RC18. Primak, S.L., Lo Vetri, J., "Modal Estimates to Describe the Topological Attributes of Multidimensional Systems," IEEE International Symposium on Electromagnetic Compatibility, 1994. Symposium Record. Compatibility in the Loop. 22-26 Aug. 1994, Pages:89 - 94.

MODAL ESTIMATES TO DESCRIBE THE TOPOLOGICAL ATTRIBUTES OF MULTIDIMENSIONAL SYSTEMS.

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Abstract -- A method of constructing steady state modal estimates of multidimensional systems with harmonic external excitation is used to derive the fuzzy electromagnetic topology attributes within upper and lower frequency bounds of the external excitation. These attributes can be used within the electromagnetic topology formalism to estimate the electromagnetic environments inside complex systems like aircraft, ships, helicopters, etc. Modal estimates for the specific example of a power line filter and a multi-conductor transmission line are determined.

INTRODUCTION

When considering the phenomena of electromagnetic interactions in electronic systems one often faces the overwhelming problem of complexity. As well as the wide variety of sources and frequencies for incident electromagnetic fields one is often concerned with the multitude of paths of the electromagnetic propagation into the system. For practical systems one may be confronted with hundreds or thousands of potential signal penetration paths. Some efforts to reduce the dimensionality of these problems were considered in [1-3]. In [2], the concept of fuzzy electromagnetic attributes was introduced and implemented into the HardSys/HardDraw software. In order for this software to be useful, one needs a database of approximate topological attributes for the different components in the systems being considered. In this paper we present a method for obtaining such approximate attributes from state space models of system components. This approach also allows a reduction in the number of calculations required to obtain useful data.

MODAL ESTIMATES OF SYSTEMS

Consider the system described by the multivariable state equation,

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t); \quad \mathbf{y}(t) = \mathbf{C}\mathbf{x}(t) \tag{1}$$

which is driven by an external excitation which is also the input of a continuous finite-dimensional autonomous system and is described by the matrix equation

$$\dot{\mathbf{z}}(t) = \Gamma\mathbf{z}(t); \quad \mathbf{u}(t) = \mathbf{P}\mathbf{z}(t); \quad \mathbf{z}(0) = \mathbf{z}_0 \tag{2}$$

where, in equations (1) and (2), $\mathbf{x}(t) \in R^n$, $\mathbf{y}(t) \in R^n$, $\mathbf{z}(t) \in R^l$, and $\mathbf{u}(t) \in R^r$ are the state vector of the system, the output of the system, the state vector of the excitation and the excitation respectively. The notation R^k denotes the k-dimensional real space, and **A**, **B**, **C**, Γ, **P** are matrices of the proper size.

In steady state, modal estimates can be obtained for the above system on the basis of the assertion that in the steady state a stable system satisfies the vector matrix equation, $\mathbf{x}_s = \pi\mathbf{z}$, where $\mathbf{x}_s$ is the state vector of the system under steady-state conditions, and in general π is the transformation matrix of singular similarity, $\pi \in R^{n\times l}$, [4]. It can also be shown that the matrix π satisfies Sylvester's matrix equation

$$\pi\Gamma - \mathbf{A}\pi = \mathbf{B}\mathbf{P}. \tag{3}$$

With the aid of the above similarity relation we can write the vectors of the state and output of the system, (1), exited by the external finite-dimensional disturbance (2) under steady-state conditions, as [5, 6],

$$\mathbf{x}_s(t) = \pi\mathbf{z}(t) = \pi\exp(\Gamma t)\mathbf{z}_0 \tag{4}$$

$$\mathbf{y}_s(t) = \pi\mathbf{x}_s(t) = \mathbf{C}\pi\exp(\Gamma t)\mathbf{z}_0 \tag{5}$$

Equations (4) and (5) relate the state and output vectors, in the steady state, with the state vector of the excitation only. All information about the properties of the system, (1), is contained in the matrix π which is found using equation (3).

