# ESD Source Modeling in FDTD

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*Abstract* - Numerical modeling of the ESD event using the finite difference time domain (FDTD) technique is considered. Two methods of modeling an electrostatic discharge using FDTD are compared. The first consists of introducing the fields into the mesh from known analytic results in a region of the mesh which is homogeneous and letting the fields propagate towards the interaction of interest. The second consists of modeling the discharge current directly by discretizing the current density term in the Maxwell equations. The second offers obvious advantages in that the discharge can be modeled close to inhomogeneities such as circuit board traces and chip leads.

## INTRODUCTION

Electrostatic discharge (ESD) is a cause of great concern for the electronics industry. Mitigation of the problem is possible once the phenomenon is well understood. Unfortunately the discharge event can occur at an infinite number of places in an electronic system; each having unique geometrical conditions which determine the features of the associated propagating electromagnetic fields. Understanding the propagation of these fields on such system components as circuit boards and interconnect cables and understanding the coupling phenomena to susceptible devices such as IC chips is crucial for the design of proper mitigation procedures.

During the past twenty-five years the finite difference time domain (FDTD) technique has become a mature tool for the analysis electromagnetic fields. The applications for which the method has been used range from radar cross-section determination to nuclear electromagnetic pulse coupling problems (see [1-2]). Recently the technique has been implemented and extended for electromagnetic modelling of electronic packages (see [3]). In that work electromagnetic fields are introduced into the mesh as initial condition(s) and then propagated using the FDTD algorithm. At various points in the mesh electrical properties such as current and voltage can then be calculated from the fields.

The FDTD method is ideally suited to time domain pulse problems and can be made computationally efficient for problems in which the fields are localized in the mesh at any instant of time (see [4]). Thus the method is a logical choice for modelling ESD problems (see [5-6]).

In this paper we discuss two methods in which the fields associated with an ESD event can be modelled. The first method is to introduce known analytic fields into the mesh directly and the second is to discretize the current density source term in the Maxwell-Ampere Law directly in the FDTD update equations. The two methods are compared against each other and against the analytic solution for the case of a transient electric dipole radiating in free space.

## METHOD 1: TANGENTIAL ANALYTIC FIELDS ON A CUBICAL BOUNDARY

In this section we consider the first method of modelling a dipole source by the imposition of the tangential analytic fields on the surface of a cube surrounding the dipole. This method of source modelling is similar to that published by Ziolkowski *et al* [7]. Our FDTD code was first validated by comparing the resulting solution to the exact analytical solution for a similar problem as that considered in [7]. The results were good and the details of this validation are now explained.

An electric dipole oriented along the z-axis was considered. The dipole was located at the centre of a finite difference mesh. For this validation our three dimensional mesh had the potential to grow to 100 x 100 x 100 (i.e. $10^6$) nodes in dimension; hence a large simulation volume was considered. The use of the dynamic memory allocation techniques explained in [4] allowed us to solve this problem using a small Sun SPARCstation (i.e. IPX with 32 MByte RAM).

The mesh was rectangular, uniform and the modelled physical dimensions of each cell in the domain was $\Delta x = \Delta y = \Delta z = 1.0$ [m] while the time step was set to the Courant limit [2]. The electric dipole was enclosed in a 4 x 4 x 4 cube of the mesh. The cube was centred in the mesh and the fields associated with the dipole were represented by specifying the exact tangential magnetic fields on the surface of this 4 x 4 x 4 cube whereas the normal electric fields on the surface were calculated using the Yee's algorithm. All the components inside this cube were set to zero. A pictorial representation of the geometry of the problem is shown in figure 1.

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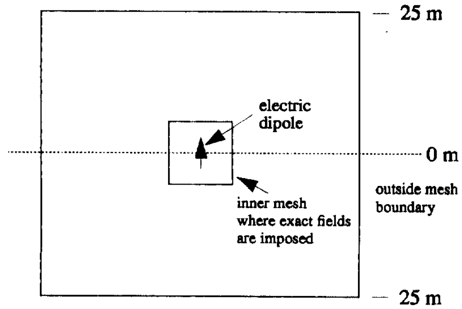


Figure 1 Cross-sectional View of the dipole problem.

