Predator-Prey Cellular Automaton

SI1336 - Simulation & Modelling

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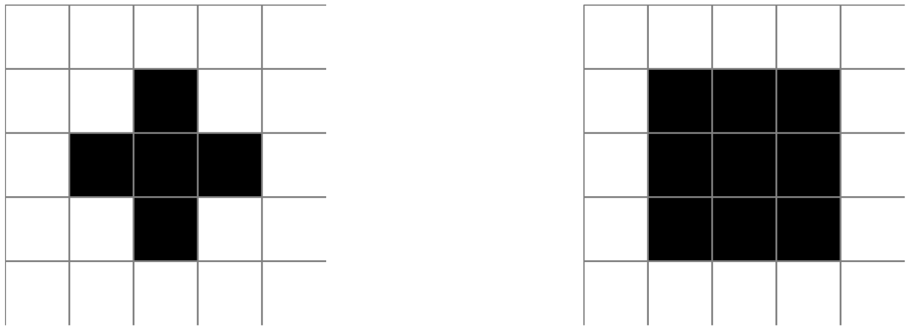
January 23, 2022

1 Introduction

In this paper a three level Predator-Prey Cellular Automaton (CA) is studied. The system was made up of a chain of one low-level prey that could be eaten by the second level species which in turn could be eaten by the top-level predator. A parameter study was conducted to understand how different parameters affects the stability and equilibrium states of the system.

It was shown that varying the creatures length of sight did not affect the stability. However, introducing environmental structure (i.e. trees) was shown to affect the steady state. In one case the low-level prey was seen to increase its steady state population with 2.6 times that without trees.



(a) von Neumann (b) Moore

Figure 1: The two typical neighborhoods (in black) in a 2D cellular automaton, both of radius 1.

2 Methods

2.1 2D Cellular automaton

A cellular automaton is a discrete structure used in various scientific fields. It consists of a two-dimensional lattice of sites together with a set of rules. Each site starts out in a pre-determined state at $t = t_0$, and the set of rules are then used to determine the site's state at time t_1 and so forth. The time development of a site most often depends on its neighbors. There are two typical neighborhoods considered in a two-dimensional CA, namely the von Neumann and Moore neighborhoods. The von Neumann-neighborhood of Manhattan distance 1 can be seen together with the Moore-neighborhood of radius 1 in figure 1.

2.2 Simulation

The main structure of the automata in this report is adapted from [1]. Let $\mathcal{L}$ denote the two-dimensional $N \times M$ lattice of sites equipped with periodic boundary conditions. Each site will always be in any of the states found in the state set $Q = \{0a, 1, 2a, 3a\}$, corresponding to a site being empty or populated by a prey, second level species or top-level predator. A configuration can be thought of as a map $\mathcal{L} \longrightarrow Q$ with $(x, y) \mapsto q \in Q$.

The rules of the CA is divided into three consecutive parts. The attack, reproduction and movement phases respectively.

2.2.1 Attack phase

The attack phase requires two parameters per species, namely the birth rate b_p and death rate d_p. Furthermore, the state set is temporarily expanded to $Q_0 = \{0a, 0b, 1, 2a, 2b, 3a, 3b\}$, where $0b$ indicates that the site became empty after the attack phase, $2b$ indicates a second level species that has eaten, and $3b$ a top-level predator that has eaten.

Now, consider a site $(x, y) \in \mathcal{L}$ in a state $q \in Q$. If the site contains a prey ($q = 1$) there are two possibilities for a new state $\tilde{q} \in Q_0$. If there are no second level species in the von Neumann-neighborhood (of radius 1) the site remains prey and $\tilde{q} = q = 1$. However, if there are any second level species in its surroundings the prey survives with probability $(1 - d_p^1)^{n^2(x,y)}$. Here d_p^i denotes the i:th species death probability and $n^j(x, y)$ is the number of the j:th species in the neighborhood of the site at position (x, y). If the prey is eaten, the state is set to $0b$.

If the site contains a second level species ($q = 2a$) there are three possible outcomes. As in the previous case, the second level creature can survive a predation attempt with probability $(1 - d_p^2)^{n^3(x,y)}$. If the attempt is successful, the new state is $0b$. In case the creature survives an attack from the top-level predator, there are two possible new states. Either the second level creature fails an attack on its prey with probability $(1 - d_p^1)^{n^1(x,y)}$ and stay in the same state $\tilde{q} = 2a$, or the attack is successful and the new state becomes $\tilde{q} = 2b$.

In case there is a top-level predator at the site, there are two possible new states. Either the predator fails an attack on the second level species with probability $(1 - d_p^2)^{n^3(x,y)}$ and stay in $\tilde{q} = 3a$, or it is successful and $\tilde{q} = 3b$. In case the site is empty, nothing happens.

