Assignment 1 - Review Problems
Due date: Friday, March 17, at Noon

Submitting the first assignment:
Option 1: Login to a CSE server and run

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give cs3121 Assignment1 1_studentID.pdf
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or

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give cs3121 Assignment1 1_studentID.doc
```

where "1_studentID.pdf " or "1_studentID.doc" is the file that contains your solutions **with your name and studentID number printed on top of the file**. For example, if your student ID is 323459 your would run:

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give cs3121 Assignment1 1_323459.pdf
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Option 2: If you do not have a CSE account or if you prefer it that way, you can also submit your assignment in either .pdf or .doc format via the web, https://cgi.cse.unsw.edu/~give/Student/give.php
Please **DO NOT** email your solutions. You should **type your solutions** because hand written notes tend not to be legible, and scanning them produces files of size which the system **CANNOT accept**.

Submit the solution to the problems ONLY - unfortunately we do not have the manpower to mark your solutions to the puzzles, but you can always ask me if your solution is correct.

1. Given an array A[1..100] which contains all natural numbers between 1 and 99, design an algorithm that runs in $O(n)$ and returns the duplicated value. (Microsoft interview question) (10 pts)

2. You are at a party attended by n people not including you, and you suspect that there might be a celebrity present. By definition, a celebrity is someone known by all of remaining $n-1$ people who on top of it does not know anyone among the remaining $n-1$ people (celebrities tend to know only other celebrities and not the rest of us). Note that if X knows who Y is, it is not necessary that also Y knows who X is. Your task is to work out if there is a celebrity present and if so, which of the n people present is the celebrity. You are allowed to ask any person X if he knows any other person Y among these n people.

   (a) Show that your task can be accomplished with asking $3n-3$ such questions; (10 pts)

   (b) Show that this task can even be accomplished with asking only $3n - \lceil \log_2 n \rceil$ such questions; (10 pts)

3. (a) Describe an $O(n \log n)$ algorithm (in the sense of the worst case performance) that, given an array S of n integers and another integer x, determines whether or not there exist two elements in S whose sum is exactly x. (10 pts)

   (b) Describe an algorithm that accomplishes the same task, but runs in $O(n)$ expected (average) time. (10 pts)

   Note that brute force does not work here, because it runs in $O(n^2)$ time.

4. Design an algorithm which sorts n integers, each smaller than n^9 and runs in time $O(n)$. (10 pts)

   Hint: review the RADIXSORT*; you might want to change the base of number representation from binary into a base which depends on n...*

5. Given n real numbers $x_1, ..., x_n$ where each x_i is a real number in the interval $[0, 1]$, devise an algorithm that runs in linear time and that outputs a permutation of the n numbers, say $y_1, ..., y_n$, such that $\sum_{i=2}^{n} |y_i - y_{i-1}| < 2$. (10 pts)

   Hint: this is easy to do in $O(n \, logn)$ time: just sort the sequence in an ascending order. In this case $\sum_{i=2}^{n} |y_i - y_{i-1}| = \sum_{i=2}^{n}(y_i - y_{i-1}) = y_n - y_1 \leq 1 - 0 = 1$. Here $|y_i - y_{i-1}| = y_i - y_{i-1}$ because all differences are non-negative, and all terms in the sum except the first and the last one cancel out. To solve this problem, one might think about tweaking the BUCKETSORT *algorithm.*

6. Given n real numbers $x_1, ..., x_n$ where each x_i is a real number in the interval $[0, 1]$, devise an algorithm that runs in linear time and that outputs a a permutation of the n numbers, say $y_1, ..., y_n$, such that $\sum_{i=2}^{n} |y_i - y_{i-1}| < 1.0001$, thus improving on the previous problem. (10 pts)

7. Suppose that you are taking care of n kids, who took their shoes off. You have to take the kids out and it is your task to make sure that each kid is wearing a pair of shoes of the right size (not necessarily their own, but one of the same size). All you can do is to try to put a pair of shoes on a kid, and see if they fit, or are too large or too small; you are NOT allowed to compare a shoe with another shoe or a foot with another foot. Describe an algorithm whose expected number of shoe trials is $O(n \log n)$ which properly fits shoes on every kid. (10 pts)

   Hint: you could try to devise a modification of the QUICKSORT *algorithm, or, better to say, a "double version" of it, with two kinds of pivots...*

8. (a) Given two arrays of n distinct integers, design an algorithm that finds out if the two arrays have an element in common and runs in time $O(n \log_2 n)$. (Microsoft interview question) (10 pts)

