MIDTERM PRACTICE PROBLEMS

1. Given an array of integers design an algorithm which determines if there are four ***distinct*** numbers *x, y, z* and *u* in the array such that $x^2 + y^2 = z^2 + u^2$ which runs in (worst case) time $O(n^2 \log n)$.

2. Assume you are given two arrays A, B, each with *n* positive numbers and the equation $x^4 - y^6 = x^2y^2 + 10$. Design an algorithm which runs in time *n* log *n* which finds if A contains a value for *x* and B contains a value for *y* that satisfy the equation.

3. Fibonacci numbers are defined by F(0) = 0; F(1) = 1; F(*n*) = F(*n-1*) + F(*n-2*) for all n ≥ 2. Thus, the sequence of F(*n*) looks as follows: 0 1 1 2 3 5 8 13 21 …
   (a) Show that
   $$\begin{pmatrix} F(n+1) & F(n) \\ F(n) & F(n-1) \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}^n$$
   (b) Find F(*n*) in $O(\log_2 n)$ many steps. Any addition or multiplication are considered a single step.

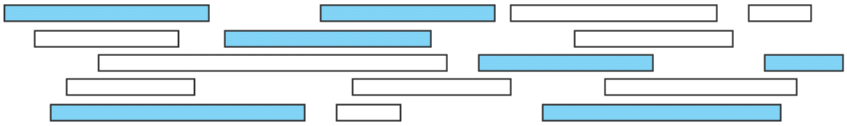
4. Assume you have a 2^n X 2^n board with one square missing at an arbitrary place of the board, and a collection of tiles consisting of three square of the form

   Design an algorithm that tiles such a board, covering all squares except the missing square.

5. Assume you are given *n* tasks each of which takes the same, unit amount of time to complete. Each task has an integer deadline and penalty associated with it which you pay if you do not complete the task in time. Design an algorithm that schedules the tasks so that the total penalty you have to pay is minimized.

6. Let X be a set of n intervals on the real line. A subset of intervals Y ⊆X is called a *tiling path* if the intervals in Y cover the intervals in X, that is, any real value that is contained in some interval in X is also contained in some interval in Y . The *size* of a tiling cover is just the number of intervals. Describe and analyze an algorithm to compute the smallest tiling path of X as quickly as possible. Assume that your input consists of two arrays XL[1 .. n] and XR[1 .. n], representing the left and right endpoints of the intervals in X.



A set of intervals. The seven shaded intervals form a tiling path.

7. Assume you are given *n* sorted arrays of different sizes. You are allowed to merge any two arrays into a single new sorted array and proceed in this manner until only one array is left. Design an algorithm that achieves this task and uses minimal total number of moves of elements of the arrays. Argue why your algorithm is optimal.

8. Assume that you got a fabulous job and you wish to repay your student loan as quickly as possible. Unfortunately, the bank "Roadrobbery" which gave you the loan has the condition that you must start by paying off $1 and then each subsequent month you must pay either double the amount you paid the previous month, the same amount as the previous month or a half of the amount you paid the previous month. On top of these conditions, your schedule must be such that the last payment is $1. Design an algorithm which, given the size of your loan, produces a payment schedule which minimizes the number of months it will take you to repay your loan while satisfying all of the bank's requirements.

9. There is a line of 111 stalls, some of which need to be covered with boards. You can use up to 11 boards, each of which may cover any number of consecutive stalls. Cover all the necessary stalls, while covering as few total stalls as possible.

10. Given two sequences of letters A and B, find if B is a subsequence of A in the sense that one can delete some letters from A and obtain the sequence B.

11. Use FFT to compute the DFT of the sequence {1,2,3,4}.