The dipole moment (see [7] and [8], p.152) of this electric dipole is given by:

$$\bar{p}(\bar{r},t) = f(t)\,\delta(\bar{r})\,\hat{a}_z \tag{1}$$

where:

$$\frac{\partial \bar{p}}{\partial t} = \bar{J} \tag{2}$$

or

$$\int_{t_0}^{t} \bar{J}\,dt + \bar{p}_0 = \bar{p}, \qquad \bar{p}_0 = \bar{p}(\bar{r}, t_0). \tag{3}$$

The electromagnetic fields associated with such an electric dipole are written as

$$\bar{E}(\bar{r},t) = \frac{\{3(\hat{r}\cdot\hat{a}_z)\hat{r} - \hat{a}_z\}}{4\pi\varepsilon_0 r^3}[f+\tau f'] + \frac{\{\hat{r}\times(\hat{r}\times\hat{a}_z)\}}{4\pi\varepsilon_0 c^2 r}[f''] \tag{4}$$

$$\bar{H}(\bar{r},t) = -\frac{(\hat{r}\times\hat{z})}{4\pi r^2}[f'+\tau f''] \tag{5}$$

where $c\tau = |\bar{r}| = r$ and $[f] = f(t-\tau)$. In rectangular coordinates the specific field components are given as [7]:

$$E_x(x,y,z,t) = \frac{xz}{4\pi\varepsilon_0 r^5}\left\{3[f+\tau f'] + \tau^2[f'']\right\} \tag{6}$$

$$E_y(x,y,z,t) = (y/x)E_x(x,y,z,t) \tag{7}$$

$$E_z(x,y,z,t) = \frac{1}{4\pi\varepsilon_0 r^5}\left\{\left(2z^2 - (x^2+y^2)\right)[f+\tau f'] - (x^2+y^2)\tau^2[f'']\right\} \tag{8}$$

$$H_x(x,y,z,t) = -\frac{y}{4\pi r^3}[f'+\tau f''] \tag{9}$$

$$H_y(x,y,z,t) = -(x/y)H_x(x,y,z,t) \tag{10}$$

$$H_z(x,y,z,t) = 0 \tag{11}$$

where $r = \sqrt{x^2+y^2+z^2}$.

As a compromise between modelling true ESD type waveforms and achieving good performance in the FDTD code, *f(t)* was set to a pure Gaussian pulse (i.e. return to zero). That is

$$f(t) = \begin{cases} 0, & t<0 \\ exp(-\alpha(t-t_0)^2), & t \ge 0 \end{cases}, \tag{12}$$

where $\alpha = 5\times10^{15}$ sec.$^{-2}$, and $t_0 = 3.5\times10^{-8}$ sec.

The computer code was executed for 50 time steps and the results were found to be in excellent agreement with the solution found directly from the analytic equations. Some of the results are shown in figures 2 and 3. The point of observation (52, 54, 58) was chosen to match the observation point chosen in [7]. As can be seen from the figures, the results are almost indistinguishable.

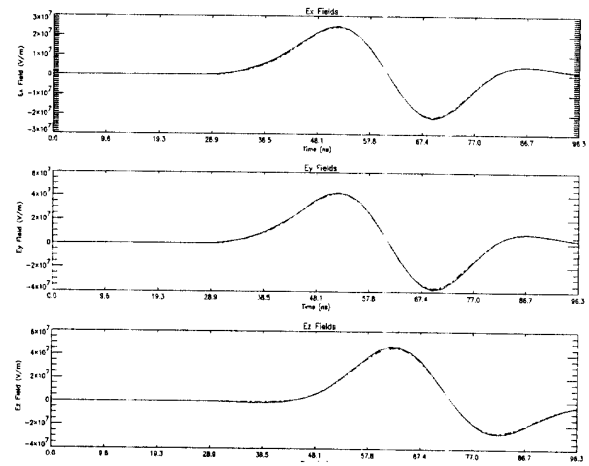


Figure 2 Comparison of Electric Fields at (52, 54, 58) ($\Delta x = \Delta y = \Delta z = 1.0$ m, $\Delta t = 1.925 \times 10^{-9}$ sec.)

