Total $\frac{38}{50}$

AM 213A HW1

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1. We can write A as $QUQ^{-1} = QUQ^{T}$ where Q and U are real matrices. If we remember that $Ax = \lambda x$

Be careful with what you are actually trying to prove

How do you know Q from Schur relates to eigen part? -6

First prove U diagonal

Then bring in eigen stuff.

$\frac{4}{10}$

$$Q^{t}AQ = \begin{pmatrix} q_1^T \\ q_2^T \\ \vdots \\ q_m^T \end{pmatrix} \begin{pmatrix} Aq_1 & Aq_2 & \dots & Aq_m \end{pmatrix}$$

$$= \begin{pmatrix} q_1^T \\ q_2^T \\ \vdots \\ q_m^T \end{pmatrix} \begin{pmatrix} \lambda_1 q_1 & \lambda_2 q_2 & \dots & \lambda_m q_m \end{pmatrix}$$

$$= \begin{pmatrix} \lambda & 0 & \dots & 0 \\ 0 & \lambda_2 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \dots & \lambda_m \end{pmatrix}$$

Fine to use comp for intuition/validation but this is a hand question.

$\frac{10}{10}$

2. Using $\epsilon = 1e-17$ and $c = 1e18$ resulted in $x = 0.00000$ and $y = 1.00000$, which is wrong. We should get $(1, 1)$. After multiplying the row by a large c, the algorithm pivots this row to the top and we get

$$\left(\begin{array}{cc|c} c\epsilon & c & c \\ 1 & 1 & 2 \end{array}\right) \Rightarrow \left(\begin{array}{cc|c} c\epsilon & c & c \\ 1-\frac{c\epsilon}{c\epsilon} & 1-\frac{c}{c\epsilon} & 2-\frac{c}{c\epsilon} \end{array}\right) = \left(\begin{array}{cc|c} c\epsilon & c & c \\ 0 & 1-\frac{1}{\epsilon} & 2-\frac{1}{\epsilon} \end{array}\right)$$

performing back substitution gets us $y = 1$ from the bottom row and now the top row will give us $x = 0$. If we don't multiply by some very large constant then ϵ won't be pivoted to the top and we won't run into this problem. By leaving ϵ on the bottom row we can zero it out by subtracting a very small multiply of the first row from the second row. This will leave all the other elements at play virtually unchanged.

True, but should show by hand

3. We know that $x^T A x > 0$ for all vectors. Then it must be the case that $e_i^T A e_i > 0$ for each i. But $e_i^T A e_i$ is just the i'th diagonal element of A and therefore each diagonal element of A must be positive.

$\frac{10}{10}$

4. expand, demonstrate why this is what needs to be shown. −1

(a) We need to show that $A_{21} - A_{21}A_{11}^{-1}A_{11} = 0$. Which is obvious since $A_{21} - A_{21}A_{11}^{-1}A_{11} = A_{21} - A_{21}I = A_{21} - A_{21} = 0$ What about A_{22} block?

(b) A can be upper triagularized by applying a lower triangular matrix to A or $L^-1A = U$. From this we can write

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$$L^{-1} = \begin{pmatrix} Y & 0 \\ X & W \end{pmatrix}$$

From this we know that $YA_{11} = U_{11}$, where U_{11} is A_{11} upper triagularized and thus $Y = L_{11}^{-1}$ and

$$L^{-1} = \begin{pmatrix} L_{11}^{-1} & 0 \\ X & W \end{pmatrix}$$

$$L^{-1}A = \begin{pmatrix} U_{11} & L_{11}^{-1}A_{12} \\ XA_{11} + WA_{21} & XA_{12} + WA_{22} \end{pmatrix}$$

Now we set $XA_{11} + WA_{21} = 0$ and solving for X we get $X = -WA_{21}A_{11}^{-1}$. From this we have

$$XA_{12} + WA_{22} = -WA_{21}A_{11}^{-1}A_{12} + WA_{22}$$

Which is exactly what we want if we set $W = I$. Relate this back to GE algo after N steps.

5.

(a) If we decompose A into $A_1 + iA_2$ then we have

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$$Ax = (A_1 + iA_2)x = A_1x + iA_2x = b_1 + ib_2$$

Which becomes

$$A_1x = b_1$$
$$A_2x = b_2$$

What happens w/ cross terms? −2

Which can be written as in matrix form

e.g. $(iA_2)(ix_2)$ is real,

$$\begin{pmatrix} A_1 & A_2 \end{pmatrix} x = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$$

Which is'nt square but looks correct. It really needs to be square −1

(b) Performing Gaussian elimination on a square $2m$ matrix is $O(\frac{8m^3}{3})$. For complex algebra, each multiplication will be quadrupled resulting in a $O(\frac{4m^3}{3})$, which is slightly cheaper.

storage costs?

−2