Multilevel Regression And Poststratification (A Primer)

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Often researchers would like to measure public opinion on some policy issue, but the surveys we use to do so are unrepresentative in some important way. Perhaps their respondents come from a convenience sample (Wang et al. 2015), or non-response bias skews an otherwise random sample. Or perhaps the data is representative of some larger population (i.e. a country-level random sample), but contains too few observations to make inferences about a subgroup of interest. Even the largest US public opinion surveys do not have enough respondents to make reliable inferences about lower-level political entities like states or municipalities. Conclusions drawn from low frequency observations – even in a large sample survey – can be wildly misleading (Ansolabehere, Luks, and Schaffner 2015).

This presents a challenge for public opinion research: how to take unrepresentative survey data and adjust it so that it is useful for our particular research question. In this chapter, I will demonstrate a method called **multilevel regression and poststratification** (MRP). Using this approach, the researcher first constructs a model of public opinion (multilevel regression) and then reweights the model's predictions based on the observed characteristics of the population of interest (poststratification). In the sections to follow, I will describe this approach in detail, and will accompany this explanation with replication code in the `R` statistical language.

MRP was first introduced by Gelman and Little (1997), and in the subsequent decades it has helped address a diverse set of research questions in political science. These range from generating election forecasts using unrepresentative survey data (Wang et al. 2015) to assessing the responsiveness of state (Lax and Phillips 2012) and local policymakers (Tausanovitch and Warshaw 2014) to their constituents' policy preferences.

In the following sections, I will illustrate how MRP can improve estimates of small area public opinion. Our running example will be drawn from the Cooperative Election Study (Schaffner, Ansolabehere, and Luks 2021), a 50,000+ respondent study of voters in the United States. The 2020 wave of the study includes a question asking respondents whether they support a policy that would "decrease the number of police on the street by 10 percent, and increase funding for other public services." Since police reform is a policy issue on which US local governments have a significant amount of autonomy, it would be useful to know how opinions on this issue vary from place to place without having to conduct separate, costly surveys in each area.

As we will see, the accuracy of our MRP estimates depends critically on whether the first-stage model makes good out-of-sample predictions. The best first-stage models are *regularized* (Gelman 2018) to avoid both over- and under-fitting to the survey data. Regularized ensemble models (Ornstein 2020) with group-level predictors tend to produce the best estimates, especially when trained on large datasets.

Running Example

To demonstrate how MRP works, we'll consider an example where we know the true population-level estimands, and can explore how various refinements to the method can improve predictive accuracy. This approach mirrors Buttice and Highton (2013), who use disaggregated ressponses from large-scale US survey of voters as the target. The Cooperative Election Study data is available here, and we'll be using a tidied version of the dataset created by the `R/cleanup-ces-2020.R` script.[1]

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[1] All replication code and data will be made available on a public repository. Throughout, I will use R functions from the "tidyverse" to make the code more human-readable.

```
library(tidyverse)
library(ggrepel)

load('data/CES-2020.RData')
```

This tidied version of the data only includes the 33 states with at least 500 respondents. First, let's plot the percent of CES respondents who supported "defunding" the police[2] by state.

```
truth <- ces %>%
  group_by(abb) %>%
  summarize(truth = mean(defund_police))

truth %>%
  mutate(abb = fct_reorder(abb, truth)) %>%
  ggplot(mapping = aes(x=truth, y=abb)) +
  geom_point(alpha = 0.7) +
  labs(x = 'Percent Who Support Policy', y = 'State') +
  theme_minimal()
```

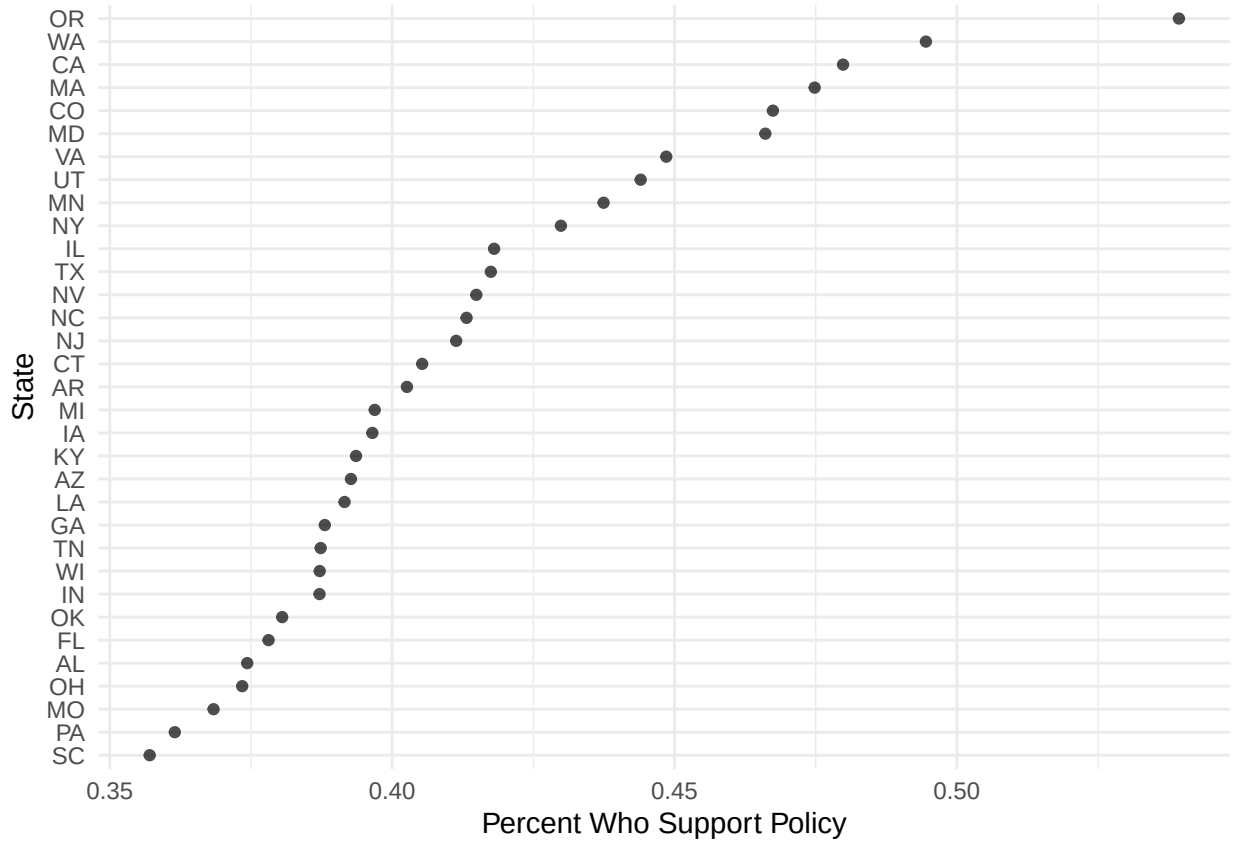

Figure 1: The percent of CES respondents in each state who support reducing police budgets. These are our target estimands.

