MAC2311: Calculus 1 - Section 1

Test 3   March 19, 2015

Name: _____

> Answer each question in the space provided on the question sheets. If you run out of space for an answer, continue on the back of the page. Credit will only be given if you clearly show all of your work. Calculators may be used for this test.

Question	Points	Score
1	2	
2	3	
3	5	
4	7	
5	8	
6	5	
7	6	
8	2	
9	9	
10	9	
Total:	56	

1. [2 points] Find the differential dy if $y = \ln(\sin x)$.

2. [3 points] Find the critical numbers of the function $f(x) = x^{3/7}(x - 5)$.

3. [5 points] Find the absolute minimum and absolute maximum values of $f(x) = x^3 - 3x$ on the interval $[0, 2]$.

4. (a) [5 points] Find the linearization $L(x)$ of the function $f(x) = \sqrt[9]{x - 4}$ at $a = 5$.

(b) [2 points] Approximate the number $\sqrt[9]{0.5}$ using the linearization $L(x)$ from part (a). (Write your answer as a reduced fraction or as a decimal number rounded to two places.)

5. [8 points] Let $f(x) = \dfrac{x}{x+1}$.

(a) [1 point] On what interval(s) is f continuous? (Write your answer using interval notation.)

(b) [2 points] On what interval(s) is f differentiable? (Write your answer using interval notation.)

(c) [5 points] Does f satisfy the hypotheses of the Mean Value Theorem on the interval $[-5, -2]$? If so, find all numbers c that satisfy the conclusion of the Mean Value Theorem.

6. [5 points] The area between the graphs of $\sin x$ and $\cos x$ on the interval $[\pi/4, x]$ is

$$A = \sqrt{2} - \cos x - \sin x$$

for $\pi/4 \le x \le 5\pi/4$. Suppose x is a function of t and changes at the rate $\sqrt{3}+1$. At what rate is A changing with respect to t when $x = \pi/3$?

7. [6 points] Use the graph of $dy/dx = f'(x)$, the derivative of $y = f(x)$, below to answer the following questions.

dy/dx = f '(x)
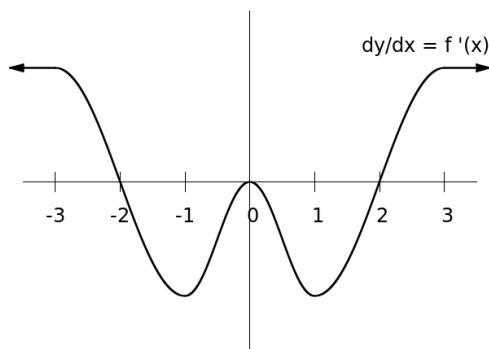
(a) [2 points] On what interval(s) is f decreasing? (Write your answer using interval notation.)

(b) [1 point] At what x-value(s) does f have a local minimum?

(c) [2 points] On what interval(s) is f concave downward? (Write your answer using interval notation.)

(d) [1 point] Does f have any inflection points? If so, state the x-coordinate(s) of the inflection point(s) of f.

8. [2 points] Suppose f'' is continuous on $(-\infty, \infty)$.
   (a) [1 point] If $f'(-2) = 0$ and $f''(-2) = -1$, what can you say about f?
      A. At $x = -2$, f has a local maximum.
      B. At $x = -2$, f has a local minimum.
      C. At $x = -2$, f has neither a maximum nor a minimum.
      D. More information is needed to determine if f has a maximum or minimum at $x = -2$.
   (b) [1 point] If $f'(0) = 0$ and $f''(0) = 0$, what can you say about f?
      A. At $x = 0$, f has a local maximum.
      B. At $x = 0$, f has a local minimum.
      C. At $x = 0$, f has neither a maximum nor a minimum.
      D. More information is needed to determine if f has a maximum or minimum at $x = 0$.

9. [9 points] Let $f(x) = xe^{-x}$.

   (a) [2 points] On what interval(s) is f increasing? (Write your answer using interval notation.)

   (b) [1 point] At what x-value(s) does f have a local maximum?

   (c) [2 points] On what interval(s) is f concave upward? (Write your answers using interval notation.)

   (d) [1 point] Does f have any inflection points? If so, state the inflection point(s) of f.

   (e) [3 points] Evaluate $\lim\limits_{x \to \infty} xe^{-x}$.

10. [9 points] Evaluate the following limits.

(a) [5 points] $\lim\limits_{x \to 1^-} \left(\dfrac{x}{\ln x} - \dfrac{1}{x-1} \right)$.

(b) [4 points] $\lim\limits_{x \to 0^+} (1 - 3x)^{1/x}$.