Deep Reinforcement Learning

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Fall 2017

Slides are based on Fei Fei Li and colleagues lectures, cs231n, Stanford 2017
and some from Surguy Levin lectures, cs294-112, Berkeley 2016.

Supervised Learning

- Data: (x, y)
  - x is data
  - y is label

- Goal: Learn a function to map x -> y

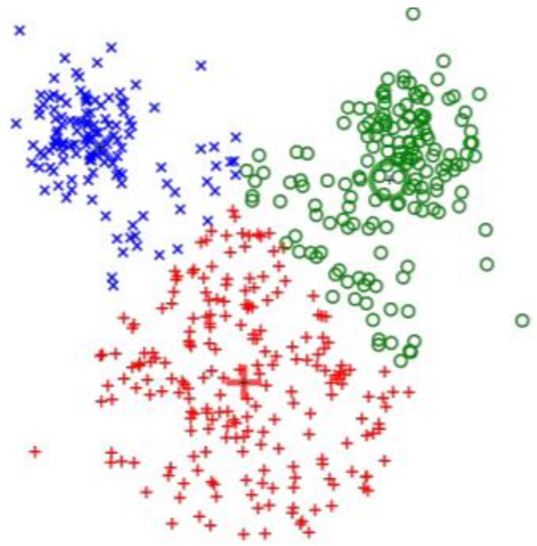
- Examples: Classification, regression, object detection, semantic segmentation, image captioning, etc.

Unsupervised Learning

- Data: x
  - Just data, no labels!

- Goal: Learn some underlying hidden structure of the data

- Examples: Clustering, dimensionality reduction, feature learning, density estimation, etc.



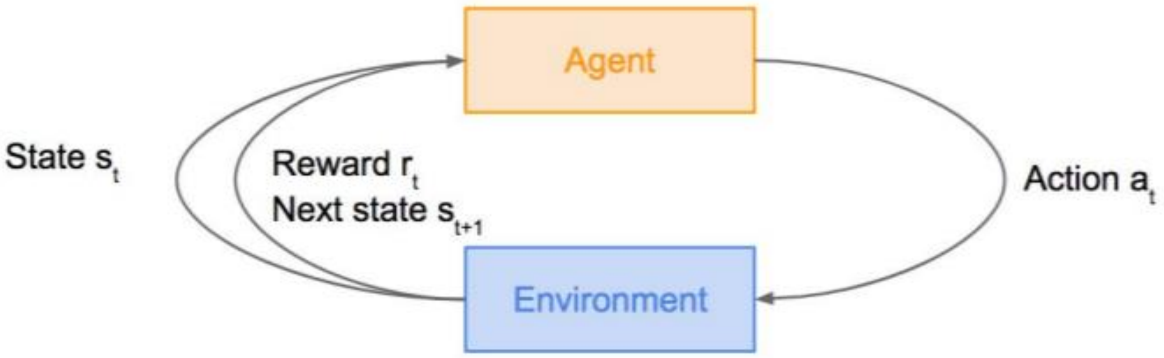
K-means clustering

Reinforcement Learning

- **Goal**: Learn how to take actions in order to maximize reward
  - Concerned with taking sequences of actions

- Described in terms of agent interacting with a previously unknown environment, trying to maximize cumulative reward

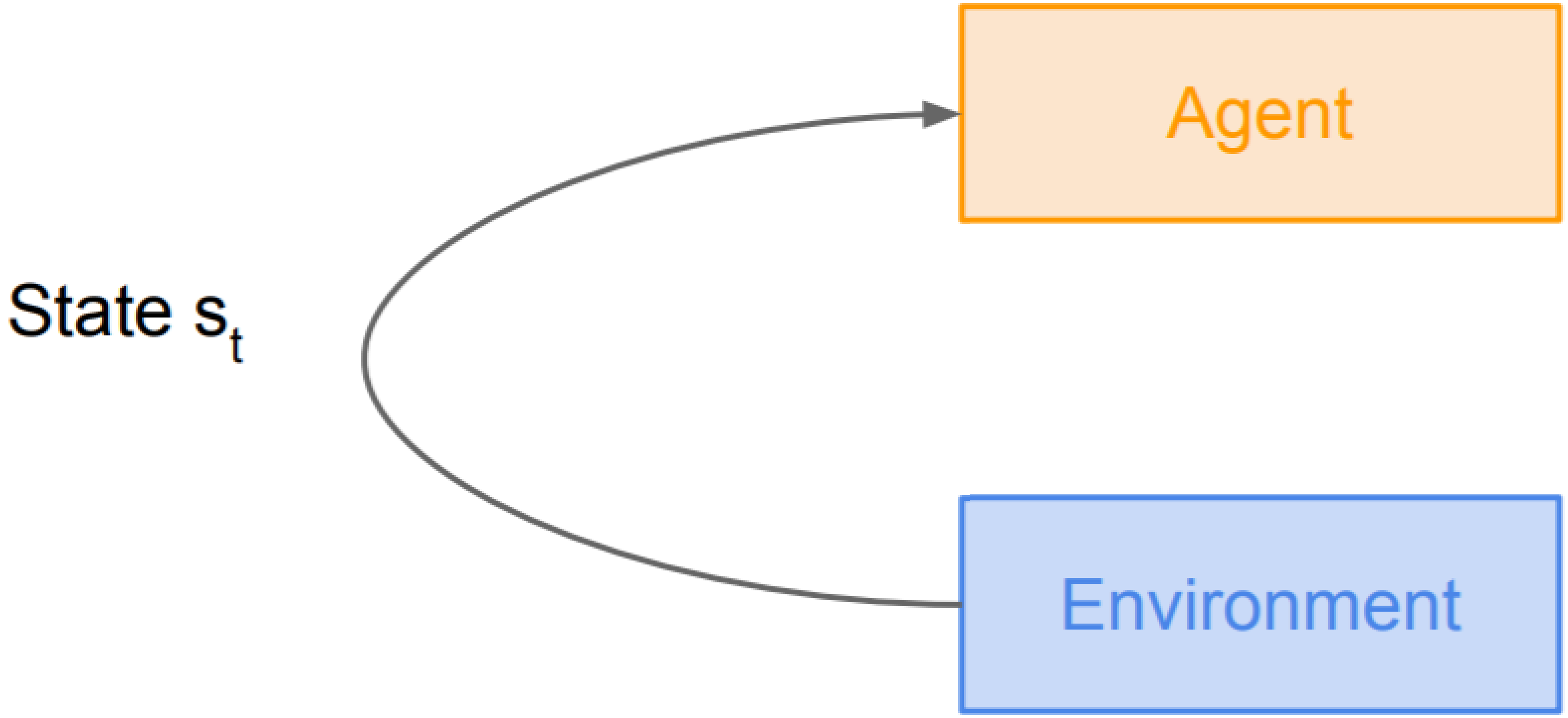
an **agent** interacting with an **environment**, which provides numeric **reward** signals



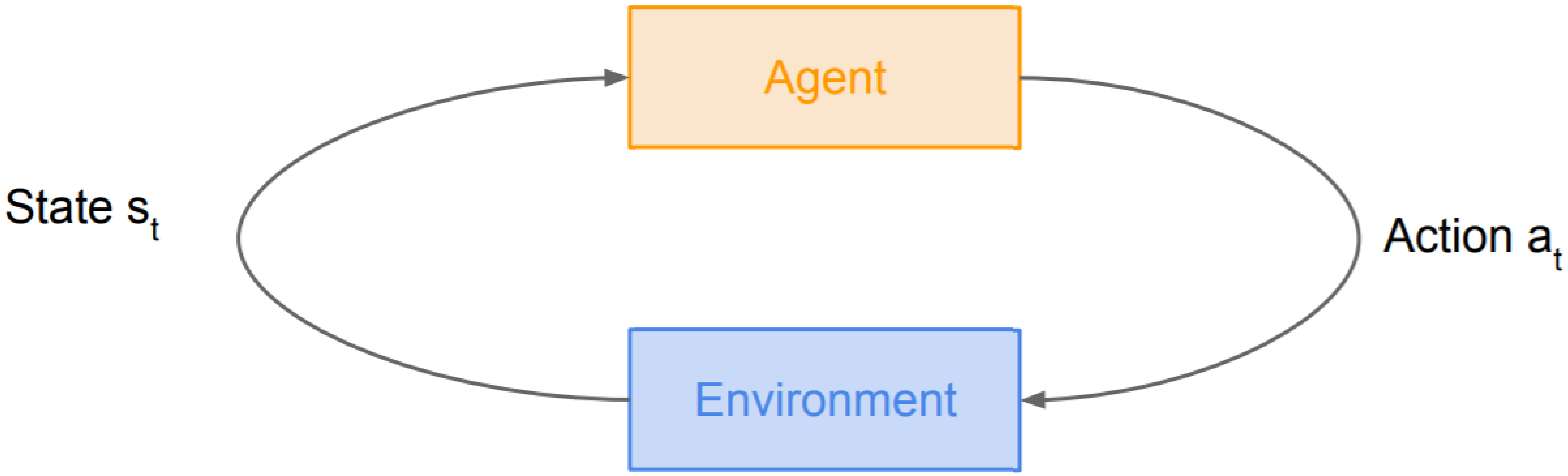
Overview

- What is Reinforcement Learning?
- Markov Decision Processes
- Q-Learning
- Policy Gradients