2.2.2 Reproduction phase

During the reproduction phase, the species reproduce and die naturally. Again, consider a site $(x, y) \in \mathcal{L}$. If the site is a low-level prey, nothing happens. However, if the site contains a second or top-level creature, they die with probability d_p^i, $i = (2, 3)$ and the site state is updated to $0a$.

The only states allowing for a reproduction is $0a$ or $0b$ i.e. the empty sites. If the site is empty, there are two possible outcomes depending on its state. Consider the case $0a$, this means the site is empty without any creature having been eaten in it during the attack phase. If either there are second level species or no low-level species in its von Neumann-neighborhood, no reproduction occurs. If none of these conditions are satisfied there is a prey reproduction with probability $1 - (1 - b_p^1)^{n^1(x,y)}$.

Now consider the state $0b$, namely a site in which either a prey of second level creature was eaten. In this case, either a second level creature or predator can reproduce. The probability for a reproduction of any of these are $1 - (1 - b_p^i)^{n_b^i(x,y)}$, where $n_b^i(x, y)$ indicates the number of fed creatures of level i in the neighborhood of (x, y). In the case that both the second and top-level species successfully reproduces, one of them are randomly chosen with equal probability.

2.2.3 Movement phase

As described in [1], let $n_N^{(r)}$, $n_E^{(r)}$, $n_S^{(r)}$, and $n_W^{(r)}$ denote the number of a certain species found in any of the four Moore quadrants of radius r corresponding to the directions north, east, south and west in regard to the considered site (x, y). With this number the intention of movement can be set for each species level. The low-level prey favors movement away from the second level creatures and the top-level predator intend to move toward it. For the second level species, the intention is set so that if there are more preys than predators in its neighborhood it will intend to move in the direction of maximum preys, and if there are more predators it will intend to move in the direction of the quadrant containing a minimum number of predators. If multiple quadrants are intended by any creature then one of them are chosen randomly. If there is no intended move, the low-level prey will stand still until the next time step. Since the other two species need to feed in order to reproduce, they will perform a movement in a random direction as if they were seeking for food.

When all intentions are set, the movements are performed in the intended von Neumann-neighborhood direction. If multiple creatures want to move into the same site, a random choice is performed.

2.2.4 Adding environmental structure

To model the effect of environmental obstacles, another type of site state is introduced, namely $q = -1$. These can be thought of as non-reproductive stationary trees. If a creature wants to move into a site that contains such obstacle, the movement is denied. Furthermore, no creature reproduction is allowed at any such site. The sight of the creatures is also affected by the trees. No species is allowed to see anything in the line of sight of a tree. This is modeled by not counting creatures that is positioned behind any tree in the straight line from the current site in the four Moore quadrants. For example, consider a site (x, y) with a species of length of sight $r > 1$. If there is a tree at $(x - 1, y - 1)$, this means that no creatures positioned along the line $(x - i, y - i)$, $0 < i \leq r$, is considered, as shown in figure 2.

2.3 Parameter study

The first thing to study is the validity of the model. This will be done by evaluating the model with only one species present. If the initial configuration only contains low-level species, they should reproduce and reach a stable relative population of 100% at some later time. If there are only second level or top-level species added to the lattice then they should die out due to starvation.

The parameter study is performed in a few steps. First, the initial configuration is set up. It is computed from the given initial density of each species. The corresponding creatures are then randomly assign to the lattice. After this, the simulation runs for some time in order to record the

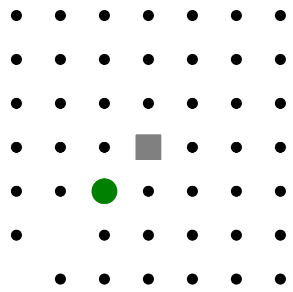


Figure 2: If a tree (green dot) is located in the neighborhood of a creature (gray box) it will cover the line of sight. Only the black dots can be seen by the creature (given that it has a length of sight of $r = 3$).

steady state populations. To do this, a final time has to be set. This will be done by running a few test simulations to see at what time one can be sure that the steady state has been reached.

The parameters to study are:

- The effect of lattice size on equilibrium
- The effect of initial species densities on the equilibrium population sizes
- The creatures length of sight vs. equilibrium population size i.e. the effect of r in $n_k^{(r)}$, $k = (N, E, S, W)$ described in section 2.2.3
- The effect of environmental obstacle density on the equilibrium population sizes

2.3.1 Statistical accuracy

Since the simulation is based on stochastic processes, it will be carried out multiple times in order to say something about the statistical accuracy. Assuming that the steady state population distribution x_i of the i:th species is the outcome of a normal distributed random variable $X_i \sim N(\mu_i, \sigma_i)$ we can estimate the mean μ_i with the sample mean $\bar{x}_i$ and the standard deviation σ_i by the sample deviation s_i.

