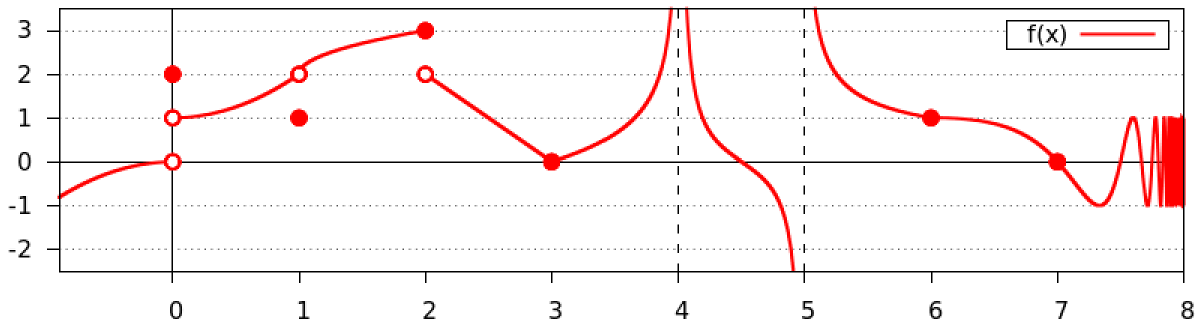
Quiz 1: Sections 2.1-2.3, 2.5

1. Use the following graph of $f(x)$ to answer the questions below it. If an answer does not exist and is not infinite, answer "DNE". For "True or False" questions, circle either "True" or "False".



(a) $f(0) = 2$

(b) $\lim_{x\to 0^-} f(x) = 0$

(c) $\lim_{x\to 0^+} f(x) = 1$

(d) $\lim_{x\to 0} f(x)$ DNE

(e) $\lim_{x\to 1} f(x) = 2$

(f) $\lim_{x\to 4} f(x) = \infty$

(g) $\lim_{x\to 5} f(x)$ DNE

(h) $\lim_{x\to 8} f(x)$ DNE

(i) True or **False**: f is continuous at 1.

(j) **True** or False: f is continuous from the left at 2.

(k) True or **False**: f is continuous from the right at 2.

(l) **True** or False: f is continuous at 3.

(m) True or **False**: f is continuous on $[0, 1]$.

(n) **True** or False: f is continuous on $(3, 4)$.

(o) **True** or False: f is continuous at $(1, 2]$.

2. The slope of the line tangent to $f(x) = x^2$ at the point $(1, 1)$ is $m = \lim_{h\to 0} \frac{(1+h)^2-1}{h}$.

(a) Evaluate $\lim_{h\to 0} \frac{(1+h)^2-1}{h}$.

$$\lim_{h\to 0} \frac{(1+h)^2-1}{h} = \lim_{h\to 0} \frac{1+2h+h^2-1}{h} = \lim_{h\to 0} \frac{h(2+h)}{h} = \lim_{h\to 0}(2+h) = 2.$$

(b) Write the equation of the tangent line. (Hint: "point-slope" form of the equation for a line is $y - y_0 = m(x - x_0)$).

$$\begin{aligned} y - 1 &= 2(x-1) \\ y - 1 &= 2x - 2 \\ y &= 2x - 1 \end{aligned}$$

3. Evaluate $\lim_{x \to 1} \cos(x^3 + x^2 - x - 1)$.

$$\lim_{x \to 1} \cos(x^3 + x^2 - x - 1) = \cos\left(\lim_{x \to 1} [x^3 + x^2 - x - 1]\right) = \cos(0) = 1$$

4. Use the squeeze theorem to show that $\lim_{x \to 0} x^2 \sin^2 \frac{\pi}{x} = 0$. (Hint: $0 \leq x^2 \sin^2 \frac{\pi}{x} \leq x^2$)

$$\begin{aligned} 0 \leq x^2 \sin^2 \frac{\pi}{x} &\leq x^2 \\ \lim_{x \to 0} 0 \leq \lim_{x \to 0} x^2 \sin^2 \frac{\pi}{x} &\leq \lim_{x \to 0} x^2 \\ 0 \leq \lim_{x \to 0} x^2 \sin^2 \frac{\pi}{x} &\leq 0 \end{aligned}$$

Therefore, $\lim_{x \to 0} x^2 \sin^2 \frac{\pi}{x} = 0$.