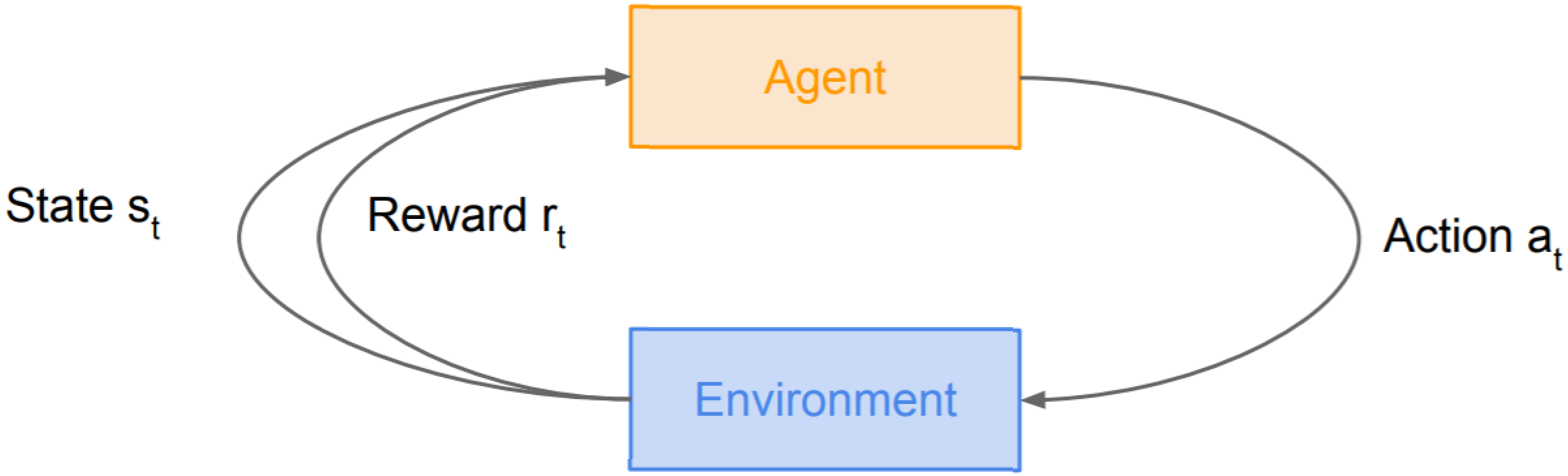
Reinforcement Learning



Reinforcement Learning



Reinforcement Learning



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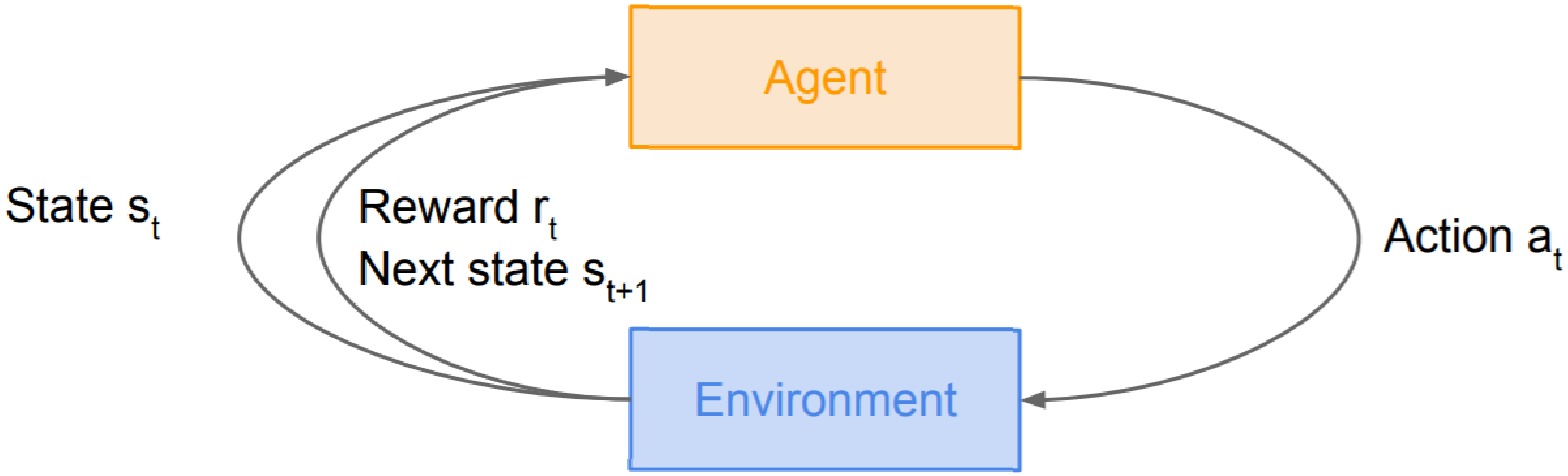
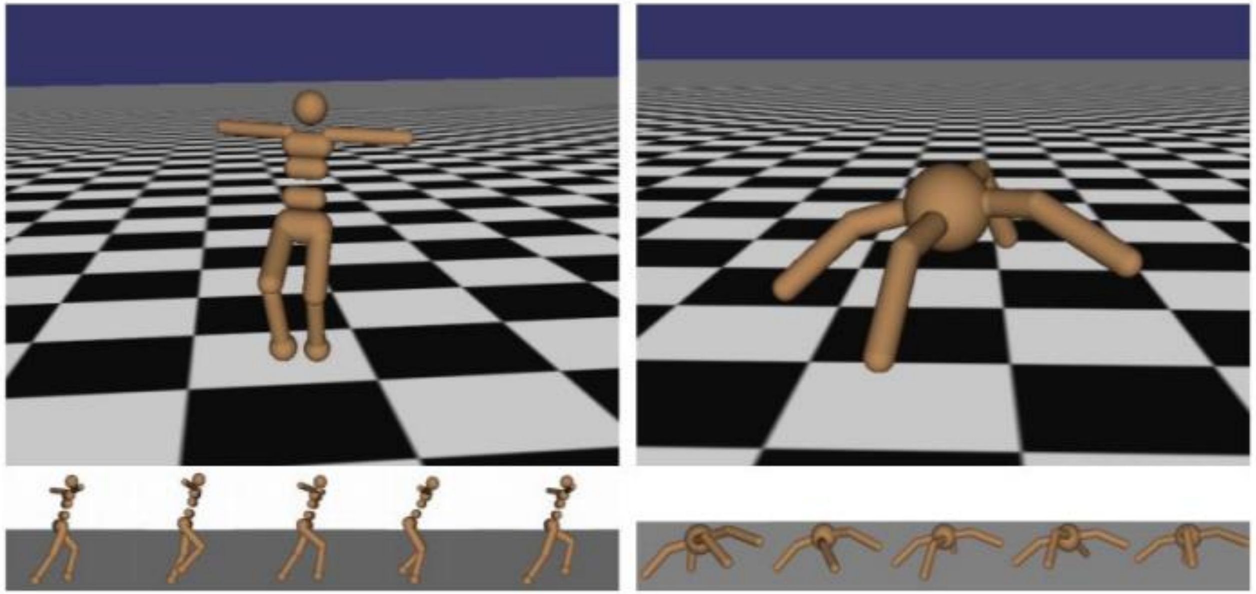


Robot Locomotion



Objective: Make the robot move forward

State: Angle and position of the joints
Action: Torques applied on joints
Reward: 1 at each time step upright + forward movement

Motor Control and Robotics

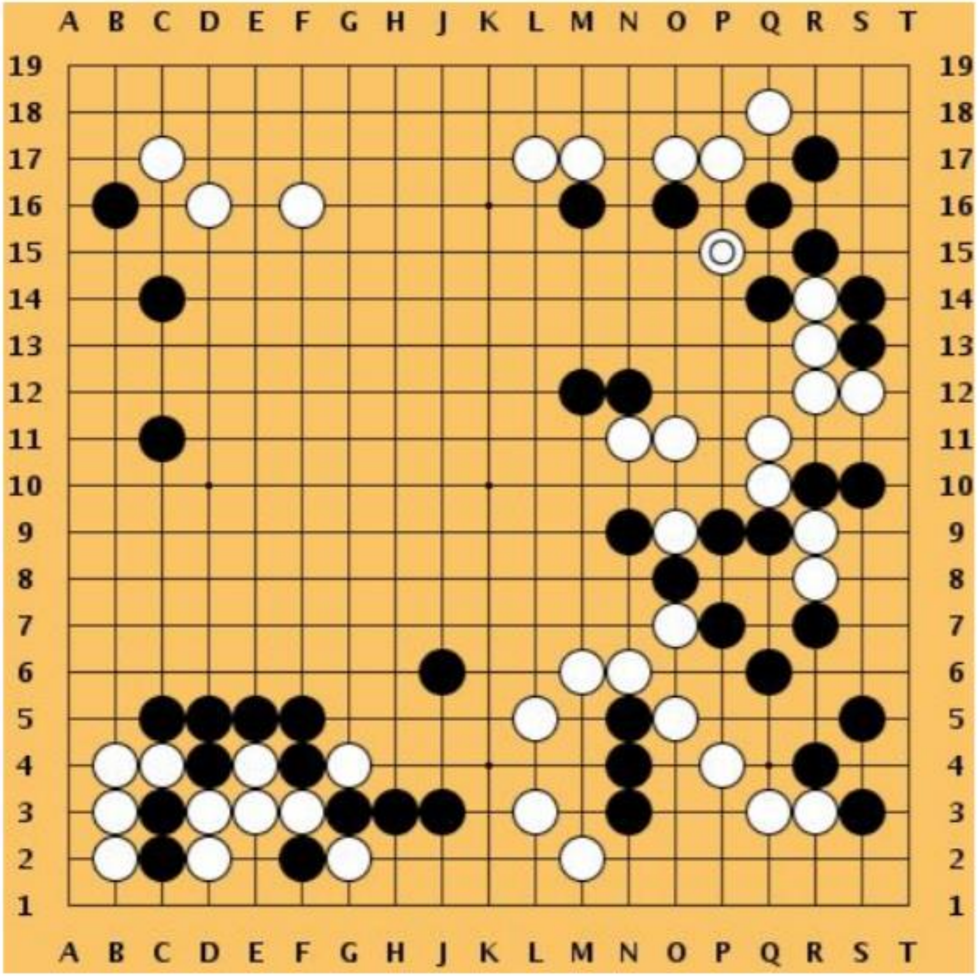
- Robotics:
  - Observations: camera images, joint angles
  - Actions: joint torques
  - Rewards: stay balanced, navigate to target locations, serve and protect humans

Atari Games

Objective: Complete the game with the highest score

State: Raw pixel inputs of the game state
Action: Game controls e.g. Left, Right, Up, Down
Reward: Score increase/decrease at each time step

Go



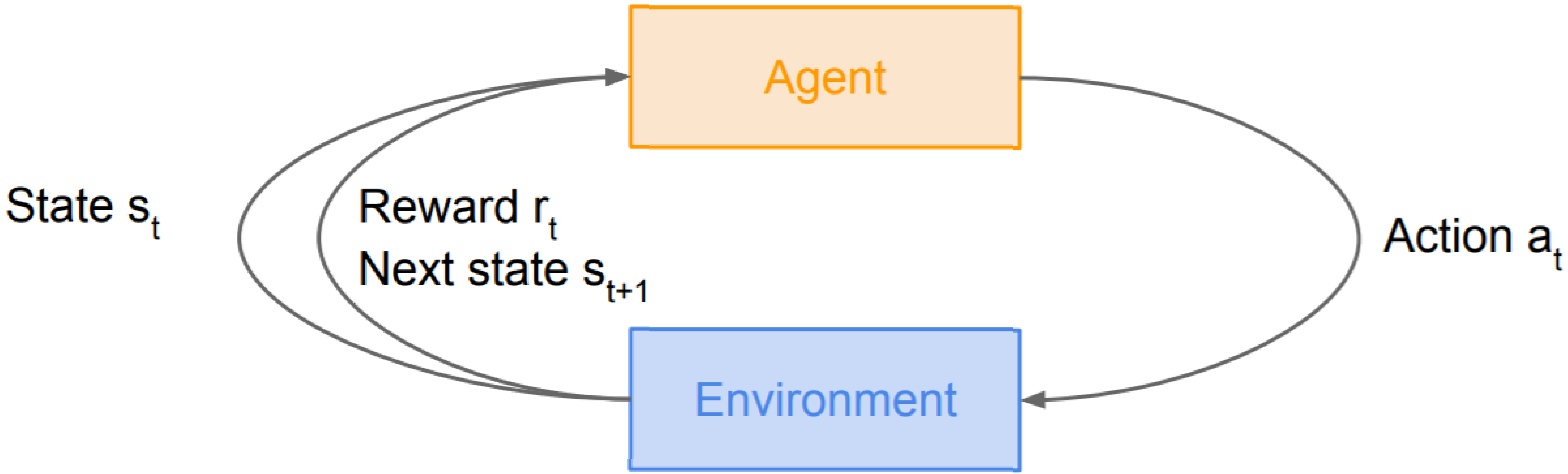
Objective: Win the game!

State: Position of all pieces
Action: Where to put the next piece down
Reward: 1 if win at the end of the game, 0 otherwise

How Does RL Relate to Other Machine Learning Problems?

- Differences between RL and supervised learning:
  - You don't have full access to the function you're trying to optimize
    - must query it through interaction.
  - Interacting with a stateful world: input x_t depend on your previous actions

How can we mathematically formalize the RL problem?



Markov Decision Process

- Mathematical formulation of the RL problem
- **Markov property**: Current state completely characterises the state of the world

Markov Decision Process

- At time step t=0, environment samples initial state $s_0 \sim p(s_0)$

- Then, for t=0 until done:
  - Agent selects action a_t
  - Environment samples reward $r_t \sim R(.\,|s_t, a_t)$
  - Environment samples next state $s_{t+1} \sim P(.\,|s_t, a_t)$
  - Agent receives reward r_t and next state s_{t+1}

- A policy π is a function from S to A that specifies what action to take in each state

- Objective: find policy π^* that maximizes cumulative discounted reward:

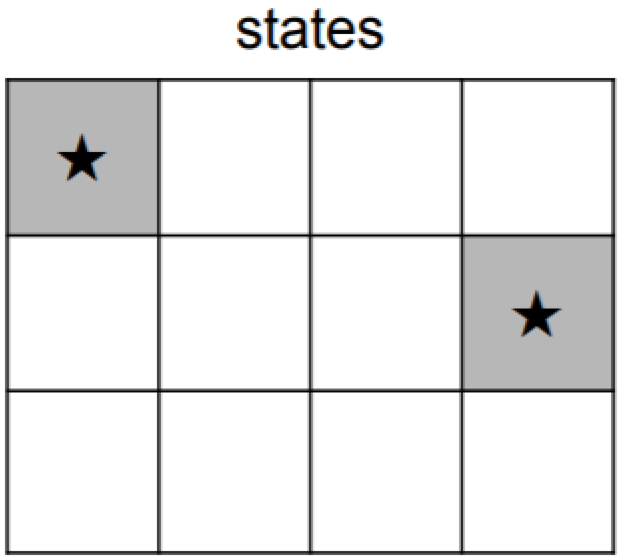
$$r_0 + \gamma r_1 + \gamma^2 r_2 + \cdots = \sum_{k=0}^{\infty} \gamma^k r_k$$

A simple MDP: Grid World

actions = {

1. right
2. left
3. up
4. down

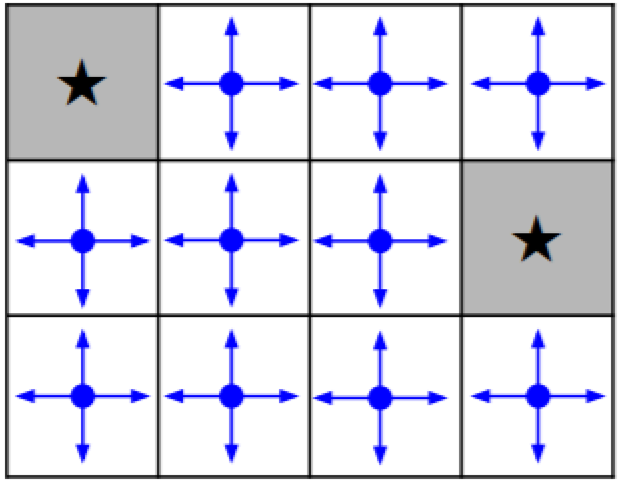
}



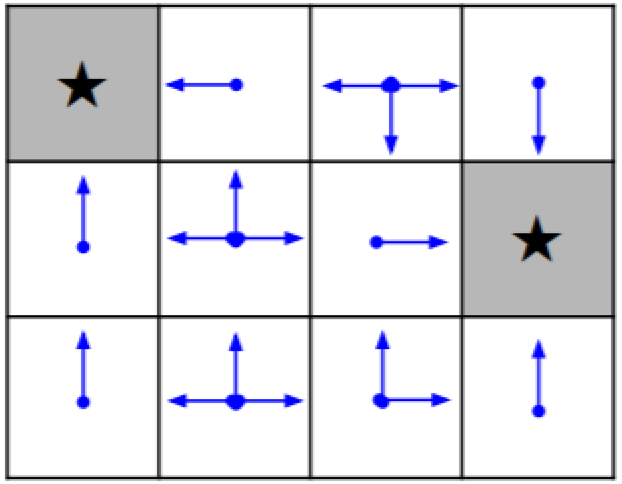
Set a negative "reward" for each transition (e.g. $r = -1$)

Objective: reach one of terminal states (greyed out) in least number of actions

A simple MDP: Grid World



Random Policy



Optimal Policy

The optimal policy π^*

- We want to find **optimal policy** $\boldsymbol{\pi}^*$ that maximizes the sum of rewards.
- How do we handle the randomness (initial state, transition probability…)?
  - Maximize the expected sum of rewards!

$$\pi^* = \arg\max_{\pi} \mathbb{E}\left[\sum_{t\geq 0} \gamma^t r_t | \pi\right]$$

$$s_0 \sim p(s_0)$$

$$a_t \sim \pi(\cdot|s_t)$$

$$s_{t+1} \sim p(\cdot|s_t, a_t)$$

Definitions: Value function and Q-value function

How good is a state?