3 Results & discussion

3.1 Validation of model

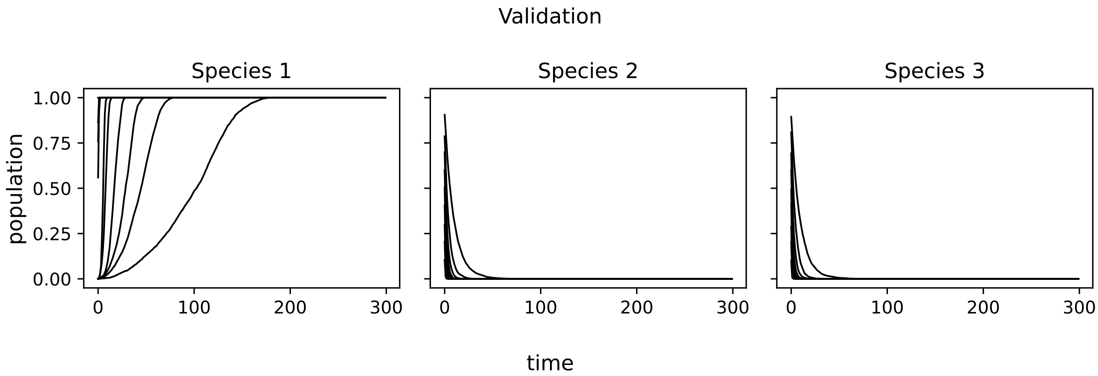


Figure 3: Validation of the three species when only one species is introduced at the beginning on a 64×64 lattice with birth and death probabilities in the range (0.1,1.0) with steps of .1. A larger species number refers to a creature higher up in the chain.

In figure 3, the three different cases of a lonely species can be seen. As expected, the low-level species population increases and reaches a steady state of 100%. In the center and right sub-figures the lonely predators can be seen to die out due to starvation. This tells us that the model works the way it was intended to.

In order to narrow the scope of this project, a set of death and birth probabilities giving interesting results was used for all studies, specifically

$$\begin{aligned} b_p &= (.9, .6, .5) \\ d_p &= (.9, .2, .01), \end{aligned} \tag{1}$$

where the first entries are the low-level species probabilities and the rest is sorted in increasing order.

3.2 Initial density

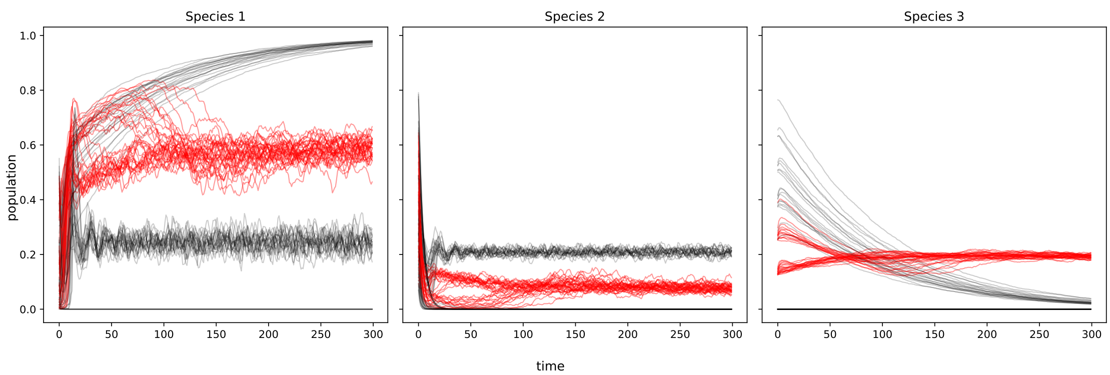


Figure 4: Relative population sizes vs. time for the three species for different initial densities with length of sight $r = 1$. The simulations for which all species survive is marked in red.

As seen in figure 4, there is only one steady state where all species survive using the probabilities in (1). Hence, the steady state is dependent on the initial density. However, it seems as if a steady state where all three species survive is found, it should be a unique state. An example configuration of this type of steady state is shown in figure 5.

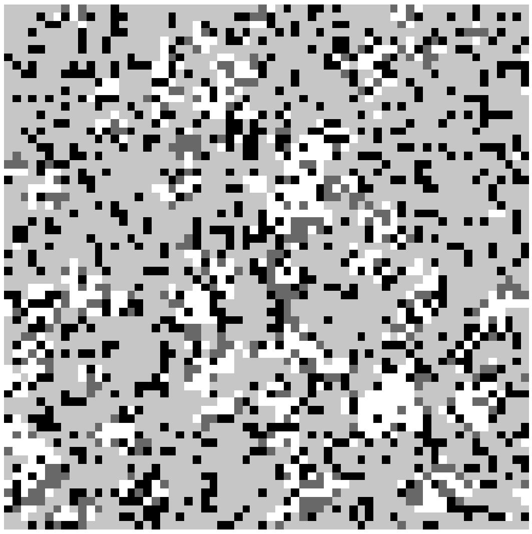


Figure 5: End time configuration for one simulation of initial density (.05,.05,.05) that renders a steady state where all species survives. A white pixel indicates an empty spot and a darker shade indicates a higher level creature.

