AM 213A HW3

Joseph Moore

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Part 1

a)

- The first ten singular values are as follows,

$$
\begin{aligned}
\sigma_1 &= 663180.202318\\
\sigma_2 &= 85706.595735\\
\sigma_3 &= 62129.025680\\
\sigma_4 &= 34664.633004\\
\sigma_5 &= 31861.792296\\
\sigma_6 &= 21872.721620\\
\sigma_7 &= 19628.442780\\
\sigma_8 &= 18434.937653\\
\sigma_9 &= 13693.815446\\
\sigma_{10} &= 12815.208252.
\end{aligned}
$$

The k^{th} singular values are as follows,

$$
\begin{aligned}
\sigma_{20} &= 7528.024652\\
\sigma_{40} &= 5489.124664\\
\sigma_{80} &= 3948.779979\\
\sigma_{160} &= 2668.223578\\
\sigma_{320} &= 1515.865932\\
\sigma_{640} &= 821.893126\\
\sigma_{1280} &= 513.568032\\
\sigma_{2560} &= 179.115035
\end{aligned}
$$

The very last singular value is $\sigma_{3355} = 16.724645$

- The matrix of singular values corresponds to the level of image compression depicted below.

$\Sigma_{\sigma_{20}}$ creates the image

$\Sigma_{\sigma_{40}}$ creates the image

$\Sigma_{\sigma_{80}}$ creates the image

$\Sigma_{\sigma_{160}}$ creates the image

$\Sigma_{\sigma_{320}}$ creates the image

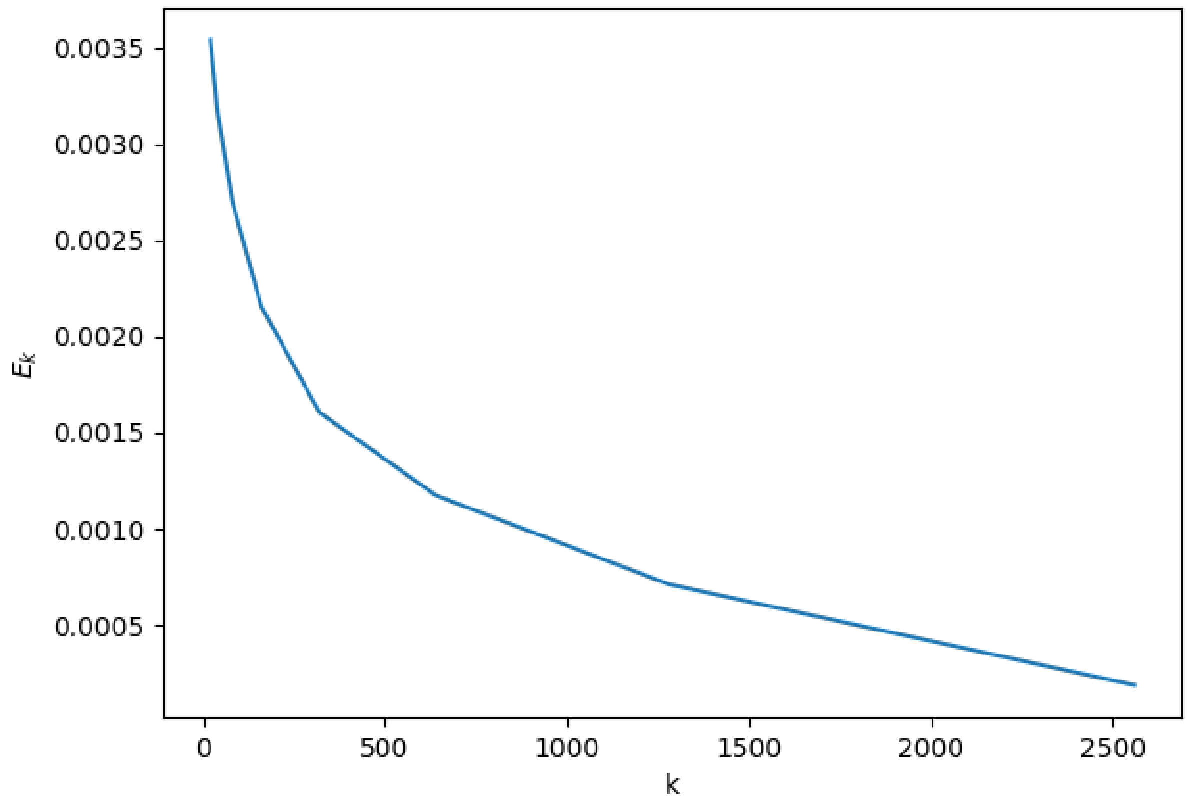
$\Sigma_{\sigma_{640}}$ creates the image

$\Sigma_{\sigma_{1280}}$ creates the image

$\Sigma_{\sigma_{2560}}$ creates the image

$\Sigma_{\sigma_{3355}}$ is the all the singular values and thus is the original image

- $$E_{20} = \frac{||A - A_{\sigma_{20}}||_F}{mn} = 0.003543$$
  $$E_{40} = \frac{||A - A_{\sigma_{40}}||_F}{mn} = 0.003166$$
  $$E_{80} = \frac{||A - A_{\sigma_{80}}||_F}{mn} = 0.002703$$
  $$E_{160} = \frac{||A - A_{\sigma_{160}}||_F}{mn} = 0.002156$$
  $$E_{320} = \frac{||A - A_{\sigma_{320}}||_F}{mn} = 0.001604$$
  $$E_{640} = \frac{||A - A_{\sigma_{640}}||_F}{mn} = 0.001173$$
  $$E_{1280} = \frac{||A - A_{\sigma_{1280}}||_F}{mn} = 0.000711$$
  $$E_{2560} = \frac{||A - A_{\sigma_{2560}}||_F}{mn} = 0.000187$$



As the number of singular values increases the image becomes closer to the original. The error falls like $\frac{1}{k}$ and appears to asymptotically approach zero. This tells us that we can get most of the information we need from the first few singular values. The higher we up k the less information is added to our image. At $k = 1280$ the error is below 10^{-3}. However, from the graph we can tell that the error drops below 10^{-3} closer to $k = 1000$. With far less than half the total number of singular values we have a very small error.

b) Both the Gauss-Jacobi and the Gauss-Seidel use an iterative method to solve $Ax = b$ for x. The Gauss-Jacobi simply splits up A into its diagonal and remaining elements like $A = D + R$ and then follows $x^{k+1} = D^{-1}(b - Rx^k)$ until x converges to a result. Gauss-Jacobi converges when $\rho(D^{-1}R) < 1$. Gauss-Jacobi further splits R into upper and lower triangular matrices so that $A = L + D + U$. Gauss-Seidel follows $x^{k+1} = D^{-1}(b - Lx^{k+1} - Ux^k)$ and converges for any symmetric, positive definite matrix.

Part 2

1.

7.

a)

b)