Bounds, in the Euclidean norms $\|\mathbf{x}_s(t)\|$, $\|\mathbf{y}_s(t)\|$, can be obtained using the estimating inequalities for the state and output spaces [5, 6] given by

$$\alpha_{x-}(t) \le \frac{\|\mathbf{x}_s(t)\|}{\|\mathbf{z}_0\|} \le \alpha_{x+}(t), \quad \alpha_{y-}(t) \le \frac{\|\mathbf{y}_s(t)\|}{\|\mathbf{z}_0\|} \le \alpha_{y+}(t) \quad \forall t \tag{6}$$

where $\alpha_x(t) \in \sigma_\alpha\{\pi \exp(\Gamma t)\}$, $\alpha_y(t) \in \sigma_\alpha\{\mathbf{C}\pi \exp(\Gamma t)\}$, $\sigma_\alpha\{(\bullet)\}$ is the algebraic spectrum of the singular values of the matrix $(\bullet)$; $\alpha_{x+}(t)$, $\alpha_{y+}(t)$ are the largest singular values of the matrices $\pi \exp(\Gamma t)$, $\mathbf{C}\pi \exp(\Gamma t)$ and $\alpha_{x-}(t)$, $\alpha_{y-}(t)$ are the smallest singular values of these matrices.

For harmonic excitation the singular values $\alpha_x(t)$ and $\alpha_y(t)$of the matrices $\pi \exp(\Gamma t)$, $\mathbf{C}\pi \exp(\Gamma t)$ are stationary in time, i.e. $\alpha_x(t) = \alpha_x$ and $\alpha_y(t) = \alpha_y$. For the general case of a harmonic multifrequency external signal, the state matrix of the excitation (2) can be written in diagonal block form [5, 6]

$$\Gamma = diag\{\Gamma_{ii}, i = \overline{1, m}\} \tag{7}$$

where the diagonal matrix blocks of minimum dimension are

$$\Gamma_{ii} = \begin{bmatrix} 0 & -\omega_i \\ \omega_i & 0 \end{bmatrix} \tag{8}$$

Here ω_i is the frequency of the external harmonic signal that excites the i-th input of system.

We now confine ourselves to the case of single frequency excitation (by a harmonic signal) of all the system inputs, so that $\omega_i = \omega$ for $i = \overline{1, m}$ in (7) and (8). Then the modal performance estimate, of the processes observed in the state space of system (1) that is expressed by the relation

$$M_+(\omega) = \alpha_{y+}(\omega) \tag{9}$$

will be an upper bound of the amplitude-frequency characteristic of the input-output characteristic of the multidimensional system (1), whereas the modal estimate

$$M_-(\omega) = \alpha_{y-}(\omega) \tag{10}$$

is a lower bound of this characteristic.

APPLYING MODAL ESTIMATES TO TOPOLOGICAL ATTRIBUTES

The topological description of an electromagnetic system using fuzzy logic attributes was presented in [2]. The electromagnetically relevant attributes of an electrical system can be isolated by decomposing the system into its corresponding electromagnetic topology and its dual graph or interaction sequence diagram [1]. The next step in modeling the electromagnetic system is to approximate the propagation of electromagnetic energy from one volume to another. Fuzzy electromagnetic attributes are introduced for each electromagnetic component in the topology as well as for the interaction paths between components. These components approximate the propagation of the electromagnetic disturbance throughout the topology and represent the electromagnetic knowledge which is known about a system.

It is easy to see that if one has the transfer function of a path in a system then the fuzzy topological attributes for that path can be derived immediately. Because we are dealing with fuzzy logic attributes we do not need to know the exact values of the transfer function and we can use the modal estimates introduced in the previous section and calculated for each value ω_i in the specified frequency range.

In such a way we can introduce the following procedure to evaluate the fuzzy logic attributes for multidimensional electronic systems:

1) Decompose the system according to its electromagnetic topology and determine the corresponding interaction sequence diagram [1];

2) Quantize the frequency range $[\omega_i ; \omega_{i+1}], i = 1,\ldots, N$, where N is the number of the ranges [2];

3) Determine the state space equations, in the form of (1), for the specific interaction path being investigated;

4) Construct and solve Silvester's equation with respect to the matrix π;

5) Calculate the spectra $\sigma_\alpha\{\mathbf{C}\pi\}$ of singular values for every range $[\omega_i ; \omega_{i+1}]$

6) In the spectrum of the singular values determine the largest singular values α_{y+} and the smallest α_{y-} singular value for each range $[\omega_i ; \omega_{i+1}]$.

7) Using the estimates of equations (9) and (10) choose the fuzzy level for the path being investigated in each frequency range as the trapezoidal membership function given by

$$FLA = [\min_\omega(\alpha_{y-}(\omega)), \max_\omega(\alpha_{y-}(\omega)), \min_\omega(\alpha_{y+}(\omega)), \max_\omega(\alpha_{y+}(\omega))] \tag{11}$$

where the *maximum* and *minimum* value are found over the whole range $[\omega_i ; \omega_{i+1}]$.