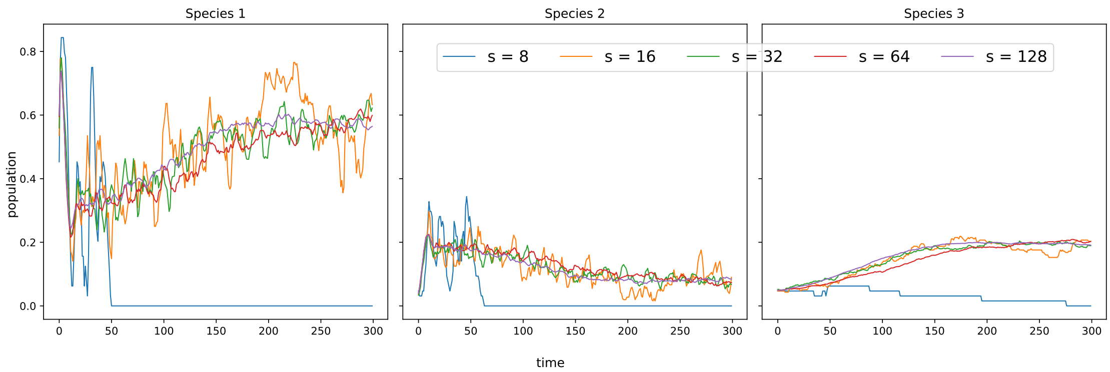


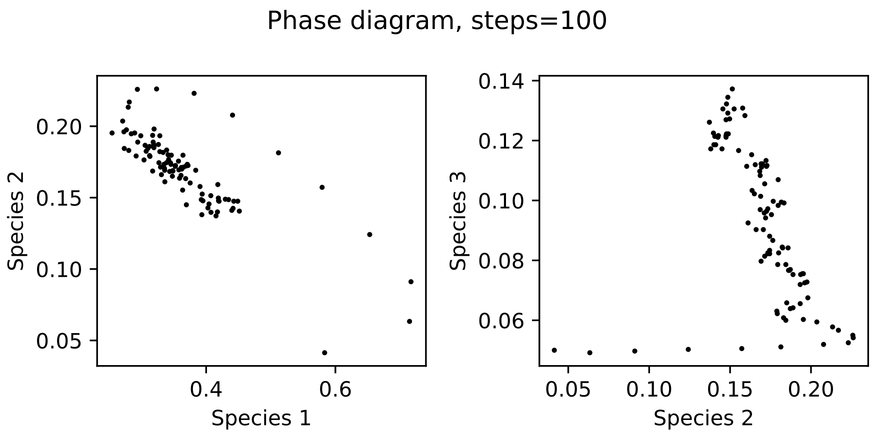
Figure 6: Populations for the three species for different linear lattice sizes, s. Here the initial density used was $d = (.05, .05, .05)$, and length of sight $r = 1$.

3.3 Lattice size

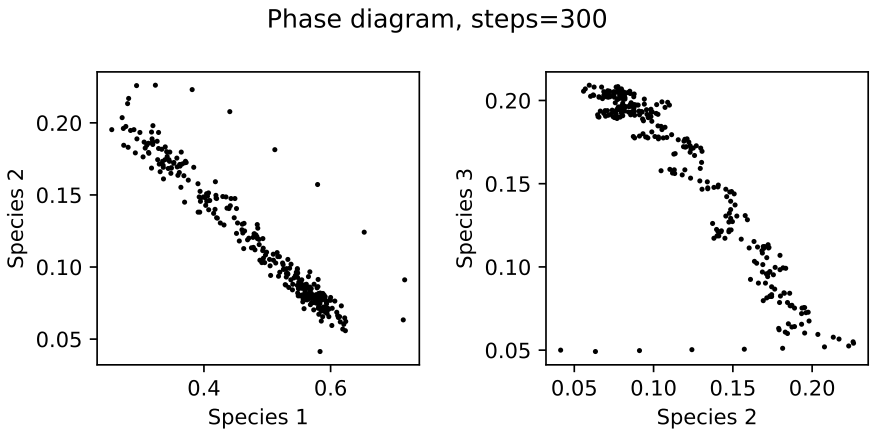
In figure 6, the effect of lattice size on the steady state can be seen. For a linear lattice size of $s = 8$, the finite size effects take over and the prey die out leading to a steady state of an empty grid. However, for sizes larger than 8, all simulations converge to the same distribution. For larger sizes the oscillations are smaller and seem to converge to a much smoother curve. The oscillations in population size are greater for the low-level species (Species 1 in figure 6). This is probably due to the large birth and death probability for this species. Meaning it more often gets eaten and reproduces in larger clumps.

