Solutions to Dummit and Foote

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Chapter 4

Group Actions

4.1 Group Actions and Permutation Representations

Proposition 4.1.2. *If G is a group acting on a set $A \ni a$. Then the cardinality of the orbit of a, $|O_a| = |G : G_a|$.*

Proof. Notice that every element of G_a acts trivially on a. Therefore $h(g(a)) = h(a)$ for all $g \in G_a$. Hence the coset hG_a acts on a and gives $h(a)$. Hence we see that $|O_a| \leq |G : G_a|$.

Now conversely if hG_a and kG_a acts on a to give the same output, i.e $h(a) = k(a)$, then $h^{-1}k(a) = a$. Therefore $h^{-1}k \in G_a$ and hence $hG_a = kG_a$. Hence we get $|O_a| = |G : G_a|$. □

3 *S*olution: Show that if $\sigma(a) = a$ for some $\sigma \in G$, then $\sigma \in G_x$ for all $x \in A$ by the Abelianess. This contradicts $G \leqslant S_A$.

Now for the other one use the above proposition along with $G_a = \{e\}$.