APPLICATIONS

Power Line Filter

As an example let us investigate the propagation of electromagnetic energy through the low-pass filter shown in figure 1. The matrices $\mathbf{A}, \mathbf{B}, \mathbf{C}, \Gamma, \mathbf{H}, \mathbf{P}$ for the state space representation of this filter, in the form (1), are

$$\mathbf{A}=\begin{bmatrix} -R_1/L_1 & -1/L_1 & 0 & 0 & 0 & 0 \\ 1/C_2 & 0 & 0 & 0 & -1/C_2 & 0 \\ 0 & 0 & -R_3/L_3 & -1/L_3 & 0 & 0 \\ 0 & 0 & 1/C_4 & 0 & 0 & -1/C_4 \\ 0 & 1/L_{n2} & 0 & 0 & -R_{n2}/L_{n2} & 0 \\ 0 & 0 & 0 & 1/L_{n4} & 0 & -R_{n4}/L_{n4} \end{bmatrix}$$

$$\mathbf{B}=\begin{pmatrix} 1/L_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1/L_3 & 0 & 0 & 0 \end{pmatrix}^T$$

$$\mathbf{C}=\begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix} \quad \mathbf{D}=\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

where $R_{n2}=R_{n4}=50[\Omega]$, $R_1=R_3=0.1[\Omega]$, $L_1=L_3=0.01[H]$, $L_5=L_6=5.0\cdot10^{-6}[H]$, and $C_2=C_4=20\cdot10^{-6}[F]$. The amplitude of the transfer function along with its estimates are shown in figure 2. The fuzzy logic attributes are presented in the table 1.

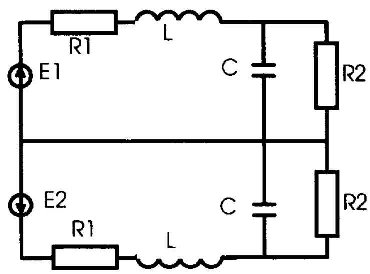

Fig. 1. Principal scheme of the power line filter.

Table 1. Fuzzy attributes for Power Line Filter of Figure 1

Frequency range [Hz]	Shielding effectiveness [dB]
$0-5.0\cdot10^3$	[0, 10, 15, 22]
$5.0\cdot10^3-10^5$	[10, 15, 30, 32]
more than 10^5	[30, 35, 60, 75]

Multiconductor Transmission Line

Although many EMC type problems manifest themselves in electronic circuits and can be formulated as lumped circuit equations, as in the filter example considered above, there are many types of EMC problems which have spatially distributed parameters as opposed to lumped. A good example of such systems are multiconductor cables. Consider the case where we are interested in estimating the transfer function of a multiconductor transmission system from the generator to the load. The first step is to derive the system of ordinary differential equations for the MTL with a passive load on one side and source on the other side of the MTL (see figure 3), where it is assumed that the source can have a source impedance.

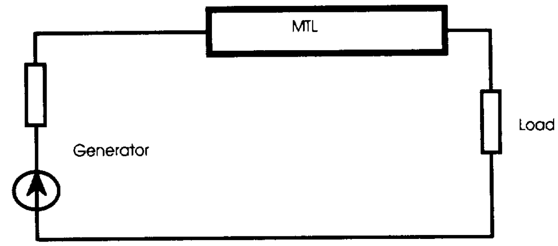

Fig. 3. Scheme of the N-conductor transmission line with load on both sides and generator on left side.

The transmission line equations for an N-conductor transmission line (i.e. N lines and one reference) can be written in the form [7]:

$$\begin{aligned} \frac{\partial \mathbf{I}}{\partial x} &= -\mathbf{G}'\mathbf{V} - \mathbf{C}'\frac{\partial \mathbf{V}}{\partial t} \\ \frac{\partial \mathbf{V}}{\partial x} &= -\mathbf{L}'\frac{\partial \mathbf{I}}{\partial t} - \mathbf{R}'\mathbf{I} \end{aligned} \tag{12}$$

or

$$\begin{aligned} \mathbf{C}'\frac{\partial \mathbf{V}}{\partial t} &= -\mathbf{G}'\mathbf{V} - \frac{\partial \mathbf{I}}{\partial x} \\ \mathbf{L}'\frac{\partial \mathbf{I}}{\partial t} &= -\frac{\partial \mathbf{V}}{\partial x} - \mathbf{R}'\mathbf{I} \end{aligned} \tag{13}$$

where $\mathbf{V}=\mathbf{V}(x,t)$ is the column vector of the line voltages, $\mathbf{I}=\mathbf{I}(x,t)$ is the column vector of the line currents, and $\mathbf{G}'$, $\mathbf{C}'$, $\mathbf{L}'$, $\mathbf{R}'$ are the per-unit conductance, capacitance, inductance and resistance matrices respectively. All matrices are of size $N\times N$.

