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A Higher-Order Mode Transmission Line Model of the Shielding Effectiveness of Enclosures with Apertures

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Abstract: The development of efficient phenomenological models which give accurate estimates for the coupling of electromagnetic energy through apertures and into enclosures is important for the EMC design of almost all electronic sub-systems. It has been previously shown that a simple analytical transmission line formulation gives good predictions of the shielding effectiveness of a rectangular enclosure with apertures. In this work the model is extended to include higher-order transverse electric (TE) and transverse magnetic (TM) cavity modes in the transmission line (TL) model. The loading effect of electronic components within the enclosure is modelled by the introduction of losses. The model remains very simple and computationally efficient. The finite difference time-domain (FDTD) method is used to validate the results.

Introduction

Electromagnetic shielding is one of the standard approaches that prevents coupling of undesired radiated electromagnetic energy into equipment otherwise susceptible to it. Shielding effectiveness can be found by numerical methods or analytical approaches. An analytical approach to the shielding effectiveness estimation of a rectangular enclosure with apertures provides a cost-effective alternative to numerical methods, thus saving significant computing resources It has been previously shown that for a simple system topology an analytical transmission line formulation gives good predictions of the shielding effectiveness of a rectangular enclosure with apertures [1], [2], [4]. However, by design, this formulation does not consider higher-order transverse electric and magnetic modes propagating within the enclosure and is therefore limited to lower frequencies. This limitation has motivated further research with the objective to extend the applicability of the model to the estimation of the shielding effectiveness when higher-order modes are present, *i.e.*, to higher frequencies.

Theoretical Investigation

A rectangular enclosure with dimensions comparable to the disturbing energy's wavelength range acts mainly as a cavity resonator, supporting TE and TM waves. The distribution of the field inside some simple closed metal cavities can be understood as that existing in transmission line structures with short-circuited ends which give rise to standing waves inside the enclosure. In a rectangular cavity with perfect conducting walls the distribution of the electromagnetic field can be modelled as a sum of TE_{mnp} and TM_{mnp} modes to any of the coordinate axis. These modes are the only possible solutions to Maxwell's equations that satisfy the boundary conditions of zero tangential electrical fields on all walls [6].

A rectangular aperture in an empty rectangular enclosure of Figure 1 is represented by the equivalent circuit of Robinson *et al.* [1]. The shunt impedance of the aperture can be modelled as

$$Z_{ap} = \frac{1}{2}\frac{l}{a} jZ_{os}\tan\frac{k_0 l}{2}, \tag{1}$$

where $Z_0 = 377\Omega$ is the free-space impedance and $k_0 = 2\pi/\lambda$ is the propagation constant, and Z_{os} is the characteristic impedance of the transmission line stub which models the aperture (as in [1]).

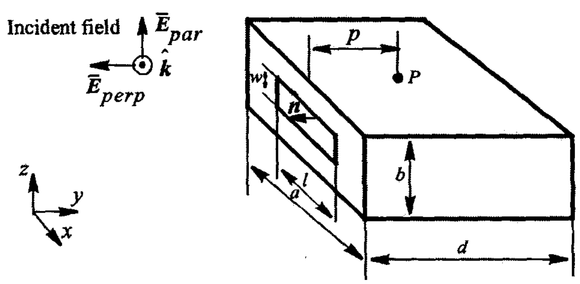

Figure 1: Rectangular enclosure with a slot

In Figure 2(a) a radiating source in the equivalent circuit is represented by voltage V_0 and impedance Z_0. Combining Z_0, V_0 and Z_{ap} according to Thevenin's theorem gives an equivalent source voltage

$$V_s = (V_0 Z_{ap})/(Z_0 + Z_{ap}) \tag{2}$$

with a source impedance

$$Z_s = (Z_0 Z_{ap})/(Z_0 + Z_{ap}) \ . \tag{3}$$

In the rectangular conducting enclosure of Figure 1, we consider TE^z_{m0} and TM^x_{m1} waveguide modes oriented with its electric and magnetic fields in the z and x directions respectively and propagating in the y direction. For the dominant mode the condition that E_z be zero at $y = 0$ and $y = d$, as required by the perfect conductor walls, is satisfied if the dimension d is half the guide wavelength λ_g, that is

$$d = \frac{\lambda_g}{2} = \frac{\lambda}{2\sqrt{1-(\lambda/2a)^2}} \ , \tag{4}$$

