3.4: The Chain Rule

The Chain Rule: If g is differentiable at x and f is differentiable at $g(x)$, then

$$[f(g(x))]' = f'(g(x)) \cdot g'(x).$$

In another notation, the chain rule is

$$\frac{df}{dx} = \frac{df}{dg} \cdot \frac{dg}{dx}.$$

Example 1. *Find $f'(x)$ if $f(x) = \sqrt{x^2+1}$.*

Example 2. *Differentiate*

(a) $y = \sin(x^2)$

(b) $y = \sin^2 x$

The Power Rule combined with the Chain Rule: If n is a real number and g is differentiable, then

$$\frac{d}{dx}[g(x)]^n = n[g(x)]^{n-1} \cdot g'(x).$$

Example 3. *Differentiate*

(a) $y = (x^3-1)^{100}$

(b) $f(x) = \frac{1}{\sqrt[3]{x^2+x+1}}$

(c) $g(t) = \left(\frac{t-2}{2t+1}\right)^9$

Example 4. *Differentiate* $y = (2x+1)^5(x^3-x+1)^4$

Example 5. *Differentiate* $y = e^{\sin x}$

The derivative of the exponential function $f(x) = a^x$ for $a > 0$ is

$$\frac{d}{dx}(a^x) = a^x \ln a.$$

Example 6. *Differentiate* $y = 2^x$

Example 7. *If $f(x) = \sin(\cos(\tan x))$, find $f'(x)$.*

Example 8. *Differentiate* $e^{\sec 3\theta}$.

Example 9. *Prove the quotient rule by differentiating f/g in the form $f \cdot g^{-1}$.*