MAC2312: Calculus 2 - Section 3

Quiz 3: 7.3 Trigonometric Substitution — May 19, 2015

1. Evaluate $\displaystyle\int \frac{x^3}{\sqrt{x^2+4}}\,dx$.

   A. $\frac{1}{6}(x^2+4)^{3/2} - 2\sqrt{x^2+4} + C$

   B. $2\sqrt{x^2+4} - \frac{1}{6}(x^2+4)^{3/2} + C$

   C. $\frac{1}{3}(x^2+4)^{3/2} - 4\sqrt{x^2+4} + C$

   D. $4\sqrt{x^2+4} - \frac{1}{3}(x^2+4)^{3/2} + C$

$$\begin{aligned}
\int \frac{x^3}{\sqrt{x^2+4}}\,dx &= \int \frac{(2\tan x)^3}{\sqrt{(2\tan x)^2+4}} \cdot 2\sec^2\theta\; d\theta && x = 2\tan\theta \\
&&& dx = 2\sec^2\theta\; d\theta \\
&= 8\int \frac{\tan^3\theta}{\sqrt{\sec^2\theta}}\sec^2\theta\; d\theta \\
&= 8\int (\sec^2\theta - 1)\sec\theta\tan\theta\; d\theta \\
&= 8\int (u^2-1)\; du && u = \sec\theta \\
&&& du = \sec\theta\tan\theta\; d\theta \\
&= \frac{8}{3}u^3 - 8u + C \\
&= \frac{1}{3}(x^2+4)^{3/2} - 4\sqrt{x^2+4} + C && x = 2\tan\theta \Rightarrow \sec\theta = \frac{1}{2}\sqrt{x^2+4}
\end{aligned}$$