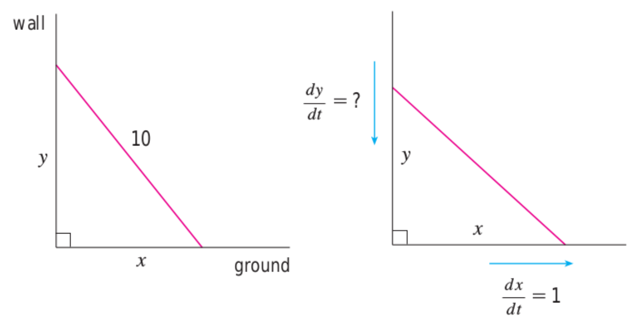
3.9: Related Rates

If two quantities are related, then a change in one will accompany a change in the other. Using differentiation, we can find an equation that relates the rates of change of two quantities. A general approach to solve related rates problems is to

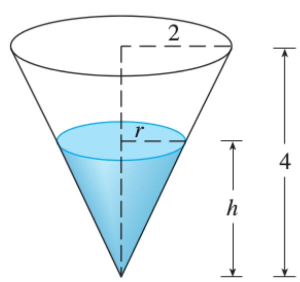
1. Identify given and unkown quantities,
2. Write an equation that relates the various quantities of the problem,
3. Use the chain rule to differentiate both sides with respect to t, and
4. Substitute the given information into the resulting equation and solve for the unkown quantity.

Example 1. *Air is being pumped into a spherical balloon so that its volume increases at a rate 100 cm*^{3}*/s. How fast is the radius of the balloon increasing when the diameter is 50 cm?*

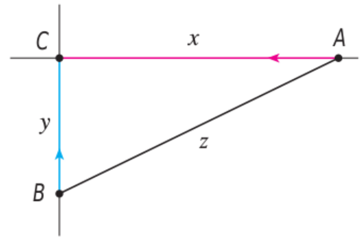
Example 2. *A ladder 10 ft long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a rate of 1 ft/s, how fast is the top of the ladder sliding down the wall when the bottom of the ladder is 6 ft from the wall?*



Example 3. *A water tank has the shape of an inverted circular cone with a base radius 2 m and height 4 m. If water is being pumped into the tank at a rate of 2 m*^{3}*/min, find the rate at which the water level is rising when the water is 3 m deep.*



Example 4. *Car A is traveling west at 50 mi/h and car B is traveling north at 60 mi/h. Both are headed for the intersection of the two roads. At what rate are the cars approaching each other when car A is 0.3 mi and car B is 0.4 mi from the intersection?*

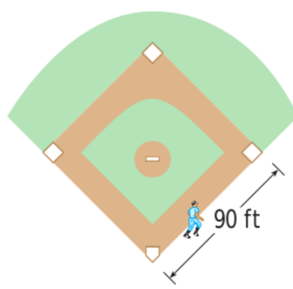


Example 5.

(a) *If A is the area of a circle with radius r and the circle expands as time passes, find dA/dt in terms of dr/dt.*

(b) *Suppose oil spills from a ruptured tanker and spreads in a circular pattern. If the radius of the oil spill increases at a constant rate 1 m/s, how fast is the area of the spill increasing when the radius is 30 m?*

Example 6. *A baseball diamond is a square with side 90 ft. A batter hits the ball and runs toward first base with a speed of 24 ft/s. At what rate is his distance from second base decreasing when he is halfway to first base?*



Example 7. *A cylindrical tank with radius 5 m is being filled with water at a rate of 3 m^3/min. How fast is the height of the water increasing?*

Example 8. *Two sides of a triangle are 4 m and 5 m in length and the angle between them is increasing at a rate of 0.06 rad/s. Find the rate at which the area of the triangle is increasing when the angle between the sides of fixed length is $\pi/3$.*

Example 9. *The height of a triangle is increasing at a rate of 1 cm/min while the area of the triangle is increasing at a rate of 2 cm^2/min. At what rate is the base of the triangle changing when the height is 10 cm and the area is 100 cm^2?*