   (b) Design an algorithm which accomplishes the same task in **average** time $O(n)$. (10 pts)

9. Assume you have an array of 2^n distinct integers.

   (a) Find the largest and the smallest number using $3 \cdot 2^{n-1} - 2$ comparisons only. (10 pts)

   (b) Find the largest and the second largest number using $2^n + n - 2$ comparisons only. (10 pts)

10. Read the review material from the class website on asymptotic notation and basic properties of logarithms, pages 38-44 and then determine if $f(n) = \Omega(g(n))$, $f(n) = O(g(n))$ or $f(n) = \Theta(g(n))$ for the following pairs: (20 pts)

$f(n)$	$g(n)$
n^2	$(n - 2\log_2 n)(n + \cos n)$
$(\log_2 n)^2$	$\log_2(n^{\log_2 n}) + 2\log_2 n$
n^{100}	$2^{n/100}$
n^{100}	$2^{n^{1/100}}$
$\sqrt{n}$	$2^{\sqrt{\log_2 n}}$
n^{100}	$2^{(\log_2 n)^2}$
$n^{1.001}$	$n \log_2 n$
$n^{(1+\sin(\pi n/2))/2}$	$\sqrt{n}$

Try your best to solve as many of the above problems as you can, but do not get frustrated if you find some of them too hard - you will improve your problem solving skills in the course of this class. Below are a few puzzles for you to further sharpen your problem solving skills. Enjoy!!

PUZZLES!!!

1. Alice and Bob play a game with N lights in a row. Some of the lights are ON, some are OFF. They take turns (Alice goes first) picking any light bulb that is ON, and turning it OFF and also toggling the states of all lights to the right of that bulb. A player loses when they cannot make a move (i.e. if it is their turn and all lights are off).

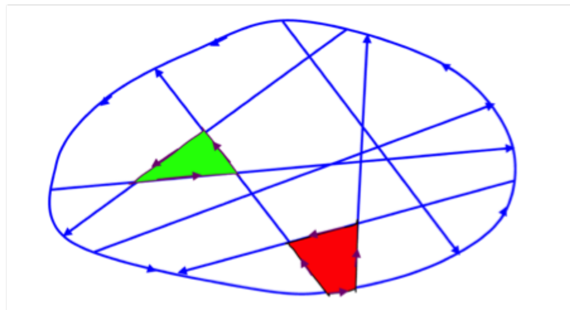
   (a) Show that the game eventually ends, no matter how Alice and Bob play.

   (b) Given a configuration of lights, can you figure out whether Alice or Bob will win?

   Hint: (a) Think in terms of binary numbers; (b) Show that the outcome does not depend on how they play, only on the initial configuration of lights; actually on the state of one light only. What do you think which light is the decisive one?

2. Tom and his wife Mary went to a party where nine more couples were present. Not every one knew everyone else, so people who did not know each other introduced themselves and shook hands. People who knew each other from before did not shake hands. Later that evening Tom got bored, so he walked around and asked all other guests (including his wife) how many hands they had shaken that evening, and got 19 different answers. How many hands did Tom shake?

3. In Elbonia all cities have a circular one-way highway around the city; see the map. All streets in the cities are one-way, and they all start and end on the circular highway



(see the map). A city block is a part of the city that is not intersected by any street. Design an algorithm that, given a map of a city, finds a block that can be circumnavigated while respecting all one-way signs (find one such block, not all such blocks).

4. There are five pirates who have to split 100 bars of gold. They all line up and proceed as follows:

   (a) The first pirate in line gets to propose a way to split up the gold (for example: everyone gets 20 bars)

   (b) The pirates, including the one who proposed, vote on whether to accept the proposal. If the proposal is rejected, the pirate who made the proposal is killed.

   (c) The next pirate in line then makes his proposal, and the 4 pirates vote again. If the vote is tied (2 vs 2) then the proposing pirate is still killed. Only majority can accept a proposal. The process continues until a proposal is accepted or there is only one pirate left. Assume that every pirate :

   (d) above all wants to live;

   (e) given that he will be alive he wants to get as much gold as possible;

   (f) given maximal possible amount of gold, he wants to see any other pirate killed, just for fun;

   (g) each pirate knows his exact position in line;

   (h) all of the pirates are excellent puzzle solvers.

   Question : What proposal should the first pirate make?

   Hint: assume first that there are only two pirates, and see what happens. Then assume that there are three pirates and that they have figured out what happens if there were only two pirates and try to see what they would do. Further, assume that there are four pirates and that they have figured out what happens if there were only three pirates, try to see what they would do. Finally assume there are five pirates and that they have figured out what happens if there were only four pirates.

5. On a circular highway there are n petrol stations, unevenly spaced, each containing a different quantity of petrol. It is known that the total quantity of petrol on all stations is enough to go around the highway once, and that the tank of your car can hold enough fuel to make a trip around the highway. Prove that there always exists a station among all of the stations on the highway, such that if you take it as a starting point and take the fuel from that station, you can continue to make a complete round trip around the highway, never emptying your tank before reaching the next station to refuel.