Note that these values likely overstate the percent of the total population that support such a policy, as self-identified Democrats are over-represented in the CES sample. But they will nevertheless serve as the

[2]Obviously that phrase means different things to different people. In this case, we'll stick with the CES proposed policy of reducing police staffing by 10% and diverting those expenditures to other priorities.

"true" population-level parameters that we will try to estimate with MRP.

Draw a Sample

Suppose that we did not have access to the entire CES dataset, but only to a random sample of 1,000 respondents. How good of a job can we do at estimating those state-level means?

```
sample_data <- ces %>%
  slice_sample(n = 1000)
```

```
sample_summary <- sample_data %>%
  group_by(abb) %>%
  summarize(estimate = mean(defund_police),
            num = n())

sample_summary
```

```
## # A tibble: 33 x 3
##    abb   estimate   num
##    <chr>    <dbl> <int>
##  1 AL       0.55     20
##  2 AR       0         4
##  3 AZ       0.438    16
##  4 CA       0.435    85
##  5 CO       0.478    23
##  6 CT       0.375     8
##  7 FL       0.402    87
##  8 GA       0.346    26
##  9 IA       0.308    13
## 10 IL       0.28     50
## # ... with 23 more rows
```

In a sample with only 1,000 respondents, there are several states with very few (or no) respondents. Notice, for example, that this sample includes only four respondents from Arkansas, of whom zero support reducing police budgets. Simply disaggregating and taking sample means is unlikely to yield good estimates, as you can see by comparing those sample means against the truth.

```
# a function to plot the state-level estimates against the truth
compare_to_truth <- function(estimates, truth){

  d <- left_join(estimates, truth, by = 'abb')

  ggplot(data = d,
         mapping = aes(x=estimate,
                       y=truth,
                       label=abb)) +
  geom_point(alpha = 0.5) +
  geom_text_repel() +
  theme_minimal() +
  geom_abline(intercept = 0, slope = 1, linetype = 'dashed') +
  labs(x = 'Estimate',
       y = 'Truth',
       caption = paste0('Correlation = ', round(cor(d$estimate, d$truth), 2), ', Mean Absolute Error =
}

compare_to_truth(sample_summary, truth)
```

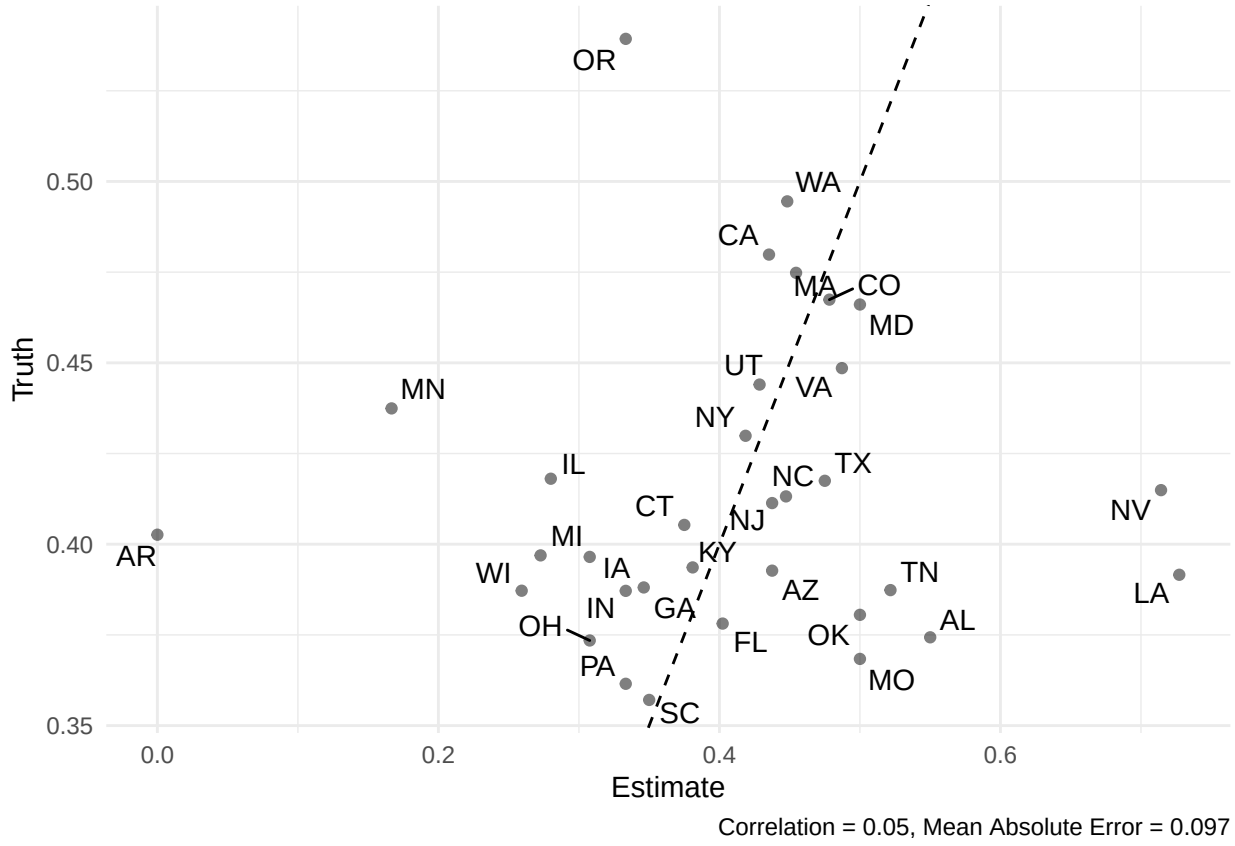

Figure 2: Esimates from disaggregated sample data

These are clearly poor estimates of state-level public opinion. The four respondents from Arksansas simply do not give us enough information to adequately measure public opinion in that state. But one of the key insights behind MRP is that the respondents from Arkansas are not the only respondents who can give us information about Arkansas! There are other respondents in, for example, Missouri, that are similar to Arkansas residents on their observed characteristics. If we can determine the characteristics that predict support for police reform using the entire survey sample, then we can use those predictions – combined with demographic information about Arkansans – to generate better estimates. The trick, in essence, is that our estimate for Arkansas will be borrowing information from similar respondents in other states.

