NATIONAL UNIVERSITY OF SINGAPORE

CS1101S — PROGRAMMING METHODOLOGY

(AY2018/2019 SEMESTER 1)

MIDTERM ASSESSMENT (ADAPTED TO AY2020/21 IN 9/2020)

Time Allowed: **1 Hour 45 Minutes**

SOLUTIONS

INSTRUCTIONS

1. This assessment paper contains **EIGHT (8)** questions and comprises **EIGHTEEN (18)** printed pages, including this page.
2. The full score of this paper is **75 marks**.
3. This is a **CLOSED BOOK** assessment, but you are allowed to bring in one A4 sheet of notes (handwritten or printed on both sides).
4. Answer **ALL** questions **within the space provided** in this booklet.
5. Where programs are required, write them in the **Source §2** language.
6. Write legibly with a **pen or pencil**. **Untidiness will be penalized**.
7. Do not tear off any pages from this booklet.
8. Write your **Student Number** below **USING A PEN**. Do not write your name.
9. Also write down your **Studio Group Number** in the provided box, if you can remember it.

(write with a pen)

Student No.:

Studio Group No. (leave blank if cannot remember):

This portion is for examiner's use only

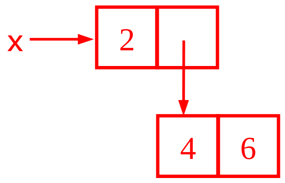
Q#	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**Σ**
MAX	10	4	8	9	10	10	12	12	75
SC									

Question 1: Box-and-Pointer Diagrams [10 marks]

Draw the box-and-pointer diagram for the value of x after the evaluation of each of the following programs. Clearly show where x is pointing to.

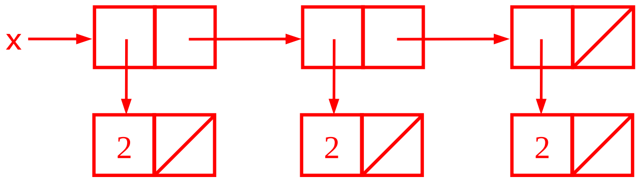
1A. [2 marks]

```
const x = pair(2, pair(4, 6));
```



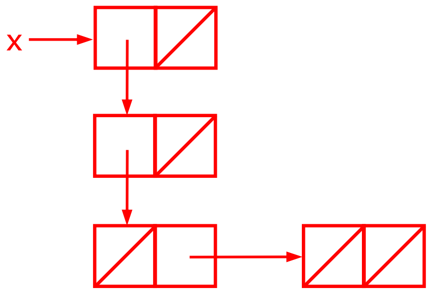
1B. [2 marks]

```
const p = list(2);
const q = pair(2, list());
const x = list(p, q, pair(2, null));
```



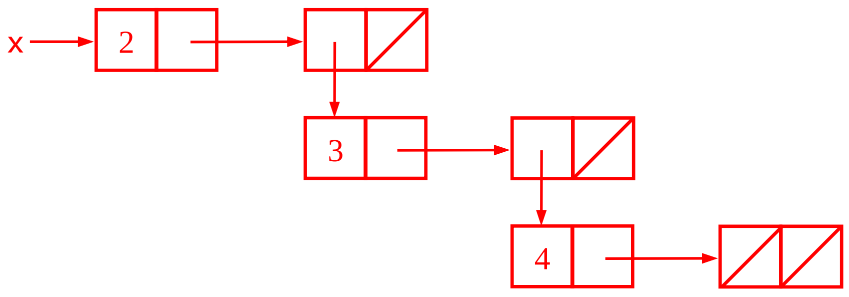
1C. [3 marks]

```
const x = list(list(list(null, null)));
```



1D. [3 marks]

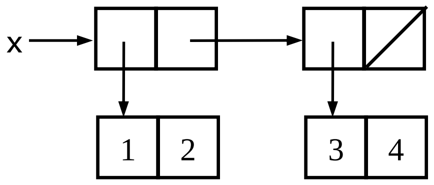
```
const x = accumulate(list, null, list(2, 3, 4));
```



Question 2: Making Pairs [4 marks]

Write a Source §2 program that produces exactly the pairs shown in each of the following box-and-pointer diagrams. At the end of the execution of your program, the constant x must refer to the pair as shown in the diagram.

