Digital Signal Processing

Zusammenfassung

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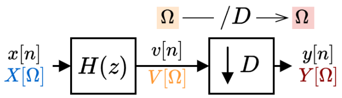
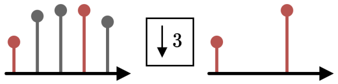
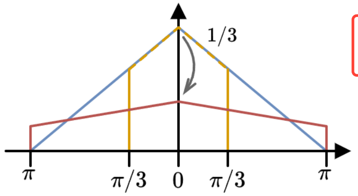
Sampling Rate Conversion

Decimation

Reducing sampling rate by an **Integer Factor** D







Sampling Theorem $f_{\max} < \frac{f_s}{2}$
Don't forget!

$$H(z) = \begin{cases} 1 \text{ if } \Omega \in [-\pi/D, \pi/D] \\ 0 \text{ otherwise} \end{cases}$$

For $\Omega \in [-\pi, \pi]$

Decimated Frequency: $\boxed{F_Y = F_X/D \leftrightarrow \Omega_Y = \Omega_{X,V}/D}$

Ideally filtered

$$Y(z) = \frac{1}{D}\sum_{d=0}^{D-1} V\left(\frac{\Omega}{D} - 2\pi \cdot \frac{d}{D}\right)$$

General Formula

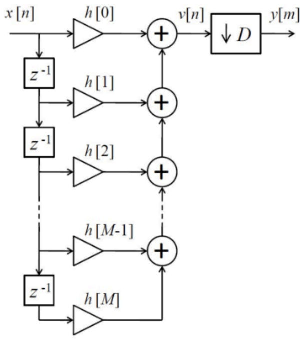
$$Y(z) = \frac{1}{D}\sum_{d=0}^{D-1} V\left(\frac{\Omega}{D} - 2\pi \cdot \frac{d}{D}\right)$$

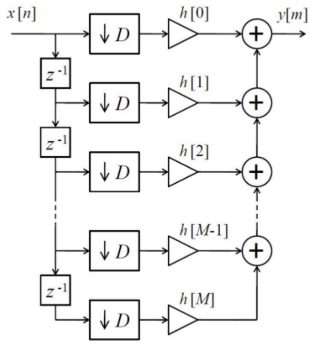
Direct Implementation

FIR Filter of order M produces full signal $v[n]$ + downsampler discards $D-1$ samples afterwards → **inefficient!**

Efficient Implementation

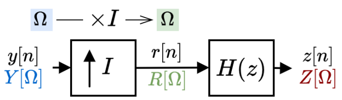
Downsampling beforehand allows the multiplier to operate at the reduced sampling rate → **much better!**

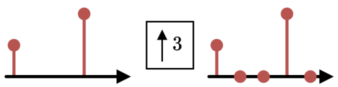


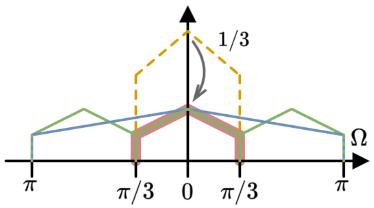


Interpolation

Increase sampling rate by an **Integer Factor** I







$$r[n] = \begin{cases} y[n/I] \text{ if } n \in [0, \pm I, \pm 2I, ...] \\ 0 \text{ otherwise} \end{cases}$$

Interpolated Frequency:

$\boxed{F_{Z,R} = I \cdot F_Y \leftrightarrow \Omega_{Z,R} = I \cdot \Omega_Y}$

Interpolation Formula

$$R(\Omega) = \sum_{n=-\infty}^{\infty} r[n]e^{-j\Omega \cdot n} = \sum_{n=-\infty}^{\infty} y[m]e^{-j\Omega \cdot I \cdot m} = Y(I\Omega)$$

Low Pass Filter

For $\Omega \in [-\pi, \pi]$

$$H(z) = \begin{cases} I \text{ if } \Omega \in [-\pi/I, \pi/I] \\ 0 \text{ otherwise} \end{cases}$$

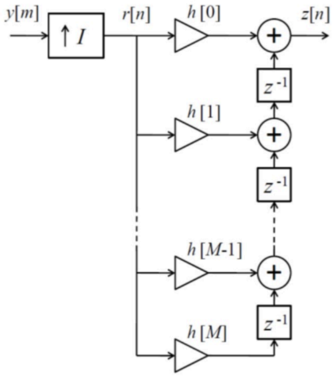
Lowpass-filter uses Ω and NOT Ω !

