Document Title

Joel Sleeba
University of Houston
joelsleeba1@gmail.com

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Chapter 1

Fourier Series on the Circle

1.1 Motivation and Heuristics

1.1.1 Motivations from Physics

The Vibrating String

Exercise 1.1.1. *Use Induction. Show that* $\sin^3(x)$ *can be written as a sum of three sines.*

Exercise 1.1.2. *Use contradiction. Integrate over* $[0, \pi]$.

Exercise 1.1.3. *Use the same logic as above.*

Exercise 1.1.4. *Easy*

The Heat Flow in Solids

1.1.2 Absolutely Convergent Trigonometric Series

Exercise 1.1.7. *Use triangle inequality on the definition of limit. Then use the given hint and the absolute convergence of the sequence again. Now use dominated convergence theorem for the counting measure.*

Exercise 1.1.8. *Same method as above the exercise using induction.*

Corollary 1.1.9. *Use the fact that*

$$\int_{\mathbb{T}} g(\theta+\phi)e^{in\theta}\ d\theta = \int_{\mathbb{T}} g(\pi)e^{in(y-\phi)}\ dy = e^{-ix\phi}\int_{\mathbb{T}} g(\pi)e^{iny}\ dy$$

to get that the n*-th Fourier coefficient for* $g(\theta-\phi)$ *is* $e^{-in\phi}D_n$ *where* D_n *is the* n*-th Fourier coefficient of* $g(x)$.