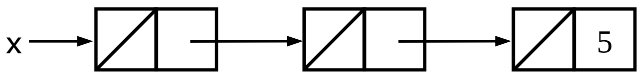
2A. [2 marks]



```
// SOLUTION 1:
const x = list(pair(1, 2), pair(3, 4));

// SOLUTION 2:
const x = pair(pair(1, 2), pair(pair(3, 4), null));
```

2B. [2 marks]



```
const x = pair(null, pair(null, pair(null, 5)));
```

Question 3: Let's Be Logical [8 marks]

3A. [3 marks]

Consider the following function

```
function hoo(f, g, h, x) {
    if (f(x) || (g(x) && h(x))) {
        return 100;
    } else {
        return 50;
    }
}
```

Rewrite the function hoo without using the keyword **if** and without any logical operators (&&, ||, !). Your function must have the same order of evaluations and produce the same result as the original.

```
function hoo(f, g, h, x) {

    return f(x) ? 100
                : g(x) ? (h(x) ? 100 : 50) : 50;







}
```

3B. [5 marks]

Consider the following program

```
function gee(n) {
    return n <= 0 ? true : (false || gee(n - 1));
}
gee(4);
```

(i) What kind of process does the program give rise to?
(ii) Justify your answer by showing the evaluation steps (hint: the substitution model).

(i) Iterative process.

(ii) There is no deferred operation in the evaluation, therefore it is an iterative process.

```
gee(4)
→ false || gee(3)
→ gee(3)
→ false || gee(2)
→ gee(2)
→ false || gee(1)
→ gee(1)
→ false || gee(0)
→ gee(0)
→ true
```

Question 4: Recursive vs Iterative Processes [9 marks]

The `get_sublist` function takes as arguments two integer numbers, `start` and `end`, and a list `L`, and returns a list containing the element(s) of `L` from position `start` to position `end`, including both. The first element of `L` is at position 0. You can assume that 0 ≤ `start` ≤ `end` < `length(L)`.

Example :

```
const L = list(11, 12, 13, 14, 15, 16, 17, 18);
get_sublist(3, 5, L);  // returns list(14, 15, 16).
```

Complete the following two implementations of `get_sublist` that give rise to **(A)** a **recursive process** and **(B)** an **iterative process**. Both implementations' runtime should have an order of growth of O(*n*), where *n* is the length of the list `L`.

4A. [4 marks] Recursive version.

```
function get_sublist(start, end, L) {

    function helper(pos, ys) {
        if (pos < start) {

            return helper(pos + 1, tail(ys));

        } else if (pos <= end) {

            return pair(head(ys), helper(pos + 1, tail(ys)));

        } else { return null; }
    }
    return helper(0, L);
}
```

4B. [5 marks] Iterative version.

```
function get_sublist(start, end, L) {

    function helper(pos, ys, result) {

        // SOLUTION 1:
        if (pos < start) {
            return helper(pos + 1, tail(ys), result);
        } else if (pos <= end) {
            return helper(pos + 1, tail(ys), pair(head(ys), result));
        } else {
            return reverse(result);
        }

        // SOLUTION 2:
        return pos < start
            ? helper(pos + 1, tail(ys), result)
            : pos <= end
                ? helper(pos + 1, tail(ys), pair(head(ys), result))
                : reverse(result);
    }
    return helper(0, L, null);
}
```

Question 5: The Benefits of Being Sorted [10 marks]

5A. [5 marks]

We represent a ***set*** of numbers using a list of **distinct** numbers **sorted** in **ascending order**. Complete the function, `is_subset(S, T)`, to determine whether set S is a subset of set T, which is true only if every element in S is also an element of T. We assume that $0 \leq N_S$ and $0 \leq N_T$, where N_S and N_T are the number of elements in S and T respectively. If $N_S = 0$, then S is always a subset of T. Your program should exploit the fact that both lists are sorted; its runtime should have an order of growth of $O(N_S + N_T)$.

```
function is_subset(S, T) {

    if (is_null(S)) {
        return true;
    } else if (is_null(T)) {
        return false;
    } else if (head(S) < head(T)) {
        return false;
    } else if (head(S) === head(T)) {
        return is_subset(tail(S), tail(T));
    } else {
        return is_subset(S, tail(T));
    }
}
```

5B. [5 marks]

Complete the following function, `super_merge`, that takes in a list `L` of **one or more** lists of numbers, where the numbers in each list are in **ascending order**, and returns a single list of all the numbers from `L`, sorted in **ascending order**. For example:

```
const L = list(list(1, 3, 4, 7, 7), list(2, 8), list(), list(3, 5, 6));
super_merge(L);  // returns list(1, 2, 3, 3, 4, 5, 6, 7, 7, 8).
```