The method proceeds in three steps.

Step 1: Fit a Model

First, we fit a model of our outcome, using observed characteristics of the survey respondents as predictors. To demonstrate, let's fit a simple logistic model including only four demographic predictors: gender, education, race, and age.

```
model <- glm(defund_police ~
               gender + educ + race + age,
             data = sample_data,
             family = 'binomial')
```

Step 2: Construct the Poststratification Frame

The poststratification stage requires the researcher to know (or estimate) the joint frequency distribution of predictor variables in each state. This information is stored in a "poststratification frame," a matrix where each row is a unique combination of characteristics, along with the observed frequency of that combination. Often, one constructs these frequency distributions from Census micro-data (Lax and Phillips 2009). For our example, I will compute them directly from the CES.

```
psframe <- ces %>%
  count(abb, gender, educ, race, age)

head(psframe)
```

```
## # A tibble: 6 x 6
##   abb   gender educ   race    age     n
##   <chr> <chr>  <chr>  <chr> <dbl> <int>
## 1 AL    Female 2_year Black    26     1
## 2 AL    Female 2_year Black    27     2
## 3 AL    Female 2_year Black    29     1
## 4 AL    Female 2_year Black    31     1
## 5 AL    Female 2_year Black    34     2
## 6 AL    Female 2_year Black    35     2
```

Step 3: Predict and Poststratify

With the model and poststratification frame in hand, the final step is to generate frequency-weighted predictions of public opinion. For each cell in the poststratification frame, append the model's predicted probability of supporting police defunding.

```
psframe$predicted_probability <- predict(model, psframe, type = 'response')
```

Then the poststratified estimates are the frequency-weighted means of those predictions.

```
poststratified_estimates <- psframe %>%
  group_by(abb) %>%
  summarize(estimate = weighted.mean(predicted_probability, n))
```

Let's see how these estimates compare with the known values.

```
compare_to_truth(poststratified_estimates, truth)
```

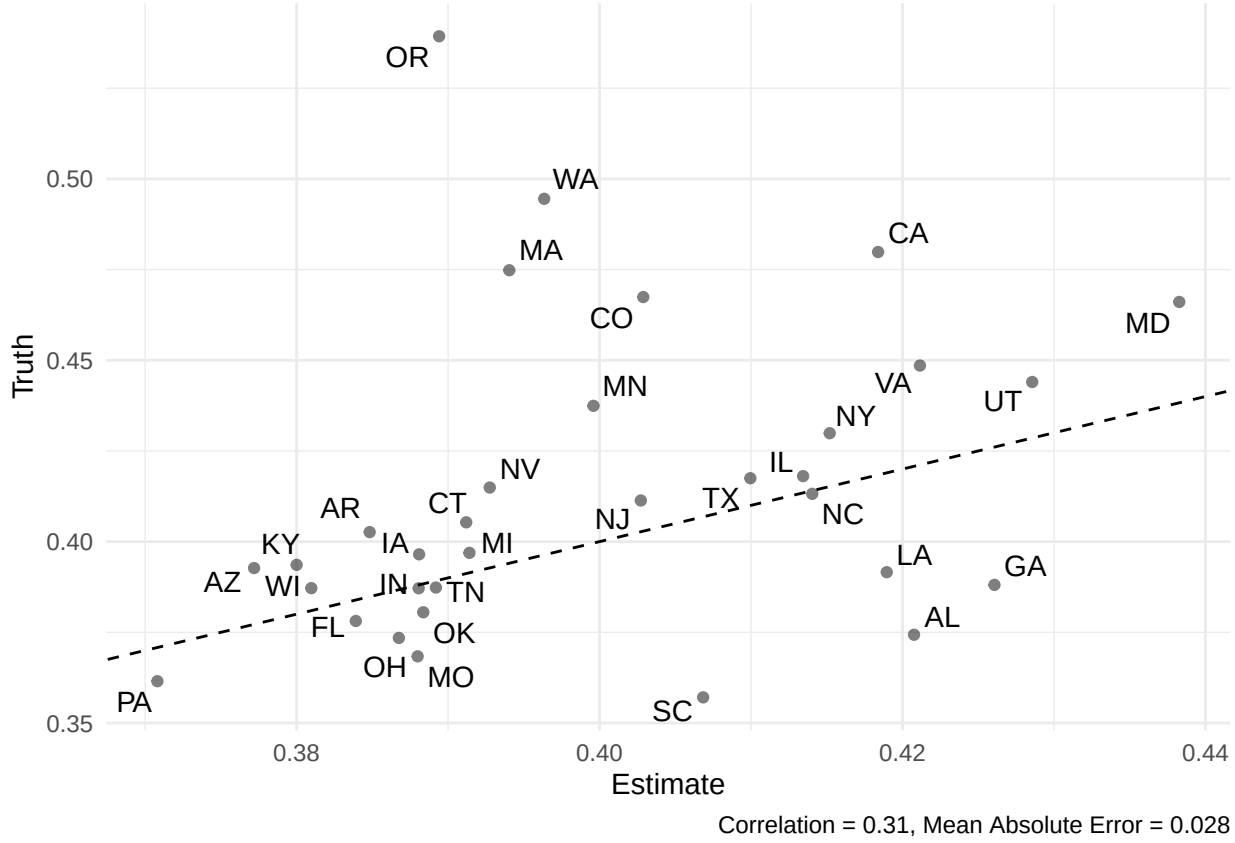

Figure 3: Underfit MRP estimates from complete pooling model

These estimates, though still imperfectly correlated with the truth, are much better than the previous estimates from disaggregation. Notice, in particular, that the estimate for Arkansas went from 0% to roughly 39%, reflecting the significant improvement that comes from using more information than the four Arkansans in our sample can provide.

