MAC2311: Calculus 1 - Section 1

Quiz 4: Sections 4.7-4.9; 5.1-5.4 April 9, 2015

Name: ______________________________

1. [5 points] A box with an open top is to be constructed from a rectangular piece of cardboard, 8 ft long by 3 ft wide, by cutting out a square from each of the four corners and bending up the sides. Find the side length of the square that yields the largest volume that such a box can have.

$$\begin{aligned} V(x) &= (8-2x)(3-2x)x \qquad \text{for } 0 \le x \le \frac{3}{2} \\ &= 24x - 22x^2 + 4x^3 \end{aligned}$$

$$\begin{aligned} V'(x) = 24 - 44x + 12x^2 &= 0 \\ 3x^2 - 11x + 6 &= 0 \\ (3x-2)(x-3) &= 0 \\ x &= \frac{2}{3}, 3 \end{aligned}$$

V has an absolute maximum at $x = 2/3$ since $V(0) = 0$, $V(3/2) = 0$, and $V(2/3) = 200/27$ ft^3. Or, by the second derivative test, V has an absolute maximum at $x = 2/3$ since $V''(x) = -44+24x$, $V''(2/3) = -28 < 0$ and $V''(3) = 28 > 0$.

2. [7 points] A ball is thrown upward with a speed of 64 ft/s from the edge of a cliff 80 ft above the ground.

(a) [4 points] Find its height above the ground t seconds later.
(Hint: the *downward* acceleration due to gravity is 32 ft/s^2.)

$$\begin{aligned} a(t) &= -32 \\ v(t) &= -32t + C \\ h(t) &= -16t^2 + Ct + D \end{aligned}$$

$v(0) = C = 64$ and $h(0) = D = 80$, so $h(t) = -16t^2 + 64t + 80$ ft.

(b) [1 point] When does it reach its maximum height?

$$\begin{aligned} v(t) = -32t + 64 &= 0 \\ t &= 2 \text{ s} \end{aligned}$$

(c) [2 points] When does it hit the ground?

$$\begin{aligned} h(t) = -16t^2 + 64t + 80 &= 0 \\ t^2 - 4t - 5 &= 0 \\ (t+1)(t-5) &= 0 \\ t &= 5 \text{ s} \end{aligned}$$

3. [2 points] Use Part 1 of the Fundamental Theorem of Calculus to find the derivative of the function

$$g(x) = \int_7^{\tan x} \frac{dt}{1+t^2}.$$

$$\begin{aligned} g'(x) &= \frac{1}{1+\tan^2 x} \frac{d}{dx}(\tan x) \\ &= \frac{1}{\sec^2 x} \sec^2 x = 1 \end{aligned}$$

4. [6 points] Let $f(x) = x^2$.

   (a) [4 points] Estimate the area under the graph of f from $x = 0$ to $x = 6$ using the Midpoint Rule with three rectangles.

   With $n = 3$ rectangles, the subintervals are $[0, 2], [2, 4]$, and $[4, 6]$ each of width $\Delta x = \frac{b-a}{n} = \frac{6-0}{3} = 2$. The midpoints are $\bar{x}_1 = 1$, $\bar{x}_2 = 3$, and $\bar{x}_3 = 5$.

$$M_3 = \sum_{k=1}^{3} f(\bar{x}_k)\Delta x = 2(1 + 9 + 25) = 70.$$

   (b) [2 points] Find the area under the graph of f from $x = 0$ to $x = 6$.

$$\int_0^6 x^2 \, dx = \left[\frac{x^3}{3}\right]_{x=0}^{6} = \frac{6^3}{3} - \frac{0^3}{3} = 72.$$