The **value function** at state s, is the expected cumulative reward from following the policy from state s:

$$V^{\pi}(s) = \mathbb{E}\left[\sum_{t\geq 0} \gamma^t r_t | s_0 = s, \pi\right]$$

How good is a state-action pair?

The **Q-value function** at state s and action a, is the expected cumulative reward from taking action a in state s and then following the policy:

$$Q^{\pi}(s, a) = \mathbb{E}\left[\sum_{t\geq 0} \gamma^t r_t | s_0 = s, a_0 = a, \pi\right]$$

Value function for policy π

$$V^{\pi}(s) = E\{\sum_{k=0}^{\infty} \gamma^k r_t \,|s_0 = s, \pi\}$$

$$Q^{\pi}(s,a) = E\{\sum_{k=0}^{\infty} \gamma^k r_t \,|s_0 = s, a_0 = a \,, \pi\,\}$$

- $V^{\pi}(s)$: How good for the agent to be in the state s when its policy is π
  - It is simply the expected sum of discounted rewards upon starting in state s and taking actions according to π

$$V^{\pi}(s) = E[r + \gamma V^{\pi}(s')|s, \pi]$$

Bellman Equations

$$Q^{\pi}(s,a) = E[r + \gamma Q^{\pi}(s',a')|s, a, \pi]$$

Bellman optimality equation

$$V^*(s) = \max_{a \in \mathcal{A}(s)} E[r + \gamma V^*(s')|s, a]$$

$$Q^*(s, a) = E\left[r + \gamma \max_{a'} Q^*(s', a')\,|s, a\right]$$

Bellman equation

The optimal Q-value function Q* is the maximum expected cumulative reward achievable from a given (state, action) pair:

$$Q^*(s,a) = \max_{\pi} \mathbb{E}\left[\sum_{t\geq 0} \gamma^t r_t | s_0 = s, a_0 = a, \pi\right]$$

Q* satisfies the following **Bellman equation**:

$$Q^*(s,a) = \mathbb{E}_{s' \sim \mathcal{E}}\left[r + \gamma \max_{a'} Q^*(s',a') | s, a\right]$$

Intuition: if the optimal state-action values for the next time-step Q*(s',a') are known, then the optimal strategy is to take the action that maximizes the expected value of $r + \gamma Q^*(s', a')$

Optimal policy

The optimal policy **π*** corresponds to taking the best action in any state as specified by Q*

- It can also be computed as:

$$\pi^*(s) = \underset{a \in \mathcal{A}(s)}{\operatorname{argmax}}\, Q^*(s, a)$$

Solving for the optimal policy: Value iteration

Value iteration algorithm: Use Bellman equation as an iterative update

$$Q_{i+1}(s,a) = \mathbb{E}\left[r + \gamma \max_{a'} Q_i(s',a') | s, a\right]$$

Q_i will converge to Q^* as i -> infinity

Solving for the optimal policy: Q-learning algorithm

Initialize $\hat{Q}(s, a)$ arbitrarily

Repeat (for each episode):

Initialize s

Repeat (for each step of episode):

e.g., greedy, ε-greedy

Choose a from s using a policy derived from $\hat{Q}$

Take action a, receive reward r, observe new state s'

$$\hat{Q}(s, a) \leftarrow \hat{Q}(s, a) + \alpha \left[r + \gamma \max_{a'} \hat{Q}(s', a') - \hat{Q}(s, a) \right]$$

$s \leftarrow s'$

until s is terminal

Problem

- Not scalable.
  - Must compute Q(s,a) for every state-action pair.
    - it computationally infeasible to compute for entire state space!

- Solution: use a function approximator to estimate Q(s,a).
  - E.g. a neural network!

Solving for the optimal policy: Q-learning

Q-learning: Use a function approximator to estimate the action-value function

$$Q(s, a; \theta) \approx Q^*(s, a)$$

function parameters (weights)

If the function approximator is a deep neural network => **deep q-learning**!

Solving for the optimal policy: Q-learning

Remember: want to find a Q-function that satisfies the Bellman Equation:

$$Q^*(s,a) = \mathbb{E}_{s' \sim \mathcal{E}} \left[r + \gamma \max_{a'} Q^*(s', a') | s, a \right]$$

Forward Pass

Loss function: $L_i(\theta_i) = \mathbb{E}_{s,a \sim \rho(\cdot)} \left[(\boxed{y_i} - Q(s, a; \theta_i))^2 \right]$

Iteratively try to make the Q-value close to the target value (yi) it should have (according to Bellman Equations).

$$y_i = \mathbb{E}_{s' \sim \mathcal{E}} \left[r + \gamma \max_{a'} Q(s', a'; \theta_{i-1}) | s, a \right]$$

Backward Pass

Gradient update (with respect to Q-function parameters θ):

$$\nabla_{\theta_i} L_i(\theta_i) = \mathbb{E}_{s,a \sim \rho(\cdot); s' \sim \mathcal{E}} \left[(r + \gamma \max_{a'} Q(s', a'; \theta_{i-1}) - Q(s, a; \theta_i)) \nabla_{\theta_i} Q(s, a; \theta_i) \right]$$

[Mnih et al., Playing Atari with Deep Reinforcement Learning, NIPS Workshop 2013; Nature 2015]

Case Study: Playing Atari Games (seen before)

Objective: Complete the game with the highest score

State: Raw pixel inputs of the game state
Action: Game controls e.g. Left, Right, Up, Down
Reward: Score increase/decrease at each time step

[Mnih et al., Playing Atari with Deep Reinforcement Learning, NIPS Workshop 2013; Nature 2015]

Q-network Architecture

$Q(s, a; \theta)$: neural network with weights θ

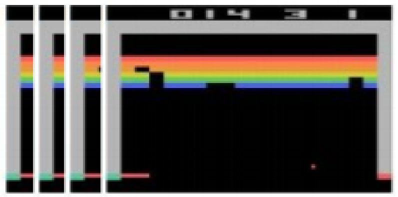
A single feedforward pass to compute Q-values for all actions from the current state => efficient!

FC-4 (Q-values)

FC-256

32 4x4 conv, stride 2

16 8x8 conv, stride 4



Current state s_t: 84x84x4 stack of last 4 frames
(after RGB->grayscale conversion, downsampling, and cropping)

Last FC layer has 4-d output (if 4 actions)
Q(st ,a1), Q(st,a2), Q(st,a3), Q(st,a4)

Number of actions between 4-18 depending on Atari game

[Mnih et al., Playing Atari with Deep Reinforcement Learning, NIPS Workshop 2013; Nature 2015]

Training the Q-network: Experience Replay

- Learning from batches of **consecutive samples** is problematic:
  - Samples are correlated => inefficient learning
  - Current Q-network parameters determines next training samples
    - can lead to bad feedback loops

- Address these problems using **experience replay**
  - Continually update a replay memory table of transitions (s_t, a_t, r_t, s_{t+1})
  - Train Q-network on random minibatches of transitions from the replay memory

✓ Each transition can also contribute to multiple weight updates => greater data efficiency

✓ Smoothing out learning and avoiding oscillations or divergence in the parameters

Putting it together: Deep Q-Learning with Experience Replay

Algorithm 1 Deep Q-learning with Experience Replay

Initialize replay memory $\mathcal{D}$ to capacity N
Initialize action-value function Q with random weights
for episode $= 1, M$ **do**
  Initialise sequence $s_1 = \{x_1\}$ and preprocessed sequenced $\phi_1 = \phi(s_1)$
  for $t = 1, T$ **do**
    With probability ϵ select a random action a_t
    otherwise select $a_t = \max_a Q^*(\phi(s_t), a; \theta)$
    Execute action a_t in emulator and observe reward r_t and image x_{t+1}
    Set $s_{t+1} = s_t, a_t, x_{t+1}$ and preprocess $\phi_{t+1} = \phi(s_{t+1})$
    Store transition $(\phi_t, a_t, r_t, \phi_{t+1})$ in $\mathcal{D}$
    Sample random minibatch of transitions $(\phi_j, a_j, r_j, \phi_{j+1})$ from $\mathcal{D}$
    Set $y_j = \begin{cases} r_j & \text{for terminal } \phi_{j+1} \\ r_j + \gamma \max_{a'} Q(\phi_{j+1}, a'; \theta) & \text{for non-terminal } \phi_{j+1} \end{cases}$
    Perform a gradient descent step on $(y_j - Q(\phi_j, a_j; \theta))^2$ according to equation 3
  end for
end for

[Mnih et al., Playing Atari with Deep Reinforcement Learning, NIPS Workshop 2013; Nature 2015]

Putting it together: Deep Q-Learning with Experience Replay

Algorithm 1 Deep Q-learning with Experience Replay

$\boxed{\begin{array}{l}\text{Initialize replay memory } \mathcal{D} \text{ to capacity } N \\ \text{Initialize action-value function } Q \text{ with random weights}\end{array}}$ → Initialize replay memory, Q-network

for episode $= 1, M$ **do**
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Initialize action-value function Q with random weights
for episode $= 1, M$ **do** — Play M episodes (full games)
  Initialise sequence $s_1 = \{x_1\}$ and preprocessed sequenced $\phi_1 = \phi(s_1)$
  for $t = 1, T$ **do**
    With probability ϵ select a random action a_t
    otherwise select $a_t = \max_a Q^*(\phi(s_t), a; \theta)$
    Execute action a_t in emulator and observe reward r_t and image x_{t+1}
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  end for
end for

Initialize state (starting game screen pixels) at the beginning of each episode

[Mnih et al., Playing Atari with Deep Reinforcement Learning, NIPS Workshop 2013; Nature 2015]

Putting it together: Deep Q-Learning with Experience Replay

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    for $t = 1, T$ **do** — For each time-step of game
        With probability ϵ select a random action a_t
        otherwise select $a_t = \max_a Q^*(\phi(s_t), a; \theta)$
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  end for
end for

With small probability, select a random action (explore), otherwise select greedy action from current policy

[Mnih et al., Playing Atari with Deep Reinforcement Learning, NIPS Workshop 2013; Nature 2015]

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  end for
end for

Take the selected action observe
the reward and next state

[Mnih et al., Playing Atari with Deep Reinforcement Learning, NIPS Workshop 2013; Nature 2015]

Putting it together: Deep Q-Learning with Experience Replay

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    Store transition $(\phi_t, a_t, r_t, \phi_{t+1})$ in $\mathcal{D}$ → Store transition in replay memory
    Sample random minibatch of transitions $(\phi_j, a_j, r_j, \phi_{j+1})$ from $\mathcal{D}$
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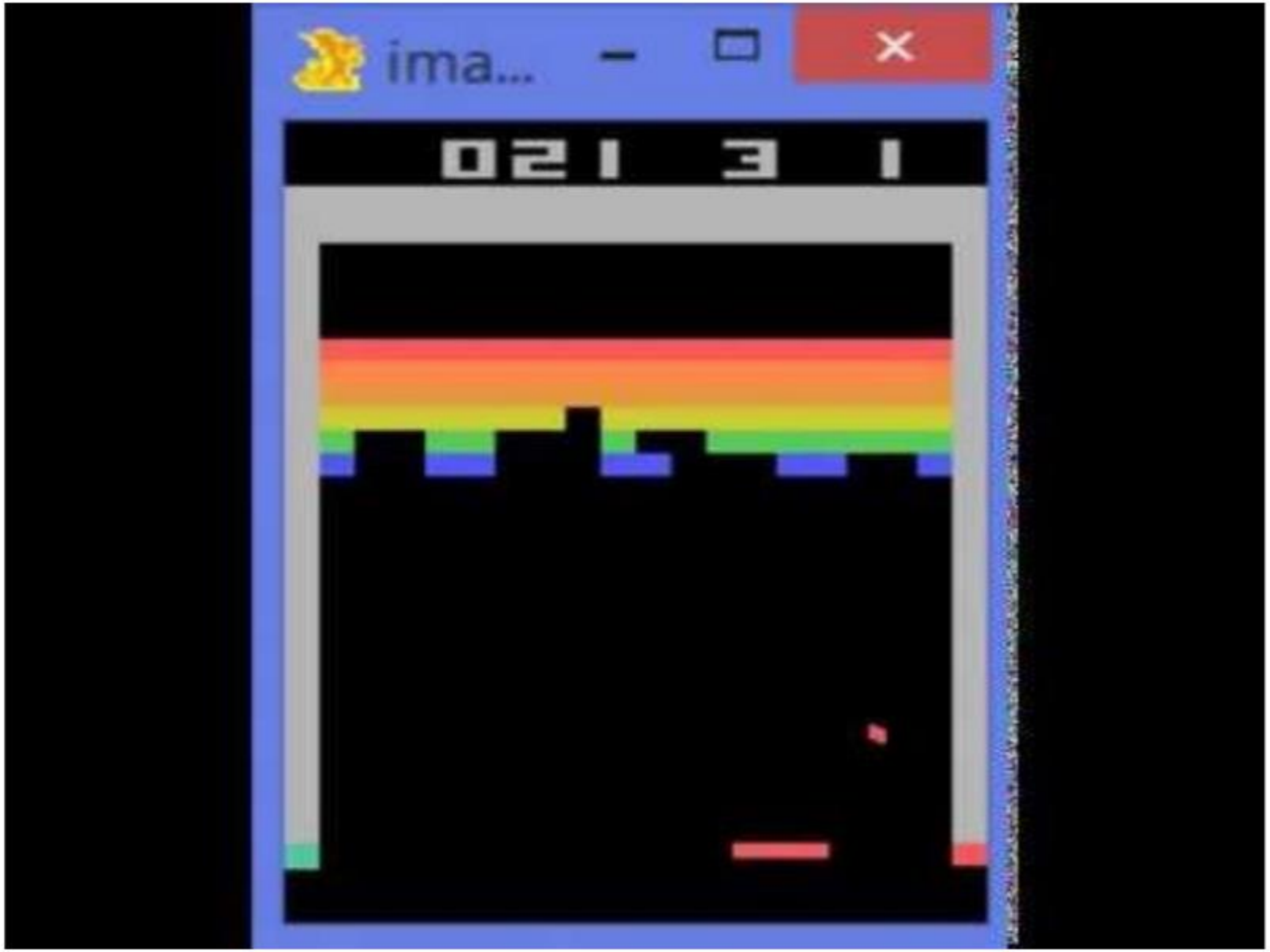
    for $t = 1, T$ **do**
        With probability ϵ select a random action a_t
        otherwise select $a_t = \max_a Q^*(\phi(s_t), a; \theta)$
        Execute action a_t in emulator and observe reward r_t and image x_{t+1}
        Set $s_{t+1} = s_t, a_t, x_{t+1}$ and preprocess $\phi_{t+1} = \phi(s_{t+1})$
        Store transition $(\phi_t, a_t, r_t, \phi_{t+1})$ in $\mathcal{D}$
        Sample random minibatch of transitions $(\phi_j, a_j, r_j, \phi_{j+1})$ from $\mathcal{D}$
        Set $y_j = \begin{cases} r_j & \text{for terminal } \phi_{j+1} \\ r_j + \gamma \max_{a'} Q(\phi_{j+1}, a'; \theta) & \text{for non-terminal } \phi_{j+1} \end{cases}$
        Perform a gradient descent step on $(y_j - Q(\phi_j, a_j; \theta))^2$ according to equation 3
    end for
end for

