TOWARDS A THEORY OF SUBMARINE MAST DETECTION

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INTRODUCTION

Complex radar targets are often modelled as a number of individual scattering elements randomly distributed throughout the spatial region containing the targets. A common assumption in this case requires independent scatterers with a uniformly distributed phase [1]. Asymptotically, this leads to a Gaussian distribution of the scattered signal or a Rayleigh distributed Radar Cross Section (RCS). However, a class of problems exists where the target must be modelled as a random scatterer and the number of scatterers is not large enough to safely apply the Large Numbers Theorem. One such problem is the detection of submarine masts in a clutter. Here we consider an alternative approach to describe the statistics of the RCS based on deterministic target response modelling and a statistical description of the target movement.

CONVENTIONAL MODEL OF AN EXTENDED TARGET

A common model for complex or extended radar targets is to consider them as a collection of randomly distributed scattering elements [1]. Each element making up the extended target is assumed to be a point isotropic scatterer (*i.e.* a scattering centre). The general statistics of the resulting radar cross-section is very difficult to derive. A physical interpretation of the scattering in the very high frequency region suggests that a deterministic target consists of a relatively small number of scattering centres (bright points) [2, 3]. In one of the models considered in [1], a radar resolution cell is assumed to contain a collection of n elemental points randomly distributed throughout the resolution cell with each scatterer position distributed independently of the position of the other scatterers. This distribution was assumed to be uniform on the surface of the scattering object. Each backscattered electric field component E_j from the j-th scatterer has a constant amplitude A_j proportional to the size or reflection strength of the j-th scatterer and a random phase ϕ_j

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uniformly distributed over the interval $[0, 2\pi]$. The superposition of the radar responses from each of the n elementary scatterers gives rise to the total backscatter electric field from the resolution cell as

$$M_n = \sum_{j=1}^{n} E_j = \sum_{j=1}^{n} A_j e^{j\phi_j}. \quad (1)$$

The overall intensity, S_n, measured by a radar (RCS) is proportional to the square of the magnitude of M_n, *i.e.* it is given by

$$S_n = \left| \sum_{j=1}^{n} A_j e^{j\phi_j} \right|^2. \quad (2)$$

Exact expressions for the Probability Density Function (PDF) of the RCS of one and two point scatterers were obtained in [1] as

$$p_1(s_1) = \delta(s_1 - A_1^2), \quad (3)$$

$$p_2(s_2) = \begin{cases} \dfrac{1}{\sqrt{((\sqrt{s_2} + A_2)^2 - A_1^2)(A_1^2 - (\sqrt{s_2} - A_2)^2)}} & \text{if } (\sqrt{s_2} - A_2)^2 < A_1^2 < (\sqrt{s_2} + A_2)^2 \\ 0 & \text{elsewhere} \end{cases} \quad (4)$$

Even more complicated results can be obtained in the case of three scatterers [1]. If the number of scatterers exceeds four, the exact solution is impossible to obtain, nevertheless, a very good approximation can be obtained through the Gram-Chalier expansion [1].

Unfortunately, assumptions of an independent scatterer with constant intensity and uniformly distributed phase has no real *physical* background, in contrast to communication applications, where such a representation was found to be realistic [4]. We will consider in detail why the aforementioned schemes cannot be applied to our specific problem, that is mast detection, as was formulated in [5, 6].

REALISTIC MODEL OF EXTENDED TARGETS

One of the possible applications of ultrawide band radar is the detection of a submarine by the detection of its masts (periscopes, communications masts, *etc.*), usually located above the sea surface, as shown in Fig. 1. The RCS of this target is a *random* function, because it depends on the sea state, which induces a rocking motion and uncertainty in the submarine location (depth). In addition, the returned signal may be embedded in a strong sea clutter. Due to these factors, detection of a submarine is a problem of detecting a random signal in clutter. The general solution is too complicated to handle analytically. However, some simplification can be made in order to reduce the complexity of the problem to a manageable size. The following are the main issues: a) determining a statistical model of RCS; b) a statistical model of the clutter; and c) finding detection algorithms. We will concentrate our attention on the first problem only. Also, we confine ourselves to the following simplified model of a submarine: we will consider only one mast and assume that it can be modelled as a cylindrical rod. In addition, only a rocking type motion, induced by the sea will be considered.