The resonant frequency of the dominant cavity mode [3] is given by

$$f_0 = \frac{\sqrt{a^2+d^2}}{2ad\sqrt{\mu\varepsilon}} \tag{5}$$

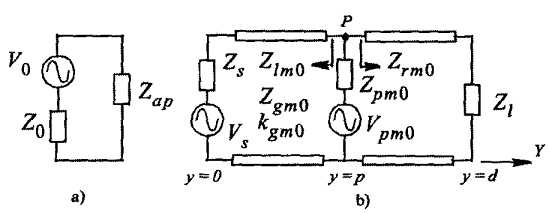

Figure 2: TL equivalent circuit based on the transmission line model of shielding

For the m-th transverse electric TE^z_{m0} mode of propagation, the enclosure is represented by a cavity formed by a shorted waveguide whose characteristic impedance [3] is

$$Z_{gm0} = Z_0/\sqrt{1-(m\lambda/2a)^2} \tag{6}$$

and the propagation constant

$$k_{gm0} = k_0\sqrt{1-(m\lambda/2a)^2} \ , \tag{7}$$

where $m = 1, 2, 3, \ldots$

For the m-th transverse magnetic TM^x_{m1} mode of propagation, the characteristic impedance is

$$Z_{gm1} = Z_0/\sqrt{1-(m\lambda/\lambda_c)^2} \tag{8}$$

and the propagation constant

$$k_{gm1} = k_0\sqrt{1-(m\lambda/\lambda_c)^2} \ , \tag{9}$$

where the cutoff wavelength and mode number are given respectively as

$$\lambda_c = \frac{2a}{\sqrt{1+(a/b)^2}} \quad \text{and} \quad m = 1, 2, 3, \ldots \tag{10}$$

When there is only one dominant mode in the enclosure then

$$Z_{gm0} = jX_{gm0} \ , \qquad (m \neq 1) \tag{11}$$

$$k_{gm0} = -j\alpha_{gm0} \ , \qquad (m \neq 1) \tag{12}$$

are imaginary. For $m = 1$, Z_{g10}, is the real characteristic impedance of the dominant mode, and

$$k_{g10} = \beta_1 - j\alpha_1 \ , \tag{13}$$

where $\beta_1 = 2\pi/\lambda_{g1}$ is the phase constant, λ_{g1} is the guide wavelength of the dominant mode and α_1 is the dominant-mode attenuation constant due to power loss in enclosure walls [3].

For typical enclosures the cutoff frequency of the dominant mode is of the order of hundreds of megahertz. This allows one to consider the ideal case and make the assumption that the wall losses are small compared to the energy which penetrates into the cavity through the aperture. For the higher frequencies associated with the higher-order modes, the effect of ohmic loss in the cavity walls must be incorporated into the model.

Based on the transmission line theory and the equivalent circuit shown in Figure 2(b), the impedance of the m-th mode Z_{rm0} at point P on the TL, viewed in the direction of the termination Z_l is

$$Z_{rm0} = \frac{Z_l + jZ_{gm0}\tan[k_{gm0}(d-p)]}{1 + jZ_{nm0}\tan[k_{gm0}(d-p)]} \ , \tag{14}$$

where $Z_{nm0} = Z_l/Z_{gm0}$ is the normalized termination impedance for the m-th mode. The losses associated with a waveguide whose walls are not perfectly conducting can be modelled using the surface impedance concept as described in [6]. The termination Z_l represents the surface impedance of the end wall given by

$$Z_l = (1+j)\sqrt{\frac{\pi f \mu}{\sigma}} \quad (15)$$

where σ is the conductivity and μ is the permeability of the enclosure material respectively. The termination Z_l includes the effect of ohmic losses in the enclosure walls into the shielding effectiveness estimation. For perfect electric conductor walls of the enclosure the surface impedance of the end wall $Z_l = 0$, and this gives an effective impedance at test location P of

$$Z_{rm0} = jZ_{gm0}\tan k_{gm0}(d-p) \quad (16)$$

The source impedance of the m-th mode Z_{lm0} at test location P on the TL, viewed in the direction of the source impedance Z_s is

$$Z_{lm0} = \frac{Z_s + jZ_{gm0}\tan(k_{gm0}p)}{1 + jZ_{nm0}\tan(k_{gm0}p)}, \quad (17)$$