But we can still do better. In the following sections, I will show how successive improvements to the first-stage model can yield more reliable poststratified estimates.

Beware Overfitting

A common instinct among social scientists building models is to take a "kitchen sink" approach, including as many explanatory variables as possible (Achen 2005). This is counterproductive when the objective is out-of-sample predictive accuracy. To illustrate, let's estimate a model with a separate intercept term for each state – a "fixed effects" model. Because our sample contains several states with very few observations, these state-specific intercepts will be over-fit to sampling variability.

```r
# fit the model
model2 <- glm(defund_police ~
                gender + educ + race + age +
                  abb,
              data = sample_data,
              family = 'binomial')

# construct the poststratification frame
psframe <- ces %>%
  count(abb, gender, educ, race, age)

# make predictions
psframe$predicted_probability <- predict(model2, psframe, type = 'response')

# poststratify
poststratified_estimates <- psframe %>%
  group_by(abb) %>%
  summarize(estimate = weighted.mean(predicted_probability, n))


compare_to_truth(poststratified_estimates, truth)
```

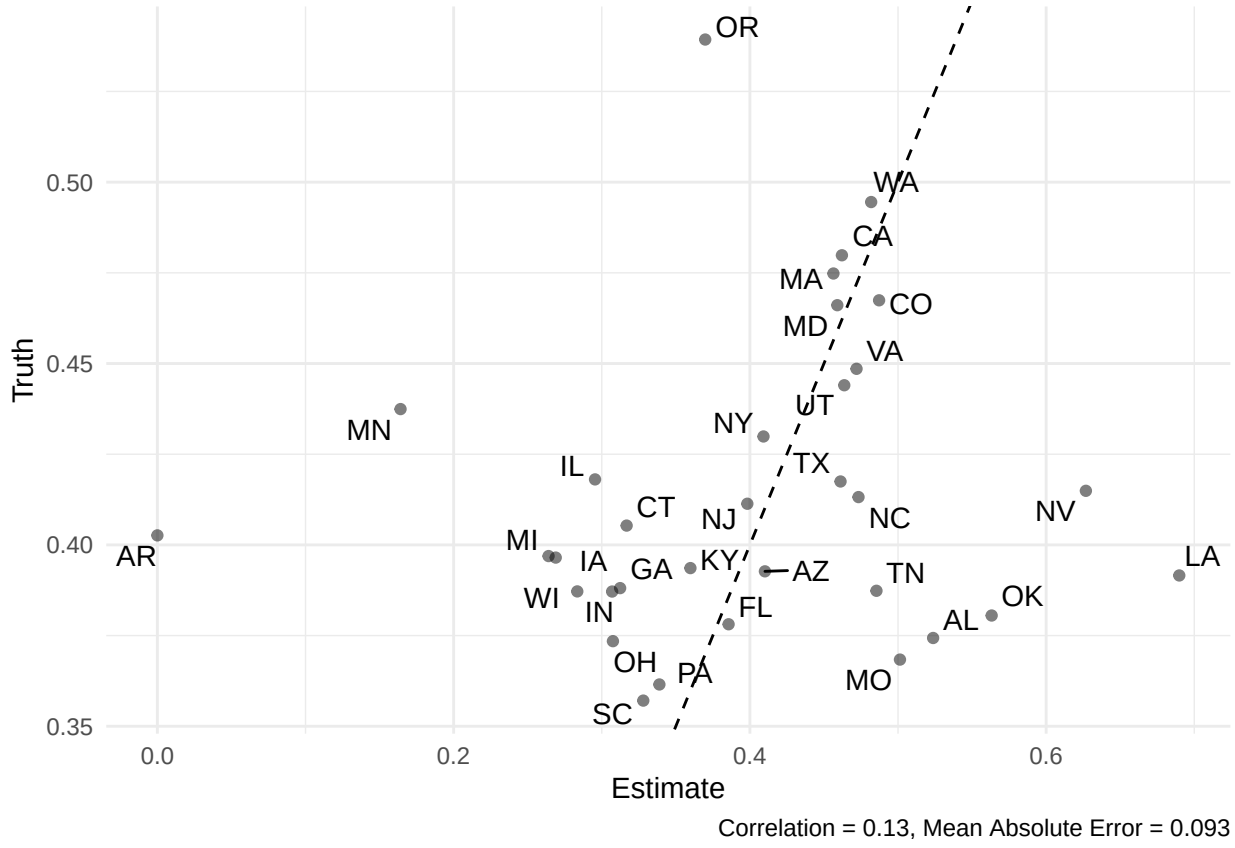

Figure 4: Overfit MRP estimates from fixed effects model

These poststratified estimates perform about as well as the disaggregated estimates from Figure 2. Because

each state's intercept is estimated separately, the over-fit model foregoes the advantages of "partial pooling" (Park, Gelman, and Bafumi 2004), borrowing information from respondents in other states. Note that the estimate for Arkansas is once again 0%.

Partial Pooling

A better approach is to estimate a multilevel model (alternatively known as "varying-intercepts" or "random effects" model), including group-level covariates. In the model below, I estimate varying intercepts by US Census division, including the state's 2020 Democratic vote share as a covariate. The result is an improvement over Figure 3 (particularly for west coast states like Oregon, Washington, and California).

```
library(lme4)

# fit the model
model3 <- glmer(defund_police ~  gender + educ + race + age +
                (1 + biden_vote_share | division),
                 data = sample_data,
                 family = 'binomial')

# construct the poststratification frame
psframe <- ces %>%
  count(abb, gender, educ, race, age, division, biden_vote_share)

# make predictions
psframe$predicted_probability <- predict(model3, psframe, type = 'response')

# poststratify
poststratified_estimates <- psframe %>%
  group_by(abb) %>%
  summarize(estimate = weighted.mean(predicted_probability, n))

compare_to_truth(poststratified_estimates, truth)
```

