

Exact Solution of Resource Constrained Routing Problems using Branch-Price-and-Cut

English Summary

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In this thesis, we are mainly concerned with exact solution of Resource Constrained Routing Problems (RCRP) by using the method referred to as Branch-Price-and-Cut.

RCRP is the class of problems where vehicles have to select routes such that a subset of customers receives the proper service. This selection usually has to be done in such a way that the total routing cost is minimized. Each of the vehicles is subject to a set of resource constraint, e.g., vehicle capacity or time usage. A route for a vehicle which does not violate the resource constraints is called resource feasible. We are primarily concerned with two RCRPs; the Elementary Shortest Path Problem with Resource Constraints (ESPPRC) and the Vehicle Routing Problem with Resource Constraints (VRP-RC). The ESPPRC is the problem of finding a minimum cost resource feasible route from an origin to a destination, where visiting each customer at most once if it pays off to do so. The VRP-RC is the problem of finding a cost-minimizing set of resource-feasible routes such that all customers are visited exactly once.

One of two prevalent solution paradigms is often used when solving RCRPs. The first of these paradigms is (meta-) heuristics, which try to identify a feasible solution, but without proving that the solution obtained is optimal. These provide an Upper Bound (UB) solution, with a solution value that is smaller than the solution value of any other feasible solution identified. The second paradigm is often referred to as exact methods, which try to identify an optimal solution. One way of proving that a feasible solution is optimal, is to identify a Lower Bound (LB) value, which is equal to the UB value. For RCRPs it is, however, often difficult to identify such an LB value.

One way of obtaining an LB is to construct a mathematical formulation of the problem. The mathematical formulations for RCRP are Integer Linear Programs (ILP) or Mixed Integer Linear Programs (MILP). It can be shown that these are hard to solve and the integer-requirement of the variables is therefore relaxed. In this way, it is possible to obtain an LB for the MILP and, consequently, the RCRPs. The LBs obtained from these formulations are, unfortunately, often of poor quality. The RCRPs often have a structure such that it is possible to construct an extremal formulation by the Dantzig-Wolfe reformulation. It divides the formulation into two associated problems called the master problem and the pricing problem, respectively. The master problem is relaxed to a Linear Program (LP) and the pricing problem generates the variables for the

master problem. This process is referred to as Column Generation. If the pricing problem is not LP-relaxed, but another stronger relaxation is used, we will often obtain an LB value from Column Generation, which is stronger than the LB value obtained from the LP-relaxation of the original problem. The solution obtained may not be feasible for the original MILP, and the Column Generation is, therefore, embedded in a Branch-and-Bound procedure. This is referred to as Branch-and-Price. It is, however, often possible to tighten the formulation further by adding additional valid inequalities in the master problem. Usually, an exponential number of such constraints exists and they have to be generated dynamically. When adding this to the Branch-and-Price, we obtain the solution method called Branch-Price-and-Cut.

The VRPRC can be formulated in an extremal form by using the Dantzig-Wolfe reformulation. This extremal form has a Set Partitioning Problem (SPP) as the master problem and an ESPPRC as the pricing problem. The prevalent way of relaxing the pricing problem is to allow customers to be visited a multiple number of times. The relaxed problem is referred to as the Shortest Path Problem with Resource Constraints (SPPRC) and can be solved by dynamic programming. The customers yielding a large pay-off will be visited many times on such routes, and the resulting relaxation of the VRPRC will tend to be weak. This can be mended by adding so-called k -cycle elimination, where revisiting a customer before at least k other customers have been visited is prohibited. Instead of relaxing the ESPPRC to a k -cycle-free SPPRC, we try to solve the ESPPRC itself. We use the Dantzig-Wolfe reformulation to construct an extremal form of the ESPPRC, where the master problem is a Set Packing Problem with one additional constraint, and the pricing problem is an SPPRC. We solve the ESPPRC by Branch-Price-and-Cut, where we identify valid inequalities and new branching strategies. By restricting the possible variables for the master problem of VRPRC to elementary resource feasible paths, we obtain a tighter bound, than we do when using non-elementary resource-feasible path. To the best of our knowledge, this approach have not been tried before. The LB-solution obtained by using ESPPRC is, however, still not always feasible and a Branch-and-Bound is needed to prove optimality of the UB-solution. We introduce a new branching strategy which we call Cut-Set Branching.

To speed up the Column Generation, we introduce the method of Primal Restriction, which serves as a primal heuristic. This method is a variation of the constraint aggregation, but where we aggregate in the pricing problem rather than in the master problem. We also show how to utilize SPPRC in collaboration with Petal Heuristics to solve certain SPPs with additional side-constraints. The Branch-Price-and-Cut method can be applied to other RCRPs and we argue how this can be done for several variations of these.

We have constructed a Branch-Price-and-Cut framework in C++ to be able to use this method on different RCRPs. The framework has been implemented for both ESPPRC and VRPRC. The implementation of ESPPRC is used as the pricing problem of VRPRC. Computational Results show that it is difficult to solve the ESPPRC. The main reason for this is that the LB obtained tends to be weak and we have to enter a full-scale Branch-and-Bound to prove that the UB is optimal. This is also reflected in the computational results for the VRPRC, where the bounds obtained seem promising, but where the execution times are large. These execution times keep us from solving a range of the Solomon test instances, and we have not obtained any previously unknown optimal solutions for these instances.