Your function can make use of the `merge` function (for merge sort) presented in the lectures and given here for your reference:

```
function merge(xs, ys) {
    if (is_null(xs)) {
        return ys;
    } else if (is_null(ys)) {
        return xs;
    } else {
        const x = head(xs);
        const y = head(ys);
        return (x < y)
               ? pair(x, merge(tail(xs), ys))
               : pair(y, merge(xs, tail(ys)));
    }
}
```

To get full marks for this part, your function must use at least one of the functions `filter`, `map` and `accumulate` in a correct and meaningful way.

```
function super_merge(L) {

// SOLUTION 1:

    return accumulate(merge, null, L);


// SOLUTION 2:

    return accumulate((x, ys) => merge(x, ys), null, L);

}
```

Question 6: Active Lists [10 marks]

An ***active list*** is a function that takes an integer number and returns an empty list or a list of length 1. It can be used as an alternative representation of a list, where it takes as argument an element's position in the list, and returns that element in a list of length 1. Note that the first element in a list is at position 0.

6A. [5 marks]

Define a function `make_active_list` that takes a list as its argument and returns an active list that represents the input list.

Example:

```
const act_list = make_active_list(list(8, 3, 5));
act_list(-1); // returns null
act_list(0);  // returns list(8)
act_list(1);  // returns list(3)
act_list(2);  // returns list(5)
act_list(3);  // returns null
```

Note that when the argument passed to `act_list` is negative, or is greater than or equal to the length of the input list to `make_active_list`, the function `act_list` should return an empty list.

```
function make_active_list(L) {

    const len = length(L);

    return pos => (pos < 0 || pos >= len)
        ? null
        : list(list_ref(L, pos));

}
```

6B. [5 marks]
Define a function `map_active_list` that takes as arguments a unary function `op` and an active list and returns an active list that represents the original list with all its elements transformed by `op`.

Example:

```
const act_list1 = make_active_list(list(8, 3, 5));
const act_list2 = map_active_list(x => x + 1, act_list1);
act_list2(-1); // returns null
act_list2(0);  // returns list(9)
act_list2(1);  // returns list(4)
act_list2(2);  // returns list(6)
act_list2(3);  // returns null
```

```
function map_active_list(op, act_list) {

// SOLUTION 1:

    function new_act_list(pos) {
        const x = act_list(pos);
        return is_null(x) ? x : list(op(head(x)));
    }
    return new_act_list;



// SOLUTION 2:

    return pos => map(op, act_list(pos));

}
```

Question 7: Binary Search Trees [12 marks]

We consider the binary search tree (BST) data structure presented in the lectures. For the subsequent parts of this question, you **must** make good use of the **binary tree abstraction**, consisting of the following functions:

- `is_empty_binary_tree(tree)` — Tests whether the given binary tree `tree` is empty.
- `is_binary_tree(x)` — Returns `true` if `x` is a binary tree and `false` otherwise.
- `left_subtree_of(tree)` — Returns the left subtree of `tree` if `tree` is not empty.
- `value_of(tree)` — Returns the value of the root node of `tree` if `tree` is not empty.
- `right_subtree_of(tree)` — Returns the right subtree of `tree` if `tree` is not empty.
- `make_empty_binary_tree()` — Returns an empty binary tree.
- `make_binary_tree_node(left, value, right)` — Returns a binary tree with left subtree `left`, value `value`, and right subtree `right`.

Do not break this binary tree abstraction in your programs.

7A. [6 marks]

Complete the function `negate_bst` that takes in a BST of numbers and returns a new BST of numbers that has all the numbers from the input BST negated. The "shape" of the result BST must be a left-right reflection of that of the input BST. For example:

```
const B = make_binary_tree_node(
    make_binary_tree_node(
        make_empty_binary_tree(),
        -3,
        make_empty_binary_tree()),
    2,
    make_binary_tree_node(
        make_empty_binary_tree(),
        5,
        make_empty_binary_tree()));

negate_bst(B);
/* returns the same tree as:
make_binary_tree_node(
    make_binary_tree_node(
        make_empty_binary_tree(),
        -5,
        make_empty_binary_tree()),
    -2,
    make_binary_tree_node(
        make_empty_binary_tree(),
        3,
        make_empty_binary_tree()));
*/
```

```
function negate_bst(bst) {

// SOLUTION 1:

    if (is_empty_binary_tree(bst)) {
        return make_empty_binary_tree();
    } else {
        return make_binary_tree_node(
            negate_bst(right_subtree_of(bst)),
            -1 * value_of(bst),
            negate_bst(left_subtree_of(bst)));
    }


// SOLUTION 2:

    return is_empty_binary_tree(bst)
        ? make_empty_binary_tree()
        : make_binary_tree_node(
            negate_bst(right_subtree_of(bst)),
            -1 * value_of(bst),
            negate_bst(left_subtree_of(bst)));

}
```

