Homework 6 Foundations of Computational Math 2 Spring 2012

Solutions will be posted Friday, 2/17/11

Problem 6.1

Consider a minimax approximation to a function $f(x)$ on $[a, b]$. Assume that $f(x)$ is continuous with continuous first and second order derivatives. Also, assume that $f''(x) < 0$ on for $a \leq x \leq b$, i.e., f is concave on the interval.

6.1.a. Derive the equations you would solve to determine the linear minimax approximation, $p_1(x) = \alpha x + \beta$, to $f(x)$ on $[a, b]$ and describe their use to solve the problem.

6.1.b. Apply your approach to determine $p_1(x) = \alpha x + \beta$ for $f(x) = -x^2$ on $[-1, 1]$.

6.1.c. How does $p_1(x)$ relate to the quadratic monic Chebyshev polynomial $t_2(x)$?

6.1.d. Apply your approach to determine $\tilde{p}_1(x) = \tilde{\alpha}x + \tilde{\beta}$ for $f(x) = -x^2$ on $[0, 1]$.

6.1.e. How could the quadratic monic Chebyshev polynomial $t_2(y)$ on $-1 \leq y \leq 1$ be used to provide and alternative derivation of $\tilde{p}_1(x)$ on $0 \leq x \leq 1$?

6.1.f. Suppose you adapt your approach to derive a constant approximation, $p_0(x)$. What points will you use as the extrema of the error?

Problem 6.2

Show that the Chebyshev polynomial of degree n can be written

$$T_n(x) = \frac{1}{2}\left[(x + \sqrt{x^2 - 1})^n + (x - \sqrt{x^2 - 1})^n\right]$$

Problem 6.3

6.3.a. Suppose you are given an arbitrary polynomial of degree 3 or less with the form

$$p(x) = \alpha_0 + \alpha_1 x + \alpha_2 x^2 + \alpha_3 x^3.$$

Show that there are unique coefficients, γ_i, $0 \leq i \leq 3$, for $p(x)$ in the representation of the form

$$p(x) = \gamma_0 T_0(x) + \gamma_1 T_1(x) + \gamma_2 T_2(x) + \gamma_3 T_3(x)$$

where $T_i(x)$, $0 \leq i \leq 3$, are the Chebyshev polynomials.

6.3.b. Is this true for any degree n? Justify your answer.

6.3.c. Consider $T_{32}(x)$, the Chebyshev polynomial of degree 32 and $T_{51}(x)$, the Chebyshev polynomial of degree 51. What is the coefficient of x^{13} in $T_{32}(x)$? What is the coefficient of x^{20} in $T_{51}(x)$?