Exercise 1 Show by induction:

a) $1^2 + 2^2 + ... + n^2 = \frac{n(n+1)(2n+1)}{6}$

b) For any $n \geq 7$, we have that $n! > 3^n$

Exercise 2 Show that the following recursive algorithm computes correctly the factorial of a number. Assume that $n \geq 0$.

Function $F(n)$
  if $n = 0$ **then**
    $s = 1$
  else
    $s = n \cdot F(n-1)$
  return s

Exercise 3 Show that the following insertion sort algorithm is able to sort a list A of n numbers in nondecreasing order.

Function $IS(n, A)$
  if $n \geq 2$ **then**
    $IS(n-1, A)$
    $i = n - 1$
    while $i \geq 1$ and $A[n] < A[i]$ **do**
      $i = i - 1$
    $i = i + 1$
    $p = A[n]$
    $j = n - 1$
    while $j \geq i$ **do**
      $A[j+1] = A[j]$
      $j = j - 1$
    $A[i] = p$

Exercise 4 Do a tail-recursive version of the following algorithm

Function $S(n)$
  if $n = 1$ **then**
    return n
  else
    return $n + S(n-1)$

Exercise 5 Read the problem *A new chess game* in Mooshak. Consider a recursive approach to solve it.