CHAPTER 2

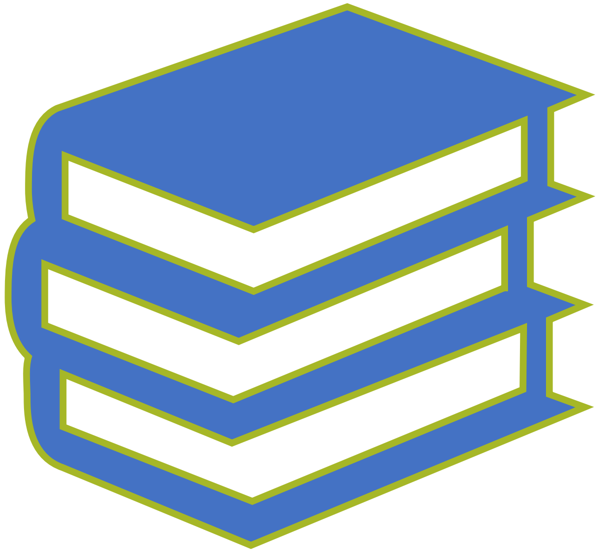
Categorical Data Analysis

Overview

1A. Rates I → 1B. Rates II → 2A. Association I

2B. Association II → 3A. Simpson's Paradox I → 3B: Simpson's Paradox II

4. Confounders



Unit 1A: Rates I

By the end of this unit you should be able to do the following:

1. Identify a categorical variable.
2. Understand and interpret tables and plots created from 1 categorical variable.

RECAP

Types of variables

Categorical variables

- **Ordinal**: Categories come with some natural ordering and numbers are often used to represent the ordering. E.g.: Happiness level
- **Nominal**: No intrinsic ordering for the variables. E.g.: Nationality

Numerical variables

- **Continuous**: One that can take on all possible numerical values in a given range or interval. E.g.: Time
- **Discrete**: One where possible values of the variable form a set of numbers with “gaps”. E.g.: Module credits

THE PROBLEM

A SNAPSHOT OF THE DATA

Size	Gender	Treatment	Outcome
Large	Male	X	Success
Large	Male	X	Success
Small	Male	Y	Success
Large	Male	Y	Failure
Small	Male	X	Success
Large	Male	Y	Success

APPLYING THE PPDAC CYCLE

In general, do treatments tend to be successful?

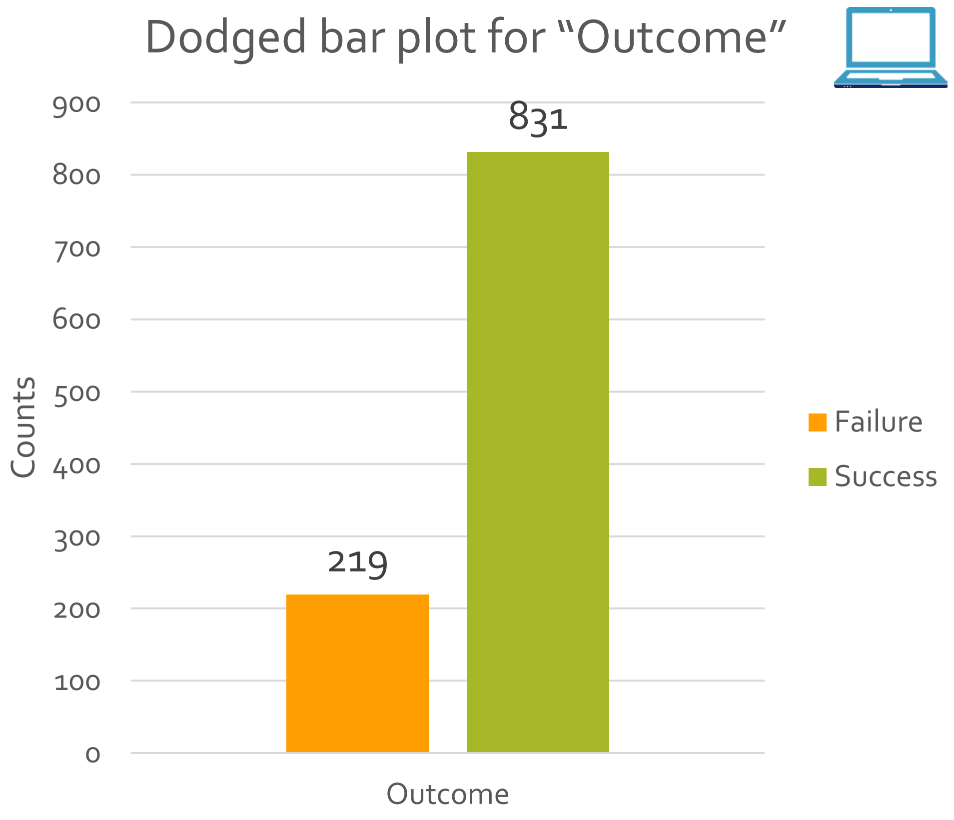
What to measure:
Variable "Outcome" tells us if the treatment was a success or not

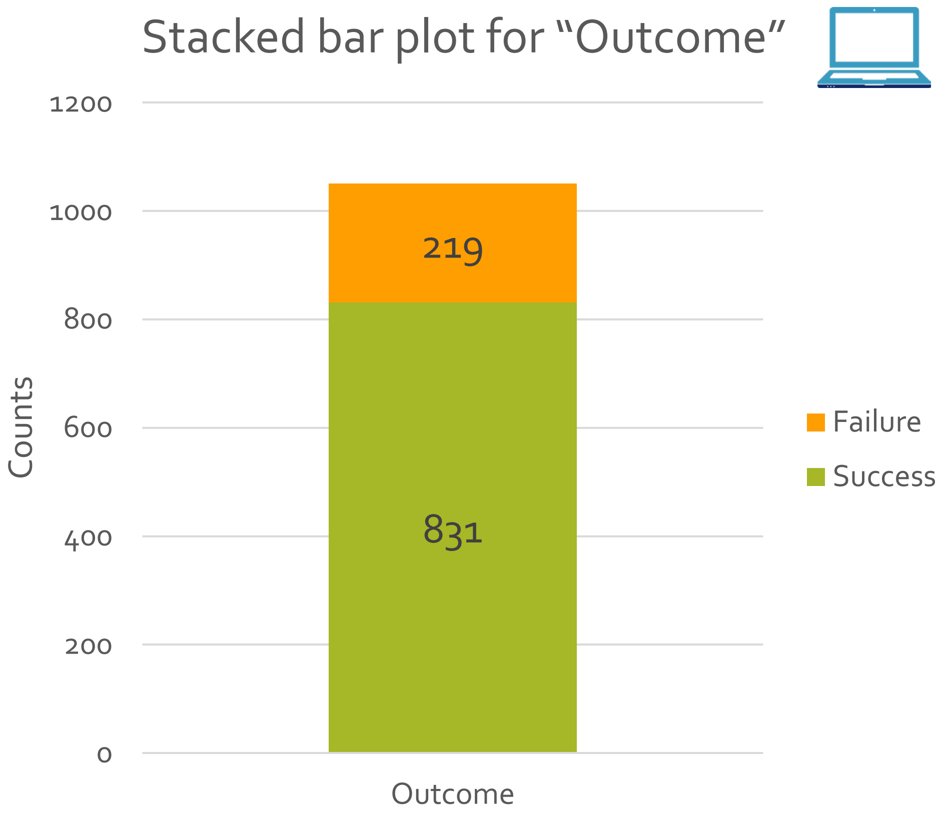
Sort the data
Plot graphs, tables of the "Outcome" variable

ANALYSING 1 CATEGORICAL VARIABLE - TABLE

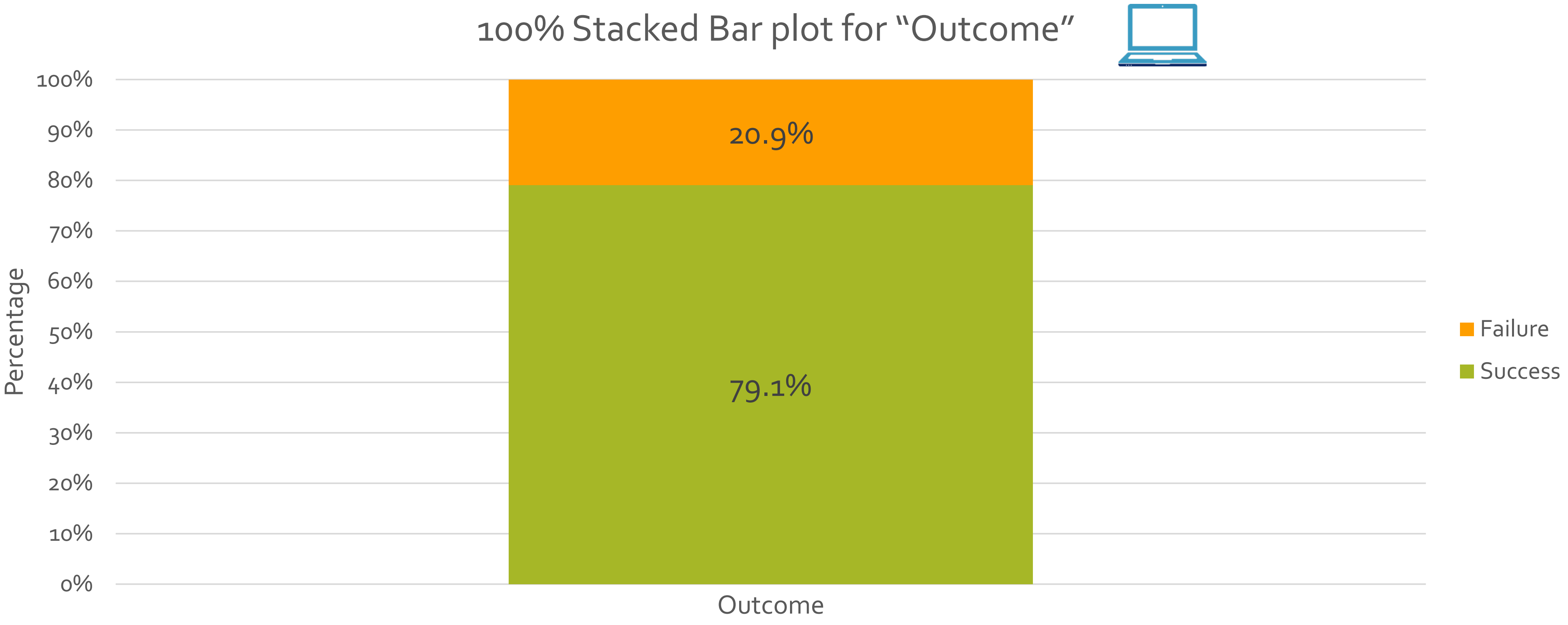
Categories of the "Outcome" variable	Count	Rate	Percentage
Success	831	$\text{rate(Success)} = \frac{831}{1050} = 0.791$	$0.791 \times 100\% = 79.1\%$
Failure	219	$\text{rate(Failure)} = \frac{219}{1050} = 0.209$	$0.209 \times 100\% = 20.9\%$
Total	1050	$\frac{1050}{1050} = 1$	$1 \times 100\% = 100\%$