We now divide the line into M equal sections of length Δl and replace each section by a lumped equivalent circuit having parameters:

$$\mathbf{R}=\mathbf{R}'\Delta l, \mathbf{G}=\mathbf{G}'\Delta l, \mathbf{L}=\mathbf{L}'\Delta l, \mathbf{C}=\mathbf{C}'\Delta l \tag{14}$$

The model for an arbitrary i-th section is shown in figure 4. It can be easily shown that this new system, which has been discretized in space, obeys the following systems of the ordinary differential equations:

$$\mathbf{L}\frac{d\mathbf{i}_1}{dt} = -(\mathbf{R}_G+\mathbf{R})\mathbf{i}_1 + \mathbf{v}_2 + \mathbf{E} \tag{15a}$$

at the generator side of the line

$$\begin{aligned} \mathbf{L}\frac{d\mathbf{i}_{2j-1}}{dt} &= \mathbf{u}_{2j-2} - \mathbf{R}\mathbf{i}_{2j-1} - \mathbf{v}_{2j} \\ \mathbf{C}\frac{d\mathbf{v}_{2j}}{dt} &= \mathbf{i}_{2j-1} - \mathbf{G}\mathbf{v}_{2j} - \mathbf{i}_{2j+1} \end{aligned} \tag{15b}$$

for an arbitrary j-th node, and

$$C\frac{dv_{2K}}{dt} = i_{2K-1} - (G + G_l)v_{2K} \tag{15c}$$

at the load side, where R_G and G_l are the source resistance of the generator and conductance of the load respectively.

Before solving this system of differential equations we normalize the different current and voltages variables using the following transformation:

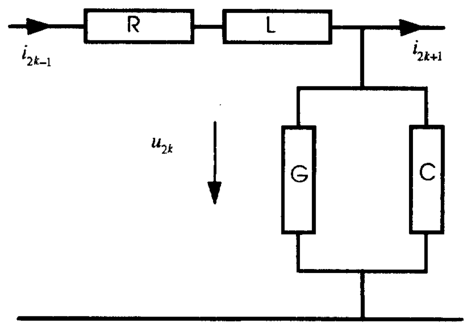

Fig. 4. Equivalent cell for the N-conductor transmission line.

$$\begin{aligned}
x_{2k-1} &= L^{-1/2} i_{2k-1}, \\
x_{2k} &= C^{-1/2} v_{2k}, \\
\omega_i &= L^{-1/2} C^{-1/2}, \\
\omega_i &= C^{-1/2} L^{-1/2}, \\
\alpha_i &= L^{-1/2} R L^{-1/2}, \alpha_{iG} = L^{-1/2} R_G L^{-1/2}, \\
\alpha_u &= C^{-1/2} G C^{-1/2}, \alpha_{ul} = C^{-1/2} G_l C^{-1/2}
\end{aligned} \tag{16}$$

This transformation is used in order to create new variables (i.e. the column vector $\mathbf{x}$), from the current and voltage variables, having the same order of magnitude. Thus we can obtain a system of ordinary differential equations in the standard form

$$\begin{aligned}
\dot{x} &= Ax + Be \\
y &= Fx
\end{aligned} \tag{17}$$

where $x = \{x_1, x_2, \cdots, x_{2M-1}, x_{2M}\}^T$ is the state vector, $e = \{L^{-1/2}E, 0, \cdots, 0\}$ is the source or input vector, y is the output vector,

$$B = \begin{bmatrix} 1_N \\ 0_N \\ \cdots \\ 0_N \end{bmatrix},$$

$F = [0_N \ \cdots \ 0_N \ 1_N]$ where $1_N, 0_N$ are the identity and zero matrices of the size $N \times N$, and the matrix A has the form

$$A = \begin{bmatrix}
-\alpha_G - \alpha & -\omega_i & 0_N & 0_N & \cdots & 0_N & 0_N \\
\omega_u & -\alpha_u & -\omega_u & 0_N & \cdots & 0_N & 0_N \\
0_N & \omega_u & -\alpha_i & -\omega_i & \cdots & 0_N & 0_N \\
\cdot & \cdot & \cdot & \cdot & \cdots & \cdot & \cdot \\
0_N & 0_N & 0_N & 0_N & \cdots & -\alpha_i & -\omega_i \\
0_N & 0_N & 0_N & 0_N & \cdots & \omega_u & -\alpha_u - \alpha_l
\end{bmatrix} \tag{18}$$

