RANDOM UPDATE TLM METHOD

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Abstract: A combination of the standard Transmission Line Matrix (TLM) method with a new interface transfer event and Glimm's Random Choice method are used to remove the dispersion inherent in the TLM method when transmission line stubs are used to model inhomogeneous media. The method gives excellent results when used on the plane wave propagation through a dielectric slab problem.

A. Introduction

A wide variety of electromagnetic phenomena involve the interaction of electromagnetic fields with material regions. As the inhomogeneous nature of the material regions increase, the efficiency and suitability of differential equation based numerical methods increase. A problem with most differential equation based methods is that numerical dispersion errors increase with material region parameters (i.e., relative permittivity, relative permeability, and conductivity). The purpose of this paper is to describe a method of reducing the numerical dispersion when applying the Transmission Line Matrix method (TLM) in material regions. It has been shown that the standard condensed node Transmission Line Matrix method [1] for approximating Maxwell's equations can be viewed as the successive solution of three one-dimensional Riemann problems and that the *voltage pulses* are equivalent to the *Riemann Invariants* (RI's) [2]. Glimm's Random Choice method was formulated for exactly such a problem [3]. Thus it follows that a combination of the two methods is appropriate.

B. Dielectric Modelling in TLM

In the modelling of an isotropic frequency- independent medium, the numerical velocity of wave propagation depends on both the direction of propagation, the frequency of excitation, and the Courant number (related to the ratio of spatial discretization, Δl to temporal discretization, Δt) [4]. For explicit time integration schemes it is generally known that the optimal Courant number ($c\Delta t/\Delta l$), in terms of minimizing numerical dispersion and anisotropy, is the upper limit of stability. Unfortunately, modeling material regions has the same effect as lowering the Courant number. For example, a simulation consisting of a free-space region and a dielectric region, can maintain an optimal Courant number in the free space region, but cannot in the dielectric region.

The standard TLM technique for modelling inhomogeneous media is to introduce transmission line stubs [1] at the cell centers which absorb part of the energy and delay its propagation through the mesh. The numerical dispersion manifests itself as ripples preceding the wavefront. From the analysis contained in [5, 6], the use of stubs produces the undesirable effect equivalent to lowering the Courant number. An example of this dispersion is shown in figure 1 for a plane wave pulse travelling in a homogeneous medium of $\varepsilon_r = 2$. A Fourier transform of the waveform is included to better see the effects of the dispersion (i.e. higher frequency components are introduced, vis-à-vis figure 2 second waveform). Of course this is not the only source of the dispersion but it is a major contributor.

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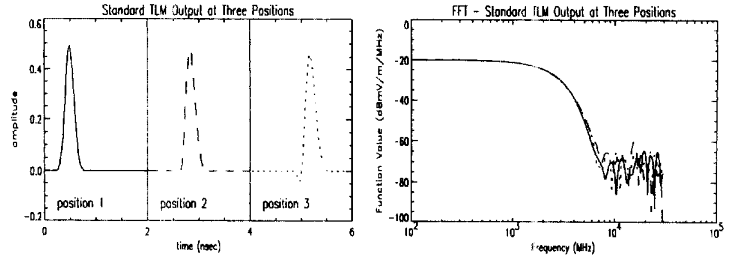

Figure 1. standard stub modelled TLM of plane wave pulse in homogeneous medium ($\varepsilon_r = 2$)

A new transfer event was formulated in [2] based on applying appropriate boundary conditions to the RI's at the boundaries between cells. For example, consider the left and right propagating RI's of both y and z polarizations where the *i'th* cell is to the left of the *(i+1)'th* cell; the new transfer event can be stated as

$$\begin{bmatrix} V_{Ly} \\ V_{Lz} \\ V_{Ry} \\ V_{Rz} \end{bmatrix}_{i+1/2}^{n+1/2^+} = \begin{bmatrix} \frac{2Y_{i+1}}{Y_{i+1}+Y_i} & 0 & \frac{Y_{i+1}-Y_i}{Y_{i+1}+Y_i} & 0 \\ 0 & \frac{2Y_{i+1}}{Y_{i+1}+Y_i} & 0 & \frac{Y_{i+1}-Y_i}{Y_{i+1}+Y_i} \\ \frac{Y_i-Y_{i+1}}{Y_{i+1}+Y_i} & 0 & \frac{2Y_i}{Y_{i+1}+Y_i} & 0 \\ 0 & \frac{Y_i-Y_{i+1}}{Y_{i+1}+Y_i} & 0 & \frac{2Y_i}{Y_{i+1}+Y_i} \end{bmatrix} \begin{bmatrix} V_{Ly} \\ V_{Lz} \\ V_{Ry} \\ V_{Rz} \end{bmatrix}_{i+1/2}^{n+1/2^-}$$

where $Y^2 = \varepsilon/\mu$ is the admittivity of the cell. Thus the inhomogeneity is handled by the transfer event at the interface only and not by stubs throughout the medium. Applying this method as formulated in [2] would require the use of a different Δl_i in each region i of different permeability or dielectric constant in order that $c\Delta t/\Delta l_i = 1$ for each region i (alternatively the Δt could be changed with obvious implications). Another alternative is to *trigger* the transfer event less than 100% of the time by the scheme discussed below.