It was found, that the most suitable analytical technique for describing the high-frequency scattering from a man-made target is the so called Geometrical Theory of

Diffraction (GTD) [7]. In the frame of this theory, a target can be represented as a set of point scatterers, with frequency response $F(\omega, \varphi)$ given as

$$F(\omega, \varphi) = A(j\omega)^{\alpha} e^{|\beta\varphi|}. \tag{5}$$

Here ω is the operating frequency of the radar, φ is the aspect angle of the scatterer with respect to the incident wave, and α is a half-integer parameter (*i.e.* $\alpha = 0, \pm 0.5, \pm 1, \pm 1.5, \ldots$), representing different types of scattering mechanisms (see [3, 7] for details). Equation (5) already shows that elementary scatterers cannot be considered as isotropic. On the other hand, GTD predicts that scattering centres must appear where a conducting body has a singularity like a sharp edge. This contradicts the assumption that scatterers are uniformly distributed on the surface of a body. Finally, the assumption that all partial sources are independent is also contradictory. In fact, the roughest model of a submarine, shown in Fig. 2, requires that the relation between the amplitude and the phase of both the real and the imaginary (*i.e.* mirrored) scattering centres are deterministically related, *i.e.* just the opposite to the standard assumption.

The previous considerations show that a different approach should be taken in order to describe the statistical properties of the RCS. Such a model should involve a deterministic analysis of the target, producing a representation of a target's RCS as a response from a set of point scatterers. This problem, at least for very high frequencies, can be solved using either the Physical Optics (PO) approximation [7] or GTD, mentioned above. Yet another option is the numerical simulation using any of a variety of methods, such as FDTD, Finite Elements, *etc.* In the second step, randomness has to be introduced through a statistical description of the position of the target under consideration. A number of models for the sea surface can be of great help. For the sake of simplicity we will use the simplest model which results in the assumption that the deflection angle γ, defined as shown in Fig. 2 is considered as a uniformly distributed random variable in the interval $[\gamma_{min}, \gamma_{max}] \in [-\pi/2, \pi/2]$. Further refinements may be made in future work.

The difference between our models and conventional ones can be shown immediately on the simplest case of a single scatterer. In this case, its RCS can be expressed according to (5) as

$$S(\omega, \varphi) = |F(\omega, \varphi)|^2 = |A(j\omega)^{\alpha} e^{|\beta\varphi|}|^2 = A^2\omega^{2\alpha}\exp(2|\beta\varphi|). \tag{6}$$

Now if the aspect angle φ is a random variable (due to the rocking motion of the boat), and is uniformly distributed on the interval $[-\Delta\varphi, \Delta\varphi]$, *i.e.*

$$p_{\varphi}(\varphi) = \begin{cases} \dfrac{1}{2\Delta\varphi} & \varphi \in [-\Delta\varphi, \Delta\varphi] \\ 0 & \varphi \notin [-\Delta\varphi, \Delta\varphi] \end{cases} \tag{7}$$

then the distribution of RCS is given by

$$p_{RCS}(S) = \begin{cases} \dfrac{1}{2S\exp(\beta\Delta\varphi)A^2\omega^{2\alpha}} & S \in [\exp(\beta\Delta\varphi)A^2\omega^{2\alpha}, A^2\omega^{2\alpha}] \\ 0 & S \notin [\exp(\beta\Delta\varphi)A^2\omega^{2\alpha}, A^2\omega^{2\alpha}] \end{cases} \tag{8}$$

which is quite different from that given by (3).

Let us now turn our attention to the case of two *independent* scatterers, each of them represented by the model given by (5),

$$F(\omega, \varphi) = A_1(j\omega)^{\alpha_1} e^{|\beta_1\varphi_1|} + A_2(j\omega)^{\alpha_2} e^{|\beta_2\varphi_2|} \tag{9}$$

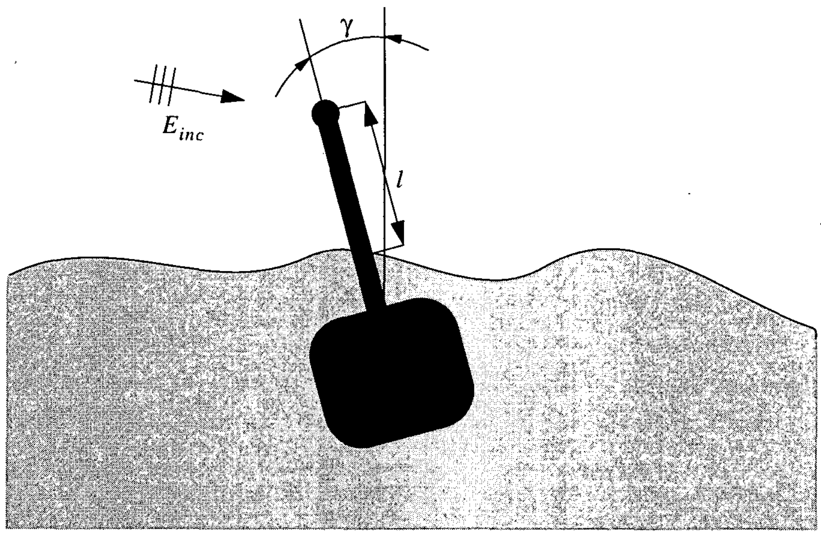

Figure 1. Typical set-up for submarine detection.