Analyzing 1 categorical variable - Plot





Analysing 1 categorical variable - Plot



Conclusion

- Table and bar plots gave us the same conclusion
- 79% success
- 21% failure
- Should go for treatment

Summary

We have learned:

- Use of tables and plots to summarize a categorical variable
- Calculation of rates

Unit 1B: Rates II

By the end of this unit you should be able to do the following:

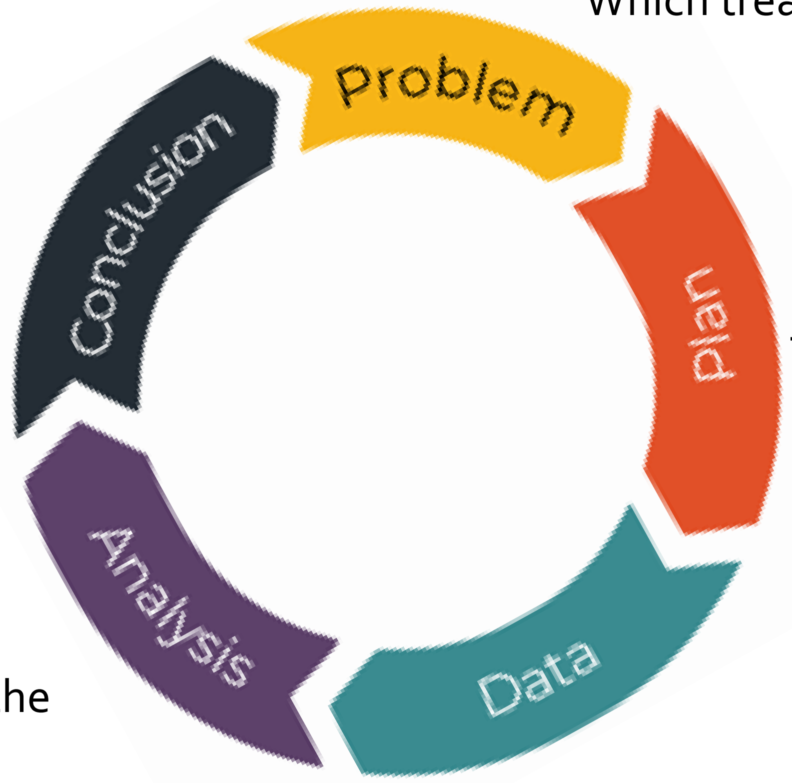
1. Understand and interpret tables and plots created from 2 categorical variables.
2. Calculate marginal, conditional and joint rates.

Size	Gender	Treatment	Outcome
Large	Male	X	Success
Large	Male	X	Success
Small	Male	Y	Success
Large	Male	Y	Failure
Small	Male	X	Success
Large	Male	Y	Success

WHICH TREATMENT TO CHOOSE?

PPDAC CYCLE – A NEW QUESTION

Which treatment is better?



Key variable of interest:
Treatment variable

Sort the data
Plot graphs, tables of the "Treatment" and "Outcome" variable

2 x 2 Table

Treatment \ Outcome	Success	Failure	Row Total
X	542	158	700
Y	289	61	350
Column Total	831	219	1050

Marginal rates / proportions / percentages

Treatment \ Outcome	Success	Failure	Row Total
X	542	158	700
Y	289	61	350
Column Total	831	219	1050

- What proportion of the total number of patients underwent Treatment Y?
- $\mathrm{rate(Y)} = \frac{350}{1050} = \frac{1}{3} = 33\frac{1}{3}\%$
- What proportion of the total number of patients had a successful treatment?
- $\mathrm{rate(Success)} = \frac{831}{1050} = 0.791 = 79.1\%$
- Calculations above are called marginal rates / proportions / percentages.

Conditional rates / proportions / percentages

Outcome / Treatment	Success	Failure	Row Total
X	542	158	700
Y	289	61	350
Column Total	831	219	1050

- If we focus on patients who undergone Treatment X, what proportion of them had a successful treatment?
- $\text{rate}(\text{Success given X}) = \frac{542}{700} = 0.774 = 77.4\%$
- Calculation above is known as a conditional proportion / percentage.
- An even shorter way of writing this is to use a vertical bar in place of given: $\text{rate}(\text{Success} \mid \text{X})$

Joint rates / proportions / percentages

Treatment \ Outcome	Success	Failure	Row Total
X	542	158	700
Y	289	61	350
Column Total	831	219	1050

- What is the proportion of patients who chose Treatment Y and had a failure?
- $\text{rate}(\text{Y and failure}) = \frac{61}{1050} = 0.0581 = 5.81\%$
- NOT a conditional rate.
- Calculation is known as a joint rate/ proportion / percentage.

Which treatment is better?

Outcome Treatment	Success	Failure	Row Total
X	542	158	700
Y	289	61	350
Column Total	831	219	1050

Treatment X has 542 successful cases.

Treatment Y has 289 successful cases.

“We should recommend Treatment X!”?

More patients choosing Treatment X as compared to Y.

Making it fair!

Compare success rate of Treatments X and Y

Given that I pick some treatment, what is the rate of success?

- $\text{rate}(\text{Success} \mid \text{X}) = \frac{542}{700} = 0.774 = 77.4\%$
- $\text{rate}(\text{Success} \mid \text{Y}) = \frac{289}{350} = 0.826 = 82.6\%$

Fair comparison

- For Treatment X, roughly 77 out of 100 patients had a successful treatment.
- For Treatment Y, roughly 83 out of 100 patients had a successful treatment.

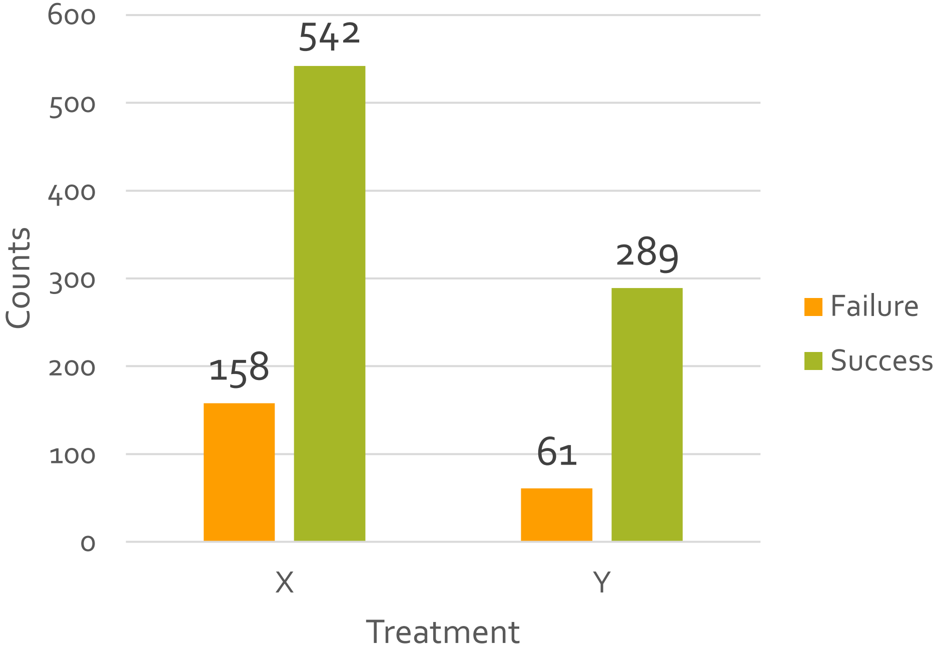
Treatment Y is better!

Table with row percentages

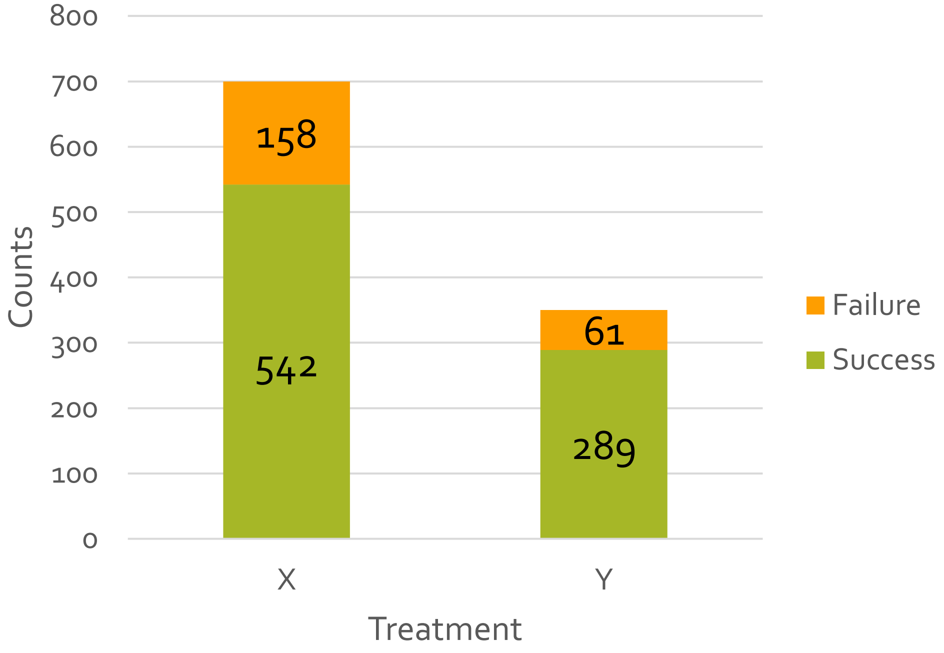
Outcome / Treatment	Success (row %)	Failure (row %)	Row Total (row %)
X	542 (77.4%)	158 (22.6%)	700 (100%)
Y	289 (82.6%)	61 (17.4%)	350 (100%)
Column Total	831 (79.1%)	219 (20.9%)	1050 (100%)