3.4 Choosing an appropriate end time

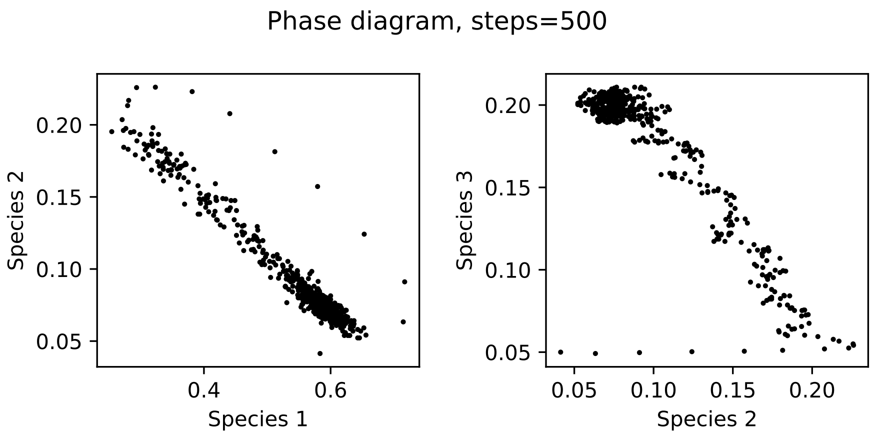
To find an appropriate final time to use in the studies, the final time was increased until the behavior of the populations seemed to have reached a steady state, as in figure 7. The dots converges to a small region of the diagram, and no big differences can be seen between sub-figures (b) and (c) meaning that a final time of 300 sufficed. It was also evident from figure 4 that 300 time steps was



(a)



(b)

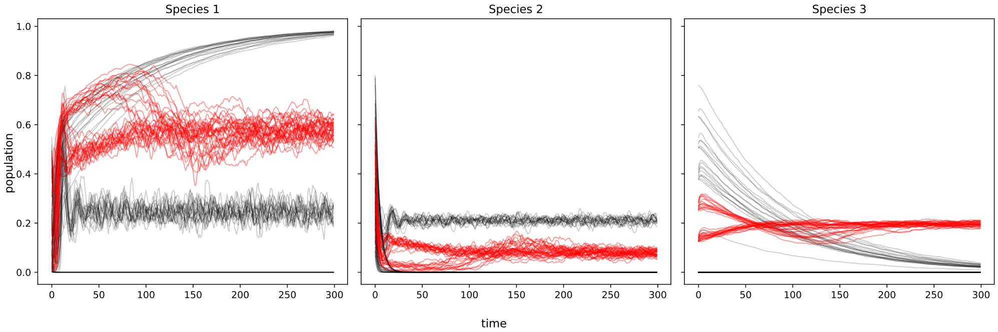


(c)

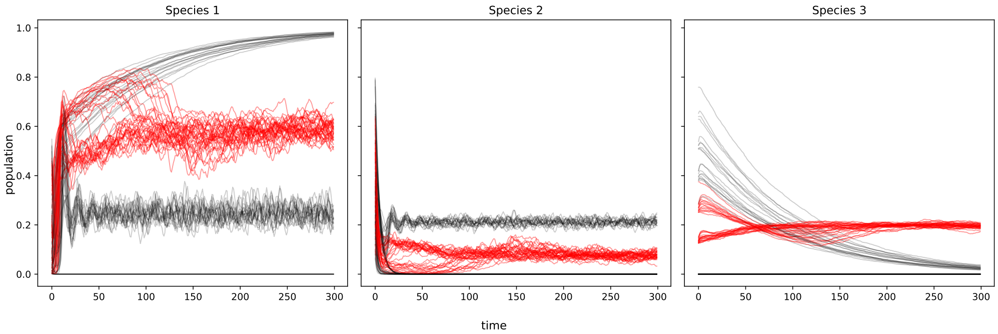
Figure 7: Projected phase diagrams obtained using 100, 300, and 500 time steps on a 64×64 lattice. Here, initial density = $(.05, .05, .05)$, and length of sight $r = 1$.

sufficient, independent of initial density.