Sample a random minibatch of transitions and perform a gradient descent step

[Mnih et al., Playing Atari with Deep Reinforcement Learning, NIPS Workshop 2013; Nature 2015]



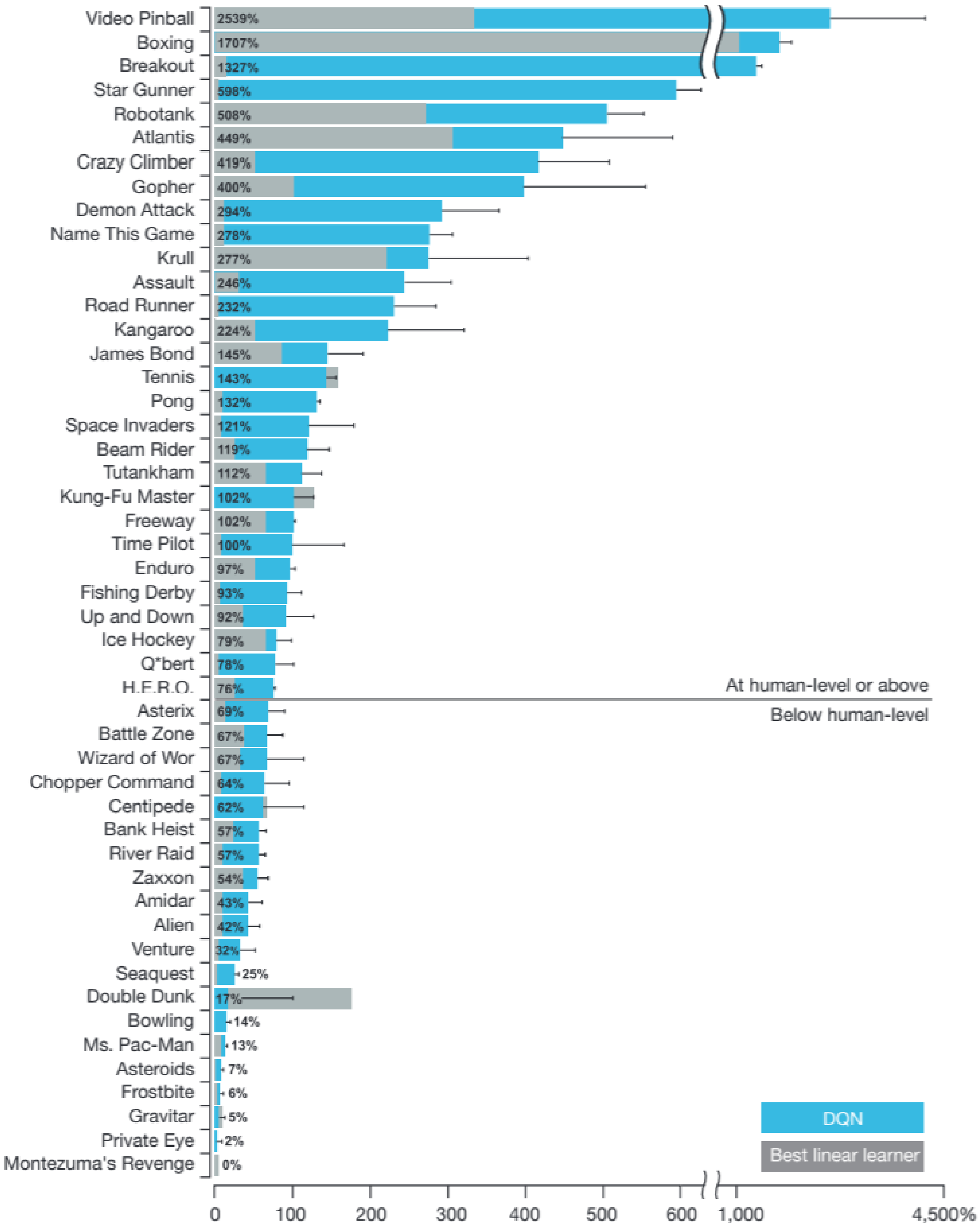
https://www.youtube.com/watch?v=V1eYniJ0Rnk

Video by Károly Zsolnai-Fehér. Reproduced with permission.

Results on 49 Games

- The architecture and hyperparameter values were the same for all 49 games.

- DQN achieved performance comparable to or better than an experienced human on 29 out of 49 games.



[V. Mnih et al., Human-level control through deep reinforcement learning, Nature 2015]

Policy Gradients

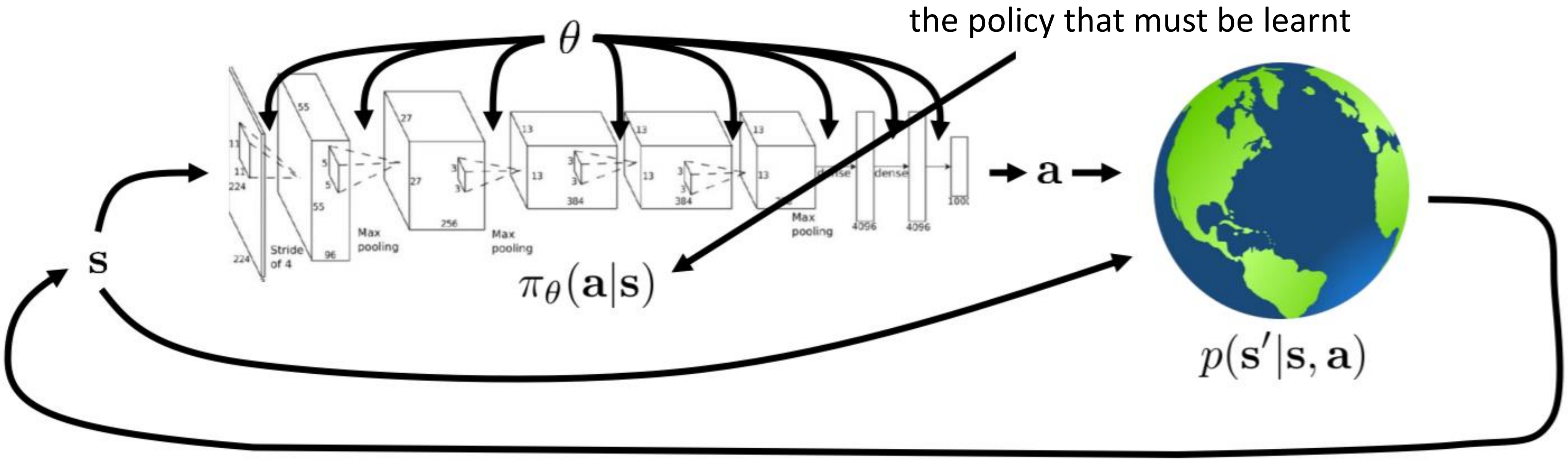
- What is a problem with Q-learning?
  - The Q-function can be very complicated!

- Hard to learn exact value of every (state, action) pair

- But the policy can be much simple

- Can we learn a policy directly, e.g. finding the best policy from a collection of policies?

The goal of RL



$$\underbrace{p_\theta(\mathbf{s}_1,\mathbf{a}_1,\ldots,\mathbf{s}_T,\mathbf{a}_T)}_{p_\theta(\tau)} = p(\mathbf{s}_1)\prod_{t=1}^{T}\pi_\theta(\mathbf{a}_t|\mathbf{s}_t)p(\mathbf{s}_{t+1}|\mathbf{s}_t,\mathbf{a}_t)$$

$$\theta^\star = \arg\max_\theta E_{\tau\sim p_\theta(\tau)}\left[\sum_t r(\mathbf{s}_t,\mathbf{a}_t)\right]$$

Policy Gradients

Formally, let's define a class of parametrized policies: $\Pi = \{\pi_\theta, \theta \in \mathbb{R}^m\}$

For each policy, define its value:

$$J(\theta) = \mathbb{E}\left[\sum_{t\geq 0} \gamma^t r_t | \pi_\theta\right]$$

We want to find the optimal policy $\theta^* = \arg\max_\theta J(\theta)$

How can we do this?

Gradient ascent on policy parameters!

REINFORCE algorithm

Mathematically, we can write:

$$J(\theta) = \mathbb{E}_{\tau \sim p(\tau;\theta)} \left[r(\tau) \right]$$
$$= \int_{\tau} r(\tau) p(\tau; \theta) \mathrm{d}\tau$$

Where r(τ) is the reward of a trajectory $\tau = (s_0, a_0, r_0, s_1, \ldots)$

REINFORCE algorithm

Expected reward: $$J(\theta) = \mathbb{E}_{\tau \sim p(\tau;\theta)} [r(\tau)]$$
$$= \int_\tau r(\tau) p(\tau;\theta) \mathrm{d}\tau$$

Now let's differentiate this: $$\nabla_\theta J(\theta) = \int_\tau r(\tau) \nabla_\theta p(\tau;\theta) \mathrm{d}\tau$$

Intractable! Gradient of an expectation is problematic when p depends on θ

REINFORCE algorithm

Expected reward: $J(\theta) = \mathbb{E}_{\tau \sim p(\tau;\theta)}\left[r(\tau)\right]$

$$= \int_\tau r(\tau)p(\tau;\theta)\mathrm{d}\tau$$

Now let's differentiate this: $\nabla_\theta J(\theta) = \int_\tau r(\tau)\nabla_\theta p(\tau;\theta)\mathrm{d}\tau$

However, we can use a nice trick: $\nabla_\theta p(\tau;\theta) = p(\tau;\theta)\frac{\nabla_\theta p(\tau;\theta)}{p(\tau;\theta)} = p(\tau;\theta)\nabla_\theta \log p(\tau;\theta)$

$$\nabla_\theta J(\theta) = \int_\tau r(\tau) \nabla_\theta p(\tau; \theta) \mathrm{d}\tau$$

$$\nabla_\theta p(\tau; \theta) = p(\tau; \theta) \frac{\nabla_\theta p(\tau; \theta)}{p(\tau; \theta)} = p(\tau; \theta) \nabla_\theta \log p(\tau; \theta)$$

$$\begin{aligned} \Rightarrow \nabla_\theta J(\theta) &= \int_\tau \left(r(\tau) \nabla_\theta \log p(\tau; \theta) \right) p(\tau; \theta) \mathrm{d}\tau \\ &= \mathbb{E}_{\tau \sim p(\tau; \theta)} \left[r(\tau) \nabla_\theta \log p(\tau; \theta) \right] \end{aligned}$$

Can estimate with Monte Carlo sampling

REINFORCE algorithm

Can we compute those quantities without knowing the transition probabilities?