Analysing 2 categorical variables - plot

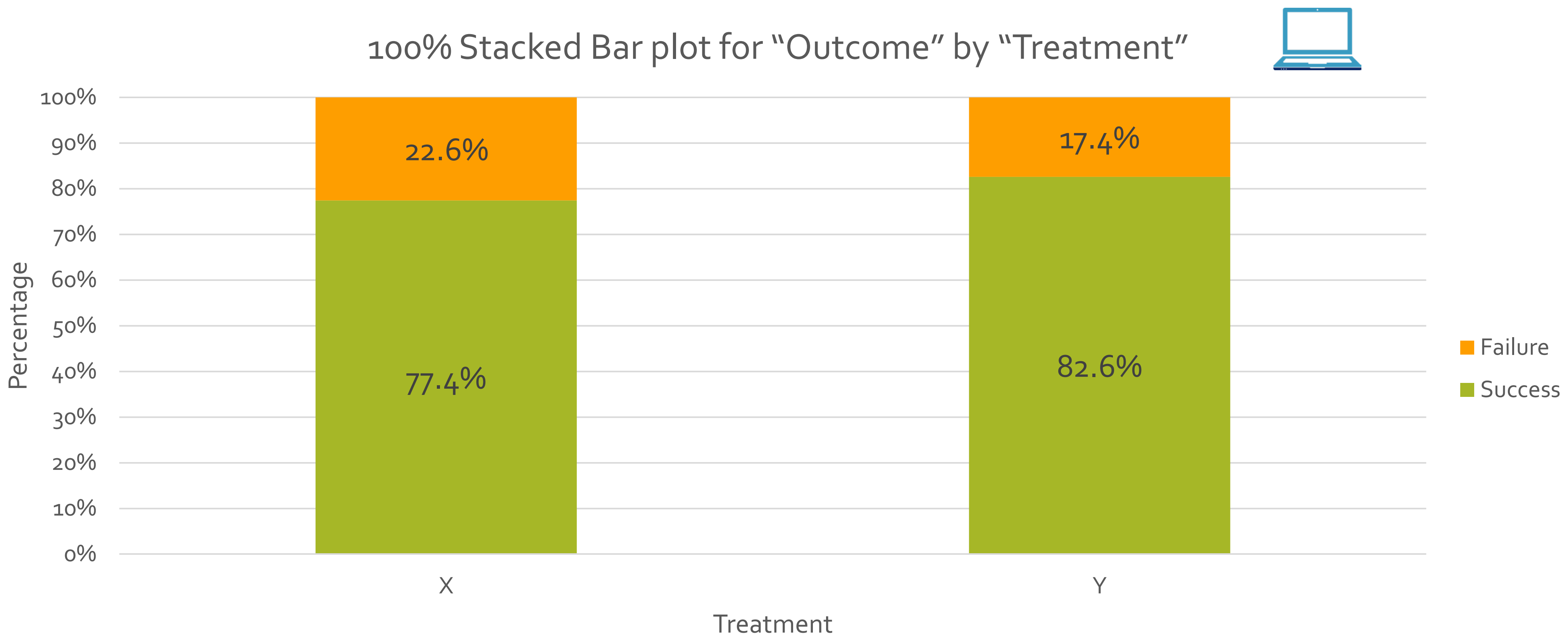
Dodged bar plot for “Outcome” by “Treatment”



Stacked bar plot for “Outcome” by “Treatment”



Analysing 2 categorical variables - plot



Summary

We have learnt how to analyse 2 categorical variables from the perspective of:

- Tables – 2x2 table
- Plots – Bar plots / 100% stacked bar plots

Unit 2A: Association I

By the end of this unit, you should be able to do the following:

1. Understand and apply association
2. Understand and apply symmetry rule

Used rates to conclude that Treatment Y is better than Treatment X.

Relationship between the type of treatment and the outcome of the treatment

"Treatment Y is positively associated to the success of the treatment."

"Treatment X is negatively associated to the success of the treatment."

Tend to see treatment Y and successful treatments go hand in hand.

Tend to see Treatment X and unsuccessful treatments go hand in hand.

Caution: Association, not causation!

Associative relationship between the 2 variables

Not sure if success of treatment is due to the treatment or not

Continuation from Unit 1

Is there an association?

Suppose we have A and B as characteristics in a population. We shall assume that some people have A, and some do not have A (labelled as NA). We assume the same about B.

Association absent	Association present	
$\text{rate}(A \mid B) = \text{rate}(A \mid NB)$	$\text{rate}(A \mid B) \neq \text{rate}(A \mid NB)$	
Rate of A is not affected by the presence or absence of B.	rate(A | B) > rate(A | NB)	rate(A | B) < rate(A | NB)
A and B are not associated.	Presence of A when B is present is stronger than when B is absent.	Presence of A when B is present is weaker than when B is absent.
	Positive association between A and B.	Negative association between A and B.

Linking back to our dataset

Checking for association between 2 variables

- Outcome of treatment
  - A: Success
  - NA: Failure
- Treatment
  - B: Treatment X
  - NB: Treatment Y

Compare

- $\text{rate}(A \mid B) = \text{rate}(\text{Success} \mid X) = 0.774$
- $\text{rate}(A \mid NB) = \text{rate}(\text{Success} \mid Y) = 0.826$

Conclusion

- rate(A | B) < rate(A | NB)
- Presence of A is weaker when B is present.
- Less successful treatments when we see Treatment X: Treatment X is negatively associated to a successful treatment.
- More successful treatments when we see Treatment Y: Treatment Y is positively associated to a successful treatment.

On Establishing Association

- Any of the following comparisons can show positive association between A and B:

rate(A | B) > rate (A | NB)
rate(B | A) > rate (B | NA)
rate(NA | NB) > rate (NA | B)
rate(NB | NA) > rate (NB | A)

- Likewise, for negative association between A and B:

rate(A | B) < rate (A | NB)
rate(B | A) < rate (B | NA)
rate(NA | NB) < rate (NA | B)
rate(NB | NA) < rate (NB | A)

- Try it – using the example in the previous slide, you can see that this relation holds true;

Eg. "If success is positively associated with treatment Y, then ..."

- "... success is negatively associated with ???"
- "... failure is positively associated with ???"
- "... failure is negatively associated with ???"

2 rules that govern rates

Suppose we have A and B as characteristics in a population. We shall assume that some people have A, and some do not have A (labelled as NA). We assume the same about B.

Symmetry rule

Basic rule on rates (to be discussed in Unit 2B: Association II)

Symmetry Rule

$$\text{rate}(A \mid B) > \text{rate}(A \mid NB) \Leftrightarrow \text{rate}(B \mid A) > \text{rate}(B \mid NA).$$

$$\text{rate}(A \mid B) < \text{rate}(A \mid NB) \Leftrightarrow \text{rate}(B \mid A) < \text{rate}(B \mid NA).$$

$$\text{rate}(A \mid B) = \text{rate}(A \mid NB) \Leftrightarrow \text{rate}(B \mid A) = \text{rate}(B \mid NA).$$

rate(A | B) > rate (A | NB) ⇔ rate(B | A) > rate(B | NA)

	B	Not B	Row Total
A	w	x	w + x
Not A	y	z	y + z
Column Total	w + y	x + z	w + x + y + z

$$\frac{w}{w+y} > \frac{x}{x+z}$$
$$w(x+z) > x(w+y)$$
$$wx + wz > xw + xy$$

$$\frac{w}{w+x} > \frac{y}{y+z}$$
$$w(y+z) > y(w+x)$$
$$wy + wz > yw + yx$$

$$wz > xy$$

$$\mathrm{rate}(A \mid B) \; > \; \mathrm{rate}(A \mid NB) \Leftrightarrow \mathrm{rate}(B \mid A) \; > \; \mathrm{rate}(B \mid NA)$$

1

2

$$\mathrm{rate}(A \mid B) \; > \; \mathrm{rate}(A \mid NB) \rightarrow \mathrm{rate}(B \mid A) \; > \; \mathrm{rate}(B \mid NA)$$

$$\mathrm{rate}(B \mid A) \; > \; \mathrm{rate}(B \mid NA) \rightarrow \mathrm{rate}(A \mid B) \; > \; \mathrm{rate}(A \mid NB)$$

$$\text{rate}(A \mid B) \; > \; \text{rate}(A \mid NB) \rightarrow \text{rate}(B \mid A) \; > \; \text{rate}(B \mid NA)$$

Rate of A given B is more than rate of A given NB.

Positive association between A and B.

More likely to see A when B is present as compared to when B is absent.

Also more likely to see B when A is present as compared to when A is absent.

Rate of B given A is more than rate of B given NA.

$$\text{rate}(B \mid A) > \text{rate}(B \mid NA) \rightarrow \text{rate}(A \mid B) > \text{rate}(A \mid NB)$$

Rate of B given A is more than rate of B given NA. → Positive association between B and A. → More likely to see B when A is present as compared to when A is absent. → Also more likely to see A when B is present as compared to when B is absent. → Rate of A given B is more than rate of A given NB.

$$\text{rate}(A \mid B) > \text{rate}(A \mid NB) \rightarrow \text{rate}(B \mid A) > \text{rate}(B \mid NA)$$

1

$$\text{rate}(B \mid A) > \text{rate}(B \mid NA) \rightarrow \text{rate}(A \mid B) > \text{rate}(A \mid NB)$$

2

$$\text{rate}(A \mid B) > \text{rate}(A \mid NB) \Leftrightarrow \text{rate}(B \mid A) > \text{rate}(B \mid NA)$$

Consequence of the symmetry rule

To identify if there is any association, check for either:

1. rate(A | B) ≠ rate(A | NB) OR

2. rate(B | A) ≠ rate(B | NA)

rate(Success | X) < rate(Success | Y):
Negative association between successful treatments and Treatment X

Check:
rate(X | Success) < rate(X | Failure)

Summary

We have learned:

- How to identify association
- Symmetry rule and its consequence on identifying association

Unit 2B: Association II

By the end of this unit, you should be able to do the following:

1. Understand and apply basic rule on rates

BASIC RULE ON RATES

The overall rate(A) will always lie between rate(A | B) and rate(A | NB).

Consequences of the basic rule on rates

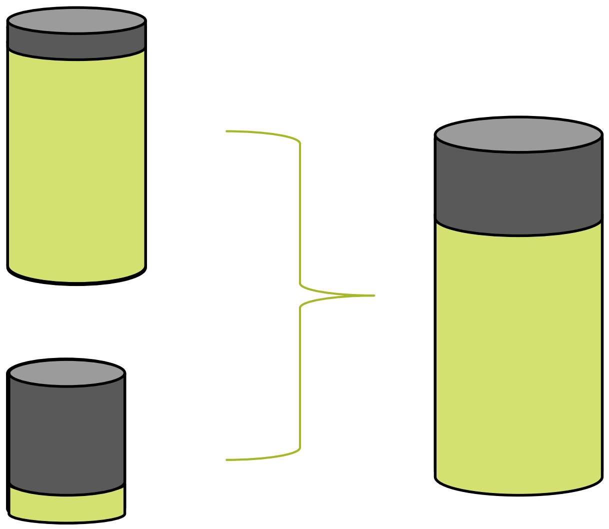
1. The closer rate(B) is to 100%, the closer rate(A) is to rate(A | B).
2. If $\mathrm{rate(B)} = 50\%$, then $\mathrm{rate(A)} = \frac{\mathrm{rate(A \mid B)+rate(A \mid NB)}}{2}$.
3. If $\mathrm{rate(A \mid B)} = \mathrm{rate(A \mid NB)}$, then $\mathrm{rate(A)} = \mathrm{rate(A \mid B)} = \mathrm{rate(A \mid NB)}$.

