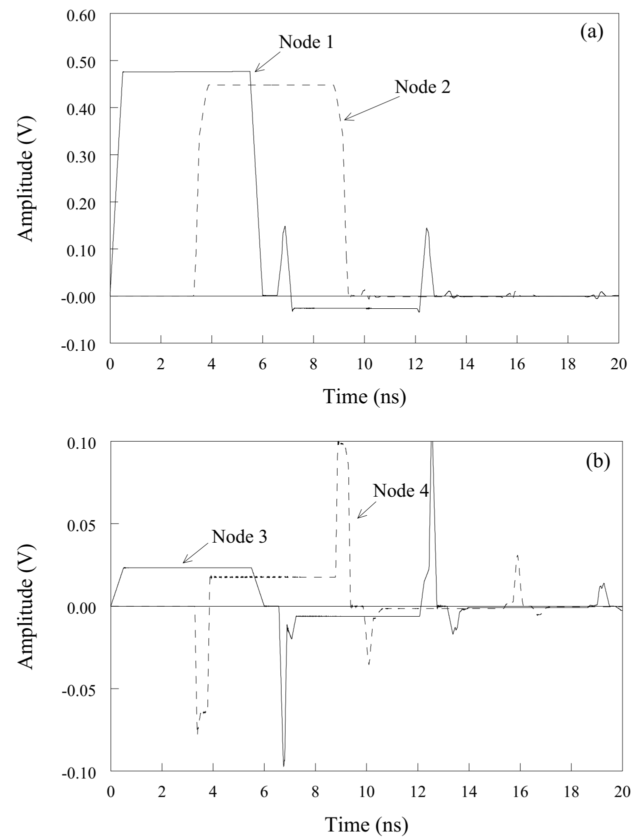

Fig. 6. Terminal voltages of the MTL shown in Fig. 4a. The excitation waveform is shown in Fig. 5.

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