Homework 10 Foundations of Computational Math 2 Spring 2011

Solutions will be posted Friday, 4/6/12

Problem 10.1

Consider the following linear multistep method:

$$y_n = -4y_{n-1} + 5y_{n-2} + h(4f_{n-1} + 2f_{n-2})$$

The method is not 0-stable.

10.1.a. Determine, p, the order of consistency of the method.

10.1.b. Determine the coefficient, C_{p+1}, in the discretization error d_n.

10.1.c. Consider the application of the method to $y' = 0$ with $y_0 = 0$ and $y_1 = \epsilon$, i.e., a perturbed initial condition. Show that $|y_n| \to \infty$ as $n \to \infty$, i.e., the numerical method is unstable.

Problem 10.2

Consider the following linear multistep method:

$$y_n = y_{n-2} + \frac{h}{3}(f_n + 4f_{n-1} + f_{n-2})$$

The method is 0-stable but it is weakly stable.

10.2.a. Determine the discretization error d_n.

10.2.b. Consider the application of the method to $y' = \lambda y$. Write the recurrence that yields y_n.

10.2.c. Show that $|y_n| \to \infty$ as $n \to \infty$, i.e., the numerical method is unstable.

Problem 10.3

Recall, our model problem

$$f = \lambda(y - F(t)) + F'(t) \quad y(0) = y_0$$
$$y(t) = (y_0 - F(0))e^{\lambda t} + F(t)$$

Take $F(t) = \sin t$ and $y(0) = 1$ and consider $y(t)$ on $0 \leq t \leq 1$.

Consider the 4 methods

- Method 1

$$y_n = -4y_{n-1} + 5y_{n-2} + h(4f_{n-1} + 2f_{n-2})$$

- Method 2 – explicit midpoint

$$y_n = y_{n-2} + 2hf_{n-1}$$

- Method 3 – Adams Bashforth two-step

$$y_n = y_{n-1} + \frac{h}{2}(3f_{n-1} - f_{n-2})$$

- Method 4 – BDF two-step

$$y_n = \frac{4}{3}y_{n-1} - \frac{1}{3}y_{n-2} + \frac{2}{3}hf_n$$

Apply the 4 methods to the model problem using exact initial conditions, i.e., $y_0 = y(0)$ and $y_1 = y(h)$. Use $h = 0.01$ for $\lambda = 10$, $\lambda = -10$, and $\lambda = -500$. Explain your results. How would reducing the stepsize h affect the results?

Note it is recommended you implement this as a simple piece of code in, e.g., MATLAB. You need not turn in your code. Simply present the solution values you observe for the required numerical integrations in support of you explanations.