Thus a system of ordinary differential equations is built from a system of partial differential equations by discretizing the spatial coordinates. It should be noted here that this method of obtaining the ODE's is equivalent to using the method of lines in a purely mathematical way.

We now apply the algorithm of section 2 to equation (14) in order to obtain the required estimates for the transfer function. Figure 5 shows the exact response (*-line), upper (solid line) and lower (dashed line) bounds of the transfer function for a two conductor line (N = 2) with the following per-unit parameter [7]:

$$L' = \begin{bmatrix} 1.1775 & 0.9034 \\ 0.9034 & 1.1738 \end{bmatrix} [\mu H/m], \quad C' = \begin{bmatrix} 52.16 & -24.34 \\ -24.40 & 51.80 \end{bmatrix} [pF/m],$$

$$R' = \begin{bmatrix} 1.06 & 0.03 \\ 0.03 & 1.27 \end{bmatrix} [\Omega/m], \quad G' = \begin{bmatrix} 1.6 & 0.05 \\ 0.05 & 1.8 \end{bmatrix} [mS/m].$$

The input resistance and output conductance for this example are chosen as

$$R_g = \begin{bmatrix} 50 & 0 \\ 0 & 50 \end{bmatrix} [\Omega], \quad G = \begin{bmatrix} 0.02 & 0.005 \\ 0.005 & 0.02 \end{bmatrix} [S]$$

and the length of the line is 2 [m]. The transfer function is found by choosing the input voltage as

$$E = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \sin(\omega t),$$

where ω is the frequency for which we wish to calculate the transfer function. In the frequency range $0 - 10^9$ [rad/s] we choose 100 points of discretization, i.e. $\Delta l = 0.03$ [m], so we have at least 10 points per wavelength. In the frequency range $10^9 - 10^{11}$ [rad/s] we use 250 points, so $\Delta l = 0.012$ m and at least 3 points per wavelength.

Once the system is solved, the following fuzzy attributes are determined:

Table 2. Fuzzy attributes for Transmission Line of Figure 3

Frequency range [rad/s]	Shielding effectiveness [dB]
$0 - 5.0 \cdot 10^6$	[3, 3, 3, 5]
$5.0 \cdot 10^6 - 2.0 \cdot 10^7$	[3, 5, 7, 10]
$2.0 \cdot 10^7 - 10^8$	[5, 10, 12, 17]
$10^8 - 10^{11}$	[7, 10, 15, 17]

It should be noted that although the frequency ranges chosen in Table 2 are very large, there is no fundamental restriction in the size of the frequency ranges. Equation (11) can be applied to any size frequency range. Of course, in some cases, the narrower the frequency ranges one chooses the better the resulting approximation will be.

The computer resources which were required to solve the reduced system are much reduced from solving full system. The resources required for the exact solution of the equations (15) is about 254 Mflops while the resources required for finding the upper and lower bounds takes only 45 MFlops.

CONCLUSIONS

The method presented in this paper allows one to find approximate topological attributes of complex systems via their state space representation in the steady state. Using scalar estimates, even for vector-input and vector output systems, leads to a reduction in the number of calculations. This approach will be useful in acquiring attributes to fill the database of the HardSys advisor [2]. We have applied the technique to two sample EMC problems: EMI filter and a multiconductor transmission line problem. The technique can also be applied to EMC problems formulated as field problems. In such a case, the reduction to a system of ordinary differential equations is accomplished via the method of lines. This case will be considered in a future paper.

It is also interesting to note that the comparison method can be formulated in the time domain and in such a way time domain bounds can be derived. This avenue will also be pursued in future work.

The actual use of this method to derive a reasonably extensive knowledge base for the EMC advisor will also be investigated. The purpose is to have a set of integrated tools for EMC analysis [8].