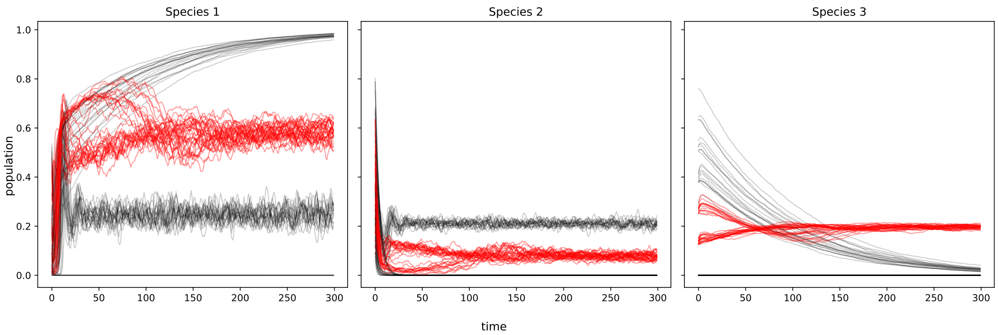
3.5 Length of sight



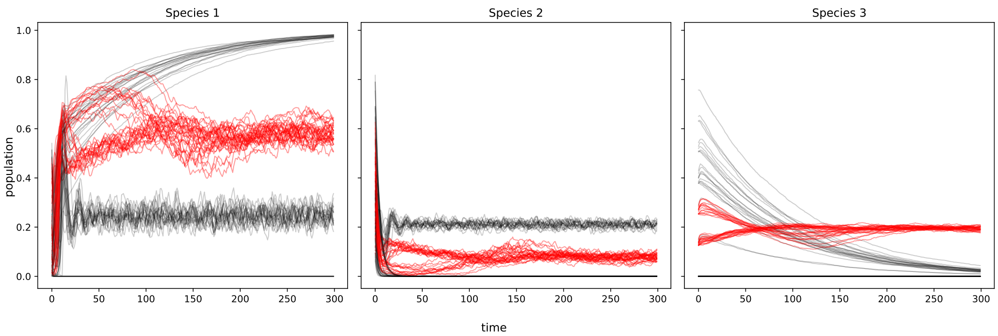
(a) $r = 1$



(b) $r = 5$



(c) $r = 10$



(d) $r_1 = 10, r_2 = 5, r_3 = 1$

Figure 8: Populations vs. time for different length of sight, r, and initial densities. The subscript in r_i indicates the length of sight of the i:th species level. If no subscript is given, all species have the same length of sight.

In figure 8, simulations with synchronous (same for all species) length of sight $r = 1, 5$, and 10 is shown together with the asynchronous case where lower level species can see longer.

There seems to be no effect of the length of sight on the steady state populations. However,

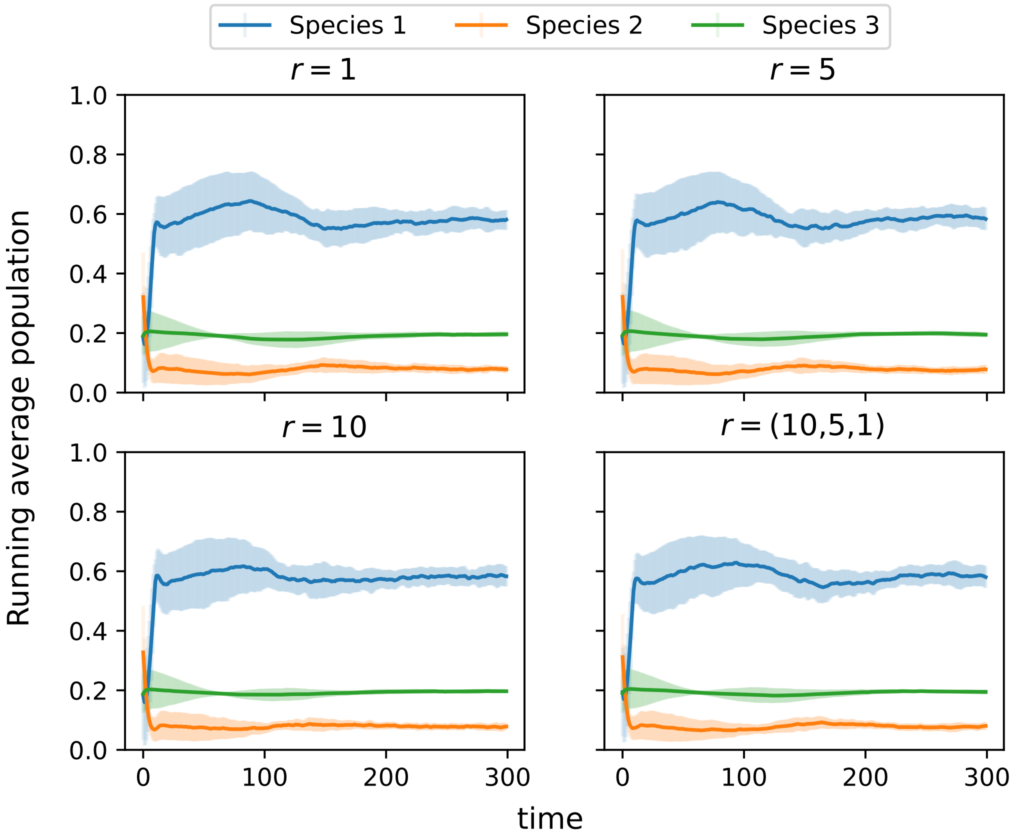


Figure 9: Running average population together with the sample standard deviation (error-bars) for the red curves shown in figure 8.

