Time Domain Finite Element Analysis of 2-D Shields

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Abstract

A two-dimensional time-domain application of a general finite element mathematical package, IMSL:PDE/PROTRAN (see [IMSL 86], [Sewell 85]), is considered for two dimensional shielding problems. The software runs on the IBM 3090 main frame computer. The shielding problem is formulated via the *magnetic vector potential* and the *Coulomb gauge*. Validation of the program is performed via a simple test problem (plane wave incident on a perfect conductor). Results are presented for the case of a plane wave with double exponential time dependence incident on the shield.

1.0 Formulation of 2-D Shielding Problems

In this section we consider the formulation of two dimensional time dependent shielding problems. Consider the specific problem shown below in figure 1 where the *initial* triangulation is also shown. The incident plane wave enters from the right hand side of the numerical boundary. The shield is assumed to be perfectly conducting (i.e. $\sigma = \infty$).

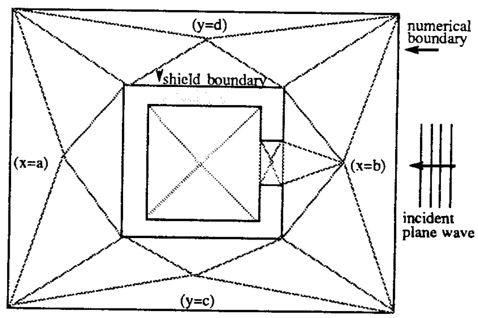

Figure 1: 2-D Shield Initial Triangulation

If the magnetic vector potential **A** is defined as $\mathbf{B} = \mu\mathbf{H} = \nabla \times \mathbf{A}$, then the electric field is given as

$$\mathbf{E} = -\frac{\partial \mathbf{A}}{\partial t} - \nabla\phi. \tag{1}$$

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The potential **A** will be a solution of the inhomogeneous wave equation. In a sourceless region, if we use the *Coulomb gauge*, $\nabla \cdot \mathbf{A} = 0$, (see: Jackson, p. 220 [Jackson 75]; and Eyges, p. 181 [Eyges 80]) then **A** will be a solution of the homogeneous wave equation. If we further assume that $\mathbf{A} = A_z \, \hat{\mathbf{a}}_z$ then the scalar wave equation arises

$$\nabla^2 A_z - \mu\varepsilon \frac{\partial^2 A_z}{\partial^2 t} = 0. \tag{2}$$

With this assumption, the only existing fields are E_z, B_x, and B_y which can be obtained in terms of the potential A_z as

$$E_z = -\frac{\partial A_z}{\partial t} \quad ; B_x = \frac{\partial A_z}{\partial y} \quad ; B_y = -\frac{\partial A_z}{\partial x} \quad . \tag{3}$$

We can thus derive initial and boundary conditions using eq.'s (3) for the partial differential equation (2). If we assume that no fields exist at time t=0 then we also have $A_z(x,y,t=0) = 0$. If the incident plane wave consists of the two components E_z, and B_y then the boundary condition on the right side of the numerical mesh (x=b) can be written as a inhomogeneous Neumann boundary condition

$$B_y(x=b,y,t) = -\frac{\partial A_z}{\partial x}, \; c \le y \le d \; ; \; t \ge 0. \tag{4}$$

The boundary conditions around the rest of the numerical mesh is approximated by homogeneous Neumann boundary conditions. This is valid as long as the time of the calculation does not proceed long enough for reflections from the shield to appear at these boundaries. At the shield boundaries we have that the electric field E_z must be equal to zero. Thus this implies a Dirichlet boundary condition on A_z.

2.0 Finite Element Results

The finite element method is a general mathematical procedure which enables one to obtain approximate solutions to differential equations. The method can be obtained via the *Method of Weighted Residuals* (see Lapidus [Lapidus 82]). Details of this method or its implementation will not be given here since the software package PDE/PROTRAN has been used (for details see [IMSL 86], [Sewell 85]).

Results are given in figure 2 for the case of a double exponential pulse incident on a perfectly conducting plane interface. The pulse has a rise time of .5 ns and a fall time of 1 ns. The program was run with 200 triangles and a time step of .1 ns. As can be seen from the figure, the numerical solution performs as expected. These results took about 40 sec. of CPU time on the IBM 3090 main frame computer.