1. The closer rate(B) is to 100%, the closer rate(A) is to rate(A | B).

- 2 Cups of bubble tea
- Let A be the level of sweetness
  - Represented by the colour “Green” in the cup.
- Let B / NB be the cups: “Cup 1” vs. “Cup 2”

Cup 1
Size: Large cup
Sweetness: 90%

Cup 2
Size: Small cup
Sweetness: 20%



Size: Cup 1 + Cup 2
Sweetness: In between 20% to 90%, but closer to Cup 1

1. The closer rate(B) is to 100%, the closer rate(A) is to rate(A | B).

Sweetness in the final cup is between Sweetness | Cup 1 and Sweetness | Cup 2

Cup 1 takes up most of the final cup.

Expect sweetness of the final cup to be nearer to the sweetness of Cup 1.

Overall rate(A) to be between rate(A | B) and rate(A | NB)

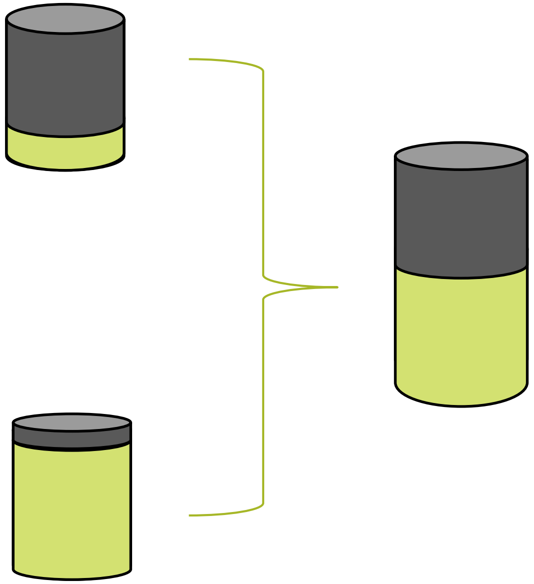
Overall rate(A) to be closer to rate(A | B) if B takes up a majority of the overall.

2. If $\text{rate}(B) = 50\%$, then

$$\text{rate}(A) = \frac{\text{rate}(A \mid B) + \text{rate}(A \mid NB)}{2}$$

Cup 1
Size: Small cup
Sweetness: 20%

Cup 2
Size: Small cup
Sweetness: 90%

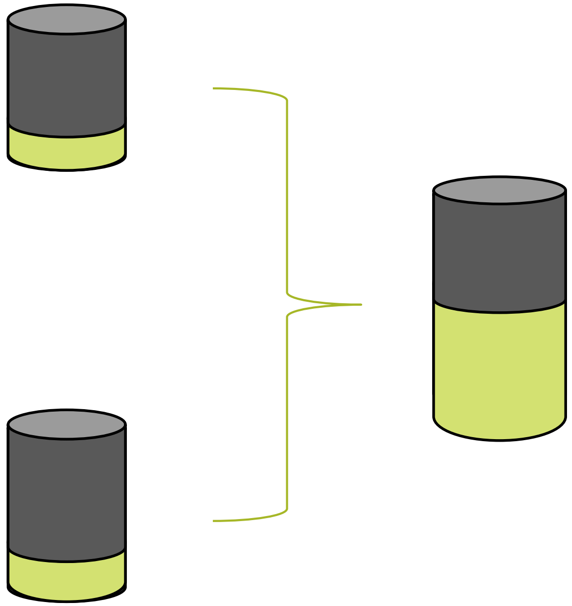


Size: Cup 1 + Cup 2
Sweetness: Exactly in between
20% to 90% = $\frac{20\%+90\%}{2} = 55\%$

3. If $\mathrm{rate}(A \mid B) = \mathrm{rate}(A \mid NB)$, then $\mathrm{rate}(A) = \mathrm{rate}(A \mid B) = \mathrm{rate}(A \mid NB)$.

Cup 1
Size: Small / Large cup
Sweetness: 20%

Cup 2
Size: Small / Large cup
Sweetness: 20%



Size: 2 Cups added together
Sweetness: Exactly 20%

Linking back to Consequences 2 and 3

If cups are of the same size, sweetness will be exactly half of the original cups.

- If Rate(B) = 50%, overall rate of A will be exactly in between the rate of A given B and the rate of A given NB.

If sweetness is the same for both cups, the sweetness of the final cup will also be the same, regardless of the sizes of the original cups.

- If rate(A | B) = rate(A | NB), then rate(A) is the same as the 2 rates.

Linking back to dataset at hand

Overall rate of successful treatments

- rate(Success) = 0.79

Groups: Treatment X and Treatment Y

- rate(Success | X) = 0.774
- rate(Success | Y) = 0.826
- rate(Success) in between the conditional rates

Overall rate of success closer to rate(Success | X)

- Treatment X takes up a majority of the treatments.
- rate(X) = $\frac{700}{1050} = 0.667 = 66\frac{2}{3}\%$
- Follows statement (1)

Summary

We have learned:

- What is the basic rule on rates
- The consequences of basic rule on rates

Unit 3A: Simpson's Paradox I

By the end of this unit you should be able to do the following:

1. Identify Simpson's paradox
2. Analysis using the slicing method

PPDAC CYCLE – A RECAP

Are the treatments are helping?

Yes. In general, there is a high rate of success.

More specifically, which treatment is better?

Treatment Y is positively associated to success rate

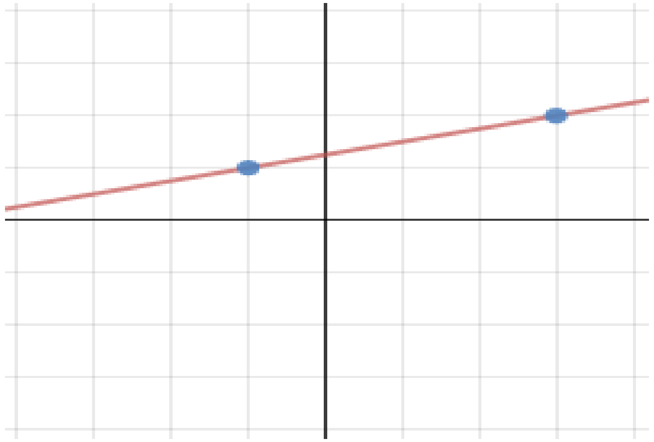
ANYTHING ELSE?

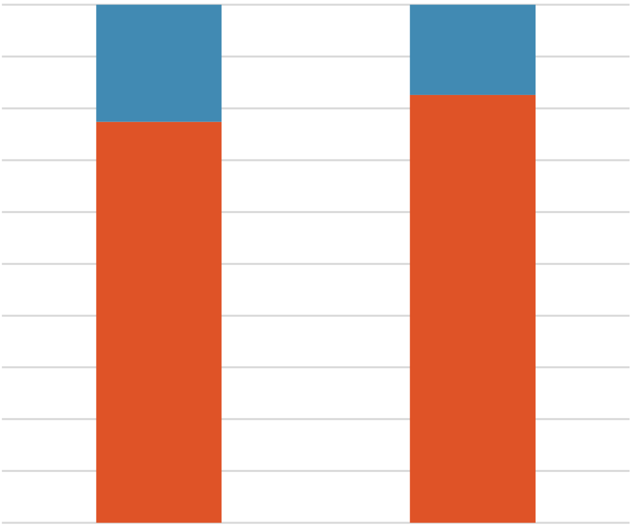
Size	Gender	Treatment	Outcome
Large	Male	X	Success
Large	Male	X	Success
Small	Male	Y	Success
Large	Male	Y	Failure
Small	Male	X	Success
Large	Male	Y	Success

WHAT ABOUT OTHER VARIABLES?

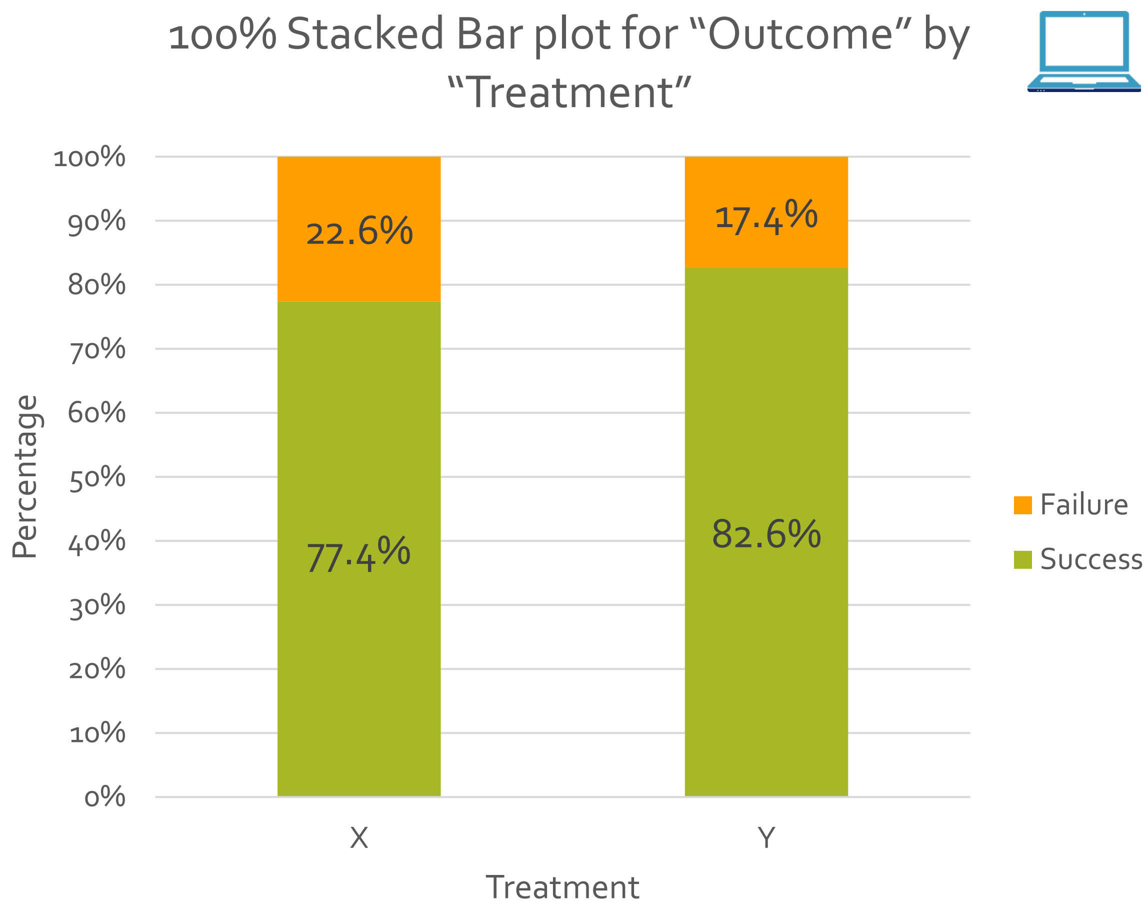
Exploring the "stone size" variable

What would be a useful visualisation?