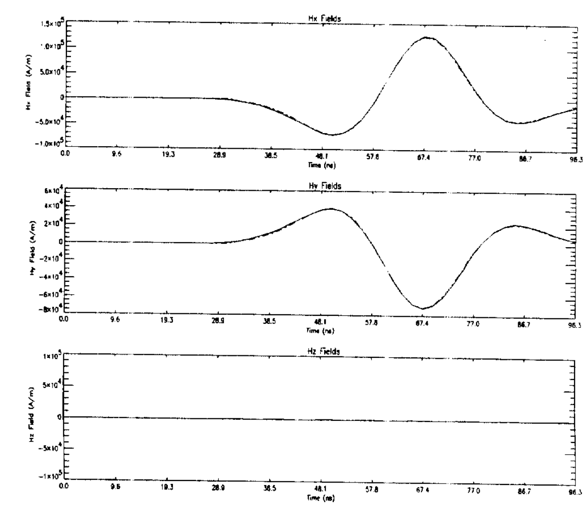


Figure 3 Comparison of Magnetic Fields at (52, 54, 58) ($\Delta x = \Delta y = \Delta z = 1.0$ m, $\Delta t = 1.925 \times 10^{-9}$ sec.)

## METHOD 2: IMPLEMENTATION OF SOURCE TERM IN FDTD

The second method which was used is the discretization of the Maxwell-Ampere Law with the imposed current density included. In rectangular coordinates these can be written as

$$\varepsilon\frac{\partial E_x}{\partial t} = \frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} - \sigma E_x - J_x, \tag{13}$$

$$\varepsilon\frac{\partial E_y}{\partial t} = \frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} - \sigma E_y - J_y, \tag{14}$$

$$\varepsilon\frac{\partial E_z}{\partial t} = \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} - \sigma E_z - J_z. \tag{15}$$

The finite difference method is now applied to these equations. For example, discretizing $E_x$ using the Yee version of Leap-Frog scheme (see [1-3, 9]),

$$\begin{aligned} E_x^{n+1}(i+\tfrac{1}{2},j,k) = \frac{1}{1+\Delta t\sigma_i e_i}\Bigg\{ & E^n{}_x(i+\tfrac{1}{2},j,k) \\ & -\frac{\Delta t}{h}e_i\left[H_z^{n+\frac{1}{2}}(i+\tfrac{1}{2},j-\tfrac{1}{2},k) - H_z^{n+\frac{1}{2}}(i+\tfrac{1}{2},j+\tfrac{1}{2},k)\right] \\ & -\frac{\Delta t}{h}e_i\left[H_y^{n+\frac{1}{2}}(i+\tfrac{1}{2},j,k+\tfrac{1}{2}) - H_y^{n+\frac{1}{2}}(i+\tfrac{1}{2},j,k-\tfrac{1}{2})\right] \\ & -\Delta t e_i\left[J_x^{n+1}(i+\tfrac{1}{2},j,k)\right]\Bigg\} \end{aligned} \tag{16}$$

where $\Delta x = \Delta y = \Delta z = h$, $e = 1/\varepsilon$. Similar expressions can be derived for $E_y$ and $E_z$.

Note that we have used the imposed current density at time n+1, i.e. $J_x^{n+1}$, to update the n+1 electric field, $E_x^{n+1}$. This is not unique, and equation (13) can also be discretized by averaging the current density term at the n+1 and n times. The expression for such a discretization becomes

$$\begin{aligned} E_x^{n+1}(i+\tfrac{1}{2},j,k) = \frac{2}{2+\Delta t\sigma_i e_i}\Bigg\{ & \frac{2-\Delta t\sigma_i e_i}{2}E_x^n(i+\tfrac{1}{2},j,k) \\ & -\frac{\Delta t}{h}e_i\left[H_z^{n+\frac{1}{2}}(i+\tfrac{1}{2},j-\tfrac{1}{2},k) - H_z^{n+\frac{1}{2}}(i+\tfrac{1}{2},j+\tfrac{1}{2},k)\right] \\ & -\frac{\Delta t}{h}e_i\left[H_y^{n+\frac{1}{2}}(i+\tfrac{1}{2},j,k+\tfrac{1}{2}) - H_y^{n+\frac{1}{2}}(i+\tfrac{1}{2},j,k-\tfrac{1}{2})\right] \\ & -\frac{\Delta t}{2}e_i\left[J_x^{n+1}(i+\tfrac{1}{2},j,k) + J_x^n(i+\tfrac{1}{2},j,k)\right]\Bigg\} \end{aligned} \tag{17}$$

It is assumed that the current densities in a Yee cell are located at the same positions as the electric fields. That is, $E_x$ and $J_x$ superposed at the same spatial location. Results based on these equations are now compared with those of method 1.

## COMPARISON OF METHOD 1 AND METHOD 2

It has been shown that the fields due to an electrostatic discharge event can be modeled as an infinitesimal current source [10]. Thus, for our comparison we chose to model the infinitesimal current source as a point source along the z-axis. Hence only one computational cell was used for the implementation of the current source.

