3.4: Real Zeros of Polynomial Functions

Supplementary Notes

Division Algorithm: For polynomials f and g, there is a unique *quotient polynomial* q and *remainder polynomial* r such that

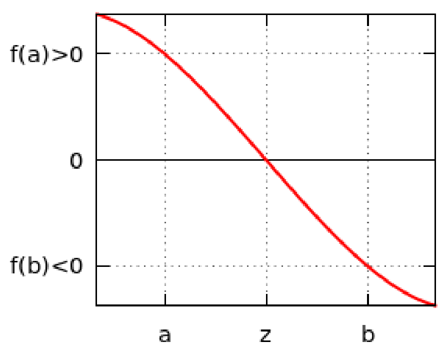
$$\underbrace{f(x)}_{\text{dividend}} = q(x)\underbrace{g(x)}_{\text{divisor}} + r(x)$$

For $f(x) = q(x)(x-z)+r(x)$, notice the remainder is $f(z)$, so $(x-z)$ is a *factor* of f when $f(z) = 0$.

Remainder Theorem: The *remainder* of f divided by $(x - z)$ is $f(z)$.

Factor Theorem: $(x - z)$ is a *factor* of f if and only if $f(z) = 0$.

Intermediate Value Theorem: For continuous f, if $f(a)$ and $f(b)$ have opposite signs, then f has a zero in (a, b).



Rational Zeros Theorem: For $f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$, if p/q in lowest terms is a *rational zero* of f, then p is a factor of a_0 and q is a factor of a_n.

Exercises

1. Two factors of $f(x) = 3x^4 + 14x^3 - 9x^2 - 38x + 24$ are $(3x - 4)$ and $(x + 2)$. Another factor is

   A. $x^2 - 4x - 3$
   B. $(x + 3)(x + 1)$
   C. $x^2 + 4x - 3$
   D. $(x - 3)(x - 1)$

2. Find k such that $f(x) = x^3 - 2kx^2 - x + 2k$ has a factor $(x + 2)$.