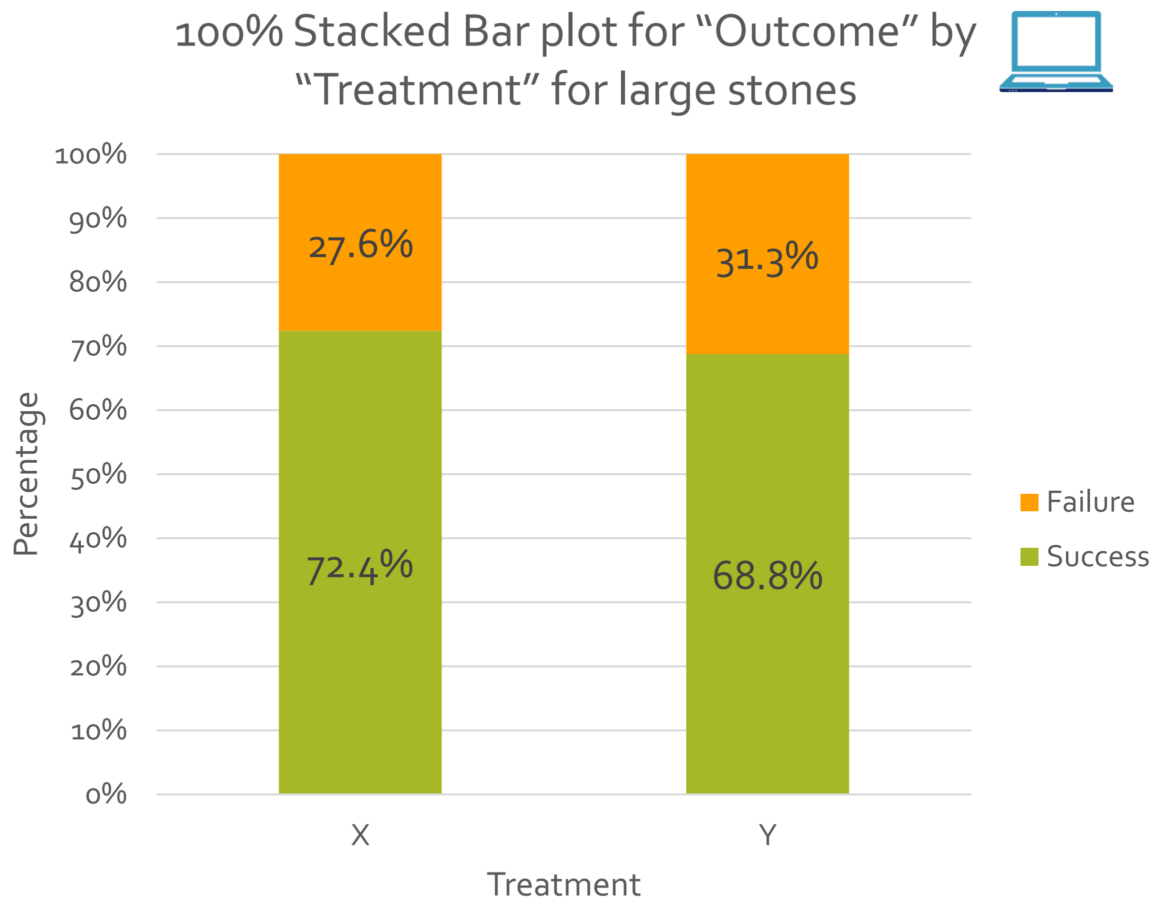
Analysing 2 categorical variables - Plot



Y Overall, Treatment Y is better

All stones	Success	Failure	Total
X	542	158	700
Y	289	61	350
Total	831	219	1050

Plot across large stones only



rate(Success | X) > rate(Success | Y)

X Across large stones, Treatment X is better

Large stones	Success	Failure	Total
X	381	145	526
Y	55	25	80
Total	436	170	606

Exercise

X

Across large stones,
Treatment X is better

$$\text{rate}(\text{Success} \mid \text{X}) = \frac{381}{526} = 0.724$$

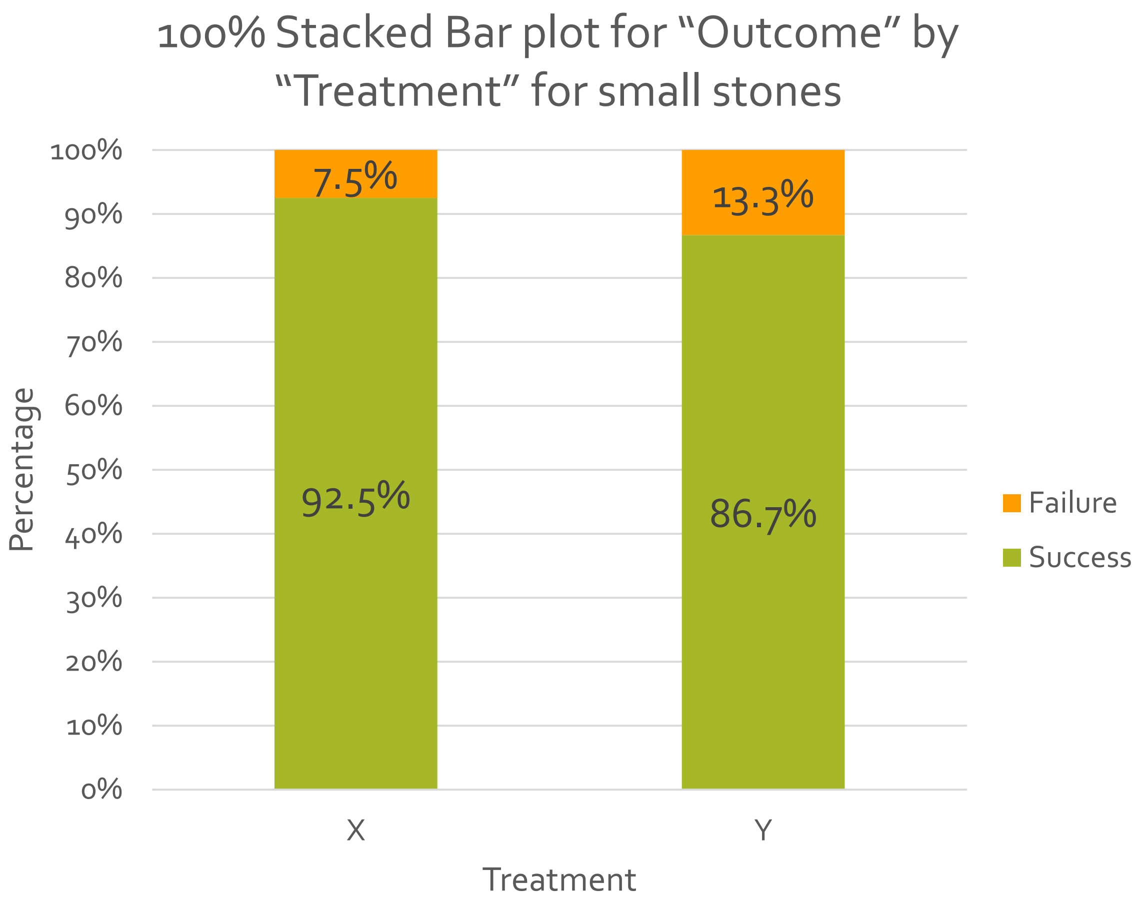
$$\text{rate}(\text{Success} \mid \text{Y}) = \frac{55}{80} = 0.688$$

Large stones	Success	Failure	Total
X	381	145	526
Y	55	25	80
Total	436	170	606

rate(Success | X) > rate(Success | Y)

Treatment X is positively associated to success

Plot across small stones only



X

Across small stones, Treatment X is better

Small stones	Success	Failure	Total
X	161	13	174
Y	234	36	270
Total	395	49	444

Analysing 3 categorical variables - plot

A paradox on our hands

Y — Overall,
Treatment Y is better

X — Across large stones,
Treatment X is better

X — Across small stones,
Treatment X is better

Unit 3B: Simpson’s Paradox II

By the end of this unit you should be able to do the following:

