

Calculus - Chapter Notes

1 Chapter 1: Basic Properties of Numbers

1. additive associativity
2. additive identify
3. additive inverse
4. additive commutativity
5. multiplicative associativity
6. multiplicative identify
7. multiplicative inverse
8. multiplicative commutativity
9. distributive law
10. trichotomy law
11. closure under addition for P
12. closure under multiplication for P

With P1-P8 very little can actually be proved. But with the introduction of P9, which is effectively a bridge between addition and multiplication, a lot more can be demonstrated. For instance, it can be shown that $(-a) \cdot (-b) = a \cdot b$. Furthermore P9 is actually the justification for almost all algebraic manipulations.

Inequality symbols such as $<$ and \leq and the absolute value are defined as one would expect in terms of P (introduced in P10-P12).

Theorem 1: For all numbers a and b , we have $|a + b| \leq |a| + |b|$

Some concluding notes from Spivak: "[though at this point in the book] we do not yet thoroughly understand numbers; we may still say that, in whatever way numbers are finally defined, they should certainly have properties P1-P12...It is still a crucial question whether P1-P12 actually account for

textitall properties of numbers. As a matter of fact, we shall soon see that they do textitnot. In the next chapter the deficiencies of P1-P12 will become quite clear, but the proper means for correcting these deficiencies is not so easily discovered [it will require all of part 2 of the book]”

2 Chapter 2: Numbers of Various Sorts