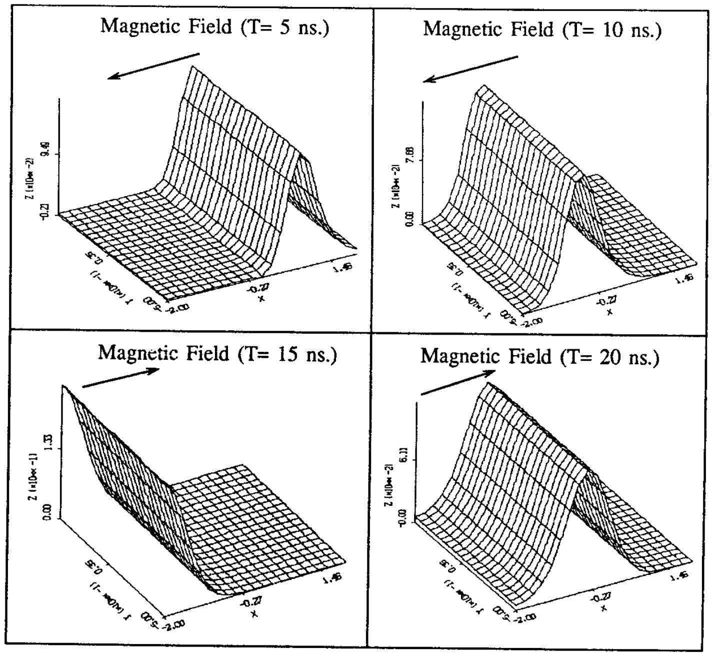

Figure 2: Test Problem (Rise-time=.5 ns, Fall-time=1 ns).

Results for the 2-D shield are shown in figure 3. Here the incident pulse had a rise time of .5 ns and a fall time of 1 ns. The shield has a thickness of 15 cm and the aperture is 30 cm wide. The outside dimensions of the shield are 80 cm square and the numerical mesh is 3 m square. For the results shown 550 triangles were used and the time step was .1 ns. As can be seen from the figure, the penetration into the shield is minimal even with such a large opening. The CPU time for these results was about 300 sec.

References

[Eyges 80] Eyges, L., **The Classical Electromagnetic Field**, Dover Publications, Inc., New York, 1980.

[Jackson 75] Jackson, J. D., **Classical Electrodynamics**, John Wiley & Sons, New York, 1975.

[Lapidus 82] Lapidus, L., and Pinder, G. F., **Numerical Solution of Partial Differential Equations in Science and Engineering**, John Wiley & Sons, New York, 1982.

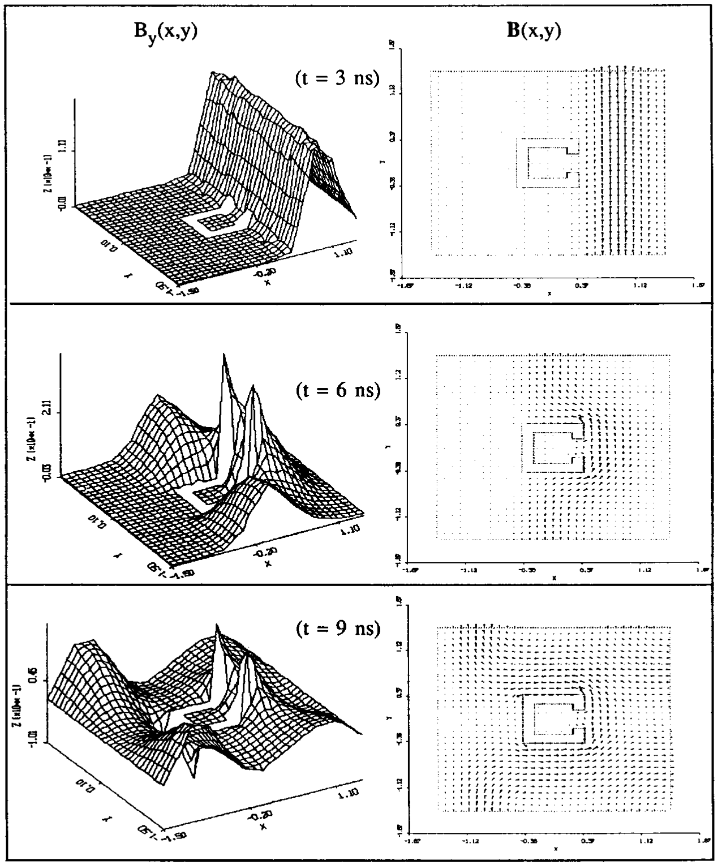

Figure 3: 2-D Shield Results B_y and **B**.

[IMSL 86] IMSL, **PDE/PROTRAN: A System for the Solution of Partial Differential Equations**, User's Manual, IMSL, Houston, Texas, 1986.

[Sewell 85] Sewell, G., **Analysis of a Finite Element Method: PDE/PROTRAN**, Springer-Verlag, New York, 1985.