1. Explain a Simpson’s paradox

A paradox on our hands

Y

Overall,
Treatment Y is better

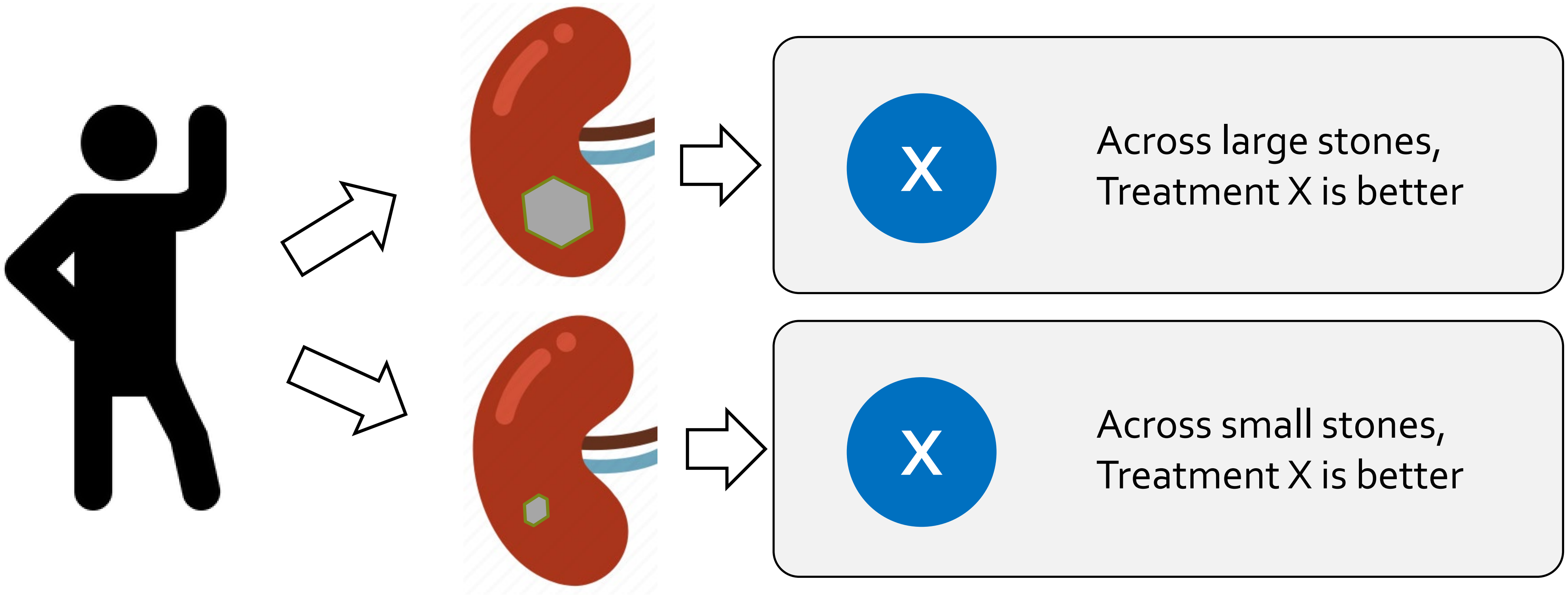
X

Across large stones,
Treatment X is better

X

Across small stones,
Treatment X is better

A paradox explained



Analysing 3 categorical variables - Table

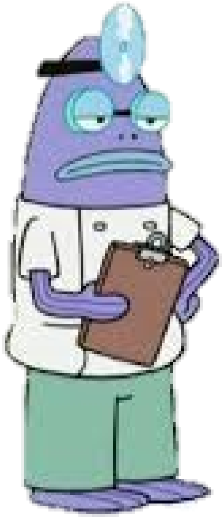
	Large stones			Small stones			Total (Large + Small)		
	Successful treatments	Total number of treatments	rate(Success) in %	Successful treatments	Total number of treatments	rate(Success) in %	Successful treatments	Total number of treatments	rate(Success) in %
X	381	526	72.4%	161	174	92.5%	542	700	77.4%
Y	55	80	68.8%	234	270	86.7%	289	350	82.6%

Analysing 3 categorical variables - Table

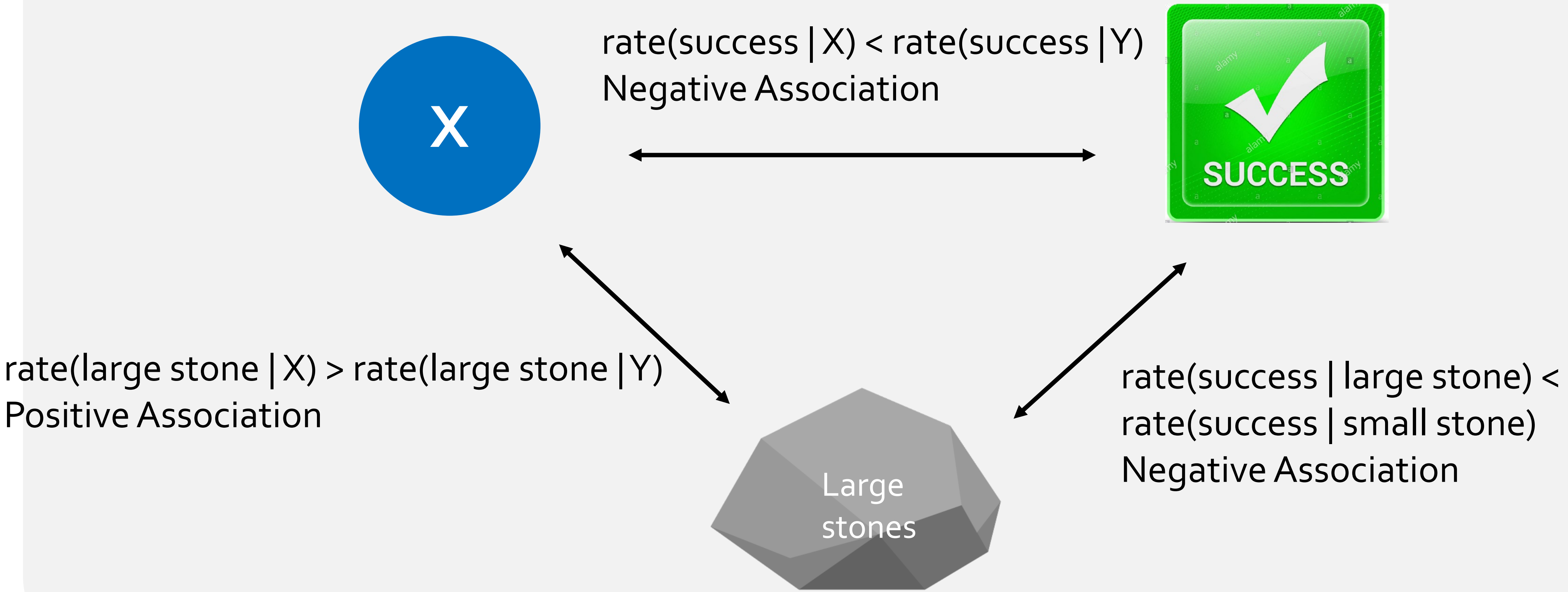
	Large stones			Small stones			Total (Large + Small)		
	Successful treatments	Total number of treatments	rate(Success) in %	Successful treatments	Total number of treatments	rate(Success) in %	Successful treatments	Total number of treatments	rate(Success) in %
X	381	526	72.4%	161	174	92.5%	542	700	77.4%
Y	55	80	68.8%	234	270	86.7%	289	350	82.6%

Analysing 3 categorical variables - Table

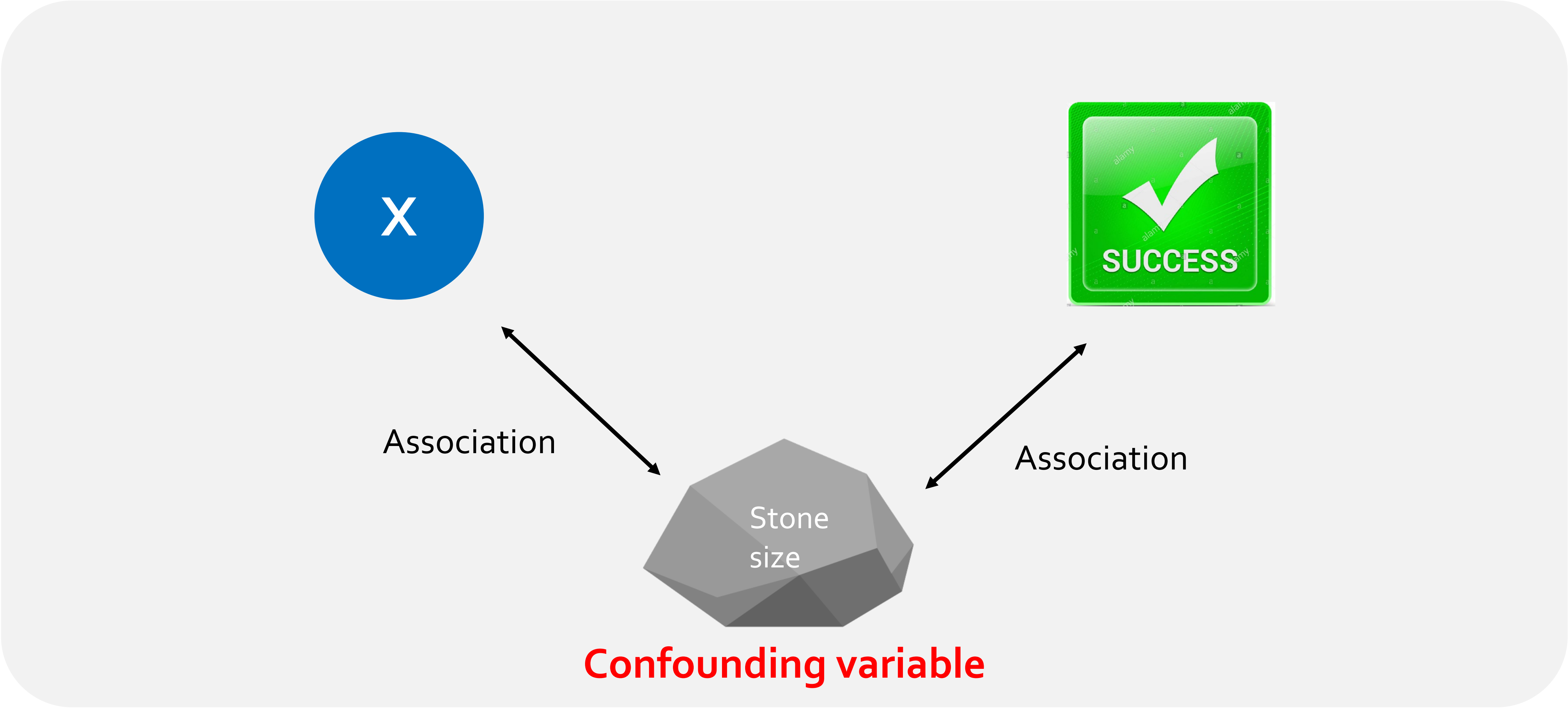
	Large stones			Small stones			Total (Large + Small)		
	Successful treatments	Total number of treatments	rate(Success) in %	Successful treatments	Total number of treatments	rate(Success) in %	Successful treatments	Total number of treatments	rate(Success) in %
X	381	526	72.4%	161	174	92.5%	542	700	77.4%
Y	55	80	68.8%	234	270	86.7%	289	350	82.6%



ANALOGY



View from Association



Simpson's paradox $\Rightarrow$ confounder
Confounder $\not\Rightarrow$ Simpson's paradox

Summary

We learnt how to analyse 3 categorical variables from the perspective of:

