AM 213A HW3

Joseph Moore

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Part 1

a)

- The first ten singular values are as follows,

$$\begin{aligned}\sigma_1 &= 663180.202318\\ \sigma_2 &= 85706.595735\\ \sigma_3 &= 62129.025680\\ \sigma_4 &= 34664.633004\\ \sigma_5 &= 31861.792296\\ \sigma_6 &= 21872.721620\\ \sigma_7 &= 19628.442780\\ \sigma_8 &= 18434.937653\\ \sigma_9 &= 13693.815446\\ \sigma_{10} &= 12815.208252.\end{aligned}$$

The k^{th} singular values are as follows,

$$\begin{aligned}\sigma_{20} &= 7528.024652\\ \sigma_{40} &= 5489.124664\\ \sigma_{80} &= 3948.779979\\ \sigma_{160} &= 2668.223578\\ \sigma_{320} &= 1515.865932\\ \sigma_{640} &= 821.893126\\ \sigma_{1280} &= 513.568032\\ \sigma_{2560} &= 179.115035\end{aligned}$$

The very last singular value is $\sigma_{3355} = 16.724645$

- The matrix of singular values corresponds to the level of image compression depicted below.

$\Sigma_{\sigma_{20}}$ creates the image

$\Sigma_{\sigma_{40}}$ creates the image

$\Sigma_{\sigma_{80}}$ creates the image

$\Sigma_{\sigma_{160}}$ creates the image

$\Sigma_{\sigma_{320}}$ creates the image

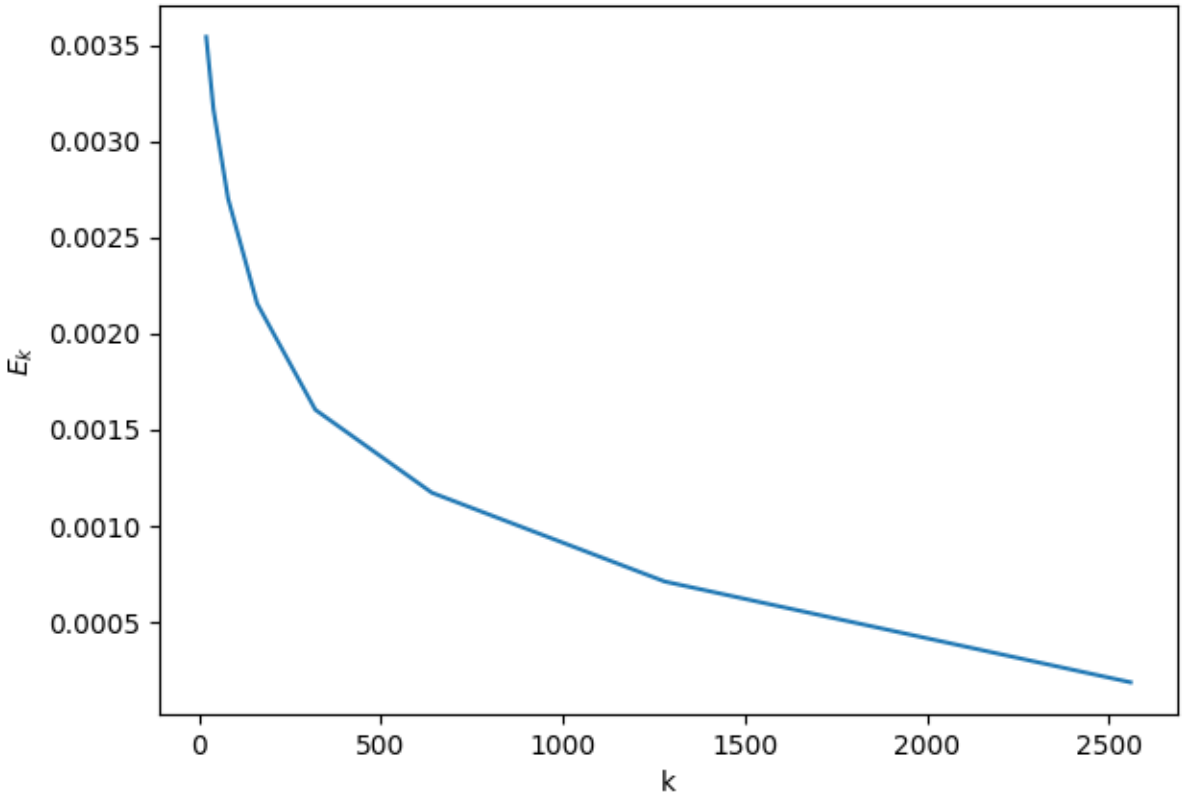
$\Sigma_{\sigma_{640}}$ creates the image

$\Sigma_{\sigma_{1280}}$ creates the image

$\Sigma_{\sigma_{2560}}$ creates the image

$\Sigma_{\sigma_{3355}}$ is the all the singular values and thus is the original image

- $$E_{20} = \frac{||A - A_{\sigma_{20}}||_F}{mn} = 0.003543$$
$$E_{40} = \frac{||A - A_{\sigma_{40}}||_F}{mn} = 0.003166$$
$$E_{80} = \frac{||A - A_{\sigma_{80}}||_F}{mn} = 0.002703$$
$$E_{160} = \frac{||A - A_{\sigma_{160}}||_F}{mn} = 0.002156$$
$$E_{320} = \frac{||A - A_{\sigma_{320}}||_F}{mn} = 0.001604$$
$$E_{640} = \frac{||A - A_{\sigma_{640}}||_F}{mn} = 0.001173$$
$$E_{1280} = \frac{||A - A_{\sigma_{1280}}||_F}{mn} = 0.000711$$
$$E_{2560} = \frac{||A - A_{\sigma_{2560}}||_F}{mn} = 0.000187$$



As the number of singular values increases the image becomes closer to the original. The error falls like $\frac{1}{k}$ and appears to asymptotically approach zero. This tells us that we can get most of the information we need from the first few singular values. The higher we up k the less information is added to our image. At $k = 1280$ the error is below 10^{-3}. However, from the graph we can tell that the error drops below 10^{-3} closer to $k = 1000$. With far less than half the total number of singular values we have a very small error.

b)

Part 2

1.

7.

a)

b)