Midterm Practice Problems 2

(1) You are running a small manufacturing shop with plenty of workers but with a single milling machine. You have to produce n items; item i requires m_i machining time first and then p_i polishing time by hand. The machine can mill only one object at a time, but your workers can polish in parallel as many objects as you wish. You have to determine the order in which the objects should be machined so that the whole production is finished as quickly as possible. Prove that your solution is optimal.

(2) You are given a set S of n overlapping arcs of the unit circle. The arcs can be of different lengths. Find a largest subset P of these arcs such that no two arcs in P overlap (largest in terms of total number of elements, not in terms of total length of these arcs). Prove that your solution is optimal.

(3) You are given a set S of n overlapping arcs of the unit circle. The arcs can be of different lengths. You have to stab these arcs with minimal number of needles so that every arc is stabbed at least once. In other words, you have to find a set of as few points on the unit circle as possible so that every arc contains at least one point (for the mathematicians: all arcs contain their end points). Prove that your solution is optimal.

(4) You are given a connected graph with weighted edges. Fins a spanning tree such that the largest weight of all of its edges is as small as possible.

(5) In Elbonia cities are connected with one way roads and it takes one whole day to travel between any two cities. Thus, if you need to reach a city and there is no a direct road, you have to spend a night in a hotel in all intermediate cities. You are given a map of Elbonia with tool charges for all roads and the prices of the

cheapest hotels in each city. You have to travel from the capital city C to a resort city R. Design an algorithm which produces the cheapest route to get from C to R.

(6) Describe all k which satisfy

$$\frac{i\,\omega_{32}^{37}}{\omega_{64}^{3}\omega_{16}^{5}} = \omega_{128}^{k}$$

(7) Compute $DFT(\langle 1, 4, 3, 5\rangle)$ in any way you like (not necessarily using FFT).

(8) Compute $IDFT(\langle 1, 4, 3, 5\rangle)$ (the Inverse Discrete Fourier Transform) in any way you like (not necessarily using IFFT). Recall that for a sequence $\langle a_0, a_1, \ldots, a_{n-1}\rangle$ and for $P(x) = a_0 + a_1 x + \ldots + a_{n-1}x^{n-1}$,

$$IDFT(\langle a_0, a_1, \ldots, a_{n-1}\rangle) = \left\langle \frac{P(1)}{n}, \frac{P(\omega_n^{-1})}{n}, \frac{P(\omega_n^{-2})}{n}, \ldots, \frac{P(\omega_n^{-(n-1)})}{n} \right\rangle$$

and that ω_n^{-k} is equal to the complex conjugate of ω_n^{k}.

(9) Assume you are given a complex number $a + b\,i$ with very large real and imaginary parts. Find its square using only two large real number multiplications.

(10) Find the product of polynomials $P(x) = a_0 + a_1 x$ and $Q(x) = a_{1000}x^{1000} + a_{1001}x^{1001}$ with only three large number multiplications.