and thus

$$
\begin{aligned}
S(\omega, \varphi) &= \left|A_1(j\omega)^{\alpha_1}e^{|\beta_1\varphi_1|} + A_2(j\omega)^{\alpha_2}e^{|\beta_2\varphi_2|}\right|^2 = \\
&A_1^2\omega^{2\alpha_1}\exp(2|\beta_1\varphi_1|) + A_2^2\omega^{2\alpha_2}\exp(2|\beta_2\varphi_2|) + \\
&2A_1A_2\omega^{\alpha_1+\alpha_2}\exp(|\beta_1\varphi_1| + |\beta_2\varphi_2|)Re\{j^{\alpha_1+\alpha_2}\}
\end{aligned} \tag{10}
$$

The complete analytical investigation of this case is rather impossible, however numerical simulations for a particular case can be easily implemented.

Finally we will consider a very special case of the geometry: when two scatterers are identical, but not independent. This setup has a very close relation to the problem of mast RCS statistics. In fact, the strongest contribution to the backscattered fields are from the edge of a mast and from its mirror image as shown in Fig. 2. This also means that the two scatterers are dependent and

$$
\begin{aligned}
S_2(\omega, \varphi) &= \left|A_1(j\omega)^{\alpha_1}e^{|\beta_1\varphi|} + \rho(\omega, P)A_1(j\omega)^{\alpha_1}e^{|\beta_1\varphi|}\right|^2 = \\
&A_1^2\omega^{2\alpha_1}|1 + \rho(\omega, P)|^2e^{|\beta_1\varphi|} = A^2e^{|\beta_1\varphi|}
\end{aligned} \tag{11}
$$

Here the reflection coefficient $\rho(\omega, P)$ is included to describe the conducting sea surface and the polarization P of the incident wave. The last equation indicates that, for this special case, the distribution of the RCS should follow the shape of the distribution derived for a single scatterer, however, its deterministic dependence on frequency is, of course, different.

Further refinement can be achieved by taking into account that the rocking motion of a boat can be well approximated by the following differential equation excited by a White Gaussian Noise (WGN) $\xi(t)$ [8]:

$$
\ddot{\varphi} + \varepsilon(1 + f(\varphi))\dot{\varphi} + \omega_0^2\varphi = K\xi(t) \tag{12}
$$

Here, $-\pi/2 < \varphi < \pi/2$ is the angular deviation of a boat's position from the vertical position, K is the intensity of the noise, ω_0 is the average frequency of fluctuation of the sea

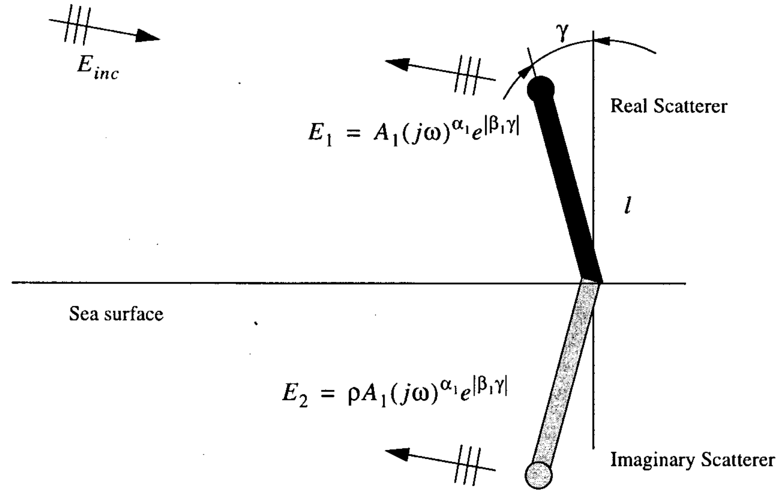

Figure 2. Rough model of a submarine mast.