We have: $p(\tau;\theta) = \prod_{t\geq 0} p(s_{t+1}|s_t, a_t)\pi_\theta(a_t|s_t)$

Thus: $\log p(\tau;\theta) = \sum_{t\geq 0} \log p(s_{t+1}|s_t, a_t) + \log \pi_\theta(a_t|s_t)$

And when differentiating: $\nabla_\theta \log p(\tau;\theta) = \sum_{t\geq 0} \nabla_\theta \log \pi_\theta(a_t|s_t)$

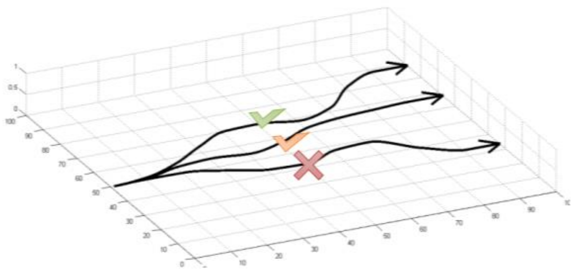
Doesn't depend on transition probabilities!

REINFORCE algorithm

$$\begin{aligned}\nabla_\theta J(\theta) &= \int_\tau \left(r(\tau)\nabla_\theta \log p(\tau;\theta)\right) p(\tau;\theta)\mathrm{d}\tau \\ &= \mathbb{E}_{\tau\sim p(\tau;\theta)}\left[r(\tau)\nabla_\theta \log p(\tau;\theta)\right]\end{aligned}$$

$$\nabla_\theta \log p(\tau;\theta) = \sum_{t\geq 0} \nabla_\theta \log \pi_\theta(a_t|s_t)$$

$$\nabla_\theta J(\theta) \approx \frac{1}{N}\sum_{n=1}^{N} r(\tau^{(n)})\, \nabla_\theta \log p\left(\tau^{(n)};\theta\right)$$



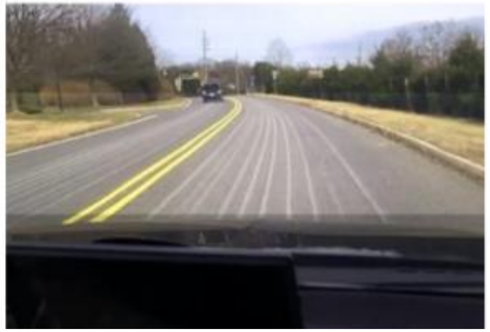
$$\nabla_\theta J(\theta) \approx \frac{1}{N}\sum_{n=1}^{N}\sum_{t\geq 0} r\left(s_t^{(n)}, a_t^{(n)}\right)\sum_{t\geq 0}\nabla_\theta \log \pi_\theta\left(a_t^{(n)}|s_t^{(n)}\right)$$

REINFORCE Algorithm

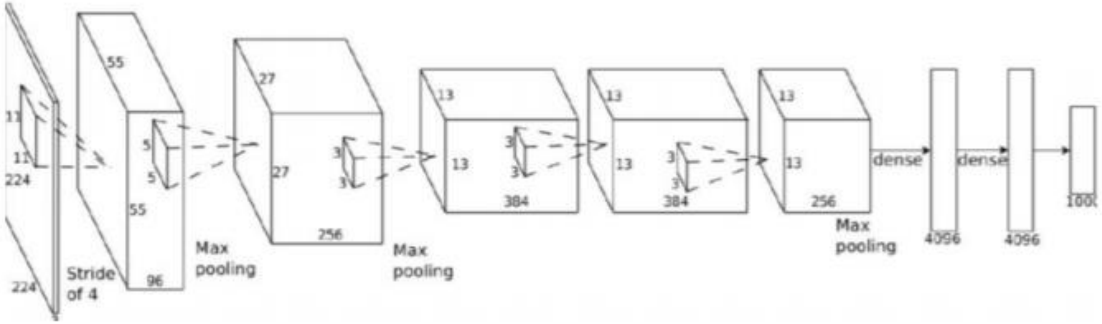
- Repeat
  - Sample $\{\tau^{(n)}\}$ from $\pi_\theta(a|s)$ (run the policy)
  - $\nabla_\theta J(\theta) \approx \frac{1}{N}\sum_{n=1}^{N}\sum_{t\geq 0} r\left(s_t^{(n)}, a_t^{(n)}\right)\sum_{t\geq 0}\nabla_\theta \log \pi_\theta\left(a_t^{(n)}|s_t^{(n)}\right)$
  - $\theta \leftarrow \theta + \alpha\nabla_\theta J(\theta)$

Evaluating the policy gradient

$$\nabla_\theta J(\theta) \approx \frac{1}{N}\sum_{n=1}^{N} \underbrace{\sum_{t\geq 0} r\left(s_t^{(n)}, a_t^{(n)}\right)}_{r\left(\tau^{(n)}\right)} \sum_{t\geq 0} \nabla_\theta \log \pi_\theta\left(a_t^{(n)} | s_t^{(n)}\right)$$

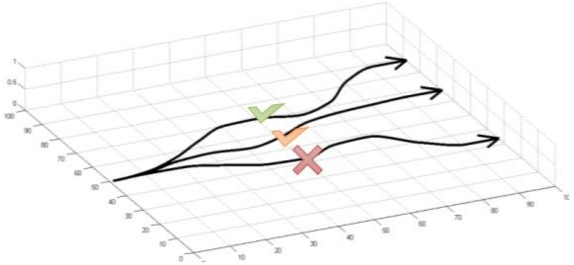


s_t



$\pi_\theta(a_t|s_t)$

a_t

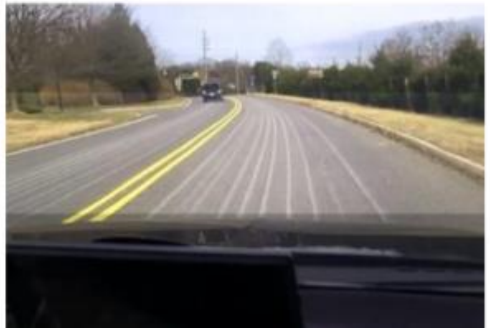


- Policy gradient:

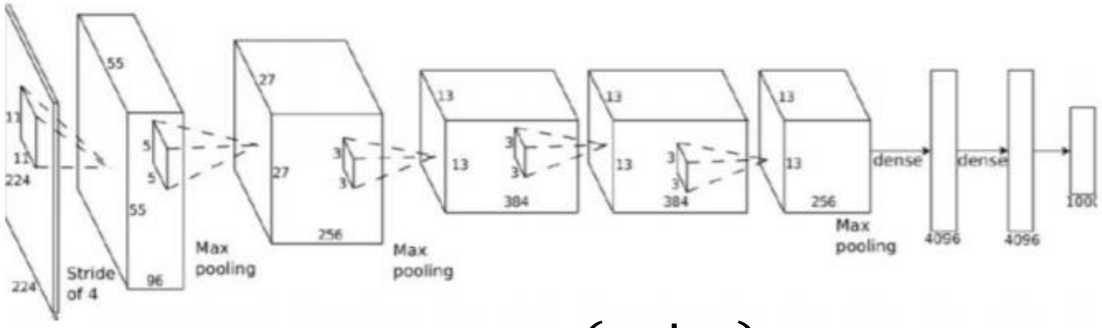
$$\nabla_\theta J(\theta) \approx \frac{1}{N} \sum_{n=1}^{N} \underbrace{\sum_{t\geq 0} r\left(s_t^{(n)}, a_t^{(n)}\right)}_{r\left(\tau^{(n)}\right)} \sum_{t\geq 0} \nabla_\theta \log \pi_\theta\left(a_t^{(n)} | s_t^{(n)}\right)$$

- Maximum Likelihood:

$$\nabla_\theta J(\theta) \approx \frac{1}{N} \sum_{n=1}^{N} \sum_{t\geq 0} \nabla_\theta \log \pi_\theta\left(a_t^{(n)} | s_t^{(n)}\right)$$



s_t

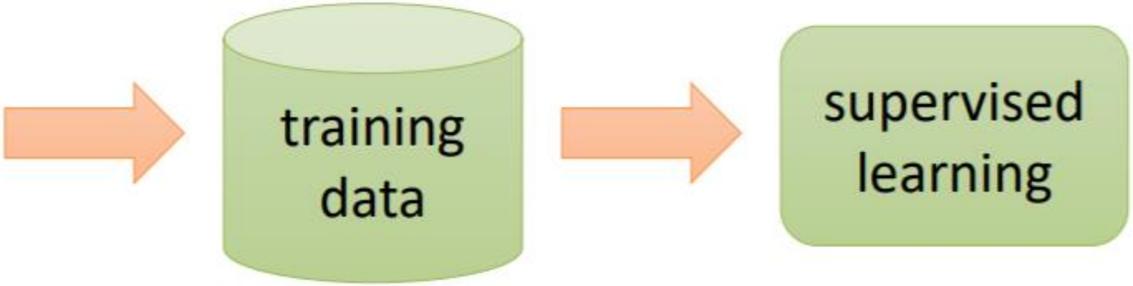


$\pi_\theta(a_t|s_t)$

a_t

$\mathbf{s}_t$

$\mathbf{a}_t$



$\pi_\theta(a_t|s_t)$

Intuition

$$\nabla_\theta J(\theta) \approx \frac{1}{N}\sum_{n=1}^{N} r\left(\tau^{(n)}\right)\sum_{t\geq 0}\nabla_\theta \log \pi_\theta\left(a_t^{(n)}|s_t^{(n)}\right)$$

Interpretation:

- If r(τ) is high, push up the probabilities of the actions seen
- If r(τ) is low, push down the probabilities of the actions seen

Might seem simplistic to say that if a trajectory is good then all its actions were good. But in expectation, it averages out!

- However, this also suffers from high variance
  - because credit assignment is really hard.

What did we just do?

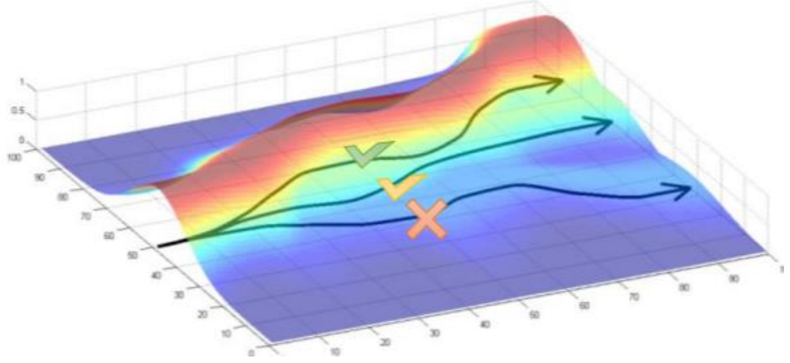
$$\nabla_\theta J(\theta) \approx \frac{1}{N}\sum_{n=1}^{N} r\left(\tau^{(n)}\right)\underbrace{\nabla_\theta \log p_\theta\left(\tau^{(n)}\right)}_{\sum_{t\geq 0}\nabla_\theta \log \pi_\theta\left(a_t^{(n)}|s_t^{(n)}\right)}$$

$$\nabla_\theta J_{ML}(\theta) \approx \frac{1}{N}\sum_{n=1}^{N} \nabla_\theta \log p_\theta\left(\tau^{(n)}\right)$$

good stuff is made more likely

bad stuff is made less likely

simply formalizes the notion of "trial and error"!



Reducing variance

$$\nabla_\theta J(\theta) \approx \frac{1}{N} \sum_{n=1}^{N} \sum_{t \geq 0} r\left(s_t^{(n)}, a_t^{(n)}\right) \sum_{t \geq 0} \nabla_\theta \log \pi_\theta \left(a_t^{(n)} | s_t^{(n)}\right)$$

- Causality:

policy at time t' cannot affect reward at time t when $t < t'$

$$\nabla_\theta J(\theta) \approx \frac{1}{N} \sum_{n=1}^{N} \sum_{t \geq 0} \left(\sum_{t' \geq t} r\left(s_{t'}^{(n)}, a_{t'}^{(n)}\right) \right) \nabla_\theta \log \pi_\theta \left(a_t^{(n)} | s_t^{(n)}\right)$$

Variance reduction

Gradient estimator: $$\nabla_\theta J(\theta) \approx \frac{1}{N}\sum_{n=1}^{N}\sum_{t\geq 0} r\left(s_t^{(n)}, a_t^{(n)}\right)\sum_{t\geq 0}\nabla_\theta \log \pi_\theta\left(a_t^{(n)}|s_t^{(n)}\right)$$

First idea: Push up probabilities of an action seen, only by the cumulative future reward from that state

$$\nabla_\theta J(\theta) \approx \frac{1}{N}\sum_{n=1}^{N}\sum_{t\geq 0}\sum_{t'\geq t} r\left(s_{t'}^{(n)}, a_{t'}^{(n)}\right)\nabla_\theta \log \pi_\theta\left(a_t^{(n)}|s_t^{(n)}\right)$$

Second idea: Use discount factor γ to ignore delayed effects

$$\nabla_\theta J(\theta) \approx \frac{1}{N}\sum_{n=1}^{N}\sum_{t\geq 0}\sum_{t'\geq t} \gamma^{t'-t} r\left(s_{t'}^{(n)}, a_{t'}^{(n)}\right)\nabla_\theta \log \pi_\theta\left(a_t^{(n)}|s_t^{(n)}\right)$$

Variance reduction: Baseline

- **Problem:** The raw value of a trajectory isn't necessarily meaningful.
  - For example, if rewards are all positive, you keep pushing up probabilities of actions.
- **What is important then**?
  - Whether a reward is better or worse than what you expect to get

- **Idea**: Introduce a baseline function dependent on the state.

$$\nabla_\theta J(\theta) \approx \frac{1}{N} \sum_{n=1}^{N} \sum_{t \geq 0} \left(\sum_{t' \geq t} \gamma^{t'-t} r\left(s_{t'}^{(n)}, a_{t'}^{(n)}\right) - b\left(s_t^{(n)}\right) \right) \nabla_\theta \log \pi_\theta \left(a_t^{(n)} | s_t^{(n)}\right)$$

Simple baseline: $b = \frac{1}{N} \sum_{n=1}^{N} r\left(\tau^{(n)}\right)$ average reward is not the best baseline, but it's pretty good!

