On the Development of a Scattered-Field Formulation for Objects in Layered Media Using the FVTD Method

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Abstract: A technique for simulating the scattering from objects in multi-layered media is presented. The formulation is efficient in that the scattered fields are only generated at the inhomogeneities, and therefore they are more easily absorbed by the boundary conditions. The incident field is created by using a 1D-FDTD solution for the plane-wave propagation through a multi-layered medium and is subsequently applied in a scattered field formulation of the FVTD method. Interpolation methods are discussed and a numerical example is given to demonstrate the technique.

Keywords: FVTD, FDTD, Numerical Methods

1. Introduction

Electromagnetic wave scattering from objects in multi-layered media is a topic with diverse applications including remote sensing of earth environments [1] and buried object detection [2]. The formulation and analysis of the multi-layered media are non-trivial tasks due to the layer interfaces that govern the propagation of the incident field. As noted in [3], a variety of analytic methods have been developed and are capable of describing the propagation through multi-layered media; however, it is challenging to utilize these methods in an existing numerical solver.

The finite-volume time-domain (FVTD) method is well-suited to model the scattering problem since the inhomogeneous subsurface objects often exhibit curved features and FVTD can be used on a conforming irregular grid [4,5]. Adding an inhomogeneity does not increase the number of unknowns that must be solved in a differential-equation-based solver [3]. With appropriate interpolation methods, FDTD solutions for the multi-layered problem [6] can be used with the FVTD method. In time-domain differential-equation-based methods, the boundary conditions are more efficient at absorbing normally incident scattered fields, and so, incorporation of the incident field into the numerical method and solving for only the scattered field allows the boundary condition to be more effective. Applying a scattered field formulation for multi-layered media in the FVTD engine provides a new tool for modeling complex layered media and has future potential to be used for modeling electromagnetic interactions in remote sensing studies.

The organization of the paper is as follows. In Section 2. we formulate the total-field/scattered-field background theory. Details of the practical application of this general concept in an FVTD numerical solver are given in Section 3. We present some numerical results in Section 4. We conclude with several remarks on the implementation and applicability of the method, along with some suggestions for our future work.

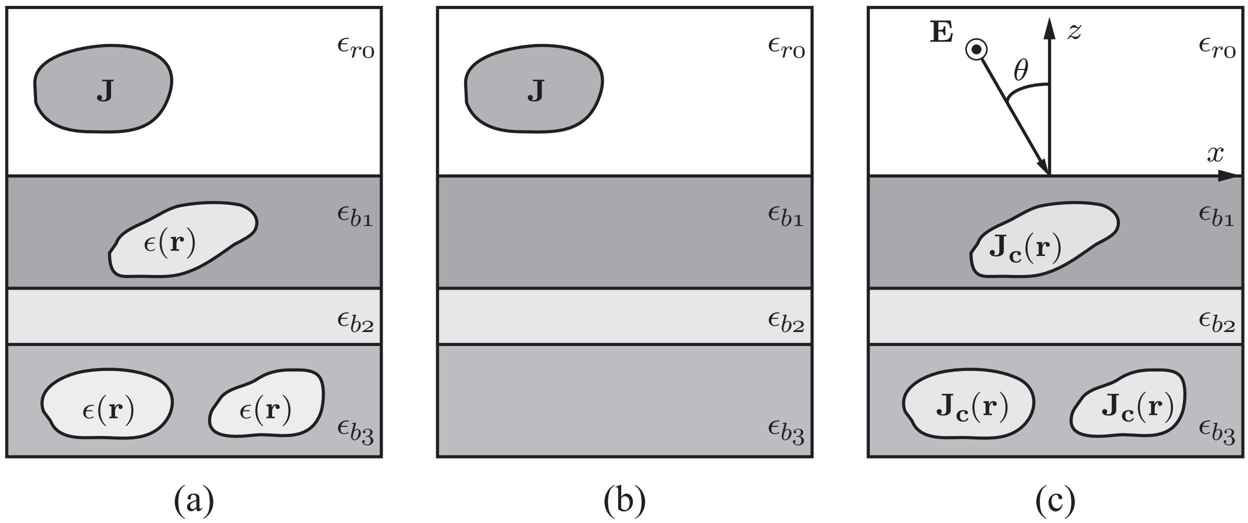


Fig. 1. Hypothetical geometry illustrating the decomposition of the fields in the total field/scattered field formulation. a) Total field geometry, b) incident field geometry, c) scattered field geometry.