7B. [6 marks]
Complete the function `accumulate_bst` that behaves like `accumulate` but can only work on BST. Note that the order of application of the input operation `op` must start from the largest value in the BST, in descending order, to the smallest value. For example, if the input BST `B` has the values 1, 2, 3, 4, 5, 6 and 7, then, regardless of the "shape" of the BST `B`, the call `accumulate_bst(pair, null, B)` should return `list(1,2,3,4,5,6,7)`, and the call `accumulate_bst((x, y) => x + y, 0, B)` should return 28.

```
function accumulate_bst(op, initial, bst) {


// SOLUTION 1:

    if (is_empty_binary_tree(bst)) {
        return initial;
    } else {
        const s = accumulate_bst(op, initial, right_subtree_of(bst));
        const t = op(value_of(bst), s);
        return accumulate_bst(op, t, left_subtree_of(bst));
    }


// SOLUTION 2:

    function listify_bst(b) {
        if (is_empty_binary_tree(b)) {
            return null;
        } else {
            const left_list = listify_bst(left_subtree_of(b));
            const value = value_of(b);
            const right_list = listify_bst(right_subtree_of(b));
            return append(left_list, pair(value, right_list));
        }
    }
    return accumulate(op, initial, listify_bst(bst));

}
```

Question 8: Permutations, Again! [12 marks]

8A. [6 marks]

Complete the function `insertions(x, ys)` that returns all possible ways to insert `x` into the list `ys`, without changing the relative order of the elements in `ys`. For example:

```
insertions(4, list(1, 2, 3));
// returns list(list(4,1,2,3), list(1,4,2,3), list(1,2,4,3), list(1,2,3,4)).
```

Your function can make use of the `take` and `drop` functions (for merge sort) presented in the lectures/reflections. They are given here for your reference:

```
// put the first n elements of xs into a list
function take(xs, n) {
    return (n === 0) ? null : pair(head(xs), take(tail(xs), n - 1));
}

// drop the first n elements from the list and return the rest
function drop(xs, n) {
    return (n === 0) ? xs : drop(tail(xs), n - 1);
}
```

```
function insertions(x, ys) {

// SOLUTION 1:
    return map(k => append(take(ys, k), pair(x, drop(ys, k))),
               enum_list(0, length(ys)));

// SOLUTION 2:
    function helper(k, result) {
        if (k < 0) {
            return result;
        } else {
            const u = append(take(ys, k), pair(x, drop(ys, k)));
            return helper(k - 1, pair(u, result));
        }
    }
    return helper(length(ys), null);

// SOLUTION 3:
    const len = length(ys);
    function helper(k) {
        return (k > len)
            ? null
            : pair(append(take(ys, k), pair(x, drop(ys, k))),
                   helper(k + 1));
    }
    return helper(0);

// SOLUTION 4:
    return is_null(ys)
        ? list(list(x))
        : pair(pair(x, ys),
               map(i => pair(head(ys), i), insertions(x, tail(ys))));
}
```

8B. [6 marks]

Complete the function `permutations` that takes as argument a list of distinct numbers and returns a list of all permutations of the input numbers. Each permutation is a list of numbers. The permutations in the result list can be in any order. For example:

```
permutations(list(1, 2, 3));
// Example result: list(list(1,2,3), list(2,1,3), list(2,3,1),
//                      list(1,3,2), list(3,1,2), list(3,2,1)).
```

To get full marks for this part, your function **must make use of the `insertions` function** from Part A in a correct and meaningful way.

```
function permutations(xs) {

// SOLUTION 1:

    if (is_null(xs)) {
        return list(null);
    } else {
        const s = permutations(tail(xs));
        const t = map(ys => insertions(head(xs), ys), s);
        return accumulate(append, null, t);
    }


// SOLUTION 2:

    return accumulate((x, ps) => accumulate((p, qs) => append(insertions(x, p),
                                                              qs),
                                            null,
                                            ps),
                      list(null),
                      xs);


// SOLUTION 3:

    return accumulate((x, ps) => accumulate(append,
                                            null,
                                            map(p => insertions(x, p), ps)),
                      list(null),
                      xs);

}
```

