Department of Mathematics University of Houston

MATH 6321

Theory of Functions of a Real Variable
Spring 2025

First name: ____________________ **Last name:** ______________________ **Points:**

Assignment 1, due Thursday, January 23, 10am

Please staple this cover page to your homework. When asked to prove something, make a careful step-by-step argument. You can quote anything we covered in class in support of your reasoning.

Problem 1

Let H be a (complex) Hilbert space and $L : H \to \mathbb{C}$ a bounded linear functional with kernel $M = \{x \in H : Lx = 0\} \neq H$. Prove that the orthogonal complement $M^{\perp}$ is a subspace of dimension one.

Problem 2

Let f be a continuous, 2π-periodic function on $\mathbb{R}$, and α an irrational number. Show that

$$\lim_{N\to\infty} \frac{1}{N} \sum_{n=1}^{N} f(2\pi n\alpha) = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(t)dt\,.$$

Hint: First examine the special case $f(t) = e^{ikt}$ with $k \in \mathbb{Z}$.

Problem 3

Compute

$$\min_{a,b,c\in\mathbb{R}} \int_{-1}^{1} |x^3 - a - bx - cx^2|^2 dx\,.$$