Sample Size Matters A Lot

The performance of MRP estimates depends strongly on the quality and size of the survey sample available. So far, we've been working with a sample of 1,000 respondents, and the estimates have been somewhat underwhelming. Suppose instead we had a sample of 5,000 respondents.

```
sample_data <- ces %>%
  slice_sample(n = 5000)

# fit the model
model3 <- glmer(defund_police ~  gender + educ + race + age +
                (1 + biden_vote_share | division),
                 data = sample_data,
                 family = 'binomial')

# construct the poststratification frame
psframe <- ces %>%
  count(abb, gender, educ, race, age, division, biden_vote_share)

# make predictions
psframe$predicted_probability <- predict(model3, psframe, type = 'response')
```

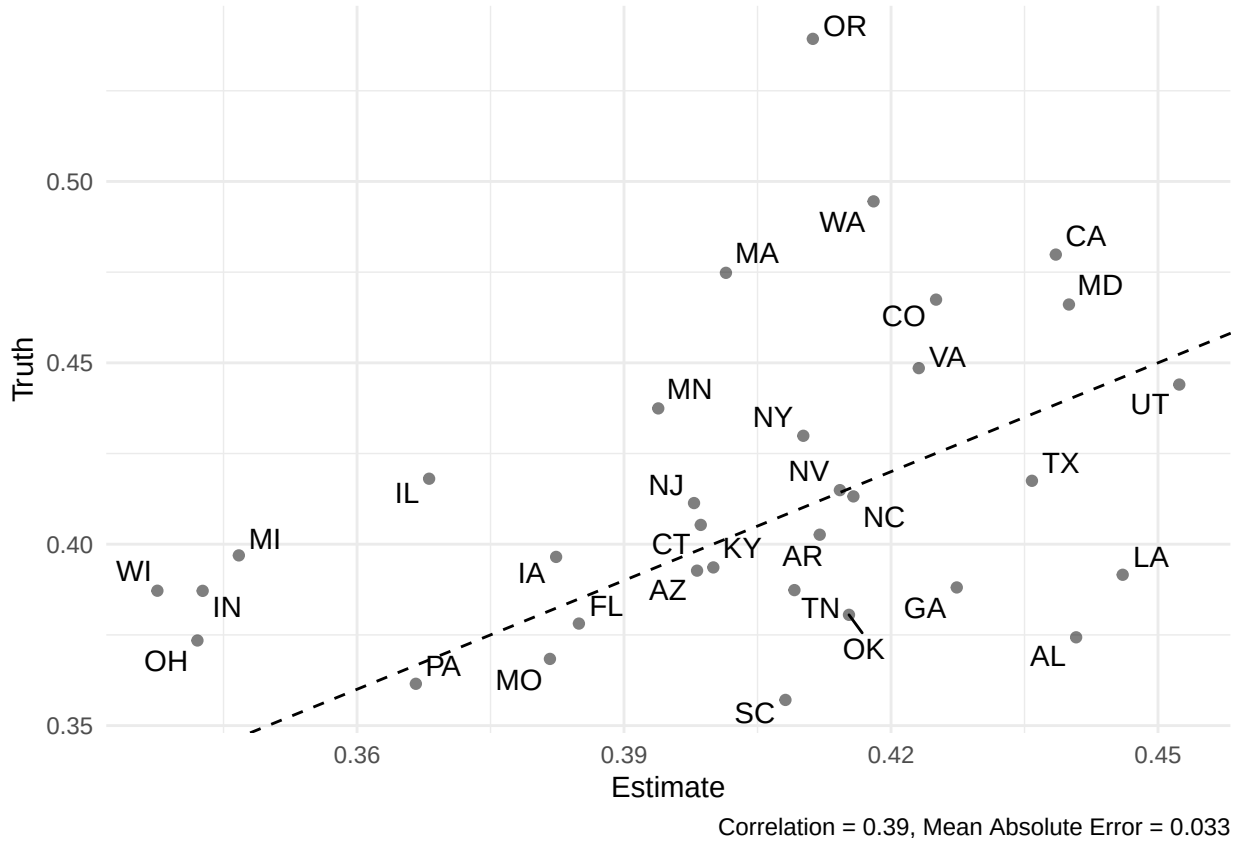

Figure 5: MRP estimates from model with partial pooling

```r
# poststratify
poststratified_estimates <- psframe %>%
  group_by(abb) %>%
  summarize(estimate = weighted.mean(predicted_probability, n))

compare_to_truth(poststratified_estimates, truth)
```

