Unit I

Supplementary Notes

3.1 Quadratic Functions

$$f(x) = \underbrace{ax^2 + bx + c}_{\text{expanded form}} = \underbrace{a(x-h)^2 + k}_{\text{vertex form}}$$

The graph of f is a *parabola* with the following properties

- opens up or down if $a > 0$ or $a < 0$
- vertex $(h, k) = (-\frac{b}{2a}, f(-\frac{b}{2a})) = (-\frac{b}{2a}, c - \frac{b^2}{4a})$
- 0, 1, or 2 x−intercepts if $b^2 - 4ac < 0, = 0$, or > 0
- y−intercept: $f(0) = c$
- vertical axis of symmetry $x = h$
- minimum value of k

3.2 Power Functions

$f(x) = a(x-h)^n + k$ click to see Supplementary Notes

3.3 Polynomial Functions

$$\begin{aligned} f(x) &= a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0 && \} \text{ expanded form} \\ &= a(x-z_1)(x-z_2)\cdots(x-z_{n-1})(x-z_n) && \} \text{ factored form} \end{aligned}$$

- degree n
- $f(z) = 0 \Leftrightarrow z$ is a *zero* of f

The graph of f has the following properties

- x−intercepts: real zeros of f
  - crosses or touches x−axis if *multiplicity* odd or even
- y−intercept: $f(0) = a_0$
- For large $|x|$, behaves like ax^n

3.4 Real Zeros of Polynomial Functions

$$\underbrace{f(x)}_{\text{dividend}} = \underbrace{q(x)}_{\text{quotient}}\,\underbrace{g(x)}_{\text{divisor}} + \underbrace{r(x)}_{\text{remainder}}$$

- $f(z)$ is the *remainder* of f divided by $(x-z)$
- $(x-z)$ *factor* of $f \Leftrightarrow f(z) = 0$
- f continuous and $f(a)$, $f(b)$ opposite sign $\Rightarrow f$ has zero in (a, b)
- p/q *rational zero* of $f \Rightarrow p$ factor of a_0 and q factor of a_n

3.5 Complex Numbers

$$a + bi \quad \text{where } i^2 = -1$$

- conjugate: $a - bi$
- powers of i:
  - $i^{2n} = (i^2)^n = (-1)^n$
  - $i^{2n+1} = (i^2)^n i = (-1)^n \cdot i$

3.6 Complex Zeros, Fundamental Theorem of Algebra

The zeros of a polynomial f with real coefficients may be real or complex; if $a + bi$ is a zero of f, so is its conjugate $a - bi$

3.7 Rational Functions

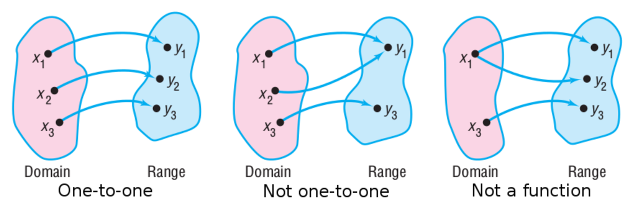
$$f(x) = \frac{g(x)}{h(x)} = \frac{a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0}{b_m x^m + b_{m-1}x^{m-1} + \cdots + b_1 x + b_0}$$

- domain: real x such that $h(x) \neq 0$
- vertical asymptotes: real x such that $h(x) = 0$ and $g(x) \neq 0$
- jump discontinuities: real x such that $h(x) = g(x) = 0$
- If $n \leq m$, horizontal asymptote:
  - $y = 0$ if $n < m$
  - $y = a_n/b_m$ if $n = m$
- If $n = m+1$, oblique asymptote: $y = q(x)$ where $g(x) = q(x)h(x) + r(x)$

3.8 Polynomial and Rational Inequalities

The sign of a function may change at a *zero* or an x-value *not in the domain* of the function

