Foundations of Computational Math II Exam 2

Take-home Exam

Open Notes, Textbook, Homework Solutions Only

Calculators Allowed

Friday 13 April, 2012

Question	Points Possible	Points Awarded
1. Approximation	25	
2. Quadrature	25	
3. GFS	25	
4. LMS Methods	25	
Total Points	100	

Name: **Alias:**

Problem 1

Suppose you are given the function $f(x)$ on $[0, 2]$:

$$f(x) = \sqrt{x}$$

1.a

Find, $p_1(x)$, the linear polynomial that is the near-minimax approximation to $f(x)$ on the interval $[0, 2]$.

1.b

Find, $q_1(x)$, the linear polynomial that is the minimax (best) approximation to $f(x)$ on the interval $[0, 2]$.

1.c

Give a bound for the error $|f(x) - p_1(x)|$ on the interval $[0, 2]$.

1.d

Give a bound for the error $|f(x) - q_1(x)|$ on the interval $[0, 2]$.

Problem 2

(25 points)

Approximate the integral

$$\int_0^2 e^x \, dx$$

using Gauss-Legendre quadrature method $I_4(f)$, i.e., using 5 points. Compare the result to using an open Newton-Cotes formula and a closed Newton-Cotes formula with the same number of points.

Problem 3

3.a

Consider $f(x) = \sin x$ on $[-1, 1]$. Determine the economized power series of degree 2 for Legendre polynomials, $\{P_i(x)\}$, and Chebyshev polynomials, $\{T_i(x)\}$, for the Taylor series of degree 4 of $f(x)$.

3.b

i. Consider the space of polynomials of degree n or less, $\mathbb{P}_n$, and the subspaces $span[P_0(x),\ P_1(x), \ldots, P_d(x)]$ and $span[T_0(x),\ T_1(x), \ldots, T_d(x)]$ where $d \leq n$. Is there any relationship between the two subspaces of $\mathbb{P}_n$?

ii. Consider an arbitrary smooth function, $f(x)$, and its Generalized Fourier Series in terms of the Legendre polynomials, $\{P_i(x)\}$, and its Generalized Fourier Series in terms of the Chebyshev polynomials, $\{T_i(x)\}$, i.e.,

$$f(x) = \sum_{i=0}^{\infty} \alpha_i P_i(x) = \sum_{i=0}^{\infty} \beta_i T_i(x).$$

Suppose you truncate each series at degree n, defining

$$f(x) \approx f_P(x) = \sum_{i=0}^{n} \alpha_i P_i(x) \quad f(x) \approx f_T(x) = \sum_{i=0}^{n} \beta_i T_i(x).$$

Are the two truncations the same? If so prove it, if not how is it possible?

Problem 4

(25 points)

Consider an explicit linear multistep method of the form

$$\alpha_0 y_n + \alpha_1 y_{n-1} + \alpha_2 y_{n-2} = h f_{n-1}$$

4.a. Is there a consistent method of this form with order at least 2? Is there more than one such method? Justify your answer.

4.b. If one or more such methods exists, choose one and determine if it is 0-stable and find the expression for its local truncation error. If there is no such method indicate how you would change the form so that one does exist.