MAC2311: Calculus 1 - Section 1

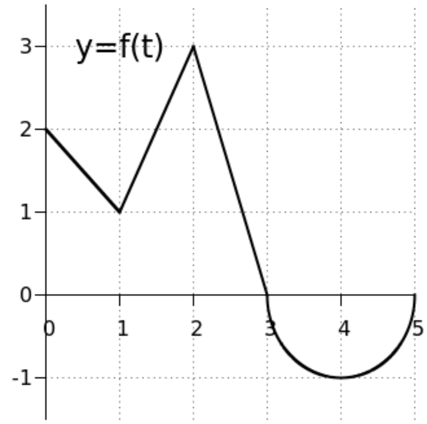
Test 4 April 16, 2015

Name: ____________________

Answer each question in the space provided on the question sheets. If you run out of space for an answer, continue on the back of the page. Credit will only be given if you clearly show all of your work. Calculators may be used for this test.

Question	Points	Score
1	7	
2	4	
3	4	
4	4	
5	4	
6	13	
7	9	
8	7	
9	6	
10	5	
Total:	63	

1. [7 points] Let $g(x) = \int_0^x f(t)\, dt$ for $0 \le x \le 5$, where f is the function whose graph is shown below. The graph of f is made up of line segments and a semicircle.



(a) [5 points] Find $g(5)$.

$$g(5) = \underbrace{\left(\frac{1}{2}\cdot 1\cdot 1 + 1\cdot 1\right)}_{\text{area under } f \text{ from 0 to 1}} + \underbrace{\left(\frac{1}{2}\cdot 1\cdot 2 + 1\cdot 1\right)}_{\text{area under } f \text{ from 1 to 2}} + \underbrace{\left(\frac{1}{2}\cdot 1\cdot 3\right)}_{\text{area under } f \text{ from 2 to 3}} - \underbrace{\frac{1}{2}\left(\pi\cdot 1^2\right)}_{\text{area above } f \text{ from 3 to 5}}$$
$$= \frac{3}{2} + 2 + \frac{3}{2} - \frac{\pi}{2} = 5 - \frac{\pi}{2}.$$

(b) [1 point] On what interval(s) is g increasing? Write your answer using interval notation.
g is increasing on $(0, 3)$

(c) [1 point] At what x-value(s) does g have an absolute maximum?
g has an absolute maximum at $x = 3$.

2. [4 points] Express the limit $\lim\limits_{n\to\infty} \sum\limits_{i=1}^{n} x_i \ln(2+x_i^2)\Delta x$ as a definite integral on the interval $[2, 3]$. Do not evaluate.

$$\int_2^3 x \ln(2 + x^2)\, dx$$

3. [4 points] Find the derivative of the function $g(x) = \int_{x^2}^{\pi} \sqrt{5+\sec(3t)}\; dt$.

$$g(x) = \int_{x^2}^{\pi} \sqrt{5+\sec(3t)}\; dt = -\int_{\pi}^{x^2} \sqrt{5+\sec(3t)}\; dt$$

By Part 1 of the Fundamental Theorem of Calculus and the chain rule,

$$\begin{aligned} g'(x) &= -\sqrt{5+\sec(3x^2)}\frac{d}{dx}(x^2) \\ &= -2x\sqrt{5+\sec(3x^2)} \end{aligned}$$

4. [4 points] Estimate the area under the graph of $f(x) = 1 + 2x^2$ from $x = -2$ to $x = 4$ using the Midpoint Rule with three rectangles.

With $n = 3$, $\Delta x = \frac{4-(-2)}{3} = 2$. The subintervals are $[-2, 0]$, $[0, 2]$, and $[2, 4]$ with midpoints $\bar{x}_1 = -1$, $\bar{x}_2 = 1$, and $\bar{x}_3 = 3$.

$$\begin{aligned} M_3 &= \sum_{k=1}^{3} f(\bar{x}_k)\Delta x \\ &= 2(3+3+19) \\ &= 50 \end{aligned}$$

5. [4 points] Use Newton's Method with initial approximation $x_1 = 5$ to find x_2, the second approximation to the root of the equation $x^2 - 5 = 0$.

$f(x) = x^2 - 5$, $f'(x) = 2x$. Newton's recursion is

$$\begin{aligned} x_{n+1} &= x_n - \frac{f(x_n)}{f'(x_n)} \\ x_{n+1} &= x_n - \frac{x_n^2 - 5}{2x_n} \end{aligned}$$

so the second approximation is

$$\begin{aligned} x_2 &= x_1 - \frac{x_1^2 - 5}{2x_1} \\ &= 5 - \frac{5^2 - 5}{2 \cdot 5} = 3 \end{aligned}$$

6. [13 points] Find the general indefinite integrals.

