Due: on July 8, before 2:00 PM.

Question 1

You're given an array A of n integers, and must answer a series of m questions, each of the form: "Given two integers, X_i and Y_i, how many elements a of the array A satisfy $a \times X_i = Y_i$". Design an *expected* $O(n+m)$ algorithm that answers all these queries.

Question 2

You are given an array S of n integers and another integer x.

a) Describe an $O(n \log n)$ algorithm that determines whether or not there exist two elements in S whose absolute difference is exactly x.

b) Describe an algorithm that acomplishes the same task, but runs in expected $O(n)$ time.

Question 3

You are an assistent news broadcaster reporting on a cycling race containing n cyclists. You have been given the task of computing the *excitement factor* of a given race, which is calculated as follows:

- The excitement factor starts at zero.
- Any time one of the first $\frac{n}{2}$ cyclists is overtaken by any other cyclist, the excitement factor increases by 1.

Unfortunately, you have only been given the starting and finishing order of the cyclists. From this data, you need to calculate the *minimum possible excitement factor* for the whole race. You may assume than n is a power of 2 and is strictly greater than 1.

Question 4

Read the review material from the class website on asymptotic notation and basic properties of logarithms, and then determine if $f(n) = \Omega(g(n))$, $f(n) = O(g(n))$ or $f(n) = \Theta(g(n))$.

	f(n)	g(n)
a)	$n!$	n^n
b)	$(\log_3 n)^2$	$\log_3\left(n^2 \times n^{\log_3 n}\right)$
c)	$\log_2 \sqrt{n}$	$\sqrt{\log_2 n}$
d)	$n\left(2n + \sin\left(\frac{\pi n}{4}\right)\right)$	n^2

Question 5

Determine the asymptotic growth rate of the solutions to the following recurrences. If possible, you can use the Master Theorem, if not, find another way of solving it.

a) $T(n) = 3T(\frac{n}{2}) + n(2 + \cos n)$

b) $T(n) = 3T(\frac{n}{4}) + n\sqrt[3]{n}$

c) $T(n) = 5T(\frac{n}{2}) + n^{\log_2 5}(1 + \sin\frac{2\pi n}{3})$

d) $T(n) = T(n-1) + \log n$