2. Problem Formulation

Let $\mathbf{E}(\mathbf{r}, t)$ be the electric field intensity, $\mathbf{H}(\mathbf{r}, t)$ is the time-domain magnetic field intensity, $\epsilon(r)$ is the permittivity, $\mu(r)$ is the permeability, and $\mathbf{J}(\mathbf{r}, t)$ is the impressed time-domain electric current. The standard incident field-scattered field decomposition gives

$$\mathbf{E}(\mathbf{r}, t) \triangleq \mathbf{E}^{inc}(\mathbf{r}, t) + \mathbf{E}^{sc}(\mathbf{r}, t) \tag{1}$$

and

$$\mathbf{H}(\mathbf{r}, t) \triangleq \mathbf{H}^{inc}(\mathbf{r}, t) + \mathbf{H}^{sc}(\mathbf{r}, t), \tag{2}$$

where the incident fields are defined to exist in a background media, with $\epsilon_b(r)$ and $\mu_b(r)$, and are produced by the impressed current $\mathbf{J}(\mathbf{r}, t)$, as shown in Fig. 1 (b). In other words,

$$\nabla \times \mathbf{E}^{inc}(\mathbf{r}, t) = -\mu_b(\mathbf{r})\frac{\partial \mathbf{H}^{inc}(\mathbf{r}, t)}{\partial t}, \tag{3}$$

$$\nabla \times \mathbf{H}^{inc}(\mathbf{r}, t) = \epsilon_b(\mathbf{r})\frac{\partial \mathbf{E}^{inc}(\mathbf{r}, t)}{\partial t} + \mathbf{J}(\mathbf{r}, t). \tag{4}$$

The scattered field is produced by the difference between the true media, $\epsilon(\mathbf{r})$ and $\mu(\mathbf{r})$, and the background. After some algebraic manipulation,

$$\nabla \times \mathbf{E}^{sc}(\mathbf{r}, t) = -\mu(\mathbf{r})\frac{\partial \mathbf{H}^{sc}(\mathbf{r}, t)}{\partial t} - \mathbf{M}_c(\mathbf{r}, t), \tag{5}$$

$$\nabla \times \mathbf{H}^{sc}(\mathbf{r}, t) = \epsilon(\mathbf{r})\frac{\partial \mathbf{E}^{sc}(\mathbf{r}, t)}{\partial t} + \mathbf{J}_c(\mathbf{r}, t), \tag{6}$$

with the contrast magnetic and electric contrast sources that are shown in Fig. 1 (c) defined as

$$\mathbf{M}_c(\mathbf{r}, t) = [\mu(\mathbf{r}) - \mu_b(\mathbf{r})]\frac{\partial \mathbf{H}^{inc}(\mathbf{r}, t)}{\partial t}, \tag{7}$$

$$\mathbf{J}_c(\mathbf{r}, t) = [\epsilon(\mathbf{r}) - \epsilon_b(\mathbf{r})]\frac{\partial \mathbf{E}^{inc}(\mathbf{r}, t)}{\partial t}. \tag{8}$$

For non-magnetic media, $\mathbf{M}_c(\mathbf{r}, t) = 0$. The incident field propagates in the layered background medium and the scattered field is generated by equivalent sources only at the inhomogeneities. The scattered field propagates outward and is easily absorbed by whatever absorbing boundary condition is being used.

3. Numerical Implementation

We now apply the TF/SF formulation into the framework of an FVTD numerical solver and utilize a numerically defined (as opposed to analytically defined) incident field source term. A technique similar to the one presented here can be used with any time-domain field solver.

