First Name: ______________________

Last Name: ______________________

Signature: ______________________

Student I.D. No.: ______________________

Math 6321 Practice Final Exam

April, 2025
80 minutes

University of Houston

Instructions:

1. Put your name, signature and I.D. No. in the blanks above.
2. There are **three problems** in this exam. Answer the questions in the spaces provided, using the backs of pages or the blank pages at the end for overflow or rough work.
3. Your grade will be influenced by how clearly you present your solutions. **Justify your solutions carefully** by referring to definitions and results from class where appropriate.
4. **This is a closed book exam.**

1. (a) State the definition of absolute continuity for a complex-valued function f on an interval $I = [a, b] \subset \mathbb{R}$, $a < b$.

   (b) State a fundamental theorem which relates the values of $f(x)$ and $f(a)$ of an absolutely continuous function f on $[a, b]$ to an expression involving an integral over $[a, x]$ for any $a \leq x \leq b$.

2. If we have σ-finite measure spaces (X, M, μ) and (Y, N, ν), and functions $f \in L^1(\mu)$, $g \in L^1(\nu)$, and we let F be the function on $X \times Y$ with values $F(x, y) = f(x)g(y)$, show that F is measurable with respect to the product algebra $M \times N$ and integrable with respect to the product measure $\mu \times \nu$.
   Hint: First prove that the function $G : X \times Y \to \mathbb{C}^2$ given by $G(x, y) = (f(x), g(y))$ is measurable. You may use that the Borel algebra on $\mathbb{C}^2$ is generated by open rectangles. Then show that F is measurable by referring to a result from class.

3. Let (X, M, μ) be a σ-finite measure space. Let S be the class of complex-valued measurable simple functions which are non-zero on a set of finite measure. Let $1 \leq p < \infty$, $C \geq 0$, and assume that Λ is a linear functional such that for each $f \in S$, $|\Lambda(f)| \leq C\|f\|_p$. What do you know about Λ? Justify your answer.