4.1 One-to-one and Inverse Functions



- f is a *function* if any *vertical* line intersects graph of f at most once
- f is *one-to-one* if any *horizontal* line intersects graph of f at most once
- f one-to-one $\Rightarrow f$ has inverse f^{-1} such that
  - $x \xrightarrow{f} y$ if and only if $x \xleftarrow{f^{-1}} y$
  - (domain of f) = (range of f^{-1}); (range of f) = (domain of f^{-1})
  - The graphs of f and f^{-1} are symmetric about the line $y = x$

Exercises

1. Write using lowercase x and y the equation of the parabola with a vertical axis and with vertex at $(-1,-2)$ and $y-$intercept -4.

2. Find the value of b which makes y a multiple of a perfect square if $y = \frac{1}{3}x^2 - 6x + b$.

3. Sketch the graph of $y = -(x + \frac{1}{2})^5$

4. Write the third degree polynomial in lowercase x and y that has zeros at -2, -3, 0 and $y(-1) = 4$. The equation may be left in factored form.

5. Select the statement that is false for $f(x) = -(x-1)(x+6)^4$

   A. f has a local minimum at $x = -6$.

   B. f has degree 5.

   C. The $y-$intercept of graph is 1296.

   D. f has two $x-$intercepts.

   E. The graph of f behaves like $y = -x^5$ for large $|x|$.

6. Find the $x-$intercepts of the graph of $f(x) = (x^2 - 900)^3$ and decide whether the graph crosses or touches the $x-$axis at each intercept.

7. Select ALL the intervals for which the Intermediat Value Theorem implies that $f(x) = 12x^3 + 4x^2 - 25x - 12$ has a zero

   A. $[-1, 0]$

   B. $[-2, -1]$

   C. $[0, 1]$

   D. $[2, 3]$

8. Find ALL (according to the Rational Zeros Theorem) potential rational zeros of the polynomial function $f(x) = 3x^4 + 8x^2 - 2x + 6$

9. Write $2i^{22} + i^{15}$ in $a + bi$ form.

10. Write $\frac{1+3i}{2-i}$ in the form $a + bi$

11. Write the polynomial with real coefficients of degree 5 having zeros of -3, $-2+i$, $1-3i$.

12. Select the statement that is false for $f(x) = -\frac{(x+1)(x+2)}{x^2+2x}$

    A. The $x-$intercept of graph is -1.

    B. The graph does not have a $y-$intercept

    C. The graph has one horizontal asymptote at $y = -1$.

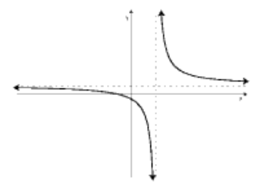
    D. The domain of f is $(-\infty, -2) \cup (-2, 0) \cup (0, \infty)$.

    E. The graph of f has two vertical asymptotes at $x = -2$ and $x = 0$.

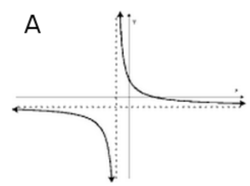
13. Find the equation (in the form $y = mx + b$, with lowercase x and y) of the oblique asymptote of $f(x) = \frac{3x^2 - x - 2}{x-2}$

14. Solve $\frac{(x+5)^2}{(2-x)(2+x)} \leq 0$

15. Select the graph of the inverse of the function shown below



A



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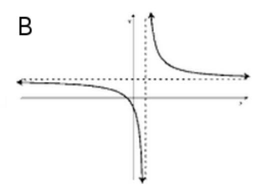

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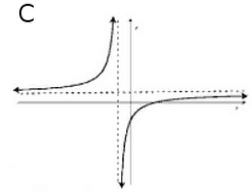

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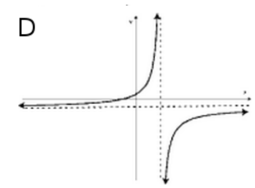

E This function has no inverse