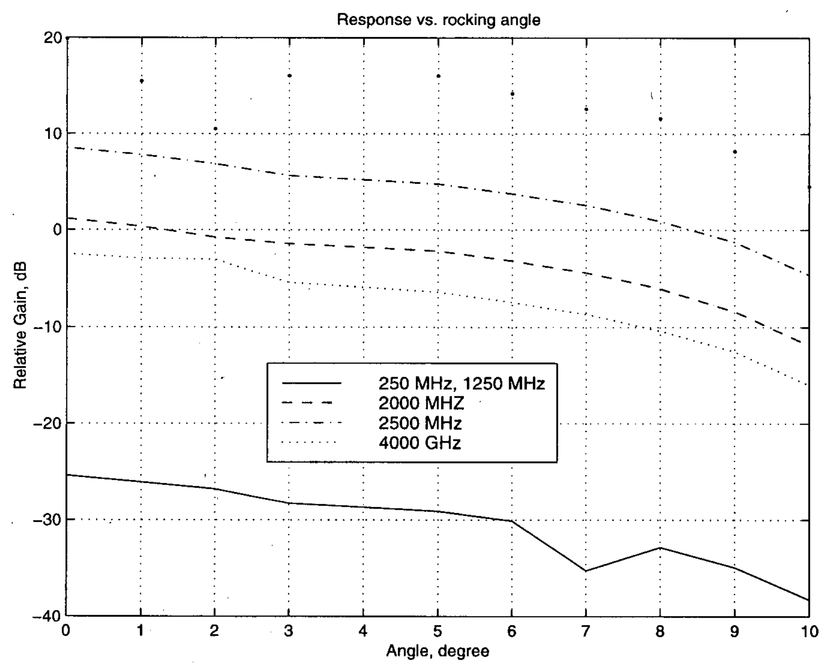

Figure 3. Dependence of the scattering response on the rocking angle. Frequency is a parameter.

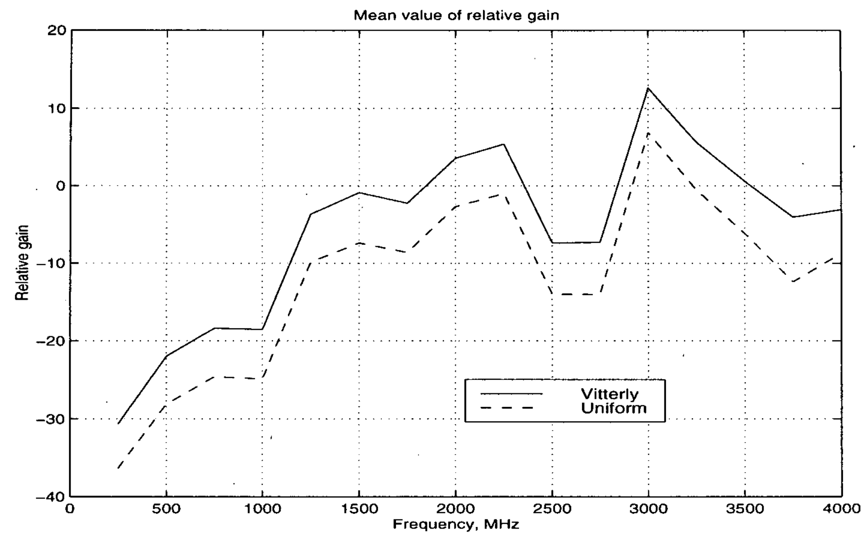

Figure 4. Mean value of the response for a uniformly distributed phase on the interval $[-10°, 10°]$ and for the Vitterbi distribution given by (13).

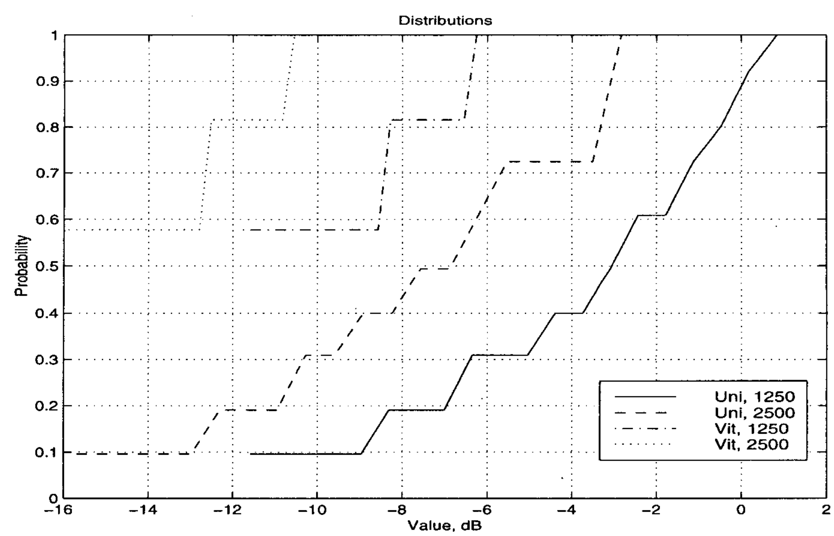

Figure 5. Cumulative distributions for two frequencies and uniform and Vitterby distributions.