How to choose the baseline?

$$\nabla_\theta J(\theta) \approx \sum_{t\geq 0} \left(\sum_{t'\geq t} \gamma^{t'-t} r_{t'} - b(s_t) \right) \nabla_\theta \log \pi_\theta(a_t|s_t)$$

- A simple baseline: constant moving average of rewards experienced so far from all trajectories

- Variance reduction techniques seen so far are typically used in “Vanilla REINFORCE”

Policy gradient in practice

- Remember that the gradient has high variance
  - This isn't the same as supervised learning!
  - Gradients will be really noisy!
- Consider using much larger batches
- Tweaking learning rates is very hard
  - Adaptive step size rules like ADAM can be OK-ish
  - policy gradient-specific learning rate adjustment methods

REINFORCE Algorithm

- Repeat
  - Sample $\{\tau^{(n)}\}$ from $\pi_\theta(a_t|s_t)$ (run the policy)
  - $\nabla_\theta J(\theta) \approx \frac{1}{N}\sum_{n=1}^{N}\sum_{t\geq 0}\gamma^{t'-t}r\left(s_t^{(n)}, a_t^{(n)}\right)\sum_{t\geq 0}\nabla_\theta \log \pi_\theta\left(a_t^{(n)}|s_t^{(n)}\right)$
  - $\theta \leftarrow \theta + \alpha\nabla_\theta J(\theta)$

$$\nabla_\theta J(\theta) \approx \frac{1}{N}\sum_{n=1}^{N}\sum_{t\geq 0}\underbrace{\left(\sum_{t'\geq t}\gamma^{t'-t}r\left(s_{t'}^{(n)}, a_{t'}^{(n)}\right)\right)}_{\text{Reward to go } Q_t^{(n)}}\nabla_\theta \log \pi_\theta\left(a_t^{(n)}|s_t^{(n)}\right)$$

How to choose the baseline?

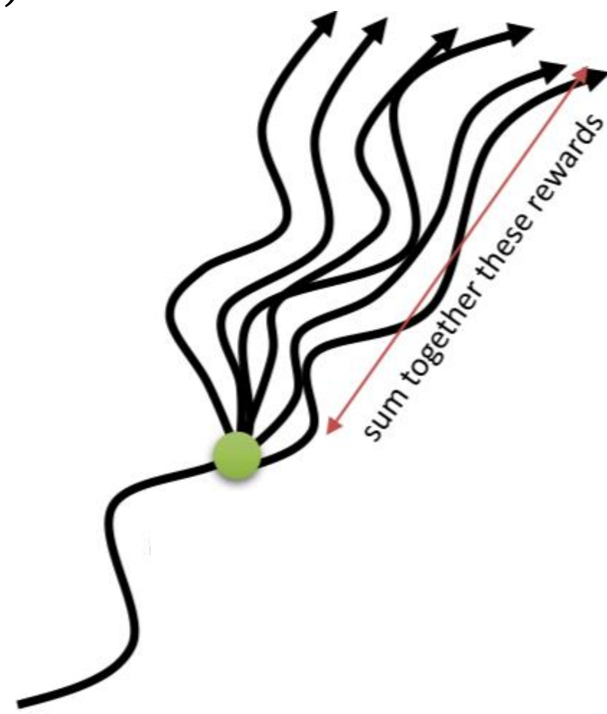
- A better baseline (to push up the probability of an action from a state):
  - if this action was better than the expected value of what we should get from that state.

$$Q^{\pi}(s_t, a_t) - V^{\pi}(s_t)$$

- We are happy with an action a_t in a state s_t if it is large
- we are unhappy with an action if it's small

Improving the policy gradient

$$\nabla_\theta J(\theta) \approx \frac{1}{N}\sum_{n=1}^{N}\sum_{t\geq 0}\underbrace{\left(\sum_{t'\geq t}\gamma^{t'-t} r\left(s_{t'}^{(n)}, a_{t'}^{(n)}\right)\right)}_{\text{Reward to go } \hat{Q}_t^{(n)}} \nabla_\theta \log \pi_\theta\left(a_t^{(n)} | s_t^{(n)}\right)$$

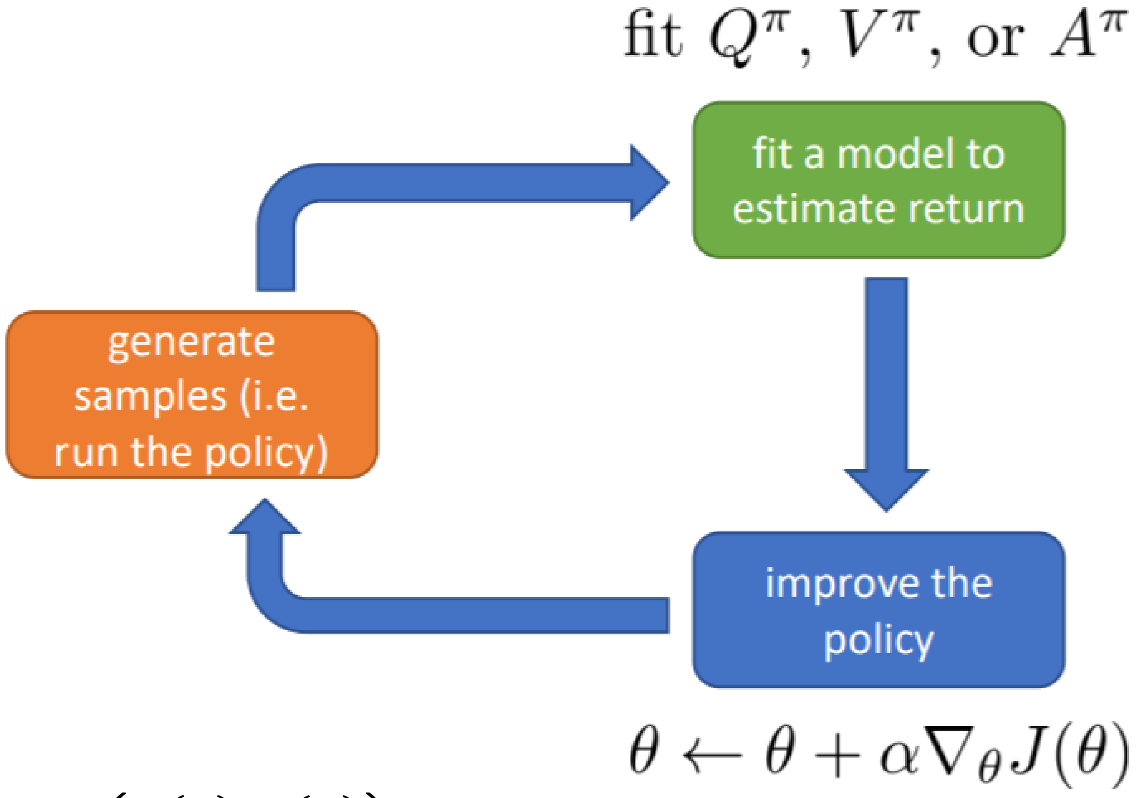


- $\hat{Q}_t^{(n)}$: estimate of expected reward if we take action $a_t^{(n)}$ in state $s_t^{(n)}$
- $Q(s_t, a_t) = \sum_{t'\geq t} E_{p_\theta}\left[\gamma^{t'-t} r(s_{t'}, a_{t'}) | s_t, a_t\right]$
  - True expected reward to go
- $V(s_t) = E_{a_t \sim \pi_\theta(a_t|s_t)} Q(s_t, a_t)$
- $\nabla_\theta J(\theta) \approx \frac{1}{N}\sum_{n=1}^{N}\sum_{t\geq 0}\left(Q\left(s_t^{(n)}, a_t^{(n)}\right) - V\left(s_t^{(n)}\right)\right)\nabla_\theta \log \pi_\theta\left(a_t^{(n)} | s_t^{(n)}\right)$

State & state-action value functions

- $Q^\pi(s_t, a_t) = \sum_{t' \geq t} E_{p_\theta}\left[\gamma^{t'-t} r(s_{t'}, a_{t'}) | s_t, a_t\right]$
  - True expected reward to go
- $V^\pi(s_t) = E_{a_t \sim \pi_\theta(a_t|s_t)} Q(s_t, a_t)$
  - Total reward from s_t
- $A^\pi(s_t, a_t) = Q^\pi(s_t, a_t) - V^\pi(s_t)$
  - How much better a_t is

Remark: we can define by the advantage function how much an action was better than expected



- $\nabla_\theta J(\theta) \approx \frac{1}{N} \sum_{n=1}^{N} \sum_{t \geq 0} A^\pi\left(s_t^{(n)}, a_t^{(n)}\right) \nabla_\theta \log \pi_\theta\left(a_t^{(n)} | s_t^{(n)}\right)$

Improving the policy gradient: summary

$$\nabla_\theta J(\theta) \approx \frac{1}{N}\sum_{n=1}^{N}\left(\sum_{t\geq 0}\gamma^{t'-t} r\left(s_t^{(n)}, a_t^{(n)}\right) - b\right)\sum_{t\geq 0}\nabla_\theta \log \pi_\theta\left(a_t^{(n)} | s_t^{(n)}\right)$$

Instead of using this unbiased, but high variance single-sample estimate, use A^π that is an estimation of expectation

$$\nabla_\theta J(\theta) \approx \frac{1}{N}\sum_{n=1}^{N}\sum_{t\geq 0} A^\pi\left(s_t^{(n)}, a_t^{(n)}\right)\nabla_\theta \log \pi_\theta\left(a_t^{(n)} | s_t^{(n)}\right)$$

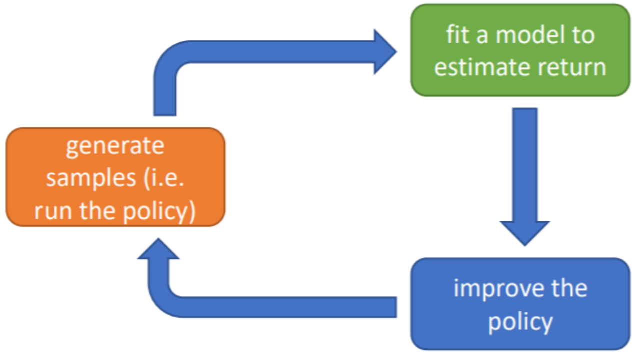
Actor-Critic Algorithm

- **Problem**: we don't know value function
- Can we learn them?
  - Yes, like Q-learning!
- We can combine Policy Gradients and Q-learning by training both an **actor** (the policy) and a **critic** (the Q-function).
  - The **actor** decides which action to take
  - the **critic** tells the actor how good its action was and how it should adjust
  - Also alleviates the task of the critic as it only has to learn the values of (state, action) pairs generated by the policy

$$A^{\pi}(s,a) = Q^{\pi}(s,a) - V^{\pi}(s)$$

Value function fitting

fit Q^π, V^π, or A^π



$\theta \leftarrow \theta + \alpha \nabla_\theta J(\theta)$

fit *what* to *what*?

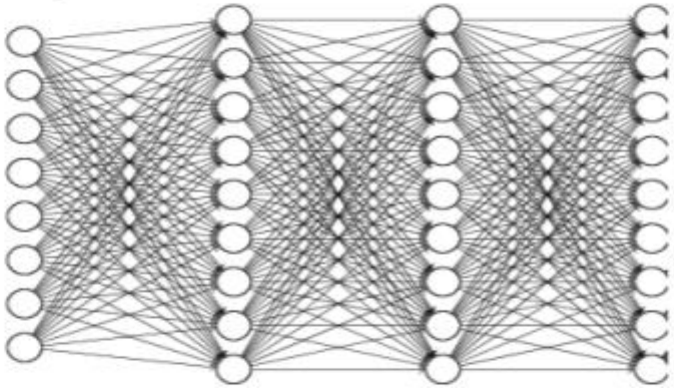
Q^π, V^π, A^π?

$$Q^\pi(\mathbf{s}_t, \mathbf{a}_t) = r(\mathbf{s}_t, \mathbf{a}_t) + \underbrace{\gamma E_{\mathbf{s}_{t+1} \sim p(\mathbf{s}_{t+1}|\mathbf{s}_t, \mathbf{a}_t)}[V^\pi(\mathbf{s}_{t+1})]}_{V^\pi(\mathbf{s}_{t+1})}$$

$$A^\pi(\mathbf{s}_t, \mathbf{a}_t) \approx r(\mathbf{s}_t, \mathbf{a}_t) + \gamma V^\pi(\mathbf{s}_{t+1}) - V^\pi(\mathbf{s}_t)$$

let's just fit $V^\pi(\mathbf{s})$!

