COMP3121/3821/9101

Midterm Exam 08s1

3 hr exam, 5 problems for all, plus 1 extra problem for COMP3821

PLEASE write **CLEARLY**, show **ALL** of your work and **EXPLAIN** everything in **DETAIL**!

The Master Theorem:
Let a, b be constants such that $a \geq 1$ and $b > 1$, let $f(n)$ be a function and let $T(n)$ be defined on natural numbers by the recurrence $T(n) = a\,T(\lfloor n/b \rfloor) + f(n)$. Then $T(n)$ can be bounded asymptotically as follows :

1) If $f(n) = O(n^{\log_b a - \varepsilon})$ for some $\varepsilon > 0$, then $T(n) = \Theta(n^{\log_b a})$

2) If $f(n) = \Theta(n^{\log_b a})$ then $T(n) = \Theta(n^{\log_b a} \log n)$

3) If $f(n) = \Omega(n^{\log_b a + \varepsilon})$ for some $\varepsilon > 0$, and if $a\,f(n/b) < c\,f(n)$ for some constant $c < 1$ and all $n \geq n_0$ for some sufficiently large n_0, then $T(n) = \Theta(f(n))$.

1. **[30pt]** In each case determine the asymptotic growth rate of the solution $T(n)$ to the recurrence.
   a. **[5pt]** $T(n) = 6\,T(n/3) + n$
   b. **[5pt]** $T(n) = T(3n/8) + \sqrt{n}$
   c. **[10pt]** $T(n) = 2T(n/2) + 2\,n \log(3\sqrt{n})$
   d. **[10pt]** $T(n) = T(n-2) + n/2 + \log n(n-1)$

2. **[20 pt]** Along the long, straight road from Loolalong to Goolagong houses are scattered quite sparsely, sometimes with long gaps between two consecutive houses. Telstra must provide mobile phone service to people that live alongside the road, and the range of Telstra's cell base station is 5km. Design an algorithm for placing the ***minimal number*** of base stations alongside the road, that is sufficient to cover all houses. Argue that your algorithm is optimal.

3. **[20 pt]** In Elbonia coin denominations are 81c, 27c, 9c, 3c and 1c. Design an algorithm that, given the amount that is a multiple of 1c, pays it with a minimal number of coins. Argue that your algorithm is optimal.

4. **[15pt]** Consider polynomial $P(x) = x^{32} + 3x^{18} + 5x^5 + 4\,x^2 + 3x + 6$. You are given a large number M. Find the value $P(M)$ using only 7 multiplications of large numbers.

5. **[15pt]** Let ω_n be the primitive complex root of unity of order n. Simplify the expression
$$(\omega_{64})^{17}\,(\omega_{128})^{30} + (\omega_{64})^{128}$$

EXTENDED CLASS 3821 **ONLY**:

6. **[20 pt]** On a long street there are apartment buildings, and in some of them you have a relative living there. You visit all relatives equally frequently; thus, you want to rent a flat in a building that has the property that the sum of the distances from that building to all of the buildings where your relatives live is minimal. Design an algorithm which, given a map with distances between the buildings on that street, finds the building where you should rent.

GOOD LUCK!!