

MAC2312: Calculus 2 - Section 3

Test 3

July 16, 2015

Name: _____

Answer each question in the space provided on the question sheets. If you run out of space for an answer, continue on the back of the page. Credit will only be given if you clearly show all of your work. Calculators may not be used for this test.

Question	Points	Score
1	8	
2	8	
3	8	
4	8	
5	8	
6	12	
7	12	
8	16	
9	10	
10	10	
11 (bonus)	–	
Total:	100	

1. (a) [4 points] Convert the point $(4, \pi/6)$ from polar coordinates to Cartesian coordinates.

(b) [4 points] Represent using polar coordinates the point whose Cartesian coordinates are $(1, -\sqrt{3})$.

2. (a) [4 points] Find a general formula for a_n , the n^{th} term, in the sequence

$$\{a_n\}_{n=1}^{\infty} = \left\{ -\frac{1}{4}, \frac{3}{16}, -\frac{5}{64}, \frac{7}{256}, -\frac{9}{1024}, \dots \right\}$$

(b) [4 points] Find the sum of the series

$$2 + \frac{6}{5} + \frac{18}{25} + \frac{54}{125} + \frac{162}{625} + \dots$$

3. [8 points] Match the graphs of the parametric equations $x = f(t)$ and $y = g(t)$ in (a)–(d) with the parametric curves labeled I–IV by filling I, II, III, and IV in the following blanks.

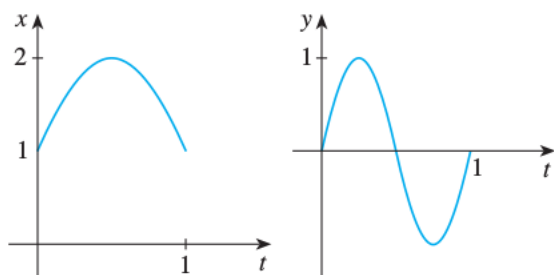
a) _____

b) _____

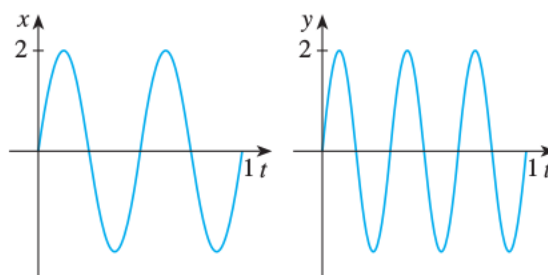
c) _____

d) _____

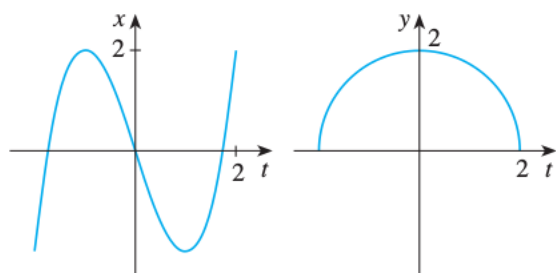
(a)



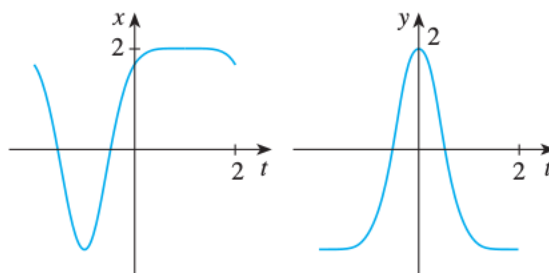
(b)



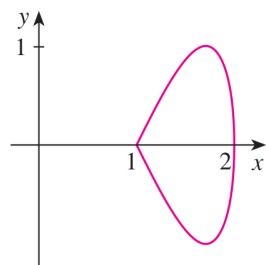
(c)



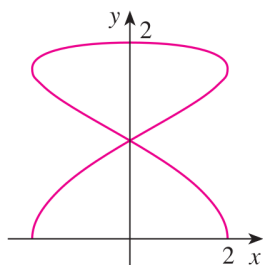
(d)



