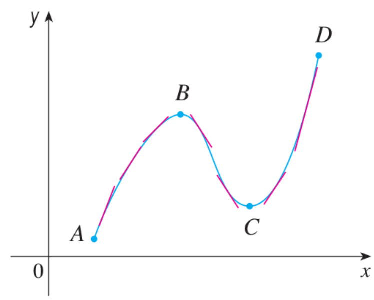
4.3: How Derivatives Affect the Shape of a Graph

What does f' say about f?

Increasing/Decreasing Test:

(a) If $f'(x) > 0$ on an interval, then f is increasing on that interval.

(b) If $f'(x) < 0$ on an interval, then f is decreasing on that interval.



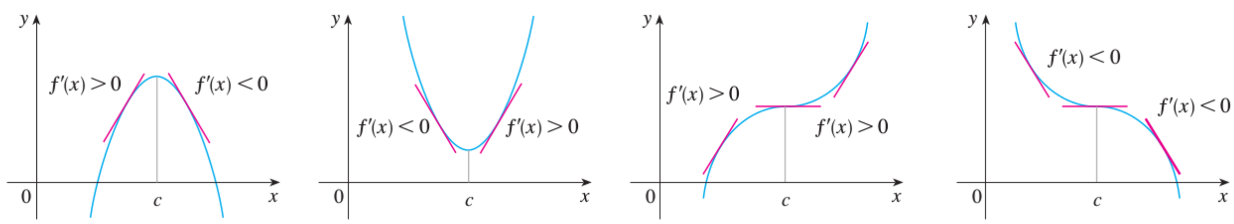
Example 1. *Find where* $f(x) = 3x^4 - 4x^3 - 12x^2 + 5$ *is increasing and where it is decreasing.*

The First Derivative Test: Suppose that c is a critical number of a continuous function f.

(a) If f' changes from positive to negative at c, then f has a local maximum at c.

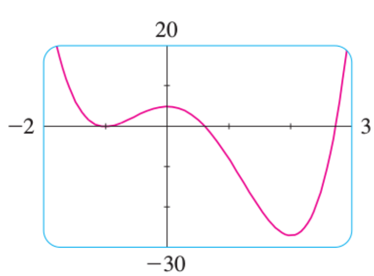
(b) If f' changes from negative to positive at c, then f has a local minimum at c.

(c) If f' does not change sign at c, then f has no local maximum or minimum at c.

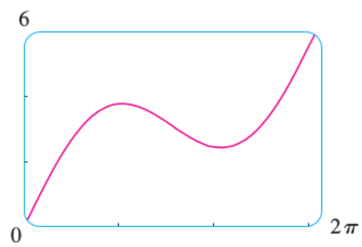


(a) Local maximum (b) Local minimum (c) No maximum or minimum (d) No maximum or minimum

Example 2. *Find the local maximum and minimum values of the function in Example 1.*

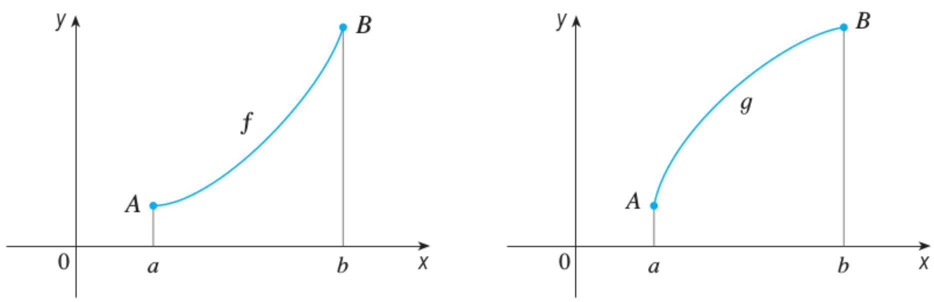


Example 3. *Find the local maximum and minimum values of the function* $g(x) = x + 2\sin x$ *on the interval* $[0, 2\pi]$.

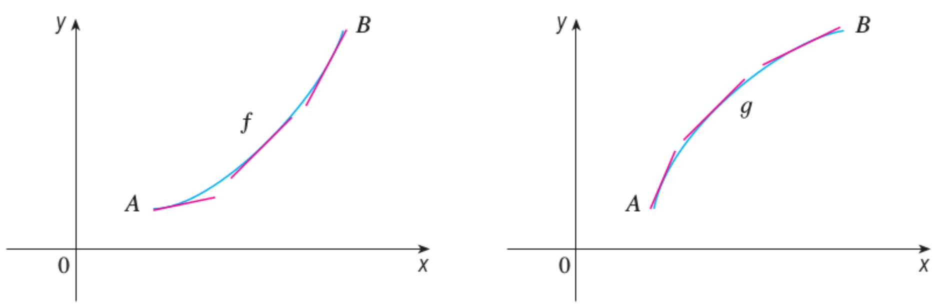


What does f'' say about f?

