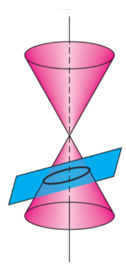
9.3: The Ellipse

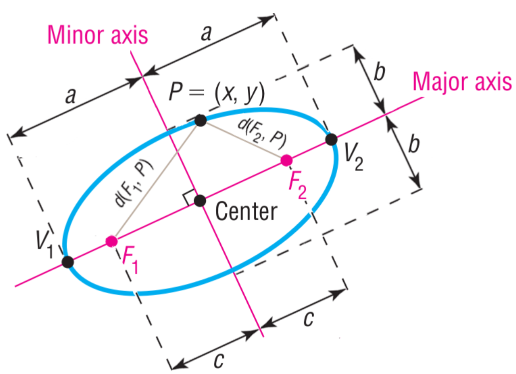
Supplementary Notes

An *ellipse* is an example of a *conic* since it is the cross-section of a cone with a plane that is neither parallel to a line on the face of the cone nor to the axis of the cone.

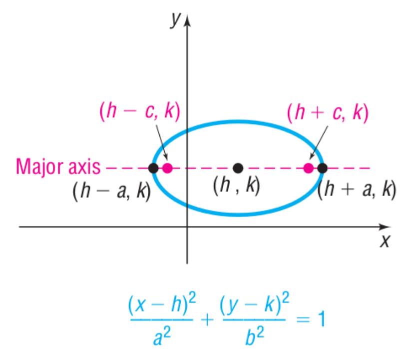


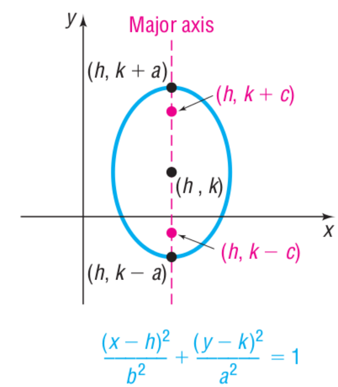
An *ellipse* is the collection of points in the xy-plane, the sum of whose distance from two fixed points F_1 and F_2, is constant. F_1 and F_2 are the *foci*. The midpoint of the line segment joining the foci is the *center*. The line containing both foci is the *major axis*, and the two points where the major axis intersects the ellipse are the *vertices*. The line perpendicular to the major axis that contains the center is the *minor axis*.

- (distance from center to vertex) $= a$
- (distance from center to ellipse along minor axis) $= b$
- (distance from center to focus) $= c$



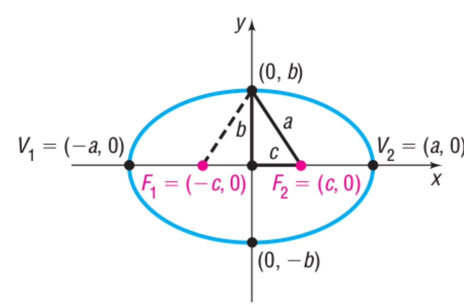
Equations of ellipses with center (h, k) and major axis parallel to a coordinate axis, $a > b > 0$			
Equation	Major Axis	Foci	Vertices
$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$	horizontal	$(h \pm c, k)$	$(h \pm a, k)$
$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$	vertical	$(h, k \pm c)$	$(h, k \pm a)$





The distances a, b, and c defined for an ellipse as above are related by

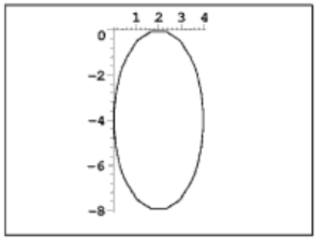
- $c^2 = a^2 - b^2$



Exercises

1. Sketch the graph of $\frac{x^2}{64} + \frac{y^2}{25} = 1$.

2. Write (using lowercase x and y) the equation of the following graph.



3. Find the vertices of the ellipse given by $\frac{(x-6)^2}{24} + \frac{(y+2)^2}{49} = 1$.

4. Find the vertices of the ellipse given by $\frac{(x+7)^2}{4} + \frac{(y+3)^2}{3} = 1$.

5. Find the foci of the ellipse given by $\frac{x^2}{36} + \frac{y^2}{27} = 1$.

6. Find the foci of the ellipse given by $\frac{(x+3)^2}{45} + \frac{(y-4)^2}{49} = 1$.

7. Write (using lowercase x and y) the equation of the ellipse with center at $(0,0)$, focus at $(0,5)$, and vertex $(0,-6)$.

8. Write (using lowercase x and y) the equation of the ellipse with center at $(1,-1)$, focus at $(1,-4)$, and vertex $(1,5)$.

9. Write (using lowercase y) the formula for x^2 in the ellipse with center at $(0,0)$, focus at $(2,0)$, and vertex $(-5,0)$.