In order to properly compare with the results of method 1, the time domain function for our current source was chosen as the derivative of (12), since $\frac{\partial \bar{p}}{\partial t} = \bar{J}$. Thus the f(t) used was:

$$f(t) = \begin{cases} 0, & t < 0 \\ -2\alpha(t-t_0)\exp(-\alpha(t-t_0)^2), & t \geq 0 \end{cases} \tag{18}$$

First, the results obtained from both the discretization schemes, i.e. equations (16) and (17), are compared against method 1 and then they are compared against the exact solution.

With the current source assumed to be located at location (50, 50, 50) in the mesh, time domain plots for the fields at the location (52, 54, 58) are shown in figures 4-11. In all these plots both the FDTD source models of method 2 are shown as a solid line. Figures 4-7 show the fields obtained from method 1 as a dash-dot line, while figures 8-11 show the exact analytic fields as a dash-dot line. As is evident from these plots the results compare very favorably. There is not much difference between the averaged and the non-averaged versions of the FDTD schemes except that the averaged version curves seem to be somewhat smoother than the non-averaged.

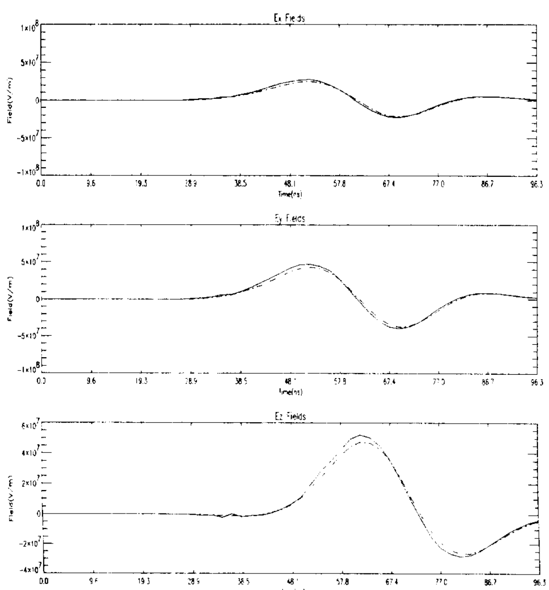


Figure 4 Method 1 vs. Method 2 Comparison of Electric Fields (Time averaging of Current Source Used)