The FVTD formulation used herein is a cell-centered, upwind, characteristic-based numerical engine for meshes consisting of first-order polyhedral elements [5]. The engine is capable of solving Maxwell's equations in the time-domain using either a total- or scattered-field formulation, the latter permitting arbitrary (i.e. non-homogeneous) background media. The implementation has been parallelized for distributed parallel environments.

A hypothetical TF/SF decomposition is shown in Fig. 1 for the case of multi-layered media. Following [6], we first calculate a 1D-FDTD solution for the incident field, which is a wave propagating through layered media at arbitrary incidence angle (Fig. 1 (b)). For completeness, the derived expressions for TE wave propagation are summarized here:

$$\frac{\partial E_y}{\partial z} = \mu_0 \frac{\partial H_x}{\partial t}, \tag{9}$$

$$\frac{\partial H_x}{\partial z} = \epsilon_0(\epsilon_r - \epsilon_{r0} sin^2\theta)\frac{\partial E_y}{\partial t}, \tag{10}$$

$$H_z = Y_0\sqrt{\epsilon_{r0}} sin\theta E_y, \tag{11}$$

where θ is the incidence angle measured between the direction of propagation and the z-axis, $Y_0 = \sqrt{\epsilon_0/\mu_0}$ is the intrinsic admittance of free space, and ϵ_{r0} is the dielectric constant of the uppermost (or 0^{th}) layer.

These equations resemble the familiar transmission line equations and with appropriate descriptions of the dielectric profile in the z-direction, they describe the propagation of a plane wave through multi-layered media. We discretize the TE equations, following the approach of [7], where the electric and magnetic field components are interleaved, and we use a central-difference approximation. We set the spatial discretization and the temporal discretization to obey the Courant stability criterion, $\delta t \leq \delta z/v_0$, where v_0 is the velocity of propagation in the layer with the highest dielectric constant. We apply a basic PML boundary condition at the upper and lower bounds of the 1D-solution grid, but also pad the solution domain to minimize the error created by the boundary (i.e. add more distance for the wave to travel than is necessary for the FVTD solution).

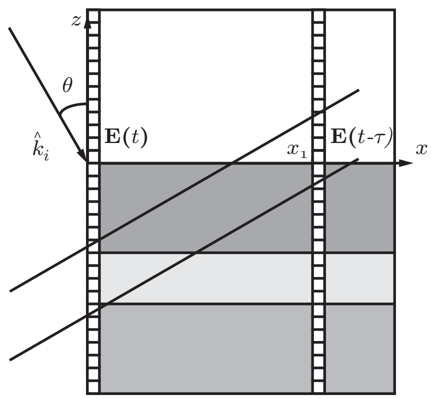
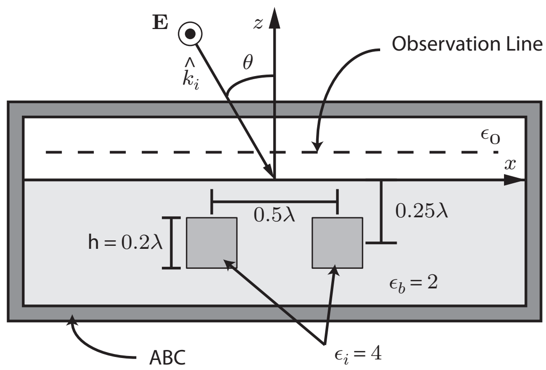


Fig. 2. Calculation of field values on the principle plane using the time-delay factor, τ. The fields calculated at x_1 are the fields calculated along the z-axis with an appropriate time-delay.

Next, the 1D-solution is interpolated into the 3D irregular mesh that is used in the FVTD computations. A principle plane containing the time history of the plane wave propagation through layered media is made to coincide with the xz-plane. To find the field values along this principle plane, we need only a time shift, as this is a property of plane wave propagation. This concept is illustrated in Fig. 2, where the principle plane is shown to coincide with the xz-plane, and $\tau = x_1 sin(\theta)/c_0$ (c_0 is speed of light in a vacuum). The solution for wave propagation along the negative z-axis is calculated and the time-delayed result is utilized to find the field value at a location x_1. To obtain a 3D representation, we utilize the invariance of the solution along the other coordinate axis (y-axis).

