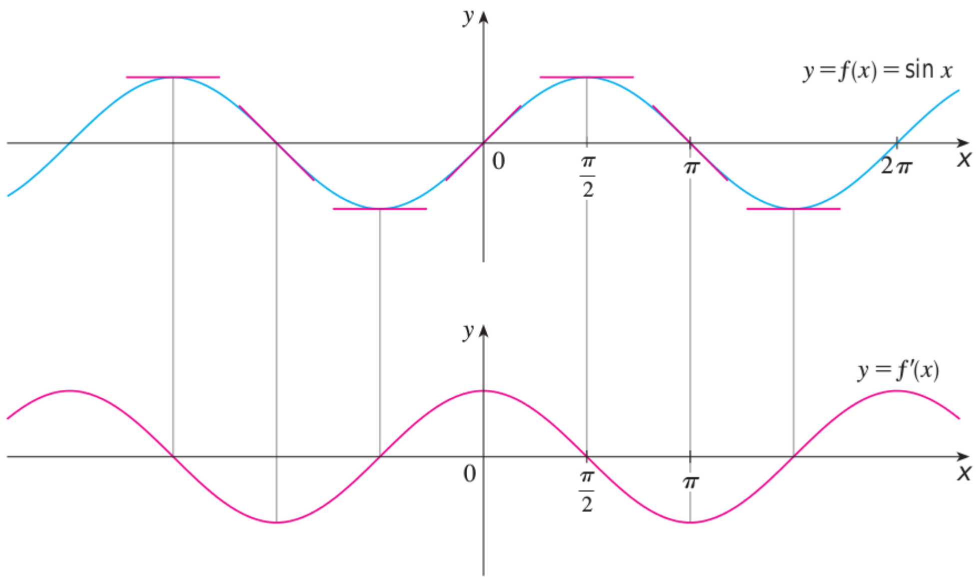
3.3: Derivatives of Trigonometric Functions

The derivative of $\sin x$ is

$$\frac{d}{dx}(\sin x) = \cos x.$$



Example 1. *Differentiate $y = x^2 \sin x$.*

The derivative of $\cos x$ is

$$\frac{d}{dx}(\cos x) = -\sin x.$$

To find the derivative of $\sin x$ and $\cos x$, we used the limits

$$\lim_{x\to 0} \frac{\sin x}{x} = 1 \text{ (see HW 2.2 \#4)} \qquad \text{and} \qquad \lim_{x\to 0} \frac{\cos x - 1}{x} = 0.$$

The derivative of $\tan x$ is

$$\frac{d}{dx}(\tan x) = \sec^2 x.$$

The derivatives of the trigonometric functions are

$\frac{d}{dx}(\sin x) = \cos x$

$\frac{d}{dx}(\cos x) = -\sin x$

$\frac{d}{dx}(\tan x) = \sec^2 x$

$\frac{d}{dx}(\csc x) = -\csc x \cot x$

$\frac{d}{dx}(\sec x) = \sec x \tan x$

$\frac{d}{dx}(\cot x) = -\csc^2 x$

Example 2. *Differentiate.*

(a) $y = \sin\theta\cos\theta$

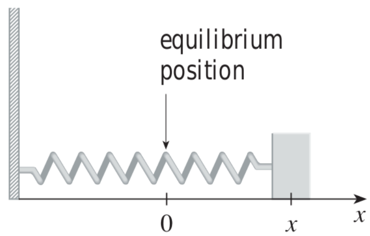
(b) $f(\theta) = \frac{\sec\theta}{1+\sec\theta}$

Example 3. *Differentiate* $f(x) = \frac{\sec x}{1+\tan x}$. *For what values of* x *does the graph of* f *have a horizontal tangent line?*

Example 4. *A mass on a spring vibrates horizontally on a smooth level surface (see the figure). Its equation of motion is* $x(t) = 8\sin t$, *where* t *is in seconds and* x *is in centimeters.*

(a) *Find the velocity and acceleration at time* t.

(b) *Find the position, velocity, and acceleration of the mass at time* $t = 2\pi/3$. *In what direction is it moving at that time?*



Example 5. *Find the 27*th *derivative of* $\cos x$.

Example 6. *A ladder 10 ft long rests against a vertical wall. Let* θ *be the angle between the top of the ladder and the wall and let* x *be the distance from the bottom of the ladder to the wall. If the bottom of the ladder slides away from the wall, how fast does* x *change with respect to* θ *when* $\theta = \frac{\pi}{3}$*?*

Example 7. *Find* $\lim_{x\to 0} \frac{\sin 7x}{4x}$.