MAC2311: Calculus 1 - Section 1

Quiz 3: Sections 3.8-3.11, 4.1-4.2 March 5, 2015

Name: ____

1. [5 points] The area (A) of a rectangle is 1 m^2 and its length $l(t)$ increases at a rate 4 m/s. At what rate is the width $w(t)$ of the rectangle changing when the length is 2 m? Use correct units in your final answer. (Hint: The area of the rectangle is $A = l(t) \cdot w(t)$.)

   Let

   $$\begin{aligned} l(t) &= \text{ length, measured in m, of the rectangle at time t, measured in s} \\ w(t) &= \text{ width, measured in m, of the rectangle at time t, measured in s} \end{aligned}$$

   We are given $\dfrac{dl}{dt} = 4$ m/s. The unknown is $\left.\dfrac{dw}{dt}\right|_{l=2}$.
   Differentiating $A = lw$ with respect to t, we have

   $$\begin{aligned} \frac{d}{dt}(A) &= \frac{d}{dt}(lw) \\ 0 &= l\frac{dw}{dt} + w\frac{dl}{dt} \\ \frac{dw}{dt} &= -\frac{w}{l}\frac{dl}{dt} = -\frac{4w}{l} \end{aligned}$$

   When $l = 2$, we solve for w by $A = lw \Rightarrow 1 = 2w \Rightarrow w = \frac{1}{2}$. Therefore, $\left.\dfrac{dw}{dt}\right|_{l=2} = -\dfrac{4(\frac{1}{2})}{2} = -1$ m/s.

2. [5 points] In the following calculations, simplify your answers.

   (a) [2 points] Find the differential dy of the function $y = x \sin x$.

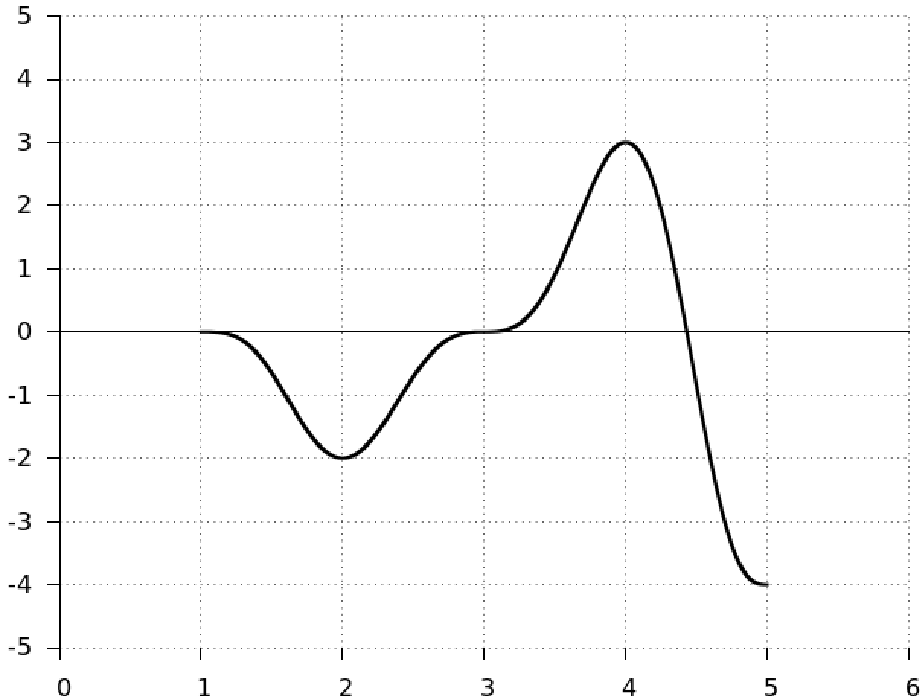
   $$\begin{aligned} dy &= \frac{dy}{dx} \cdot dx \\ dy &= (x\cos x + \sin x)dx \end{aligned}$$

   (b) [3 points] Find the linearization $L(x)$ of the function $f(x) = x \sin x$ at $a = \frac{\pi}{2}$.

   $$\begin{aligned} L(x) &= f(a) + f'(a)(x-a) \\ &= a\sin a + (a\cos a + \sin a)(x-a) \\ &= \frac{\pi}{2}\sin\frac{\pi}{2} + \left(\frac{\pi}{2}\cos\frac{\pi}{2} + \sin\frac{\pi}{2}\right)\left(x - \frac{\pi}{2}\right) \\ &= \frac{\pi}{2} + x - \frac{\pi}{2} \\ &= x \end{aligned}$$

3. [5 points] Sketch the graph of a function f that is continuous on $[1, 5]$ and has the given properties:

- absolute minimum at 5,
- absolute maximum at 4,
- local minimum at 2, and
- no local minimum or maximum at 3, but 3 is a critical number.



4. [5 points] Verify that the function $f(x) = \sqrt{x}$ satisfies the hypotheses of the Mean Value Theorem on the interval $[0, 4]$, then find all numbers c that satisfy the conclusion of the Mean Value Theorem.

$f(x) = \sqrt{x}$ is continuous on $[0, \infty)$ and $f'(x) = \frac{1}{2\sqrt{x}}$ exists on $(0, \infty)$ so f satisfies the hypotheses of the Mean Value Theorem on the interval [0,4]. Therefore, there is a number c in $(0, 4)$ such that

$$
\begin{aligned}
f'(c) &= \frac{f(4) - f(0)}{4 - 0} \\
\frac{1}{2\sqrt{c}} &= \frac{\sqrt{4} - \sqrt{0}}{4} = \frac{1}{2} \\
\sqrt{c} &= 1 \\
c &= 1
\end{aligned}
$$