$\mathbf{s}$



$\hat{V}^\pi(\mathbf{s})$

parameters ϕ

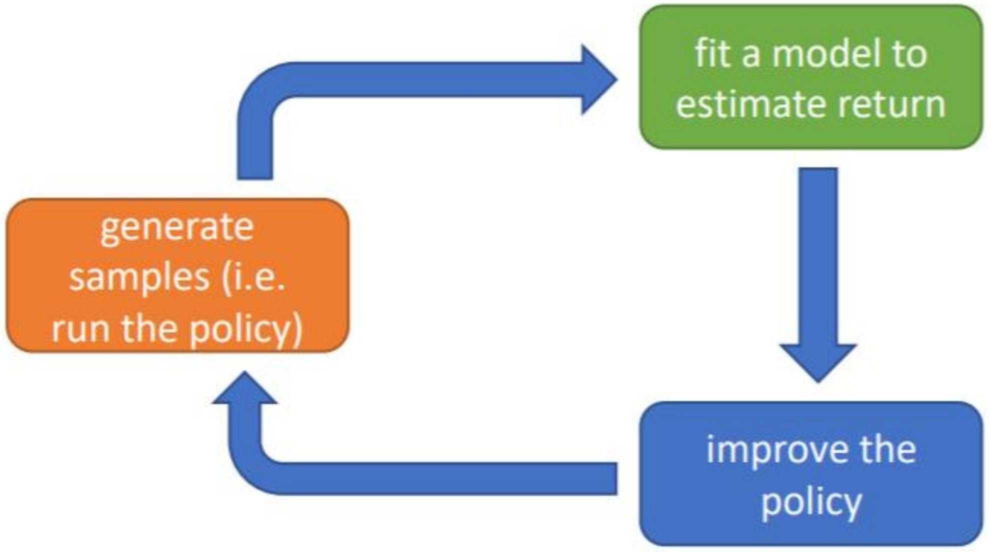
An actor-critic algorithm

batch actor-critic algorithm:

repeat

1. sample $\{\mathbf{s}_i, \mathbf{a}_i\}$ from $\pi_\theta(\mathbf{a}|\mathbf{s})$ (run it on the robot)
2. fit $\hat{V}^\pi_\phi(\mathbf{s})$ to sampled reward sums
3. evaluate $\hat{A}^\pi(\mathbf{s}_i, \mathbf{a}_i) = r(\mathbf{s}_i, \mathbf{a}_i) + \hat{V}^\pi_\phi(\mathbf{s}'_i) - \hat{V}^\pi_\phi(\mathbf{s}_i)$
4. $\nabla_\theta J(\theta) \approx \sum_i \nabla_\theta \log \pi_\theta(\mathbf{a}_i|\mathbf{s}_i)\hat{A}^\pi(\mathbf{s}_i, \mathbf{a}_i)$
5. $\theta \leftarrow \theta + \alpha \nabla_\theta J(\theta)$

fit $\hat{V}^\pi$

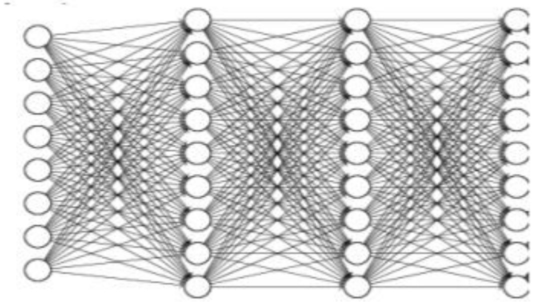


$\theta \leftarrow \theta + \alpha \nabla_\theta J(\theta)$

$$y_t \approx r(s_t, a_t) + \hat{V}^\pi_\phi(s_{t+1})$$

supervised regression: $\mathcal{L}(\phi) = \sum_t \left(\hat{V}^\pi_\phi(s_t) - y_t\right)^2$

$\mathbf{s}$

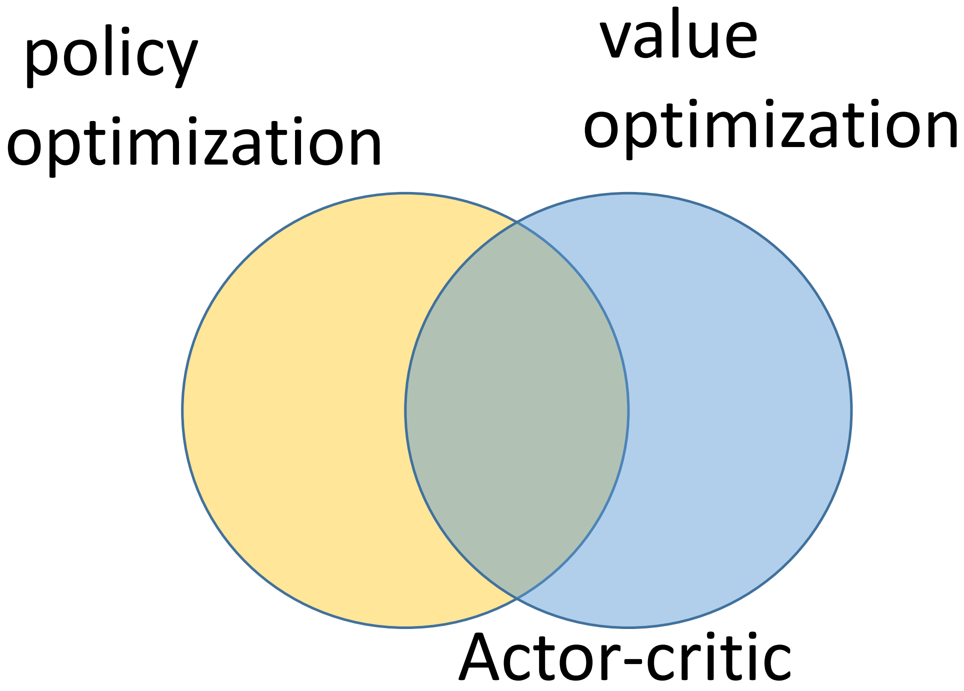


$\hat{V}^\pi(\mathbf{s})$

parameters ϕ

$$V^\pi(\mathbf{s}_t) = \sum_{t'=t}^{T} E_{\pi_\theta}\left[r(\mathbf{s}_{t'}, \mathbf{a}_{t'})|\mathbf{s}_t\right]$$

Actor-critic methods



Advantages of Policy-based RL

- Advantages
  - Better convergence properties
  - Effective in high dimensional or continuous action spaces
  - Can learn stochastic policies

- Disadvantages
  - Typically converges to a local rather than global optimum
  - Evaluating a policy is typically inefficient and high variance

RL in Other ML Problems

- Hard Attention
  - Observation: current image window
  - Action: where to look
  - Reward: classification

V. Mnih et al., “Recurrent models of visual attention”, NIPS 2014.

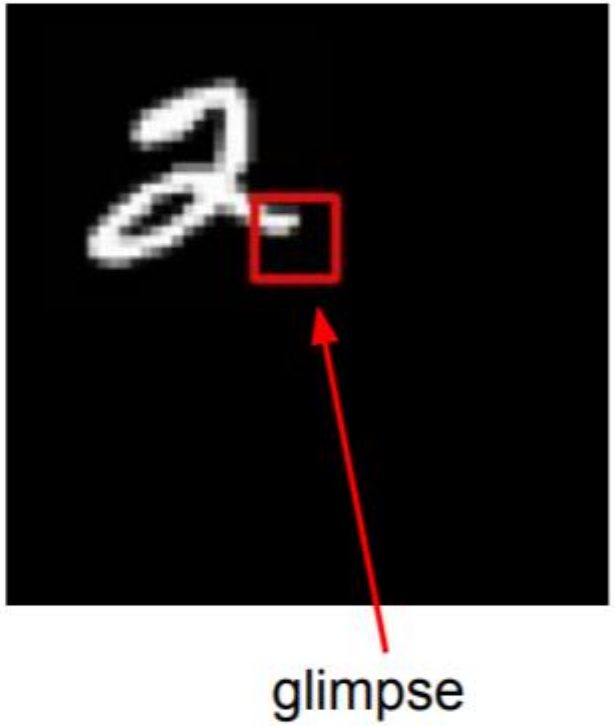
- Sequential/structured prediction, e.g., machine translation
  - Observations: words in source language
  - Actions: emit word in target language
  - Rewards: sentence-level metric, e.g. BLEU score

M. Ranzato et al., "Sequence level training with recurrent neural networks“, 2015.

REINFORCE in action: Recurrent Attention Model (RAM)

- **Objective**: Image Classification

- Take a sequence of "glimpses" selectively focusing on regions of the image, to predict class
  - Inspiration from human perception and eye movements
  - Saves computational resources => scalability
  - Able to ignore clutter / irrelevant parts of image



[Mnih et al. 2014]

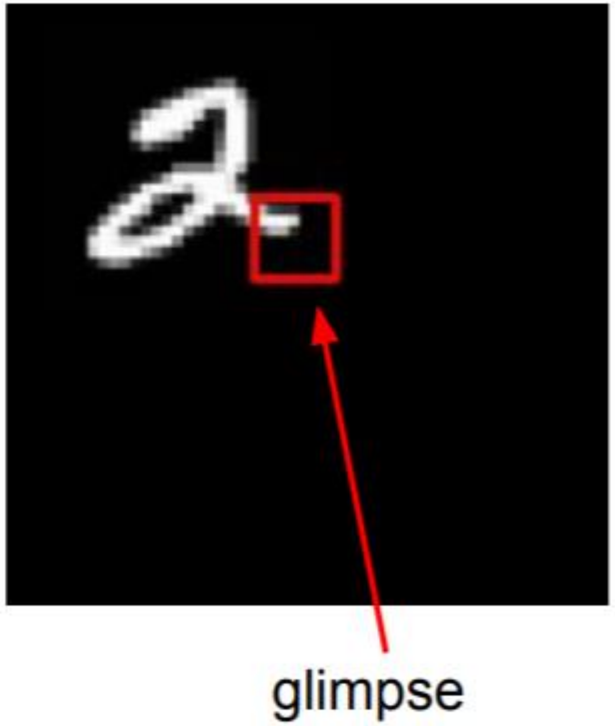
REINFORCE in action: Recurrent Attention Model (RAM)

- **Objective**: Image Classification

- **State**: Glimpses seen so far
- **Action**: (x,y) coordinates (center of glimpse) of where to look next in image
- **Reward**: 1 at the final timestep if image correctly classified, 0 otherwise

- Glimpsing is a non-differentiable operation

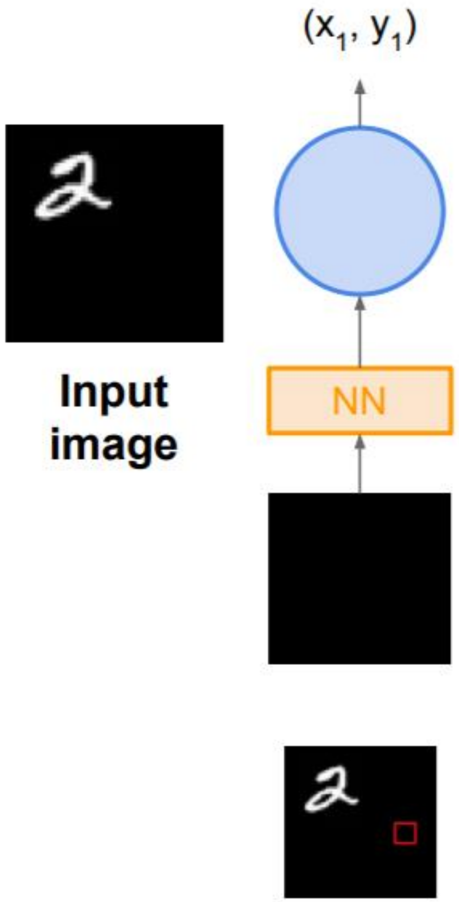
  => learn policy for how to take glimpse actions using REINFORCE



[Mnih et al. 2014]

REINFORCE in action: Recurrent Attention Model (RAM)

- Given state of glimpses seen so far, use RNN to model the state and output next action



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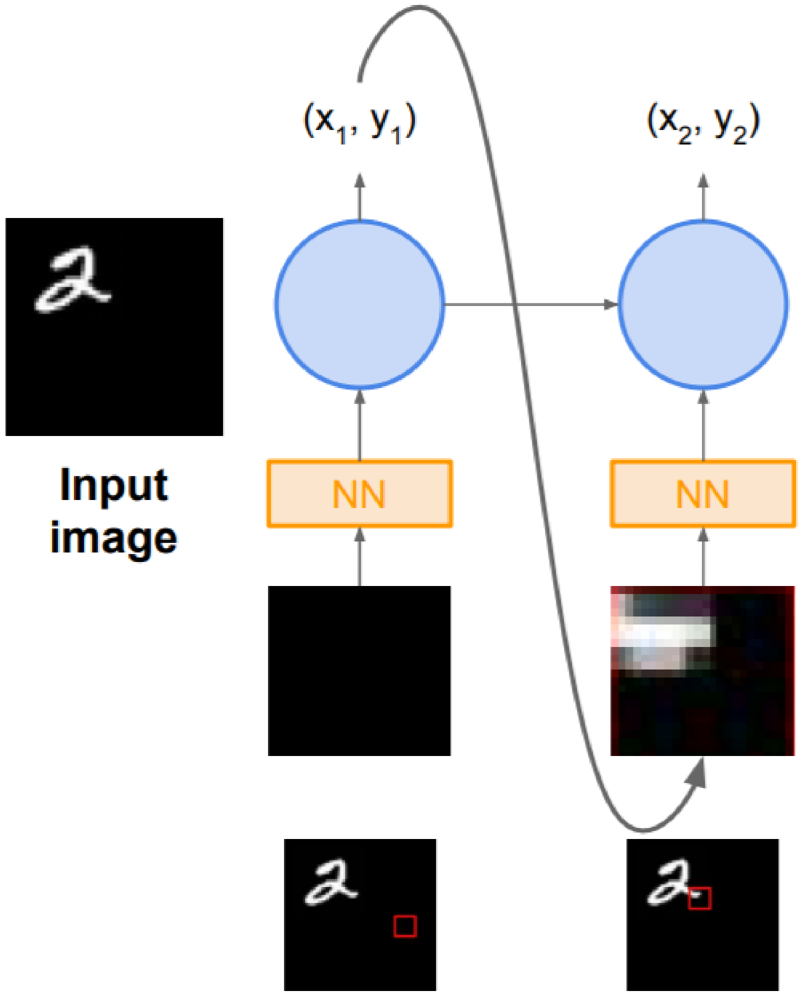