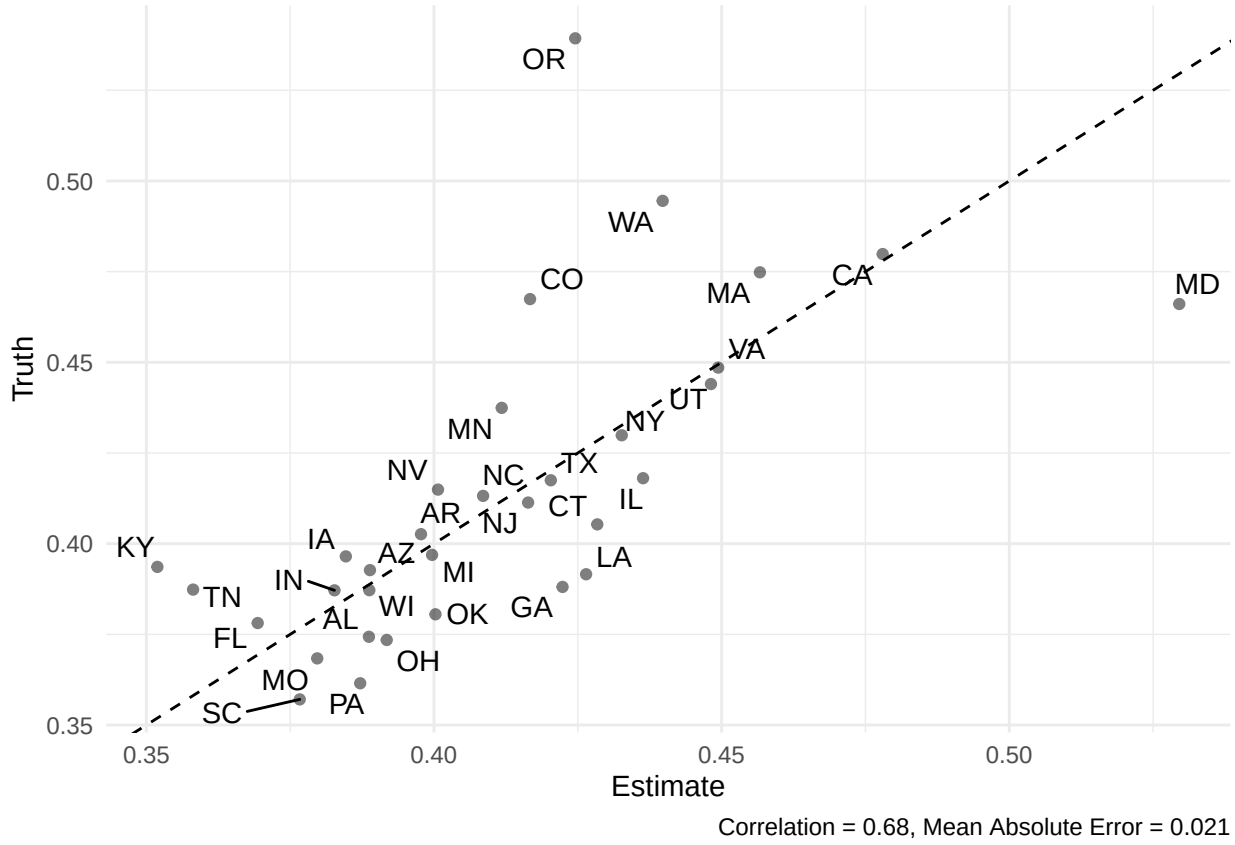

Figure 6: Poststratified estimates with a survey sample of 3,000

Stacked Regression and Poststratification (SRP)

Ultimately, the accuracy of one's poststratified estimates depends on the out-of-sample predictive performance of the first-stage model. As we've seen above, the challenge is to thread the needle between over-fitting and under-fitting. Several recent papers Broniecki, Leemann, and Wüest (2022) have shown that approaches from machine learning can help to automate this process, particularly with large survey samples.

In the code below, I'll demonstrate how an *ensemble* of models – using the same set of predictors but different methods for combining them into predictions – can yield superior performance to a single multilevel regression model. In particular, I will fit a "Super Learner" ensemble (van der Laan, Polley, and Hubbard 2007), which generates a weighted average prediction from multiple models, where the weights are based on cross-validated prediction performance. The literature on ensemble models is vast, but for good entry points I recommend Breiman (1996), Breiman (2001), and Montgomery, Hollenbach, and Ward (2012).

```r
# construct the poststratification frame
psframe <- ces %>%
  count(abb, gender, educ, race, age, division, biden_vote_share)
```

```r
library(SuperLearner)

# fit the model (an ensemble of random forest and logistic regression)
SL.library <- c("SL.ranger", "SL.glm")

X <- sample_data %>%
  select(gender, educ, race, age, division, biden_vote_share)

newX <- psframe %>%
  select(gender, educ, race, age, division, biden_vote_share)

sl <- SuperLearner(Y = sample_data$defund_police,
                   X = X,
                   newX = newX,
                   family = binomial(),
                   SL.library = SL.library, verbose = TRUE)

# make predictions
psframe$predicted_probability <- sl$SL.predict

# poststratify
poststratified_estimates <- psframe %>%
  group_by(abb) %>%
  summarize(estimate = weighted.mean(predicted_probability, n))

compare_to_truth(poststratified_estimates, truth)
```

These performance gains reflect the improvement that come from modeling "deep interactions" in the predictors of public opinion (Ghitza and Gelman 2013). If, for example, income better predicts partisanship in some states but not in others (Gelman et al. 2007), then a model that captures that moderating effect will produce better poststratified estimates than one that does not. Machine learning techniques like random forest (Breiman 2001) are especially useful for detecting and modeling such deep interactions, and stacked regression and poststratification (SRP) tends to outperform MRP in simulations, particularly for training data with large sample size (Ornstein 2020).

Synthetic Poststratification

Suppose that we did not have access to the entire joint distribution of individual-level covariates. Leemann and Wasserfallen (2017) suggest an extension of MRP, which they (delightfully) dub Multilevel Regression and Synthetic Poststratification (MrsP). Lacking the full joint distribution of covariates for poststratification, one can instead create a *synthetic* poststratification frame by assuming that additional covariates are statistically independent of one another. So long as your first stage model is linear-additive, this approach yields the same predictions as if you knew the true joint distribution![3] And even if the first-stage model is not linear-additive, simulations suggest that the improved performance from additional predictors tends to overcome the error introduced by synthetic poststratification.

Let's suppose that we want to include these predictors in the model:

- How important is religion to the respondent?
- Whether the respondent lives in an urban, rural, or suburban area
- Whether the respondent or a member of the respondent's family is a military veteran

[3] See Ornstein (2020) appendix A for mathematical proof.