Practical implementation issues include choosing an appropriate interpolation method and ensuring that the spatial discretization of the auxiliary 1D-FDTD simulation does not cause dispersion in the main FVTD simulation. If the FVTD grid were regular (as would be used in a 3D-FDTD grid), we could follow the method of [6] and increase the spatial sampling by an odd factor and thereby ensure that the interfaces were preserved. Moreover, since the auxiliary FDTD simulation takes a small fraction of the time needed for the FVTD scattering simulation, we choose the spatial discretization to be finer than that of the FVTD mesh. Cubic spline spatial interpolations and nearest-neighbor temporal interpolations are performed to find the corresponding field values in the centroids of the 3D elements (tetrahedrons) that make up the FVTD mesh.

The results of the incident field propagation are calculated and stored in arrays in memory. They are potentially accessed only during the update scheme of the FVTD simulations, which checks to see if the contrast between the background layered medium (that which is seen by the incident field) and the scattered field mesh are non-zero. A non-zero contrast indicates the presence of a contrast source (i.e. $\mathbf{J}_c(\mathbf{r}, t) \neq 0$) and a scattered field is generated at that particular mesh element. Furthermore, the results of the incident field interpolations are only used when needed and they are not inefficiently calculated and stored throughout the entire mesh.



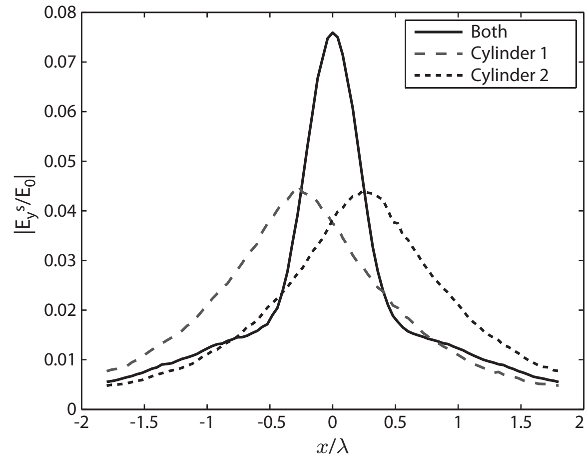


Fig. 3. Geometry for dielectric cylinders buried in a half-space. Cylinder radii are 0.1λ, separation is 0.5λ, cylinder centers are at $z = -0.25\lambda$.

4. Numerical Results and Discussion

We present an example of the scattered near-field values for buried cylinders in a lossless half-space. More examples and comparisons with literature data are presented in [9]. We create the computational geometry using a freeware mesh-generating program, GMSH [8]. The time function of the input waveform was a Gaussian derivative:

$$E_y(t) = \frac{2A(t - t_0)}{b^2} exp(-(t - t_0)^2/b^2), \tag{12}$$

where $A = 1$, $t_0 = 0.2$ ns, and $b = 70$ ps. The constants in (12) were chosen such that sufficient energy would propagate at the frequency of interest (specifically, 6 GHz).

The computational geometry for the case of two cylinders is shown in Fig. 3. We simulated the scattering from both cylinders, from cylinder 1 only (the left-hand cylinder in Fig. 3), and from cylinder 2 only (the right-hand cylinder in Fig. 3). The simulation results are presented in Fig. 3.

5. Conclusion

Through incorporating the scattered-field formulation for multi-layered media in an FVTD engine, an efficient method of modeling complex layered media has been developed. We have presented details of the method used to calculate electromagnetic scattering from objects buried in multi-layered media, which has a wide range of potential application. Our method is capable of calculating the scattering from multiple objects with minimal increase in the number of unknowns in the computation. In our future research, we intend to use the proposed method for modeling electromagnetic scattering from geophysical media.

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