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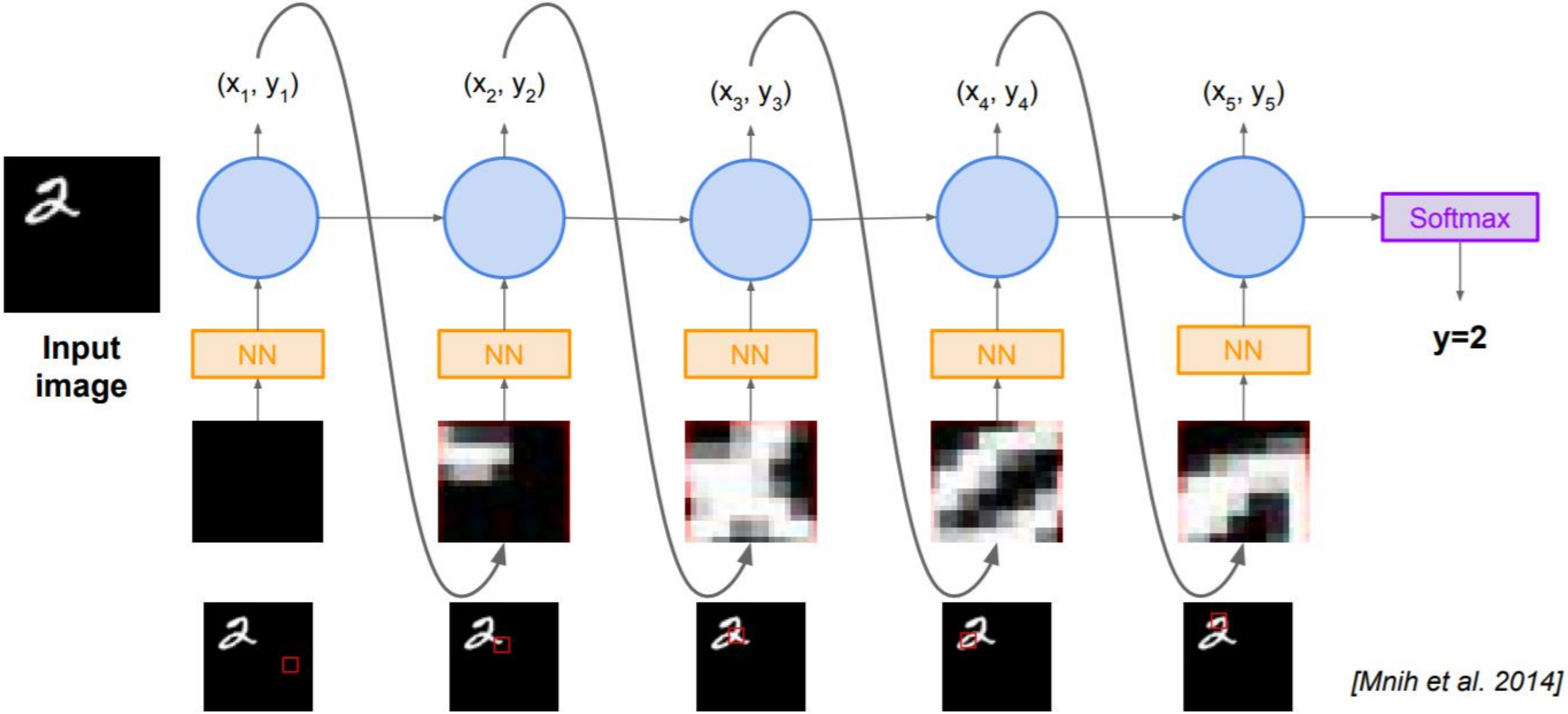

[Mnih et al. 2014]

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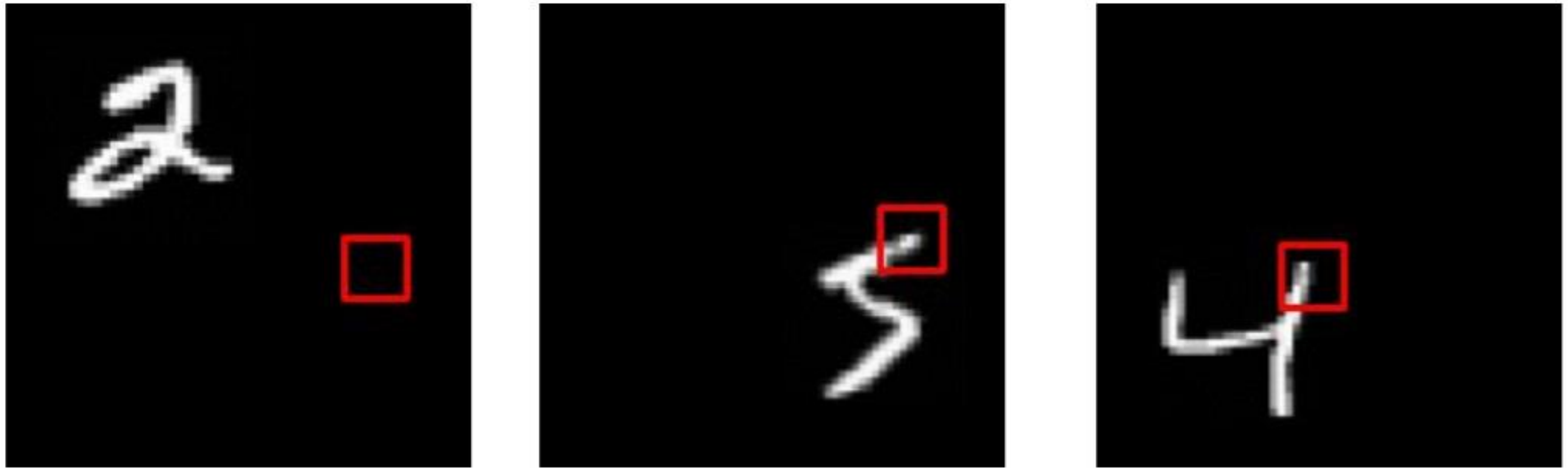

Has also been used in many other tasks including fine-grained image recognition, image captioning, and visual question-answering!

[Mnih et al. 2014]

Sequence generation: disadvantages of previous methods

- Model was trained on a different distribution of inputs from the ones encounters during test (generated by itself)
- Errors made along the way will quickly accumulate (**exposure bias**)
- The **loss function** used to train these models is at the **word level**
- Training these models to directly optimize **metrics like BLEU** (by which they are typically evaluated) is hard
  - because these are not differentiable
  - BLEU: comparing the sequence of actions from the current policy against the optimal action sequence

M. Ranzato et al., "Sequence level training with recurrent neural networks", 2015.

Sequence level evaluation

- A greedy left-to-right process which does not necessarily produce the most likely sequence according to the model

$$\prod_{t=1}^{T} \max_{w_{t+1}} p_\theta(w_{t+1}|w_t^g, \mathbf{h}_{t+1}) \leq \max_{w_1,\ldots,w_T} \prod_{t=1}^{T} p_\theta(w_{t+1}|w_t^g, \mathbf{h}_{t+1})$$

- One of the existing methods to reduce this effect is **Beam Search**
  - It pursues not only one but k next word candidates at each point.

M. Ranzato et al., "Sequence level training with recurrent neural networks", 2015.

Sequence level training

- Starts from the greedy policy and then slowly deviate from it to let the model explore and make use of its own predictions.
  - greedy policy is obtaind by maximum likelihood on training data

M. Ranzato et al., “Sequence level training with recurrent neural networks”, 2015.

Sequence level training: Loss function

$$L_\theta = -\sum_{w_1^g,\ldots,w_T^g} p_\theta(w_1^g,\ldots,w_T^g) r(w_1^g,\ldots,w_T^g) = -\mathbb{E}_{[w_1^g,\ldots w_T^g]\sim p_\theta} r(w_1^g,\ldots,w_T^g),$$

$$\frac{\partial L_\theta}{\partial \theta} = \sum_t \frac{\partial L_\theta}{\partial \mathbf{o}_t} \frac{\partial \mathbf{o}_t}{\partial \theta}$$

$$\frac{\partial L_\theta}{\partial \mathbf{o}_t} = \left(r(w_1^g,\ldots,w_T^g) - \bar{r}_{t+1}\right)\left(p_\theta(w_{t+1}|w_t^g, \mathbf{h}_{t+1}, \mathbf{c}_t) - \mathbf{1}(w_{t+1}^g)\right)$$

encourages a word choice w_{t+1}^g if $r > \bar{r}_{t+1}$, or discourages it if $r < \bar{r}_{t+1}$

- baseline $\bar{r}_t$ is estimated by a linear regressor which takes as input the hidden states h_t of the RNN

M. Ranzato et al., "Sequence level training with recurrent neural networks", 2015.

Sequence level training

Data: a set of sequences with their corresponding context.
Result: RNN optimized for generation.
Initialize RNN at random and set N^{XENT}, $N^{\text{XE+R}}$ and Δ;
for $s = T, 1, -\Delta$ **do**
  if $s == T$ **then**
    train RNN for N^{XENT} epochs using XENT only;
  else
    train RNN for $N^{\text{XE+R}}$ epochs. Use XENT loss in the first s steps, and REINFORCE (sampling from the model) in the remaining $T - s$ steps;
  end
end

XENT $$\frac{\partial L_\theta^{\text{XENT}}}{\partial \mathbf{o}_t} = p_\theta(w_{t+1}|w_t, \mathbf{h}_{t+1}, \mathbf{c}_t) - \mathbf{1}(w_{t+1})$$

XE+R $$\frac{\partial L_\theta}{\partial \mathbf{o}_t} = \left(r(w_1^g, \dots, w_T^g) - \bar{r}_{t+1}\right)\left(p_\theta(w_{t+1}|w_t^g, \mathbf{h}_{t+1}, \mathbf{c}_t) - \mathbf{1}(w_{t+1}^g)\right)$$

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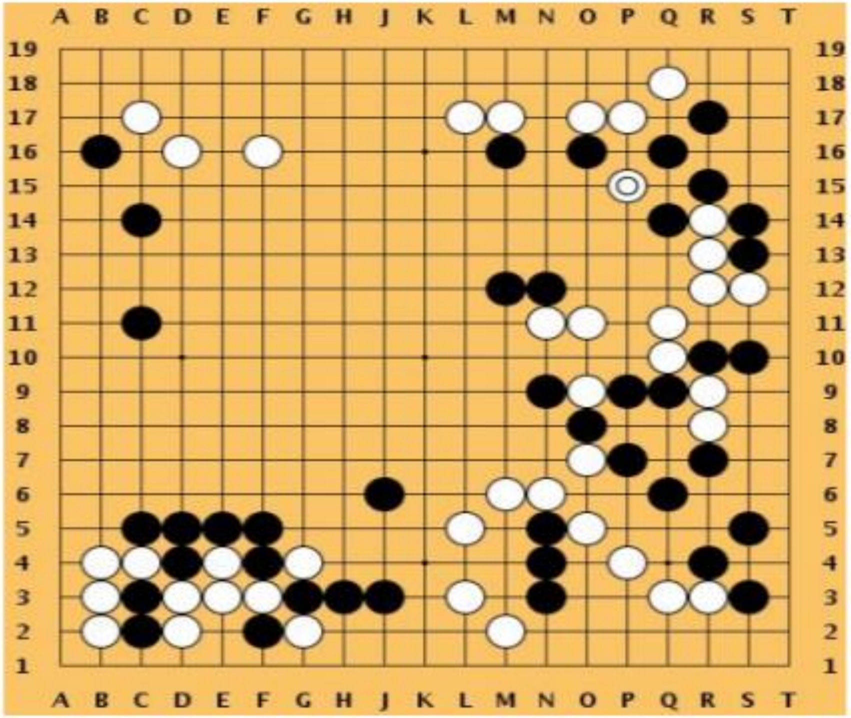
More policy gradients: AlphaGo

Overview:

- Mix of supervised learning and reinforcement learning
- Mix of old methods (Monte Carlo Tree Search) and recent ones (deep RL)



[Silver et al., Nature 2016]

More policy gradients: AlphaGo

- How to beat the Go world champion:
  - Featurize the board (stone color, move legality, bias, …)
  - Initialize policy network with supervised training from professional go games, then continue training using policy gradient
    - play against itself from random previous iterations, +1 / -1 reward for winning / losing
  - Also learn value network (critic)
  - Finally, combine policy and value networks in a Monte Carlo Tree Search algorithm to select actions by lookahead search

Summary

- Policy gradients: very general but suffer from high variance so requires a lot of samples.
  - Challenge: sample-efficiency
- Q-learning: does not always work but when it works, usually more sample-efficient.
  - Challenge: exploration
- Guarantees:
  - Policy Gradients: Converges to a local minima of $J(\theta)$, often good enough!
  - Q-learning: Zero guarantees since you are approximating Bellman equation with a complicated function approximator