Homework 7 Foundations of Computational Math 1 Fall 2012

Problem 7.1

7.1.a

Let $f(x) = x^3 - 3x + 1$. This polynomial has three distinct roots.

(i) Consider using the iteration function

$$\phi_1(x) = \frac{1}{3}(x^3 + 1)$$

Which, if any, of the three roots can you compute with $\phi_1(x)$ and how would you choose $x^{(0)}$ for each computable root?

(ii) Consider using the iteration function

$$\phi_2(x) = \frac{3}{2}x - \frac{1}{6}(x^3 + 1)$$

Which, if any, of the three roots can you compute with $\phi_2(x)$ and how would you choose $x^{(0)}$ for each computable root?

(iii) For each of the roots you identified as computable using either $\phi_1(x)$ or $\phi_2(x)$, apply the iteration to find the values of the roots. (You need not turn in any code, but using a simple program to do this is recommended.)

7.1.b

Let $\phi(x) : [a, b] \to [a, b]$ be a continuous function. Show that if $\phi(x)$ is a contraction mapping on $[a, b]$ then the sequence $\{x^{(k)}\}$ defined by $x^{(k+1)} = \phi(x^{(k)})$ is a Cauchy sequence.

Problem 7.2

Textbook, p. 283, Problem 2

Problem 7.3

Textbook, p. 283, Problem 5

Problem 7.4

Textbook, p. 283, Problem 6

Problem 7.5

Textbook, p. 284, Problem 8