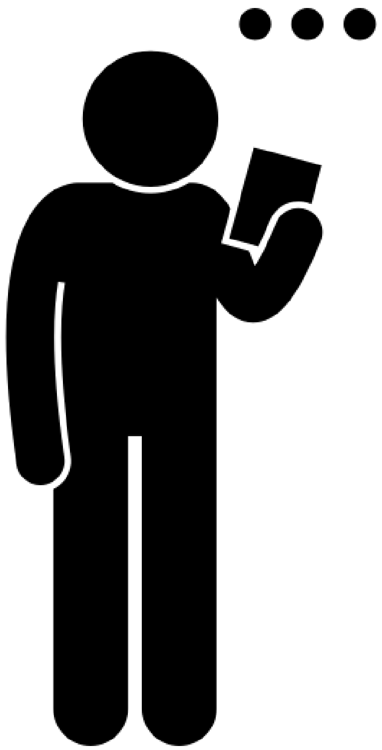
- Tables – slicing by subgroups
- Graphs – sliced bar graph

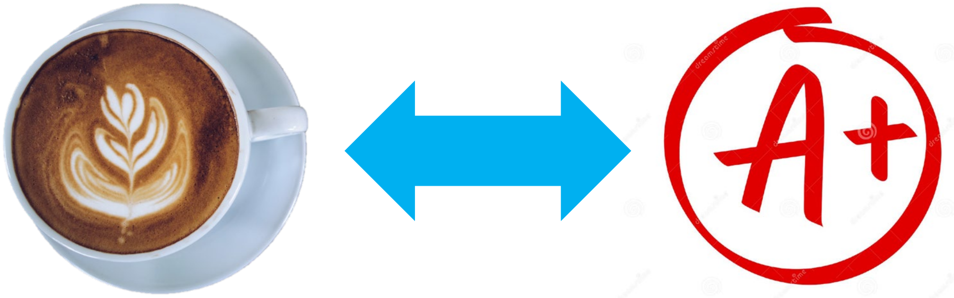
Unit 4
Confounders

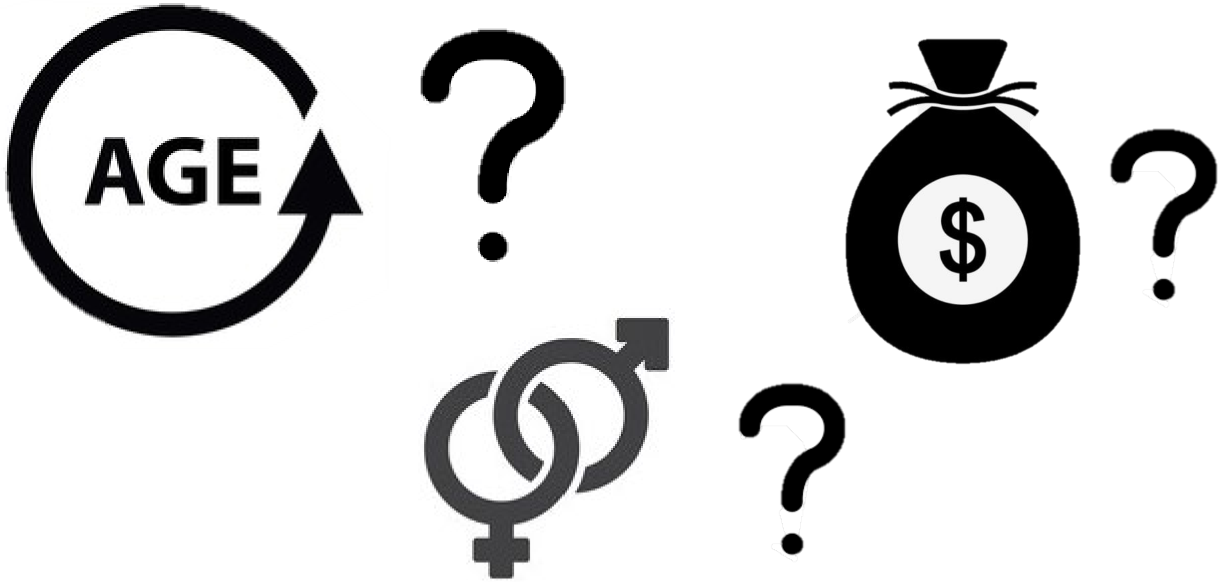
By the end of this unit you should be able to do the following:

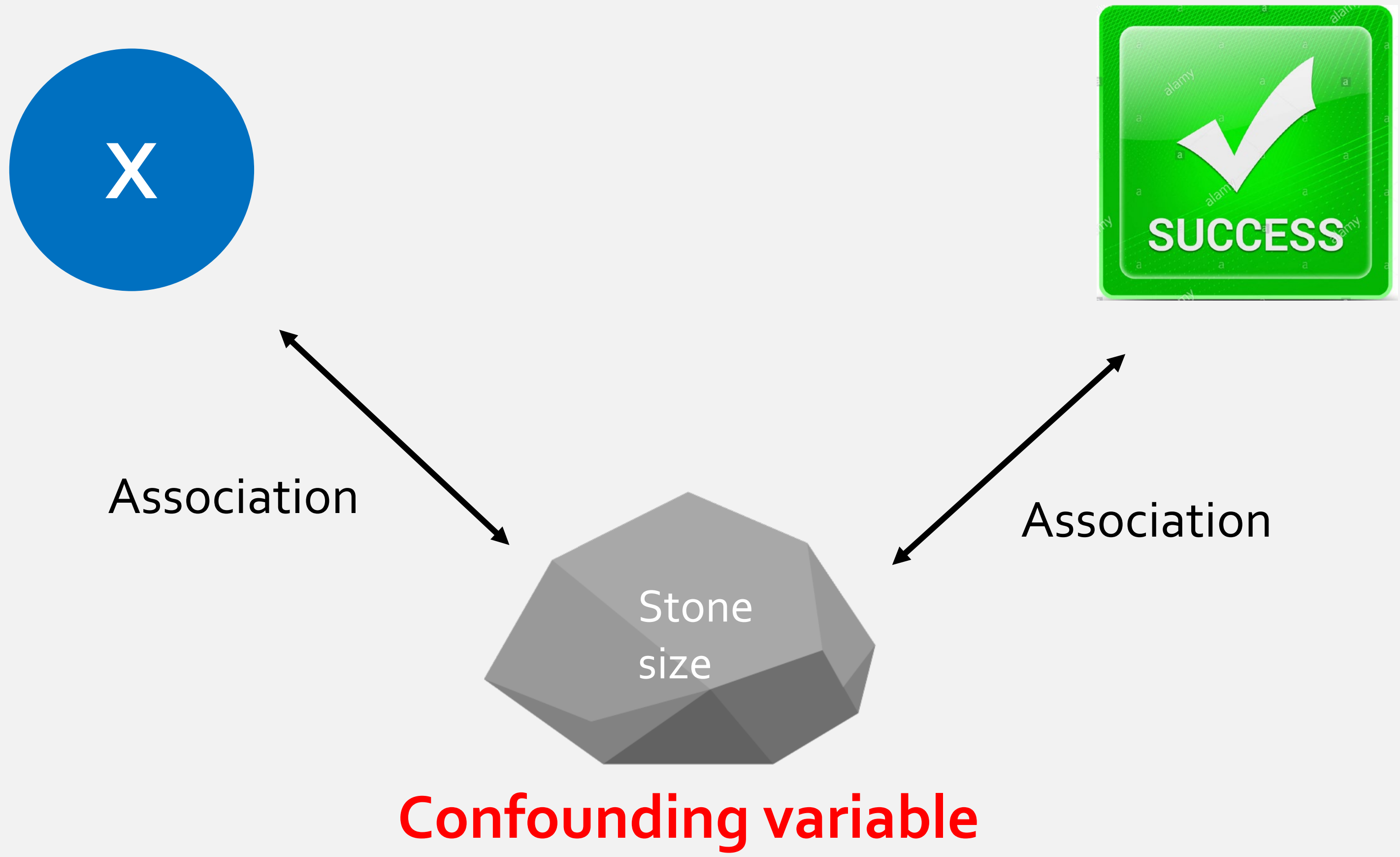
1. Define a confounder (ie. confounding variable)
2. Identify possible confounding variables in a study

Introduction









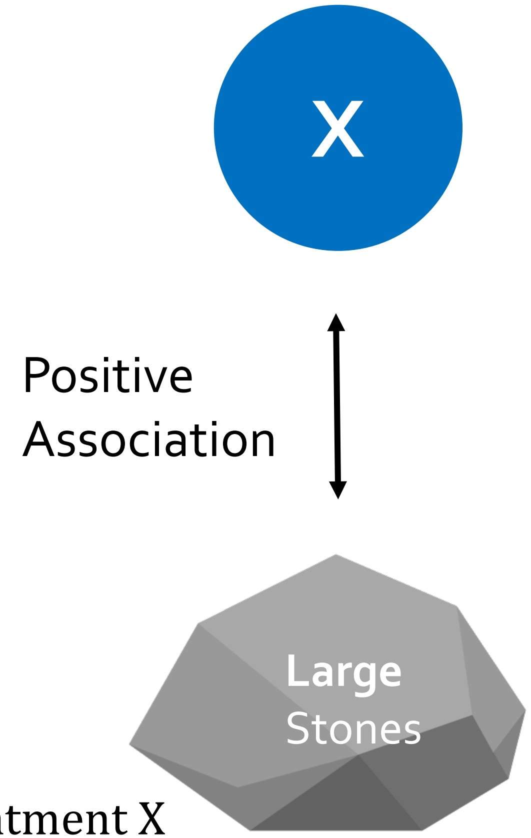
Definition:
A confounder is a third variable that is associated to both the independent and dependent variable whose relationship we are investigating

Stone size associated to treatment type

	Large	Small	Total
X	526	174	700
Y	80	270	350
Total	606	444	1050

$$\left.\begin{array}{l} \text{rate}(\text{Large} \mid X) = \dfrac{526}{700} = 0.751 \\[2ex] \text{rate}(\text{Large} \mid Y) = \dfrac{80}{350} = 0.229 \end{array}\right\}$$