——— END OF QUESTIONS ———

Appendix

List Support

The following list processing functions are supported:

- `pair(x, y)`: Makes a pair from `x` and `y`.
- `is_pair(x)`: Returns `true` if `x` is a pair and `false` otherwise.
- `head(x)`: Returns the head (first component) of the pair `x`.
- `tail(x)`: Returns the tail (second component) of the pair `x`.
- `is_null(xs)`: Returns `true` if `xs` is the empty list, and `false` otherwise.
- `is_list(x)`: Returns `true` if `x` is a list as defined in the lectures, and `false` otherwise. Iterative process; time: $O(n)$, space: $O(1)$, where n is the length of the chain of `tail` operations that can be applied to `x`.
- `list(x1, x2,..., xn)`: Returns a list with `n` elements. The first element is `x1`, the second `x2`, etc. Iterative process; time: $O(n)$, space: $O(n)$, since the constructed list data structure consists of n pairs, each of which takes up a constant amount of space.
- `length(xs)`: Returns the length of the list `xs`. Iterative process; time: $O(n)$, space: $O(1)$, where n is the length of `xs`.
- `map(f, xs)`: Returns a list that results from list `xs` by element-wise application of `f`. Recursive process; time: $O(n)$, space: $O(n)$, where n is the length of `xs`.
- `build_list(n, f)`: Makes a list with `n` elements by applying the unary function `f` to the numbers 0 to `n - 1`. Recursive process; time: $O(n)$, space: $O(n)$.
- `for_each(f, xs)`: Applies `f` to every element of the list `xs`, and then returns `true`. Iterative process; time: $O(n)$, space: $O(1)$, where n is the length of `xs`.
- `list_to_string(xs)`: Returns a string that represents list `xs` using the text-based box-and-pointer notation `[...]`.
- `reverse(xs)`: Returns list `xs` in reverse order. Iterative process; time: $O(n)$, space: $O(n)$, where n is the length of `xs`. The process is iterative, but consumes space $O(n)$ because of the result list.
- `append(xs, ys)`: Returns a list that results from appending the list `ys` to the list `xs`. Recursive process; time: $O(n)$, space: $O(n)$, where n is the length of `xs`.
- `member(x, xs)`: Returns first postfix sublist whose head is identical to `x` (===); returns `null` if the element does not occur in the list. Iterative process; time: $O(n)$, space: $O(1)$, where n is the length of `xs`.
- `remove(x, xs)`: Returns a list that results from `xs` by removing the first item from `xs` that is identical (===) to `x`. Recursive process; time: $O(n)$, space: $O(n)$, where n is the length of `xs`.
- `remove_all(x, xs)`: Returns a list that results from `xs` by removing all items from `xs` that are identical (===) to `x`. Recursive process; time: $O(n)$, space: $O(n)$, where n is the length of `xs`.
- `filter(pred, xs)`: Returns a list that contains only those elements for which the one argument function `pred` returns `true`. Recursive process; time: $O(n)$, space: $O(n)$, where n is the length of `xs`.
- `enum_list(start, end)`: Returns a list that enumerates numbers starting from `start` using a step size of 1, until the number exceeds (>) `end`. Recursive process; time: $O(n)$, space: $O(n)$, where n is the length of `xs`. For example, `enum_list(2, 5)` returns the list `list(2, 3, 4, 5)`.
- `list_ref(xs, n)`: Returns the element of list `xs` at position `n`, where the first element has index 0. Iterative process; time: $O(n)$, space: $O(1)$, where n is the length of `xs`.

- `accumulate(op, initial, xs)`: Applies binary function `op` to the elements of `xs` from right-to-left order, first applying `op` to the last element and the value `initial`, resulting in r_1, then to the second-last element and r_1, resulting in r_2, etc, and finally to the first element and r_{n-1}, where n is the length of the list. Thus, `accumulate(op, zero, list(1,2,3))` results in `op(1, op(2, op(3, zero)))`. Recursive process; time: $O(n)$, space: $O(n)$, where n is the length of `xs`, assuming `op` takes constant time.

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