(a) [4 points] $\int \left(5x^2 + 8 + \frac{3}{x^2+1}\right)\, dx$

$$\int \left(5x^2 + 8 + \frac{3}{x^2+1}\right)\, dx = \frac{5}{3}x^3 + 8x + 3\arctan x + C$$

(b) [4 points] $\int \frac{4\sqrt{x}+3}{x}\, dx$

$$\begin{aligned}\int \frac{4\sqrt{x}+3}{x}\, dx &= \int \left(4x^{-1/2} + \frac{3}{x}\right)\, dx \\ &= 4 \cdot \frac{x^{1/2}}{1/2} + 3\ln|x| + C \\ &= 8\sqrt{x} + 3\ln|x| + C\end{aligned}$$

(c) [5 points] $\int e^{\cos 17x} \sin 17x\, dx$

Let $u = \cos 17x$, then $-\frac{1}{17}du = \sin 17x$ so

$$\begin{aligned}\int e^{\cos 17x} \sin 17x\, dx &= -\frac{1}{17}\int e^u\, du \\ &= -\frac{1}{17}e^u + C \\ &= -\frac{1}{17}e^{\cos 17x} + C\end{aligned}$$

7. [9 points] Evaluate the following definite integrals.

(a) [3 points] $\int_0^{\pi/4} \sec^2 x \; dx$

$$\begin{aligned} \int_0^{\pi/4} \sec^2 x \; dx &= [\tan x]_{x=0}^{\pi/4} \\ &= \tan \frac{\pi}{4} - \tan 0 = 1 \end{aligned}$$

(b) [6 points] $\int_7^8 x\sqrt{x-7} \; dx$

Let $u = x - 7$, so $du = dx$ and $x = u + 7$. When $x = 7$, $u = 0$, and when $x = 8$, $u = 1$, so

$$\begin{aligned} \int_7^8 x\sqrt{x-7} \; dx &= \int_0^1 (u+7)\sqrt{u} \; du \\ &= \int_0^1 (u^{3/2} + 7u^{1/2}) \; du \\ &= \left[\frac{2}{5}u^{5/2} + \frac{14}{3}u^{3/2}\right]_{u=0}^1 \\ &= \frac{2}{5} + \frac{14}{3} = \frac{76}{15} \end{aligned}$$

8. [7 points] A ball is thrown upward with a speed of 64 ft/s from the edge of a cliff 80 ft above the ground.

(a) [4 points] Find its height above the ground t seconds later.
(Hint: the *downward* acceleration due to gravity is 32 ft/s^2.)

$$\begin{aligned} a(t) &= -32 \\ v(t) &= -32t + C \\ h(t) &= -16t^2 + Ct + D \end{aligned}$$

$v(0) = C = 64$ and $h(0) = D = 80$, so $h(t) = -16t^2 + 64t + 80$ ft.

(b) [1 point] When does it reach its maximum height?

$$\begin{aligned} v(t) = -32t + 64 &= 0 \\ t &= 2 \text{ s} \end{aligned}$$

(c) [2 points] When does it hit the ground?

$$\begin{aligned} h(t) = -16t^2 + 64t + 80 &= 0 \\ t^2 - 4t - 5 &= 0 \\ (t+1)(t-5) &= 0 \\ t &= 5 \text{ s} \end{aligned}$$

9. [6 points] Find the area of the region enclosed by the curves $x = 7y^2$ and $x = 1 + 6y^2$.

The two curves intersect when $7y^2 = 1 + 6y^2 \Rightarrow y = \pm 1$. Integrating with respect to y

$$\begin{aligned} A &= \int_{-1}^{1} (1 + 6y^2 - 7y^2)\ dy \\ &= \int_{-1}^{1} (1 - y^2)\ dy \\ &= \left[y - \frac{y^3}{3} \right]_{y=-1}^{1} \\ &= \left[\left(1 - \frac{1}{3}\right) - \left(-1 + \frac{1}{3}\right) \right] = \frac{4}{3} \end{aligned}$$

10. [5 points] Given that $\int_3^0 f(s)\ ds = -7$ and $\int_0^5 f(t)\ dt = 9$, find $\int_3^5 [2f(x) + 1]\ dx$.

Since $\int_3^0 f(s)\ ds = -7$, $\int_0^3 f(s)\ ds = 7$, and since $\int_0^5 f(x)\ dx = \int_0^3 f(x)\ dx + \int_3^5 f(x)\ dx$,

$$\begin{aligned} \int_3^5 f(x)\ dx &= \int_0^5 f(x)\ dx - \int_0^3 f(x)\ dx \\ &= 9 - 7 = 2, \end{aligned}$$

so

$$\begin{aligned} \int_3^5 [2f(x) + 1]\ dx &= 2\int_3^5 f(x)\ dx + \int_3^5\ dx \\ &= 2(2) + (5 - 3) = 6 \end{aligned}$$