for the synchronous cases, the deviations in population sizes is somewhat smaller for the longer sight simulations as seen in the tighter error span for $r = 10$ in figure 9 and the plot of standard deviation in figure 10. The differences are small though, and if this is a stochastic result or statistically significant is yet to be determined.

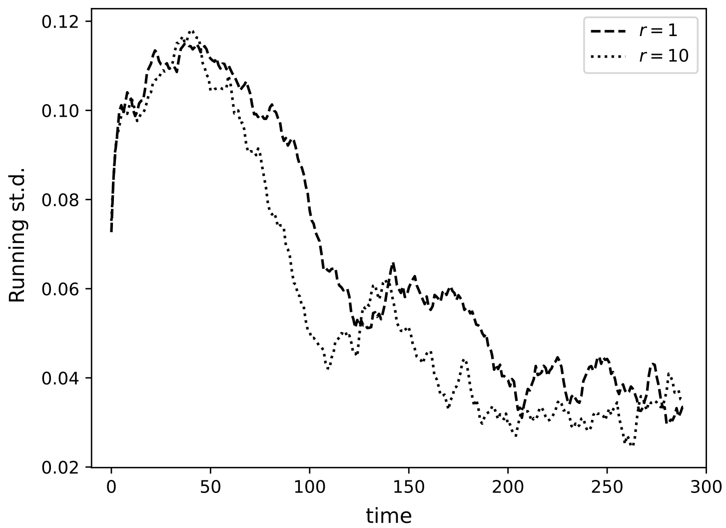
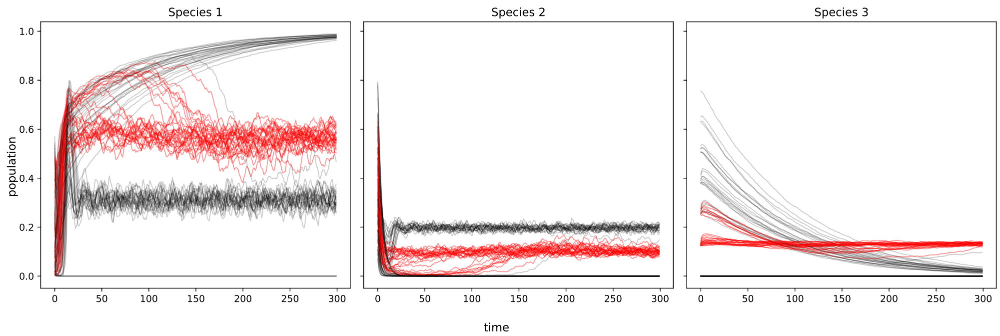
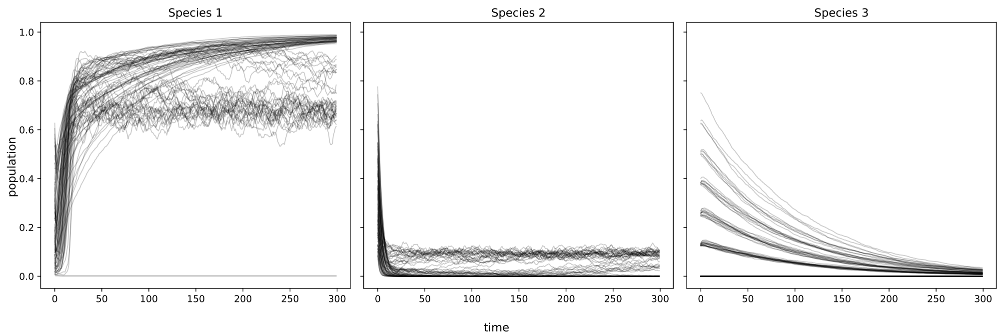


Figure 10: Running average low-level species population deviation for the length of sight $r = 1$ and $r = 10$.