surface, ε is a small parameter, and $f(x)$ is a non-linear function. Any solution of equation (12) is an almost periodic function of frequency ω_0 with a distribution of instantaneous values depending on the form of the non-linearity of $f(x)$. One of the possible distributicns for $\varphi(t)$, predicted by (12), is the so called Vitterbi distribution [9], given as

$$p_{\varphi}(\varphi) = \frac{\exp(D\cos\varphi)}{2\pi I_0(D)}, -\frac{\pi}{2} < \varphi < \frac{\pi}{2} \tag{13}$$

where the parameter D is proportional to ε. For a very small ε this distribution is almost Gaussian, and converges to a uniform one when ε approaches unity. The model implied by (13) has a more physical meaning than just a uniform distribution: the probability of large deviations from the vertical position is small, *i.e.* the angle distribution cannot be a uniform one. Combining (13) and (11) one can obtain the distribution of the RCS as

$$p_{RCS}(S) = \begin{cases} \dfrac{\exp\left(D\cos\ln\dfrac{S}{A^2}\right)}{4\pi I_0(D) S\exp\left(\dfrac{\beta\pi}{2}\right)A^2\omega^{2\alpha}} & S \in \left[\exp\left(\dfrac{\beta\pi}{2}\right)A^2\omega^{2\alpha}, A^2\omega^{2\alpha}\right] \\ 0 & S \notin \left[\exp\left(\dfrac{\beta\pi}{2}\right)A^2\omega^{2\alpha}, A^2\omega^{2\alpha}\right] \end{cases} \tag{14}$$

The models described here are very simple ones, but being analytical models they allow us to properly approximate the behaviour of the RCS of seaborne objects. More accurate approximations can only be performed numerically as described in the following section.

PROCESSING OF REAL DATA

Realistic data are much more complex than that represented by a simple or double scatterer. However similar methods can be applied to describe a target's behaviour. Two different models can be considered: frequency domain and time domain. The first can be applied to all kinds of narrowband radar, including stepped frequency and chirp waveform radars. The latter has attracted more attention recently due to the intensive development of ultra-wideband radar radiating very short pulses instead of synthesizing the time domain responses from a number of single frequency measurements.

Both methods can be implemented in the very same way. Let $R(\omega, \gamma)$ and $r(t, \gamma)$ be the frequency and time domain responses of the target under test as a function of the "rocking" angle γ (see Fig. 1). This angle itself has a certain distribution $p_{\gamma}(\varphi)$, for example given by (7) or (13). Then for every single frequency ω_j and frequency response of the target $R(\omega, \gamma)$ the corresponding histograms for phase and magnitude can be estimated as

$$p_{FM}(\rho, \omega_j) = Prob\{|R(\omega_j)| < \rho\} \tag{15}$$

$$p_{FPh}(\Phi, \omega_j) = Prob\{\angle(R(\omega_j) < \Phi|)\}. \tag{16}$$

The mean values of the magnitude and phase, as well as confidence intervals for every frequency can then be calculated from the empirical distributions (15-16) and used in detection algorithms. For the case of time domain data, the only difference is that time samples must be processed instead of frequency samples. Corresponding results are shown in Fig.'s 3-5.

CONCLUSIONS AND FUTURE WORK

This paper addresses the first steps of the complex problem of detection of fluctuating targets such as submarine masts; we have found that the models are very different from those suggested in the literature. The next step in our research will be the development of detection algorithms based on these analytical and empirical models.

Another opportunity to improve the detection algorithm is to incorporate model based approximations of targets. It was stated that the time response at any angle should contain the dominant scattering from "bright points" suggested in the previous section--a property totally ignored by most algorithms. One possible and powerful tool will be time-frequency analysis. Instead of looking at the distribution of the signal itself it is possible to look at the distribution of different time-frequency components. The data processing will be very similar to that suggested for frequency domain data. A possible advantage may be the better localization of energy in the time-frequency domain than in the frequency domain alone.

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