and the equivalent voltage V_{lm0} is

$$V_{lm0} = \frac{V_s}{\cos(k_{gm0}p) + jZ_{nm0}\sin(k_{gm0}p)} \quad (18)$$

The m-th mode voltage at test location P is

$$V_{pm0} = V_{lm0}Z_{rm0}/(Z_{lm0} + Z_{rm0}) \quad (19)$$

and the total voltage at P is now

$$V_{tp} = \sum_m V_{lm0}Z_{rm0}/(Z_{lm0} + Z_{rm0}) \quad (20)$$

The m-th mode current at test location P is

$$I_{pm0} = V_{lm0}/(Z_{lm0} + Z_{rm0}) \quad (21)$$

and the total current at test location P is now

$$I_{tp} = \sum_m V_{lm0}/(Z_{lm0} + Z_{rm0}) \quad (22)$$

In the absence of the enclosure, the load impedance at test location P is simply Z_0, so the voltage at P is $V_P^0 = V_0/2$, and the current is $I_p^0 = V_0/2Z_0$. The electric and magnetic shielding effectiveness are respectively

$$S_E = -20\log\left| V_{tp}/V_p^0 \right| = -20\log\left| 2V_{tp}/V_0 \right| \quad (23)$$

$$S_M = -20\log\left| I_{tp}/I_p^0 \right| = -20\log\left| 2I_{tp}Z_0/V_0 \right| \quad (24)$$

Thus, the expressions (23) and (24) can be used for estimating the electric S_E and magnetic S_M shielding effectiveness of the rectangular enclosure with aperture at higher-order modes. The contributions of higher-order modes are computed by omitting the $m = 1$ term in equations (6), (7) or (8), (9).

Numerical Investigation

The TL equivalent circuit of Figure 2 was used for computing the electric S_E and magnetic S_M shielding effectiveness of the rectangular enclosure with dimensions of d x a x b (0.3 m x 0.3 m x 0.12 m) having aperture with dimensions of l x w (0.025 m x 0.005 m) the transverse electric $TE_{10}^z - TE_{80}^z$ and transverse magnetic $TM_{11}^x - TM_{81}^x$ field modes were included in the analysis. The results are shown in Figure 3 and Figure 4. It is evident that each of these higher-order modes acts as a high-pass filter. The attenuation provided below cutoff, like that for a loss-free filter, is a reactive attenuation representing reflection but no dissipation [3]. The resonant frequencies of the TE^z_{m0} modes and unbounded medium wavelengths of the rectangular enclosure with aperture are given in Table 1.

Table 1. ***TE^z_{m0} and TM^x_{m1} mode resonant frequencies and unbounded medium wavelengths of the rectangular enclosure with aperture***

Mode order	1	2	3	4	5	6	7	8
TE^z_{m0}, MHz	707.11	1118.0	1581.1	2061.6	2549.5	3041.4	3535.5	4031,1
TM^x_{m1}, MHz	1346.3	1600.8	1952.6	2358.5	2795.1	3250.1	3716.5	4190.8
λ_{gm0}, m	0.4242	0.2683	0.1897	0.1456	0.1177	0.09865	0.08486	0.07442

As seen from Figure 3 and Figure 4 the enclosure shielding effectiveness is significantly reduced at the respective resonant frequencies. This means that above the frequency of a dominant mode resonance, where the density of resonances increases, the enclosure compromised by apertures, slots or slits is not very effective as a shield. However, this relates to an empty enclosure with perfect electric conductor walls.

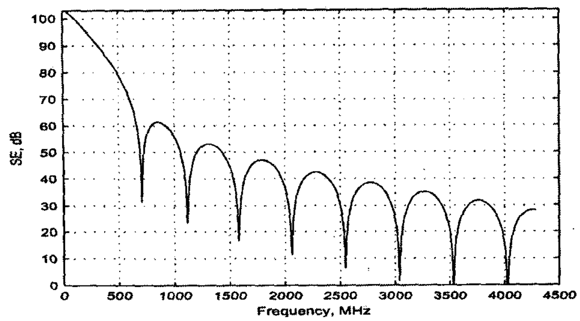

Figure 3: The electric shielding effectiveness of a rectangular enclosure with aperture obtained by multimode simulation

Introduction of Losses Into Enclosure

One can see from Figure 3 that at enclosure resonant frequencies the shielding effectiveness of an empty enclosure with aperture is small or even negative. Losses are introduced in the enclosure to mimic the loading effect of electronics during numerical modeling [7], [9]. It is assumed that the losses are uniformly distributed throughout the enclosure. Distributed losses can be modeled in transmission lines by including a complex correction factor γ in the expressions for characteristic impedance and propagation constant [3]. Applying a similar approach to the multimode model, one obtains for the m-th mode the characteristic impedance Z^{l}_{gm0}

$$Z^{l}_{gm0} = \frac{Z_0}{\sqrt{1-(m\lambda/2a)^2}}(1+\gamma_{gm0}-j\gamma_{gm0}) \tag{25}$$

and propagation constant k^{l}_{gm0}

$$k^{l}_{gm0} = k_0\sqrt{1-(m\lambda/2a)^2}(1+\gamma_{gm0}-j\gamma_{gm0}) \tag{26}$$

For a lossy TL model the equations (25) and (26) are substituted for eqs. (6) and (7).