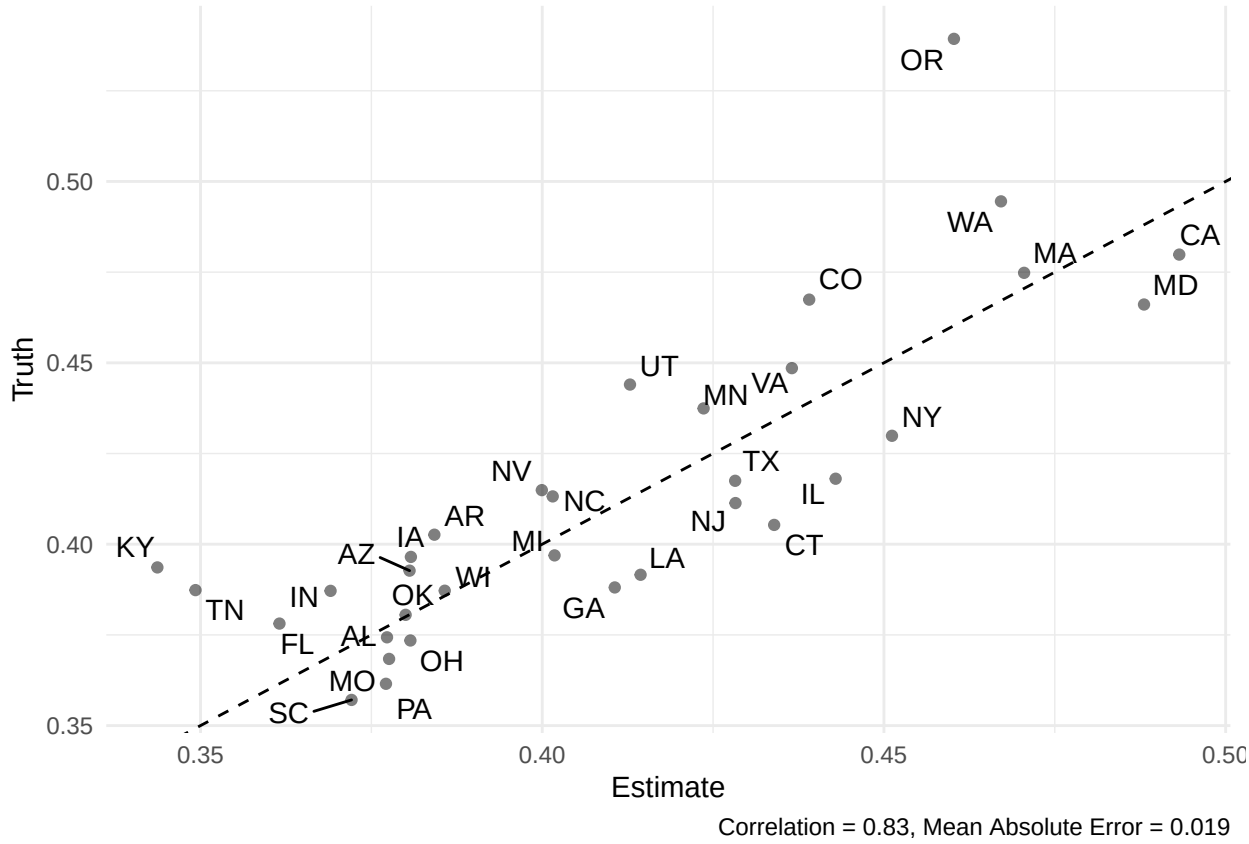

Figure 7: Estimates from an ensemble first-stage model

- Whether the respondent owns or rents their home
- Is the respondent the parent or guardian of a child under the age of 18?

These variables may be useful predictors of opinion about police policy and the first-stage model could be improved by including them. But there is no dataset (that I know of) that would allow us to compute a state-level joint probability distribution over every one of them. Instead, we would typically only know the marginal distributions of each covariate (e.g. the percent of a state's residents that are military households, or the percent that live in urban areas). To create a synthetic poststratification frame, we multiply those marginal probabilities together.[4]

CODE TO COME

Then poststratify as normal.

CODE TO COME

Best Performing

As a final exercise, suppose we had access to the entire joint distribution over those covariates, *and* our first stage model was a Super Learner ensemble...

```
psframe <- ces %>%
  count(abb, gender, race, age, educ,
        division, biden_vote_share,
        pew_religimp, homeowner, urban,
        parent, military_household)

library(SuperLearner)

# fit Super Learner
SL.library <- c("SL.ranger", "SL.glm")

X <- sample_data %>%
  select(gender, race, age, educ,
        division, biden_vote_share,
        pew_religimp, homeowner, urban,
        parent, military_household)

newX <- psframe %>%
  select(gender, race, age, educ,
        division, biden_vote_share,
        pew_religimp, homeowner, urban,
        parent, military_household)

sl <- SuperLearner(Y = sample_data$defund_police,
                   X = X,
                   newX = newX,
                   family = binomial(),
                   SL.library = SL.library,
                   verbose = FALSE)

# make predictions
psframe$predicted_probability <- sl$SL.predict
```

[4]The `SRP` package contains a convenience function for this operation (see the vignette for more information).

```r
# poststratify
poststratified_estimates <- psframe %>%
  group_by(abb) %>%
  summarize(estimate = weighted.mean(predicted_probability, n))

compare_to_truth(poststratified_estimates, truth)
```