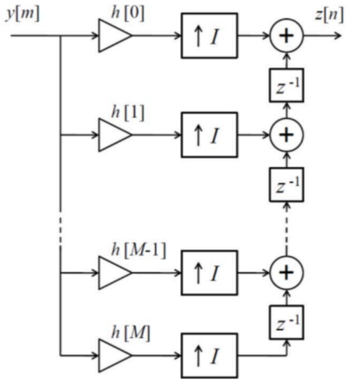
Direct Implementation

FIR or IIR Filter ; $I-1$ out of I $r[n]$ samples are zero → **inefficient!**

Efficient Implementation

Upsampling after filtering → multiplier operates at **reduced** sampling rate (F_Y) → **much better!**





Polyphase Filter Structure

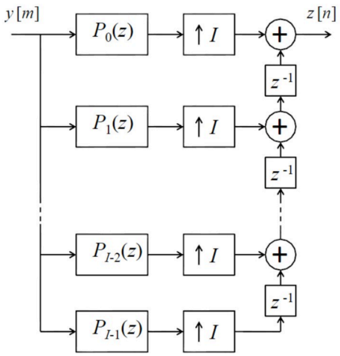
Efficient filter implementation

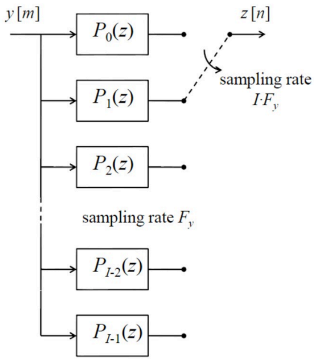
TODO Relearn

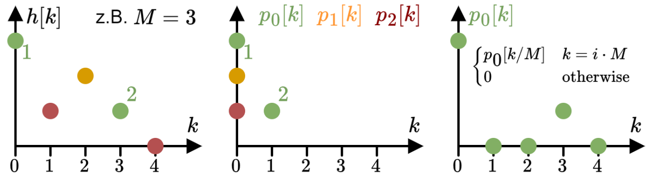
Split filter into M **downsampled** variants of the impulse resonse $h[k]$. Every variant $p_{i[k]}$ holds only every M-th coefficient („sum" of variants = $h[k]$)

$$p_i[k] = h[kM+i], \quad i = 0, 1, ..., M-1 \quad | \quad H(z) = \sum_{i=0}^{M-1} z^{-1} P_i(z^M)$$

$$P_{i(z)} = \sum_{k=-\infty}^{\infty} p_i[k]z^{-k} = \sum_{k=-\infty}^{\infty} h[kM+i]z^{-k}, \quad i = 0, 1, ..., M-1$$

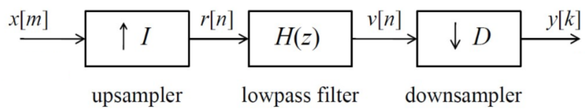






TODO include fourier transform equation by hand!

Sampling Rate Conversion



$$F_X \Rightarrow F_R = I \cdot F_X \Rightarrow F_V = F_R \Rightarrow F_Y = I/D \cdot F_X = 1/D \cdot F_V$$

Decimation ⇒ loss of information

Interpolation ⇒ higher intermediate sampling rate

$$F_H(\Omega) = \min\left(\frac{\pi}{I}, \frac{\pi}{D}\right) \qquad \frac{I}{D} = \underbrace{I \cdot \left(\frac{1}{D}\right)}_{H_I \to I \to D \to H_D} = \underbrace{\left(\frac{1}{D}\right) \cdot I}_{I \to H \to D}$$

TODO FIR-order = FIR-length

Filter Banks

TODO

Quadrature Mirror Filters

TODO

DFT Filter Banks

TODO

Random Signals

TODO

Autocorrelation and Spectrum

TODO

Spectral Shaping

TODO

Linear Models for Stochastic Processes

TODO

Spectral Density Estimation

TODO

Wiener Filters

TODO

Unconstrained Wiener Filters

TODO

The Principle of Orthogonality

TODO

Kalman Filter

TODO

Linear Predictive Coding

TODO

LMS Algorithm

TODO

The LMS Algorithm

TODO

Acoustic Echo Cancellation

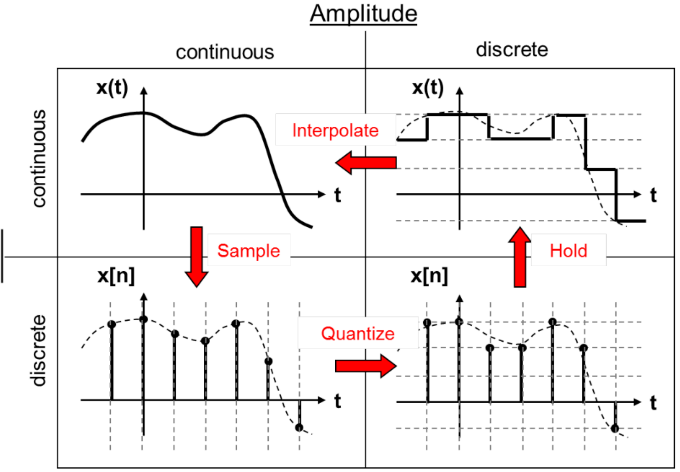
TODO

Hello World

Hello World

Digital Signal Processing (DSP)