C. Incorporation of Glimm's Random Choice Method in TLM

Glimm's method operates by propagating the RI's in a stochastic fashion [3]. This was incorporated into the TLM method by *triggering* the transfer event according to a predetermined binary trigger vector of size N (the total number of time steps). If the trigger vector contains a *1* at time step n then the transfer event is allowed otherwise the transfer event is disallowed. A different trigger vector must be determined for each region of differing dielectric constant or permeability. The trigger vector is filled with approximately $T = (c_2/c_1)N < N$, *1's* equally distributed across the vector where c_2 is the propagation speed in the dielectric region and c_1 is the free space speed. This was done by first determining a whole number ratio p_1/p_2 such that $T = N(p_1/p_2) + q$ and N/p_2 is a whole number, then filling every p_2 positions with p_1 *1's* equally distributed over the interval. The remaining $N(1-p_1/p_2)$ trigger vector positions were filled by generating $N(1-p_1/p_2)$ random variables ζ_i equally distributed between *0* and *1* and setting the *i'th* position to *1* if $\zeta_i < q/N(1-p_1/p_2)$ This method reduces the occurrences of a large number of successive *0's* in the trigger vector.

D. Homogeneous Test Problem

The method was first tested for the case of a plane wave propagating in a 3-D homogeneous media of relative permittivity ($\varepsilon_r = 2$). The results showing the time domain waveform at three different positions along the point of wave travel are given in figure 2. The first set of waveforms were smoothed by a linear interpolation algorithm at points of no-trigger. The second set uses an algorithm based on the trigger vector itself to smooth out the waveform. This algorithm consists of removing the time domain points for which the trigger value is *0* and then multiplying the time vector by N/T. It can be shown that this algorithm is exact for the homogeneous problem but is approximate for the inhomogeneous problem. Note that the second set of waveforms reproduces the exact solution to the problem.

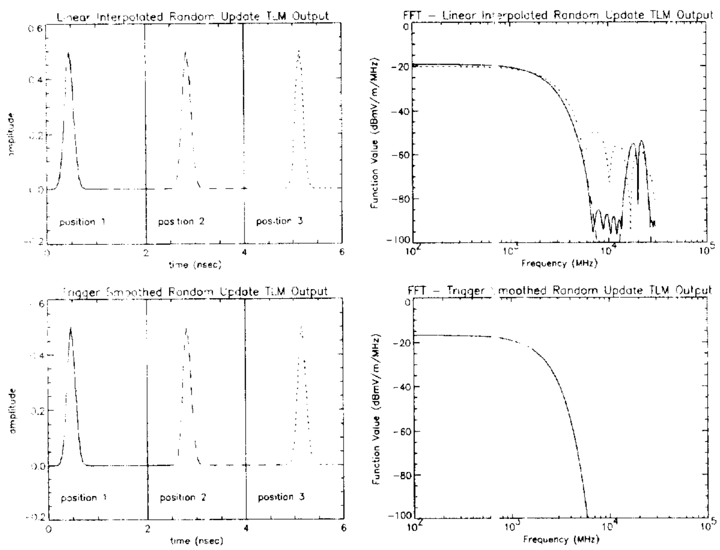

Figure 2. random update implementation with linear interpolation and trigger guided smoothing

E. Dielectric Slab Test Problem

The dielectric slab test problem was chosen to compare the standard TLM method and the new method. The results are shown in figure 4 for three different snapshots in time. As can be seen from the figure, stub modelling results in numerical dispersion whereas the random update method produces no dispersion. No smoothing was required here since these are plots of spatial waveforms. Of course the phase velocity of the wave in the random update results will only be correct on average.

F. Conclusions

A new random update TLM method has been formulated. This method gives excellent results for the simple cases tested herein. More complex inhomogeneities need and will be tried to more fully explore the method.

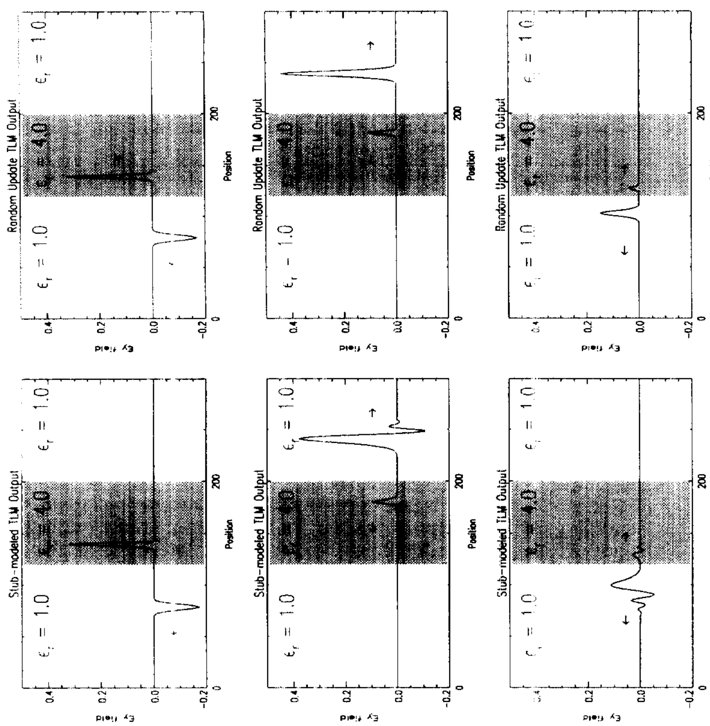

Figure 3. 3-D plane wave penetration into dielectric slab

G. References

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