Homework 11 Foundations of Computational Math 2 Spring 2012

Solutions will be posted Friday, 4/13/12

Problem 11.1

Consider the Runge Kutta method called the implicit midpoint rule given by:

$$\hat{y}_1 = y_{n-1} + \frac{h}{2} f_1$$
$$f_1 = f(t_{n-1} + \frac{h}{2}, \hat{y}_1)$$
$$y_n = y_{n-1} + h f_1$$

An alternate form of the the method is given by:

$$y_n = y_{n-1} + hf\big(\frac{t_n + t_{n-1}}{2}, \frac{y_n + y_{n-1}}{2}\big)$$

Show that the two forms are identical.

Problem 11.2

Consider the Runge Kutta method called the explicit trapezoidal rule given by:

$$\hat{y}_1 = y_{n-1} + hf(t_{n-1}, y_{n-1})$$
$$y_n = y_{n-1} + \frac{h}{2}\big(f(t_{n-1}, y_{n-1}) + f(t_n, \hat{y}_1)\big)$$

Show that the method has truncation error $O(h^2)$.