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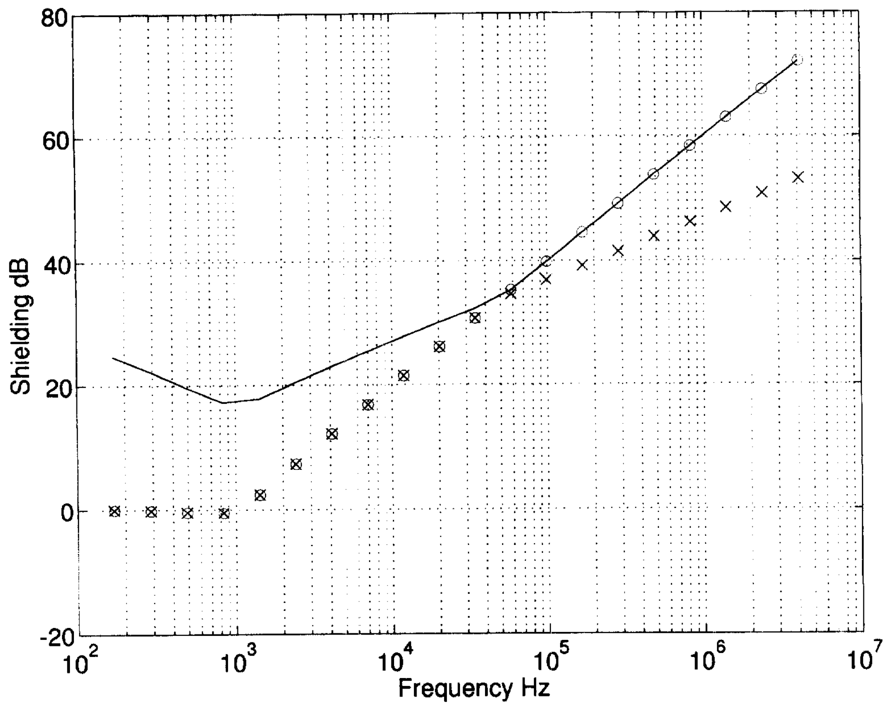

Fig. 2. Bounds and exact solution for power line filter: solid-line - upper bound, x-line - lower bound, o-line - exact solution.

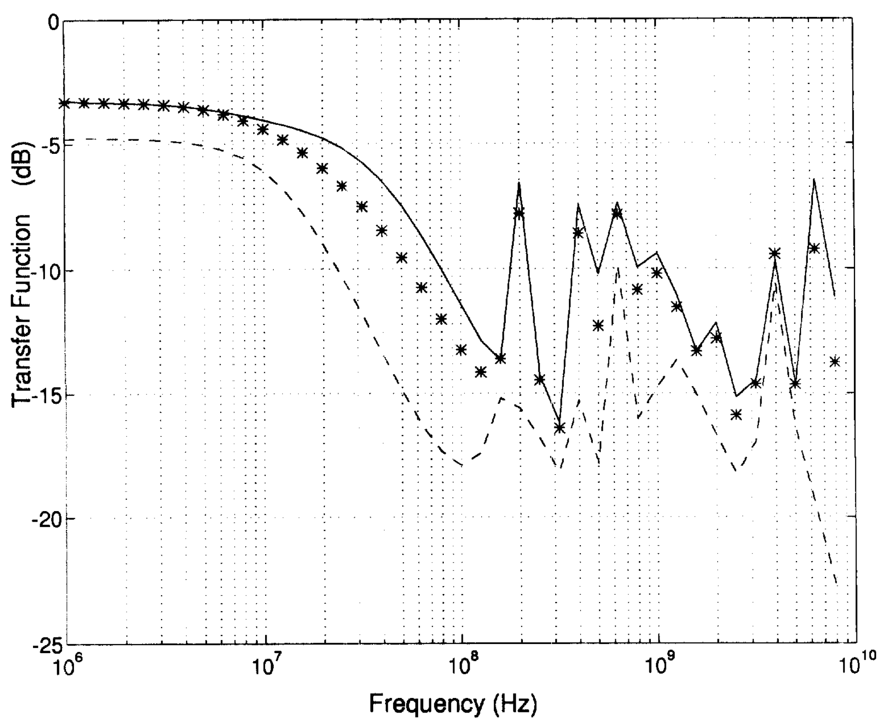

Fig. 5. Bounds and exact solution for transmission line problem: solid-line - upper bound, dashed-line - lower bound, *-line - exact solution.