The accuracy of the developed model was investigated by comparing the predictions with full-field simulations using the FDTD technique [5]. The shielding effectiveness of the rectangular enclosure with aperture of Figure 1 computed at the test location using the FDTD technique is shown in Figure 5. One can observe the shielding effectiveness reduction at higher-order modes which is consistent with that in Figure 3 plotted for the multimode model. One can come to a conclusion that the enclosure is useless at frequencies above the dominant resonance without damping higher-order modes.

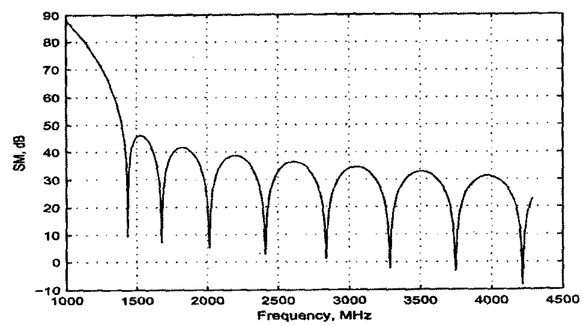

Figure 4: The magnetic shielding effectiveness of a rectangular enclosure with aperture obtained by multimode simulation

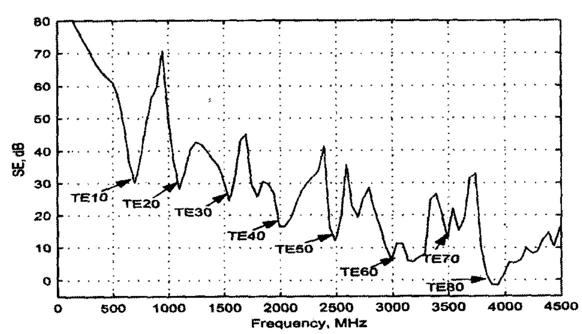

Figure 5: The electric shielding effectiveness of a rectangular enclosure with aperture obtained by FDTD simulation

It is evident from Figure 6 that increasing the losses appreciably dampens higher-order mode resonances of the empty rectangular enclosure thus improving the shielding effectiveness of a high-Q enclosure at higher frequencies. However, little effect of the introduced losses at the dominant-mode frequency is observed. The higher-order mode resonant frequencies are also lowered by incorporating the losses.

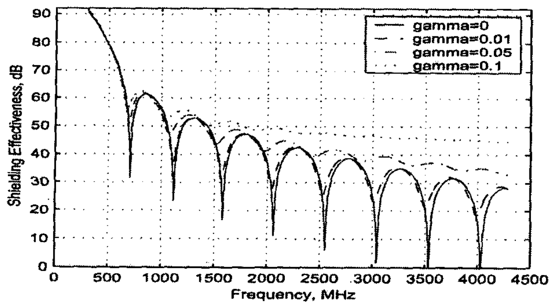

Figure 6: The electric shielding effectiveness of a rectangular enclosure with incorporated losses obtained by multi-mode simulation

Coupling of Electromagnetic Fields into a Rectangular Enclosure with an Aperture Under Resonant Conditions

When an enclosure is compromised with an aperture whose self-resonance is near the enclosure resonance, energy flows freely from outside through the aperture inside the enclosure. A condition of maximum power penetration exists when the resonant frequency of the aperture coincides with the resonant frequency of the enclosure dominant mode. A similar penetration takes place at resonant frequencies of higher-order modes excited in the enclosure as well. However, it is expected that losses introduced in the enclosure by electronics would diminish their effect to a much greater degree in relation to the dominant mode as is seen in Figure 6. These are so called aperture-cavity resonances which produce a significant drop in the shielding effectiveness [8], [9].

Now, referring to the equivalent circuit of Figure 2 and Table 1, we choose the aperture length equal to one-half of the dominant-mode wavelength $\lambda_{g10}/2$. The shielding effectiveness of the rectangular enclosure of Figure 1 for two aperture lengths is shown in Figure 7. One can observe a significant degradation of the shielding effectiveness when the aperture length is equal to one-half of the dominant mode wavelength, *i.e.*, at the aperture-enclosure resonance. The bandwidth of the negative shielding effectiveness has appreciably increased as well.