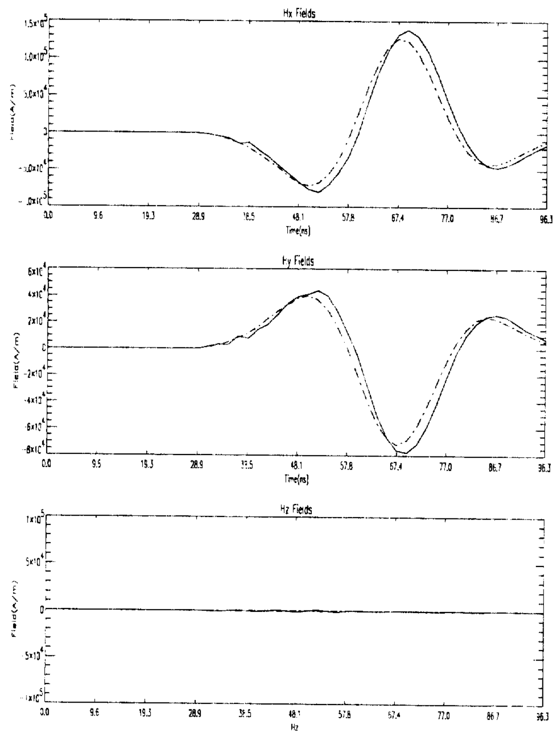


**Figure 5** Method 1 vs. Method 2 Comparison of Magnetic Fields (Time averaging of Current Source Used)

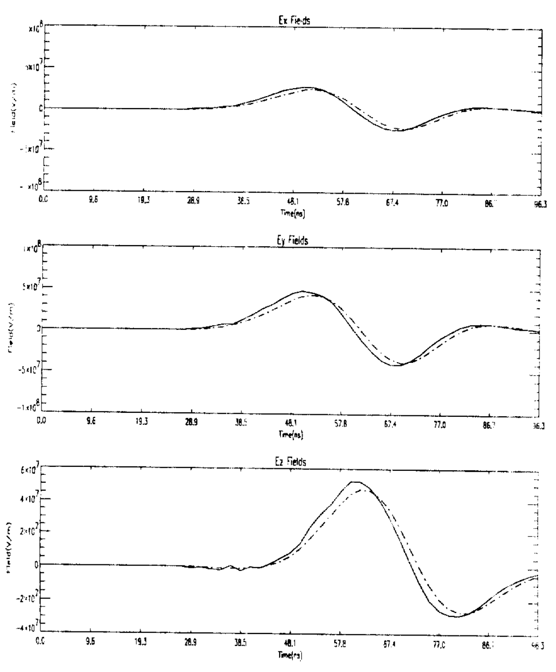


**Figure 6** Method 1 vs. Method 2 Comparison of Electric Fields (No time averaging of Current Source)

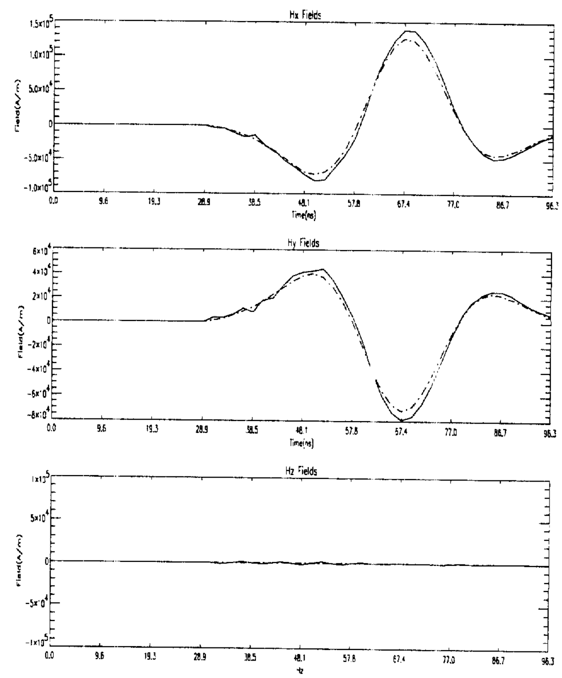


**Figure 7** Method 1 vs. Method 2 Comparison of Magnetic Fields (No time averaging of Current Source)

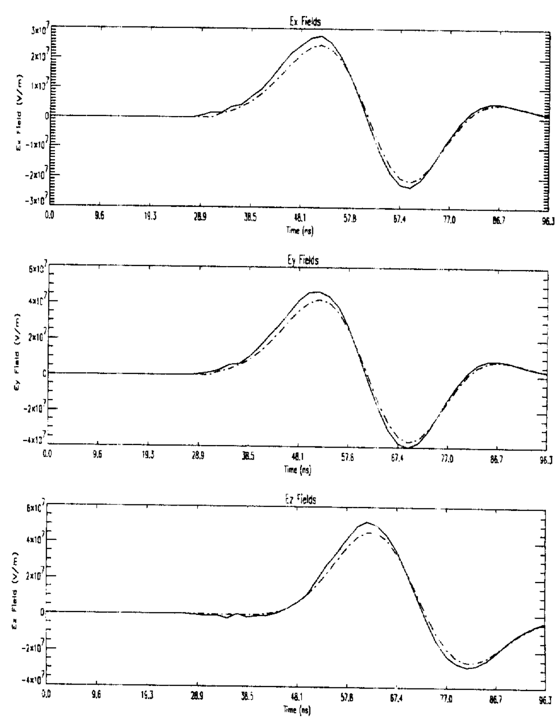


**Figure 8** Method 2 vs. Analytic Comparison of Electric Fields (Time averaging of Current Source Used)

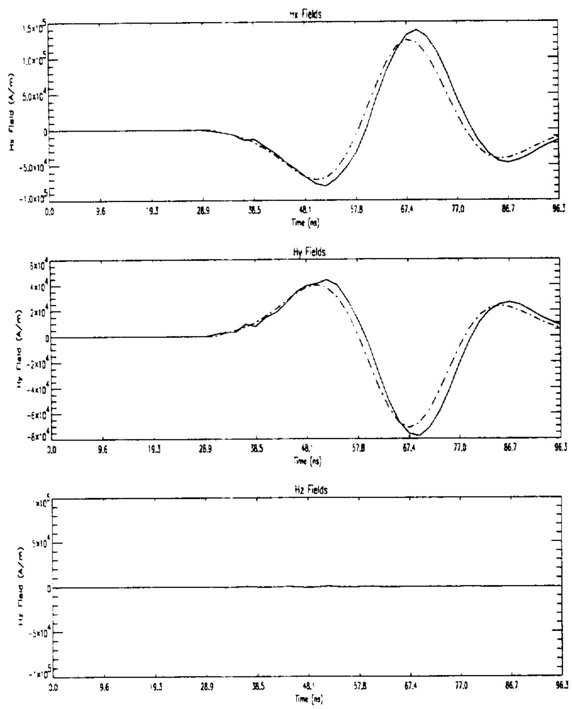


Figure 9 Method 2 vs. Analytic Comparison of Magnetic Fields (Time averaging of Current Source)