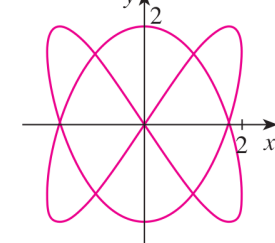
I



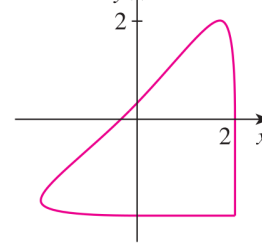
II



III



IV



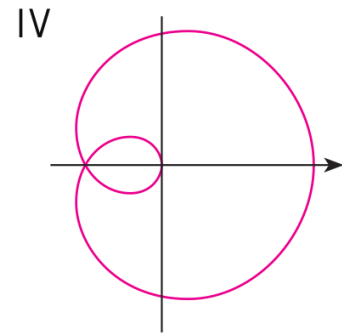
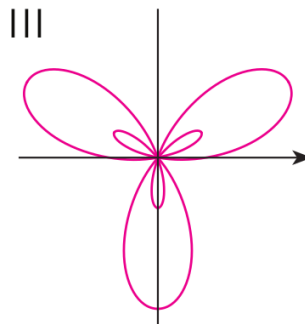
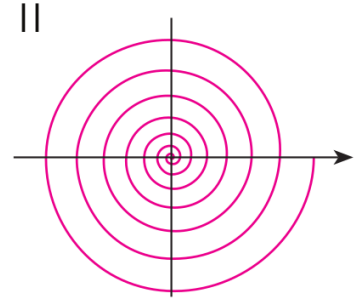
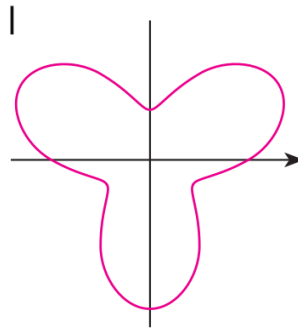
4. [8 points] Match the polar equations in (a)–(d) with the graphs labeled I–IV by filling I, II, III and IV in the following blanks.

a) $r = \theta^2$ _____

b) $r = \cos(\theta/3)$ _____

c) $r = 2 + \sin 3\theta$ _____

d) $r = 1 + 2 \sin 3\theta$ _____



5. [8 points] Set up an integral that represents the length of the curve with polar equation $r = \theta^2$ for $0 \leq \theta \leq 8\pi$.

6. [12 points] Set up an integral that represents the area of the surface obtained by rotating one arch of the cycloid with parametric equations $x = \theta - \sin \theta$, $y = 1 - \cos \theta$ about the x -axis.

7. [12 points] Set up an integral that represents the area of the region enclosed by one loop of the curve with polar equation $r = \sin(2\theta)$.

8. Consider the curve defined by the parametric equations $x = 3t^2$, $y = 6t - 2t^3$.

(a) [4 points] Find all points where the tangent is either horizontal or vertical.

(b) [4 points] For what values of t is the curve concave upward?

(c) [8 points] Find the area bounded by the y -axis and the curve for $-1 \leq t \leq 1$.

9. [10 points] Determine if the series $\sum_{n=1}^{\infty} \sqrt[n]{2}$ converges or diverges.

10. [10 points] Given that the following sequence is increasing and bounded, find its limit.

$$\{a_n\}_{n=1}^{\infty} = \left\{ 1, \sqrt{2}, \sqrt{2\sqrt{2}}, \sqrt{2\sqrt{2\sqrt{2}}}, \dots \right\}$$

11. [5 points (bonus)] Evaluate $\int_0^1 f(x) \, dx$ if

$$f(x) = \begin{cases} \frac{1}{x} & \text{if } x = 1/n \text{ for some integer } n \\ 1 & \text{otherwise.} \end{cases}$$