Joseph Murray Homework 1 1/17/17

$f_1(n) = n^2$

$f_2(n) = n^2 + 1000n$

$f_3(n) = \begin{cases} n & \text{if } n \text{ is even} \\ n^3 & \text{if } n \text{ is odd} \end{cases}$

$f_4(n) = \begin{cases} n & \text{if } n \le 100 \\ n^3 & \text{if } n > 100 \end{cases}$

Indicate for each distinct pair i, and j whether $f_i(n)$ is $O(f_j(n))$ and whether $f_i(n)$ is $\Omega(f_j(n))$

- f_1 and f_2

$f_1(n) = n^2$

$f_2(n) = n^2 + 1000n$

$f_1(n) \in O(f_2(n))$ $\exists$ $c, n_0 > 0$ such that $f_1(n) \le c f_2(n)$

ignore constants

$n^2 \in O(n^2)$

$n^2 \le c(n^2)$

let $c = 1$

$n_0 = 1$ $n^2 \le (n^2 + n)$ $\forall$ $n \ge 1$

- f_2 and f_1

$f_2(n) \in O(f_1(n))$ $\exists$ $c, n_0 > 0$ such that $f_2(n) \le c(f_1(n))$

$n^2 + 1000n \in O(n^2)$

200 $n^2 + 1000n \le cn^2$

let $c = 2$

$n^2 + 1000n \le 2n^2$

$\lim_{n \to \infty} \frac{2n^2}{n^2 + 1000n} \to 2$

$n_0 = 2$ $n^2 + 1000n \le n^2$ $\forall$ $n > c$

• f_1 and f_3

$f_1(n) = n^2$

$f_3(n) = \begin{cases} n, & \text{if } n \text{ is odd} \\ n^3, & \text{if } n \text{ is even} \end{cases}$

$f_1(n) \in O(f_3(n)$ $\exists\ c, n_0 > 0$ such that $f_1(n) \leq n f_3(n)$

$n^2 \in O(n^3)$

$n^2 \leq cn^3$

let $c = 1$

$n_0 = 1$ $n^2 \leq n^3$ $\forall\ n \geq 1$ for all even numbers

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• f_1 and f_4

$f_1(n) = n^2$

$f_4(n) = \begin{cases} n, & n \leq 100 \\ n^3, & n \geq 100 \end{cases}$

$f_1 \in O(f_4)$ $\exists\ c, n_0 > 0$ such that $f_1(n) \leq cf_4(n)$

$n^2 \leq cn^3$

let $c = 1$;

$n_0 = 1$ $n^2 \leq 1n^3$ $\forall\ n \geq 100$

• f_2 and f_3

$f_2 \in O(f_3(n))$ $\exists\ c, n_0 > 0$ such that $f_2(n) \leq cf_3(n)$

$f_2(n) = n^2 + 1000n$

$f_3(n) = \begin{cases} n, & \text{if } n \text{ is odd} \\ n^3 & \text{if } n \text{ is even} \end{cases}$

$n^2 + 1000n \leq cn^3$

let $c = 1$

$n_0 = 1$, $n^2 + 1000n \leq n^3$ $\forall\ n \leq 1$

- f_3 and f_4

$f_3 \in O(f_4)$ $\exists$ $c, n_0 > 0$ such that $f_3(n) \leq f_4(n)$.

$n^3 \leq c n^3$

let $c = 1$

$n^3 \leq 1 n^3$ $\forall n > 0$

$n_0 = 0$

$f_4 \in O(f_3)$ $\exists$ $c, n_0 > 0$ such that $f_4(n) \leq c f_3(n)$

$n^3 \leq c n^3$

let $c = 1$

$n_0 = 0$ $n^3 \leq 1 n^3$ $\forall n > 0$

1.12 A procedure matmpy

Running Time. $O'(n^3)$

3 loops, each nested $(n)(n)(n)$

B Procedure mystery

Run Time

$O(n)$

first for loop sets $i = 1$ the next loop n times
then increments i in the
so the last loop is never executed

for $i := 1$ to $n-1$ do
for $j = i+1$ to n do.
for $k := 1$ to j do (never executed n times $= 0$)

C Procedure veryodd

worst run.time $O(n^2)$

For i = 1 to n do $n - i + 1$

...

for j = 1 to n do: $n - 1 + 1$

two nested loops $n \times n$ run time

If statement with odd would be a constant value
n and would not affect $O(n^2)$

D Function recursive

worst run time $O(2^n)$,

return(recursive(n-1) + recursive(n-1))

for each n call to function recursive two
more times

$n = 1$

return (recursive (1 - 1) + recursive (1 - 1))

returns 2

$O(2^n)$ times