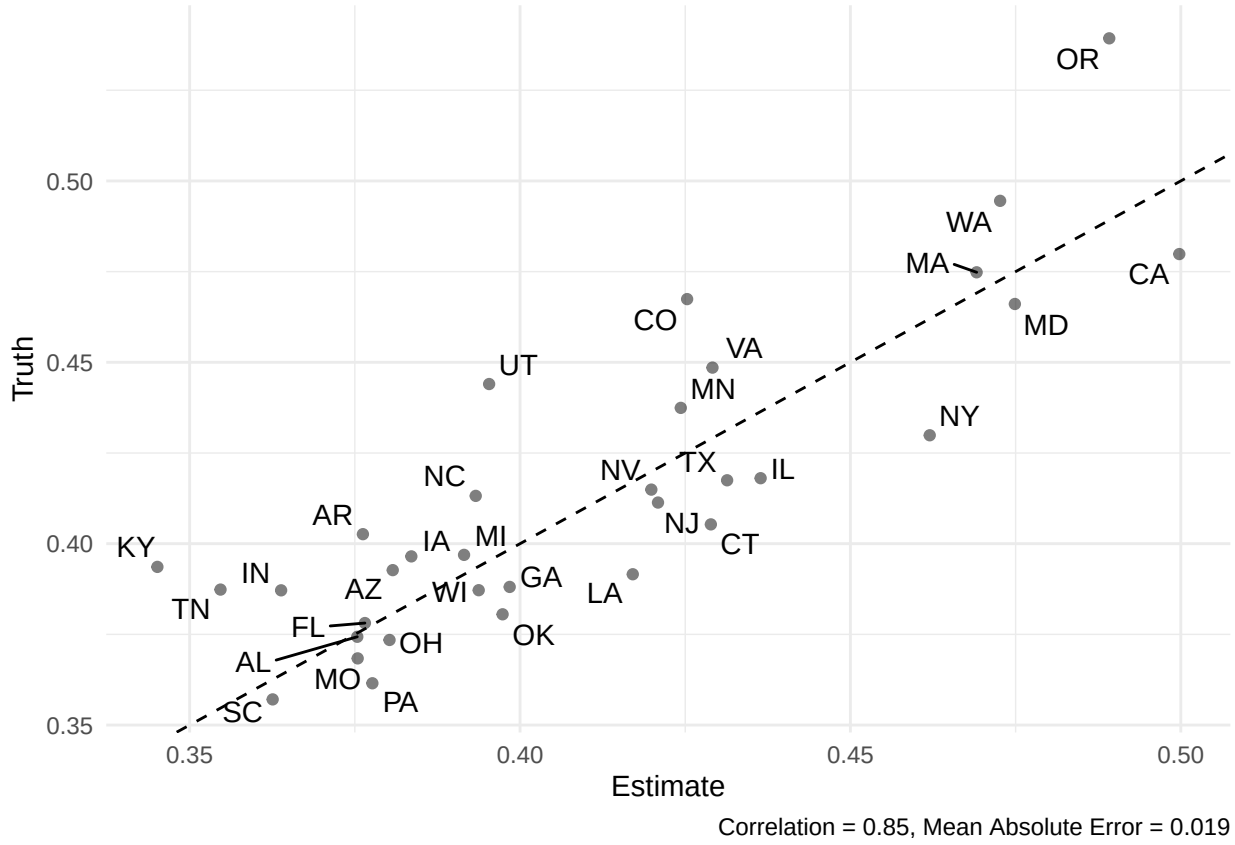

Figure 8: The best performing estimates, using an ensemble first stage model, group-level predictors, and synthetic poststratification.

Conclusion

In the code above I emphasize "do-it-yourself" approaches to MRP – fitting a model, building a poststratification frame, and producing estimates separately. But there are a number of `R` packages available with useful functions to help ease the process. In particular, I would encourage curious readers to explore the `autoMrP` package (Broniecki, Leemann, and Wüest 2022), which implements the ensemble modeling approach described above, and performs quite well in simulations when compared to existing packages.

References

Achen, Christopher H. 2005. "Let's Put Garbage-Can Regressions and Garbage-Can Probits Where They Belong." *Conflict Management and Peace Science* 22 (4): 327–39. https://doi.org/10.1080/07388940500339167.

Ansolabehere, Stephen, Samantha Luks, and Brian F. Schaffner. 2015. "The Perils of Cherry Picking Low Frequency Events in Large Sample Surveys." *Electoral Studies* 40 (December): 409–10. https://doi.org/10.1016/j.electstud.2015.07.002.

Bisbee, James. 2019. "BARP: Improving Mister P Using Bayesian Additive Regression Trees." *American Political Science Review* 113 (4): 1060–65. https://doi.org/10.1017/S0003055419000480.

Breiman, Leo. 1996. "Stacked Regressions." *Machine Learning* 24: 49–64. https://doi.org/10.17485/ijst/2016/v9i28/98380.

———. 2001. "Random Forests." *Machine Learning* 45 (1): 5–32. https://doi.org/10.1023/A:1010933404324.

Broniecki, Philipp, Lucas Leemann, and Reto Wüest. 2022. "Improved Multilevel Regression with Poststratification Through Machine Learning (autoMrP)." *The Journal of Politics* 84 (1). https://doi.org/10.1086/714777.

Buttice, Matthew K., and Benjamin Highton. 2013. "How Does Multilevel Regression and Poststratification Perform with Conventional National Surveys?" *Political Analysis* 21 (4): 449–67. https://doi.org/10.1093/pan/mpt017.

Gelman, Andrew. 2018. "Regularized Prediction and Poststratification (the Generalization of Mister p)." *Statistical Modeling, Causal Inference, and Social Science (Blog)* May 19 (https://statmodeling.stat.columbia.edu/2018/05/19/).

Gelman, Andrew, and Thomas C Little. 1997. "Poststratification into Many Categories Using Hierachical Logistic Regression." *Survey Methodology* 23 (2): 127–35.

Gelman, Andrew, Boris Shor, Joseph Bafumi, and David Park. 2007. "Rich State, Poor State, Red State, Blue State: What's the Matter with Connecticut?" *Quarterly Journal of Political Science* 2 (June 2006): 345–67. https://doi.org/10.1561/100.00006026.

Ghitza, Yair, and Andrew Gelman. 2013. "Deep Interactions with MRP: Election Turnout and Voting Patterns Among Small Electoral Subgroups." *American Journal of Political Science* 57 (3): 762–76. https://doi.org/10.1111/ajps.12004.

Lax, Jeffrey R., and Justin H. Phillips. 2009. "How Should We Estimate Public Opinion in the States?" *American Journal of Political Science* 53 (1): 107–21. https://doi.org/10.1111/j.1540-5907.2008.00360.x.

———. 2012. "The Democratic Deficit in the States." *American Journal of Political Science* 56 (1): 148–66. https://doi.org/10.1111/j.1540-5907.2011.

Leemann, Lucas, and Fabio Wasserfallen. 2017. "Extending the Use and Prediction Precision of Subnational Public Opinion Estimation." *American Journal of Political Science* 61 (4): 1003–22.

Montgomery, Jacob M, Florian Hollenbach, and Michael D Ward. 2012. "Improving Predictions Using Ensemble Bayesian Model Averaging." *Political Analysis* 20 (3): 271–91.

Ornstein, Joseph T. 2020. "Stacked Regression and Poststratification." *Political Analysis* 28 (2): 293–301. https://doi.org/10.1017/pan.2019.43.

Park, David K., Andrew Gelman, and Joseph Bafumi. 2004. "Bayesian Multilevel Estimation with Poststratification: State-Level Estimates from National Polls." *Political Analysis* 12 (4): 375–85. https://doi.org/10.1093/pan/mph024.

Schaffner, Brian, Stephen Ansolabehere, and Sam Luks. 2021. "Cooperative Election Study Common Content, 2020." Edited by YouGov and Add your team name(s) here. https://doi.org/10.7910/DVN/E9N6PH.

Tausanovitch, Chris, and Christopher Warshaw. 2014. "Representation in Municipal Government." *The American Political Science Review* 108 (03): 605–41. https://doi.org/10.1017/S0003055414000318.

van der Laan, Mark J., Eric. C. Polley, and Alan E. Hubbard. 2007. "Super Learner." *Statistical Applications in Genetics and Molecular Biology* 6 (1).

Wang, Wei, David Rothschild, Sharad Goel, and Andrew Gelman. 2015. "Forecasting Elections with Non-Representative Polls." *International Journal of Forecasting* 31 (3): 980–91. https://doi.org/10.1016/j.ijforecast.2014.06.001.