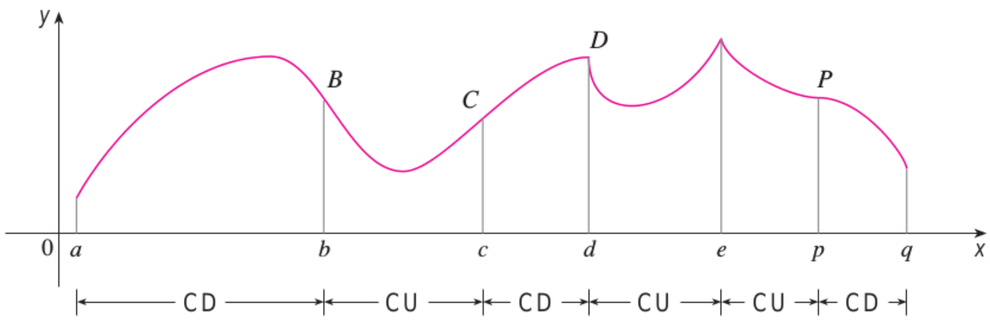
If the graph of f lies above all of its tangent lines on an interval, then it is **concave upward** on that interval. If the graph of f lies below all of its tangent lines on an interval, then it is **concave downward** on that interval.



(a) (b)



(a) Concave upward (b) Concave downward

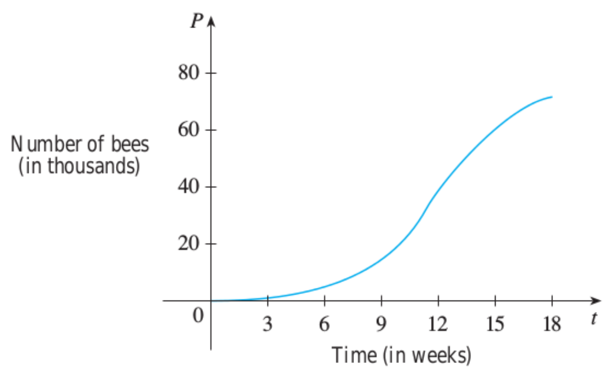


Concavity Test:

(a) If $f''(x) > 0$ for all x on an interval, then f is concave up on that interval.

(b) If $f''(x) < 0$ for all x on an interval, then f is concave down on that interval.

Example 4. *The following figure shows the population $P = f(t)$ for honeybees raised in an apiary. How does the rate of population increase change over time? When is the rate highest? Over what intervals is the graph concave upward or concave downward?*



A point on a curve $y = f(x)$ is called an **inflection point** if f is continuous at the point and the curve changes from concave upward to concave downward or concave downward to concave upward at the point.

Example 5. *Sketch a possible graph of a function f that satisfies the following conditions:*

(i) $f'(x) > 0$ *on* $(-\infty, 1)$, $f'(x) < 0$ *on* $(1, \infty)$

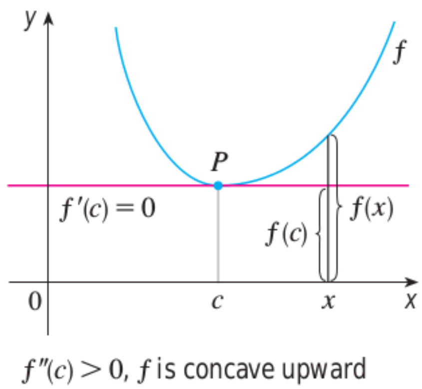
(ii) $f''(x) > 0$ *on* $(-\infty, -2)$ *and* $(2, \infty)$, $f''(x) < 0$ *on* $(-2, 2)$

(iii) $\lim_{x\to-\infty} f(x) = -2$ *and* $\lim_{x\to\infty} f(x) = 0$

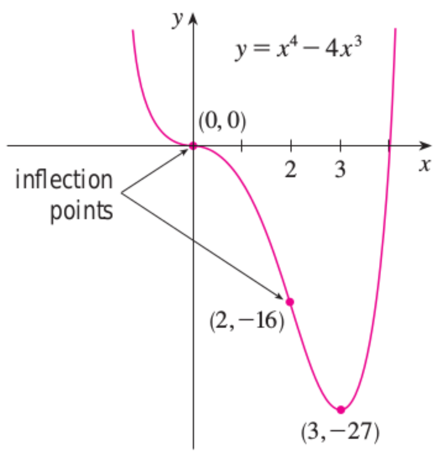
The Second Derivative Test: Suppose f'' is continuous near c

(a) If $f'(c) = 0$ and $f''(c) > 0$, then f has a local minimum at c.

(b) If $f'(c) = 0$ and $f''(c) < 0$, then f has a local maximum at c.



Example 6. *Discuss the curve $y = x^4 - 4x^3$ with respect to concavity, points of inflection, and local maxima and minima. Use the information to sketch the curve.*



Example 7. *Sketch the graph of the function $f(x) = x^{2/3}(6 - x)^{1/3}$.*