DSP concerned in the application of the following methods: ① **Signal Generation**, ② **Signal Analysis**, ③ **Signal Composition**, ④ **Signal Selection**



Pros (3 P's): **P**rogrammability, **P**arametrizability, Re-**P**eatability

Cons: additional effort for ADC & DAC, No processing of broadband HF, electro-magnetic disturbance

Signal Analysis

Sampling an Analog Signal

$$f_S = \frac{1}{T_S} \quad x(n \cdot T_S) = x[n]$$

Other Functions

causal: $x[n] = 0$ for $n < 0$

T_S: Always known!

unit impulse

$$\delta[n] = \begin{cases} 0 : n \neq 0 \\ 1 : n = 0 \end{cases}$$

step impulse

$$u[n] = \begin{cases} 0 : n < 0 \\ 1 : n \geq 0 \end{cases}$$

periodic symbols ($k = T_0/T_S$)

$$x[n] = x[n + T_0/T_S]$$
$$= \hat{X} \cdot e^{\underbrace{j2\pi \cdot f_0 \cdot n \cdot T_S}}$$
$$= \hat{X}\big(C(\underbrace{\ }) + j \cdot S(\underbrace{\ })\big)$$

expected/mean value

$$\mu_x = \frac{1}{N}\sum_{i=0}^{N-1} x[i]$$

$(\text{mean value})^2$ / avg DC power

$$\rho^2 = \frac{1}{N}\sum_{i=0}^{N-1} x[i]^2$$

variance/avg AC power

$$\sigma_x^2 = \frac{1}{N}\sum_{i=0}^{N-1}(x[i] - \mu_x)^2$$

Acoustic signals: corresponds to audible power content

power ratio

$$A_{dB} = 10 \cdot \log_{10}\left(\frac{P_1}{P_2}\right)$$

Signal-to-Noise-ratio

$$\text{SNR} = 10 \cdot \log_{10}\left(\frac{P_S}{P_N}\right)$$
$$= 20 \cdot \log_{10}\left(\frac{U_S}{U_N}\right)$$

power ratio

$$A_{dB} = 10 \cdot \log_{10}\left(\frac{P_1}{P_2}\right)$$

static correlation ($x = y \Rightarrow \uparrow R$)

$\Rightarrow$ yields new signal, quantifying the similarty of x and y

$$R = \frac{1}{N}\sum_{i=0}^{N-1} x[i]y[i]$$

linear correlation

$$r_{xy}[n] = \frac{1}{N}\sum_{i=0}^{N-1} x[i]y[i+n]$$

$$N_{xy} = N_x + N_y - 1$$

not commu.: $r_{xy}[n] \neq r_{yx}[n]$

(linear) convolution

$$z[n] = \sum_{i=-\infty}^{\infty} x[i]y[n-i]$$
$$= x[n] * y[n]$$

circular convolution

$$N_X = N_Y$$

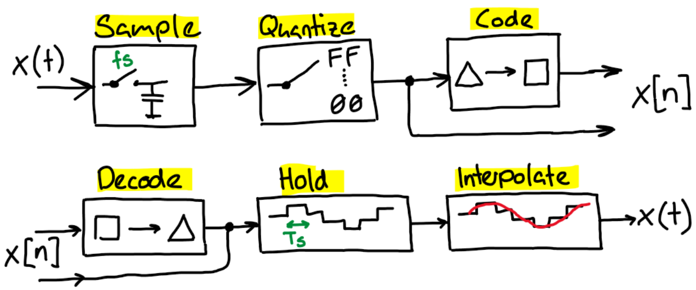
$$z[n] = x[n] \circledast_N y[n]$$

$x[i] = \{1, 1, 1, -1\}$ and $y[i] = \{1, 1, 0.5, 0.5\}$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0.5 & 0.5 & 1 \\ 1 & 1 & 0 & 0 & 0 & 0.5 & 0.5 \\ 0.5 & 1 & 1 & 0 & 0 & 0 & 0.5 \\ 0.5 & 0.5 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0.5 & 0.5 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0.5 & 0.5 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0.5 & 0.5 & 1 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \\ 1 \\ -1 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 2.5 \\ 1 \\ 0 \\ 0 \\ -0.5 \end{pmatrix}$$

$$z[n] = \{1, 2, 2.5, 1, 0, 0, -0.5\}$$

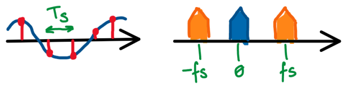
A/D & D/A Conversion



Code/Decode z.B. DFT & IDFT ; *Interpolate* z.B. Tiefpass-Filter

Sampling & Aliasing

$$x_{s(t)} = \sum_{n=-\infty}^{\infty} x(t) \cdot \delta(t - nT_s)$$



⚠ Sampling → period. Spektrum mit f_s-vielfachen *Spiegelbilder* ⚠

Bei *Bilderüberlagerung* entsteht Aliasing. **Theorem!** $\left[f_{\max} < \frac{f_S}{2}\right]$