Since $0.751 > 0.229$,
Large stones **positively** associated to treatment X

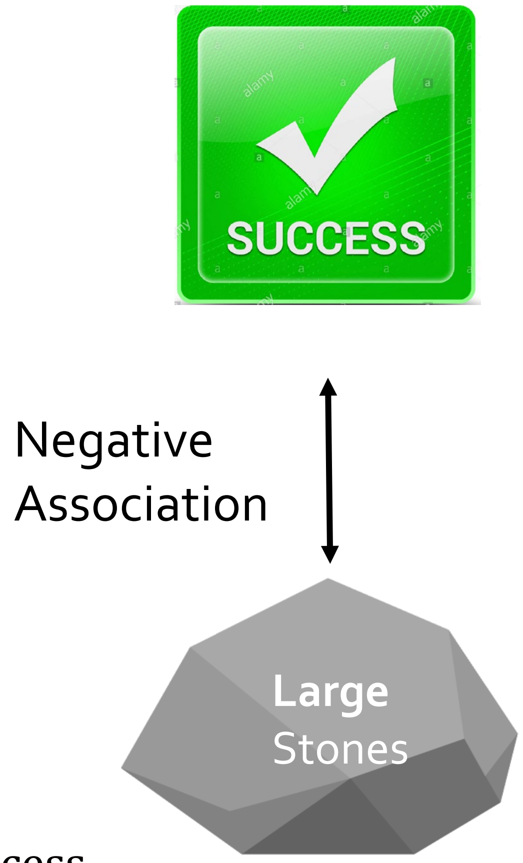


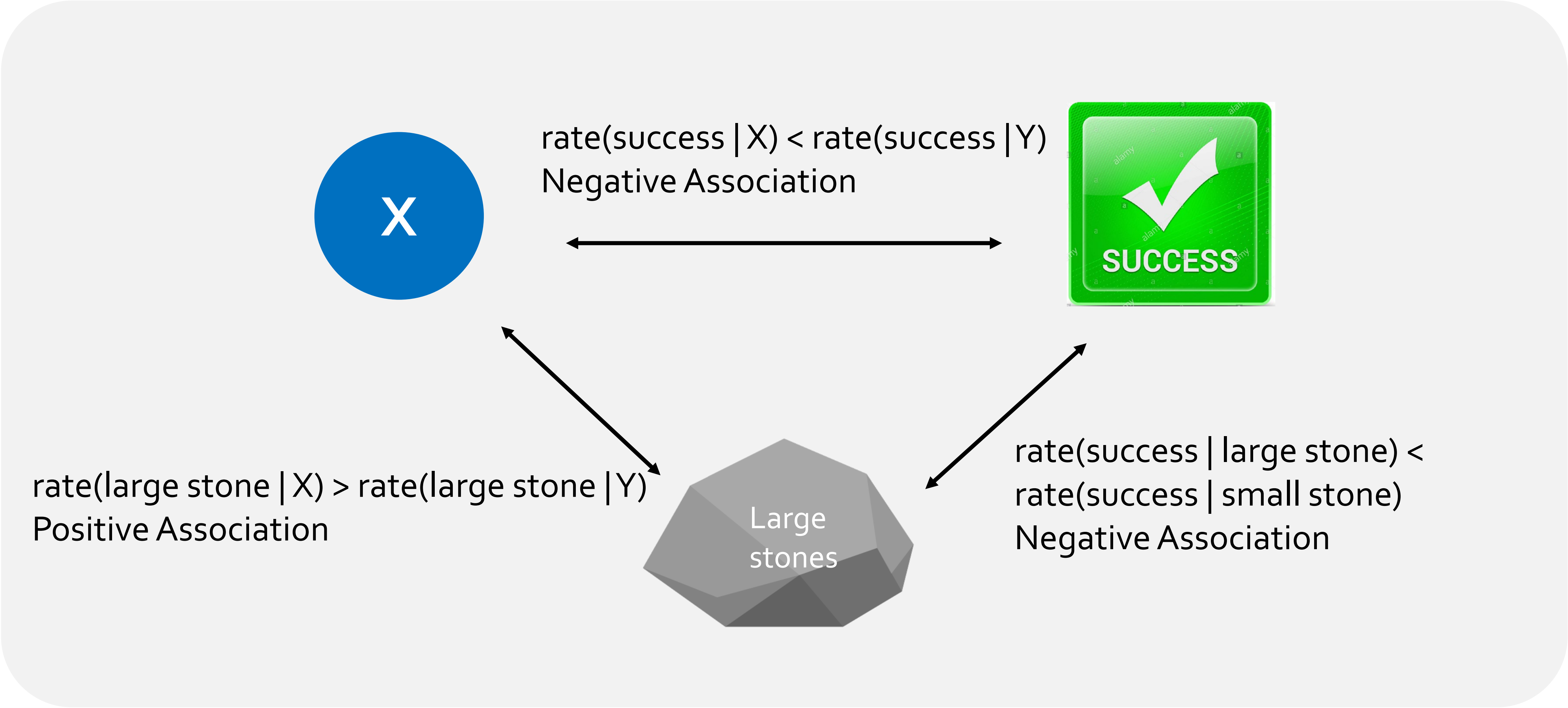
Stone size associated to success

	Success	Failure	Total
Large	436	170	606
Small	395	49	444
Total	831	219	1050

$$\left.\begin{aligned}\text{rate}(\text{Success} \mid \text{Large}) &= \frac{436}{606} = 0.719 \\ \text{rate}(\text{Success} \mid \text{Small}) &= \frac{395}{444} = 0.890\end{aligned}\right\}$$

Since $0.719 < 0.890$,
Large stones **negatively** associated to success





View from Association

Recall:

Size	Treatment Type	Outcome
Large	X	Success
Large	X	Success
Small	Y	Success
Large	Y	Failure
Small	X	Success
Large	Y	Success

X

After slicing,
Treatment X is better

Size	Treatment Type	Outcome
Large	X	Success
Large	X	Success
Small	Y	Success
Large	Y	Failure
Small	X	Success
Large	Y	Success

DO WE STILL OBSERVE SIMPSONS PARADOX?

Y

No,
Treatment Y is better

We have to measure a variable in order to check if it is a confounder!

THE PROBLEM

We must measure a variable in order to check if it is a confounder

We need to collect data on lots of variables

This is not feasible
(costly, difficult to analyse)

???

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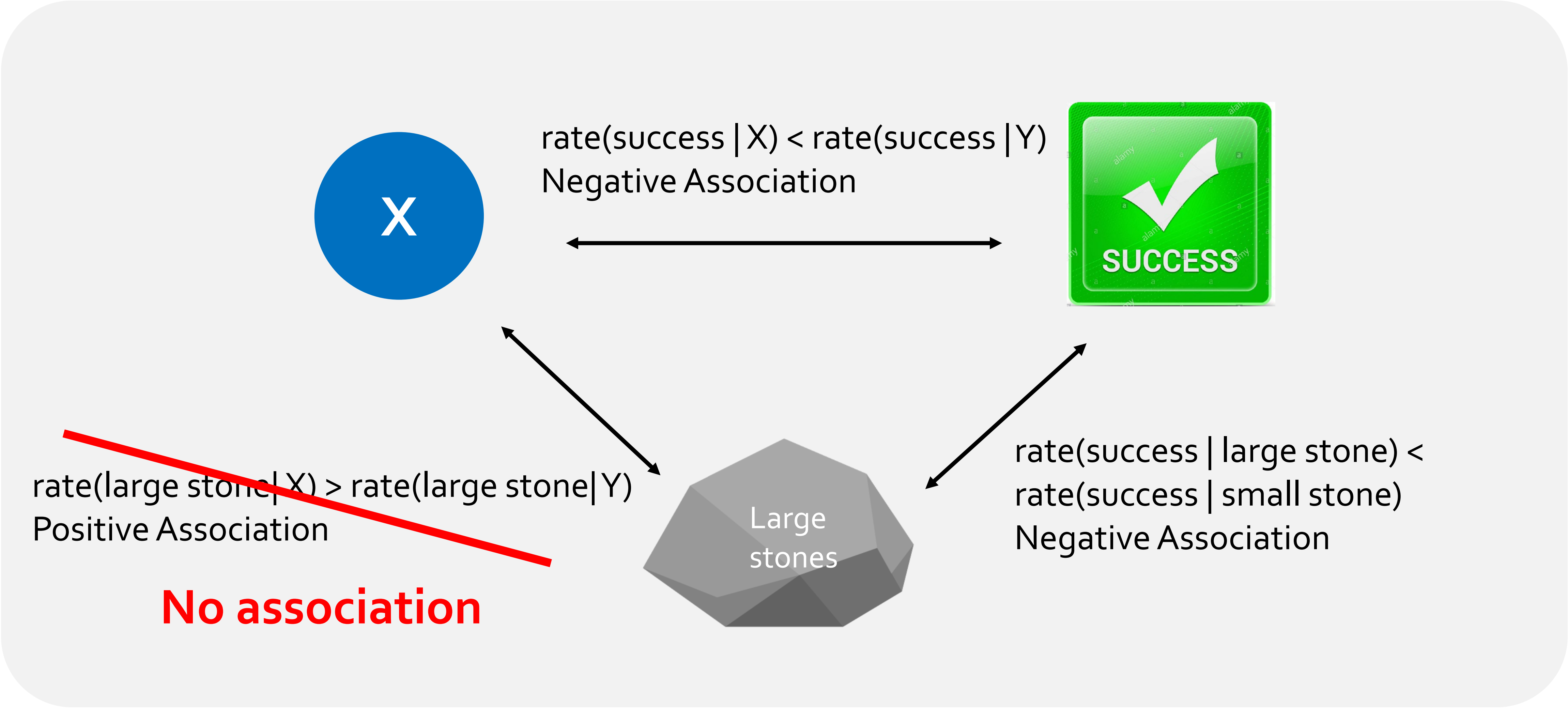
RANDOMISATION

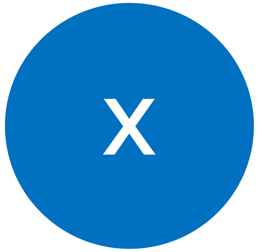


The effect of randomly assigning stone size to treatment type

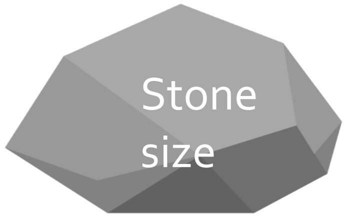
Randomisation is not always possible

Summary

Main variables



Confounding variable



(prove using association)

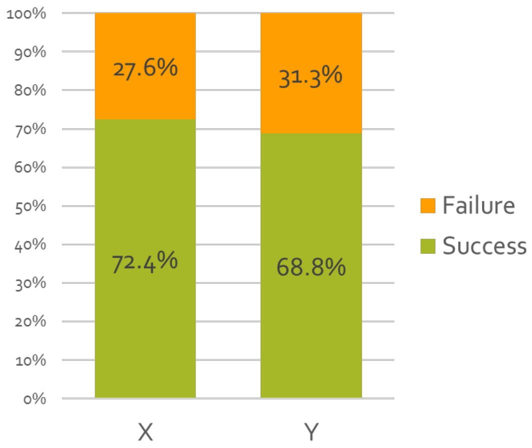
Proving Association

$\text{rate}(A \mid B) \neq \text{rate}(A \mid NB)$

OR

$\text{rate}(B \mid A) \neq \text{rate}(B \mid NA)$

OR



Chapter 2 end

We learnt how to analyse categorical variables from the perspective of:

- Tables
- Graphs