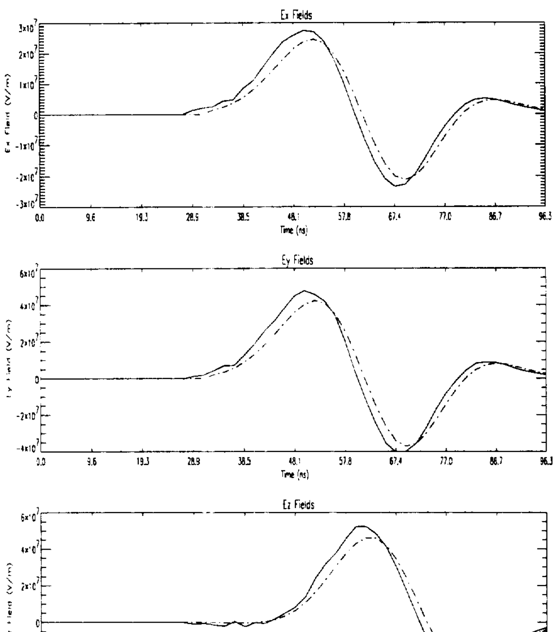


Figure 10 Method 2 vs. Analytic Comparison of Electric Fields (No time averaging of Current Source)

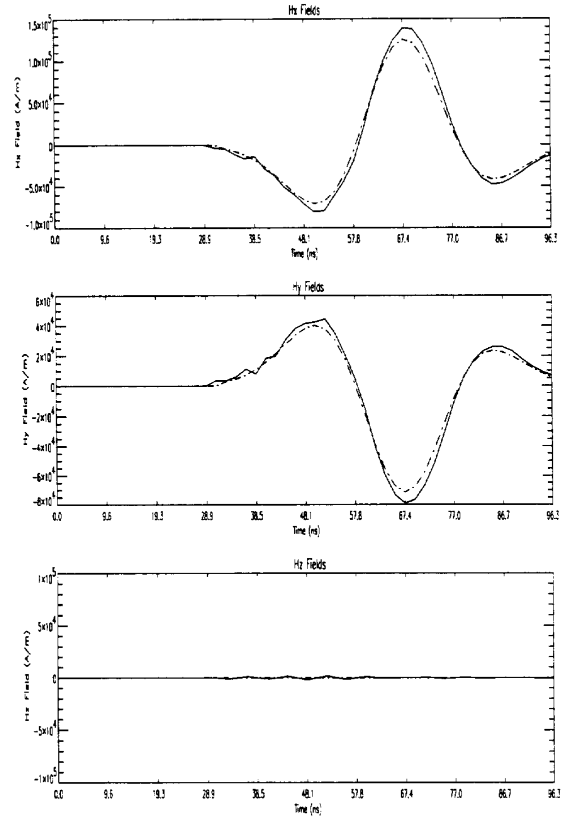


Figure 11 Method 2 vs. Analytic Comparison of Magnetic Fields (No time averaging of Current Source Used)

## CONCLUSIONS

The FDTD algorithm, with extensions for ESD current source modeling have been implemented in an object-oriented fashion similar to that reported in [4]. The code uses second order Mur absorbing boundary conditions to simulate an unbounded region or the region can be allowed to expand. This code has been used to model the ESD phenomena via an infinitesimal current source and both discretization schemes, i.e. (16) and (17), have been compared for accuracy against the exact analytic solution given by equations (4) and (5) in a homogeneous region. The solutions have also been compared against imposing the exact tangential magnetic field components on a 3 X 3 X 3 cube surrounding the current source.

The results are encouraging and it is seen that incorporating the current source density into the FDTD update equations in a simple way achieves acceptable results. With this validation complete, the use of this ESD source model on more complicated inhomogeneous structures can progress with confidence. We will be using this code and this source model to analyze the fields associated with the ESD event on such problems as discharge on a multilayer circuit board.

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