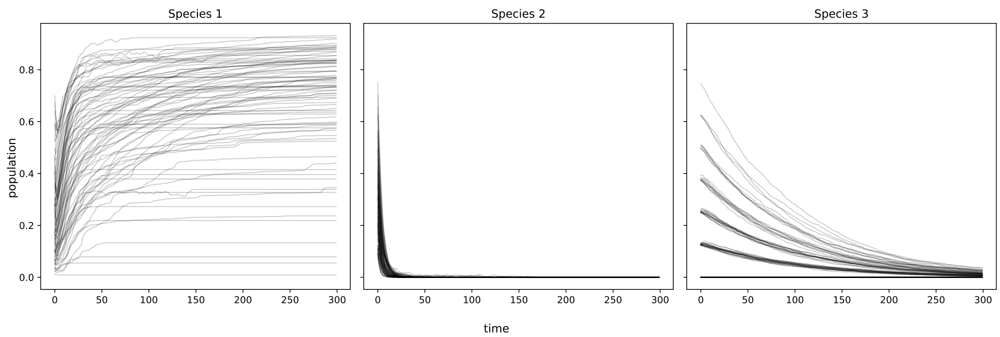
3.6 Environmental effect



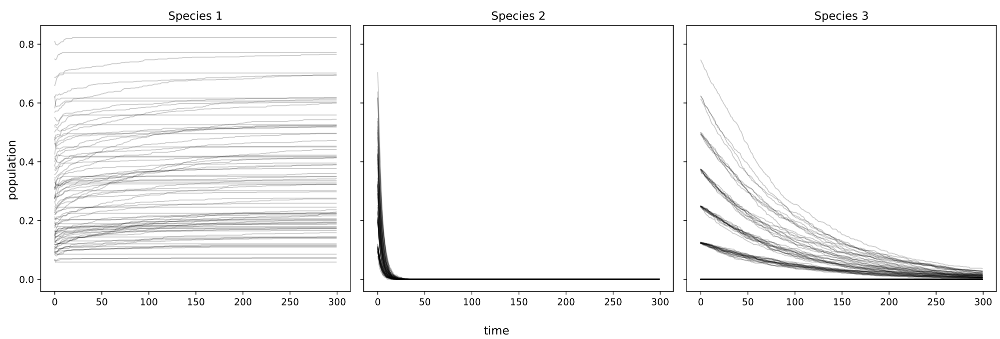
(a) $t = .1$



(b) $t = .3$



(c) $t = .5$



(d) $t = .8$

Figure 11: Populations vs. time for different tree densities, t, and initial densities. The steady states where all three species survives is colored in red. A length of sight of $r = 3$ was used for all simulations.

In figure 11, the effect of trees as environmental obstacles can be seen. For the two larger tree densities (i.e. $t = .5$ and $t = .8$) it can be noted that all higher level species die off due to starvation. Also, the low-level species sometimes reaches a steady population size lower than 100%. This can be explained by the trees covering their way and thus hindering them from reproducing in their neighboring sites. Basically, when there are too many trees, the low-level species will be blocked in sites surrounded by them. And since the low-level species can not die a natural death, they will never die out. If a higher level species is blocked, it will eventually die out due to starvation.

Looking at the two upper sub-figures in 11 is where things is starting to get interesting. Consider sub-figure (b) corresponding to 30% trees. It is clear, compared to the two-species steady states in figure 8, that the trees are favorable for the low-level species: the steady state seem to be somewhere around 65% low-level and 10% second level species, compared to the case of no trees where the steady state is 25% low-level and 20% second level species (in figure 8). An increase of 160%. The trees are blocking the sight of the predator and thus safely covering the low-level prey.

The same phenomenon can be seen in sub-figure (a) above. Here, we have two non-trivial steady states where one includes all three species. Comparing this latter state to the ones in figure 8 it can be seen that the second level species seem to benefit from the trees. Their population size is increased and the top-level predator population size is lowered. The other steady state where only the lower two species survives also show the same trend. The low-level prey benefits from the covering trees to the second level predators disadvantage.

4 Improvements

There are a few things that could improve the quality of the conducted study. First and foremost, not much can be said about how the birth and death probabilities affect the outcome of the study. For a future project it would have been interesting to study this and see for example if the effect of tree density is coupled with the birth and death probabilities.

Regarding the rules of the CA: it would have been interesting to let the creatures walk, reproduce, and eat in a Moore neighborhood and hence not restrict them in the diagonal directions. This should allow for a higher tree density before the low level species starts to get stuck as discussed in section 3.6.

Furthermore, it would probably be a more realistic setting if the second level species could have a separate natural death probability. As of now, this creature is equally probable to die off naturally as getting eaten.

Even though the environmental structure was shown to affect the outcome of the steady state, the modeling of the trees could have been made more realistic. For example, a tree closer to a creature should realistically cover a larger span of its vision. This effect was not considered here. Lastly, it would also be interesting to enable the low level species to "hide" inside the trees and see how this would affect the outcome.

References

[1] Gianpiero Cattaneo, Alberto Dennunzio, and Fabio Farina. “A Full Cellular Automaton to Simulate Predator-Prey Systems”. In: (2006).