MAC2312: Calculus 2 - Section 3

Quiz 2: 7.2 Trigonometric Integrals

May 14, 2015

1. Evaluate $\int \sin^2(\pi x)\cos^5(\pi x)\ dx$.

   A. $\frac{1}{3\pi}\sin^3(\pi x) - \frac{1}{5\pi}\sin^5(\pi x) + C$

   B. $\frac{1}{5\pi}\sin^5(\pi x) - \frac{1}{3\pi}\sin^3(\pi x) + C$

   C. $\frac{1}{\pi}\sin(\pi x) - \frac{2}{3\pi}\sin^3(\pi x) + \frac{1}{5\pi}\sin^5(\pi x) + C$

   D. $\frac{1}{3\pi}\sin^3(\pi x) - \frac{2}{5\pi}\sin^5(\pi x) + \frac{1}{7\pi}\sin^7(\pi x) + C$

$$\begin{aligned}
\int \sin^2(\pi x)\cos^5(\pi x)\ dx &= \int \sin^2(\pi x)(\cos^2(\pi x))^2\cos(\pi x)\ dx \\
&= \int \sin^2(\pi x)(1-\sin^2(\pi x))^2\cos(\pi x)\ dx \\
&= \int u^2(1-u^2)^2\frac{1}{\pi}\ du \qquad \begin{array}{c} u = \sin(\pi x) \\ du = \pi\cos(\pi x)\ dx \\ \frac{1}{\pi}\ du = \cos(\pi x)\ dx \end{array} \\
&= \frac{1}{\pi}\int (u^2 - 2u^4 + u^6)\ du \\
&= \frac{1}{3\pi}\sin^3(\pi x) - \frac{2}{5\pi}\sin^5(\pi x) + \frac{1}{7\pi}\sin^7(\pi x) + C
\end{aligned}$$