The condition of the aperture-cavity resonance requires that the dominant-mode reactance $X_d = Im(Z_{gm0})$, $m = 1$ be compensated by the sum of the equivalent source reactance $X_s = Im(Z_s)$ and the reactance due to non-propagating higher-order modes in the enclosure with aperture

$$X_{hm} = Im\left(\sum_m Z_{gm0}\right), \quad m \neq 1 \tag{27}$$

that is,

$$X_s + X_{hm} = X_d \tag{28}$$

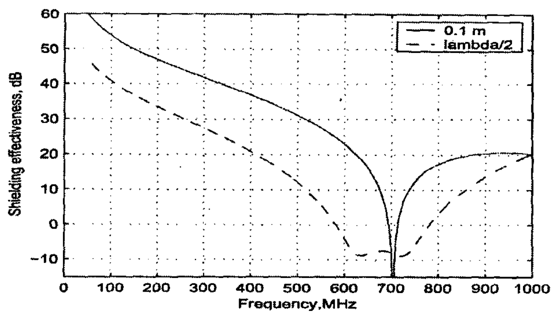

Figure 7: A shielding effectiveness degradation at the aperture-cavity resonance

Depending on the aperture length the reactance in the left-hand side of (28) can be either a positive or negative quantity. The TL simulation under the aperture-enclosure resonance condition based on the TL equivalent circuit of Figure 2, in addition to the natural enclosure resonance, revealed a pair of aperture-enclosure resonances illustrated in Figure 8. The normalized wavelength in Figure 8 is defined as

$$\lambda_n = \lambda_{g10}/d, \tag{29}$$

where λ_{g10} is the dominant mode wavelength for the unbounded medium, and d is the enclosure length.

A similar result was obtained by C. H. Liang *et al.* investigating the electromagnetic coupling of an incident plane wave through a slot-aperture backed by a lossy rectangular cavity by using a generalized network formulation based on the application of the equivalence principle. It was found that under certain conditions, apart from the cavity natural resonance, two types of aperture-cavity resonances may occur. The first aperture-cavity resonance is of the parallel type and the second one is of the series type [8].

The shielding effectiveness of the rectangular enclosure with aperture of Figure 1 under the aperture-enclosure resonant condition obtained by the TL simulation was compared with that obtained by the full-field simulation using the FDTD technique [5]. The obtained results are shown in Figure 9. It is evident that there is good agreement between the two.

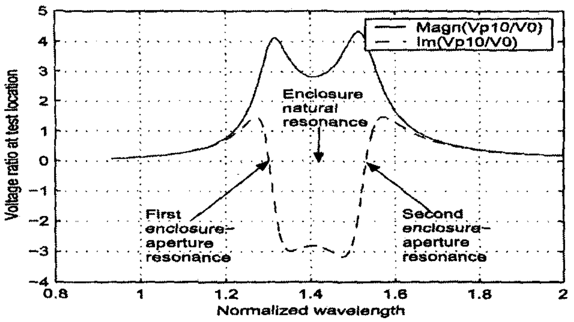

Figure 8: The magnitude and imaginary component of the voltage ratio V_{p10}/V_0 at the test location

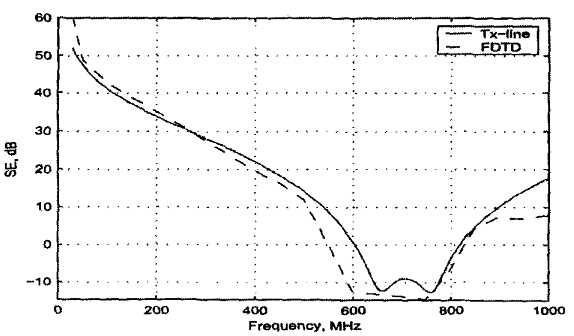

Figure 9: Shielding effectiveness of a rectangular enclosure under resonant conditions

Conclusion

The problem of EM coupling into a rectangular enclosure through an aperture has been studied based on a multimode approach which takes into account the multimode propagation inside the enclosure. The contributions of individual as well as multiple higher-order modes to the shielding effectiveness of the enclosure were considered. It is evident that losses can be easily incorporated in the TL modeling of higher-order modes and may be used to mimic the loading effect of electronics. However, for a lossy TL model contributions of higher-order modes are relatively small.

The electromagnetic coupling into a rectangular enclosure with aperture under aperture-enclosure resonant conditions has been numerically investigated using the TL model. A pair of aperture-enclosure resonances have been found which results in a significant reduction of the shielding effectiveness. The energy transfer into the enclosure may be reduced at the design stage by detuning the aperture-enclosure resonances by geometrically trimming the shape of the aperture or enclosure.

Estimation of the shielding effectiveness based on the multimode approach offers a cost-effective alternative to the FDTD technique, saving significant computing resources. Solution time is the key advantage of the developed analytical method.

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