# 4.6: Compound Interest

## Supplementary Notes

*Interest* is money paid to borrow money. *Principal* ($P$) is the amount borrowed. Annual *interest rate* ($r$) is the percentage of principal paid per year. The amount owed ($A$) after $t$ years may be compounded *periodically* $n$ times per year, e.g.

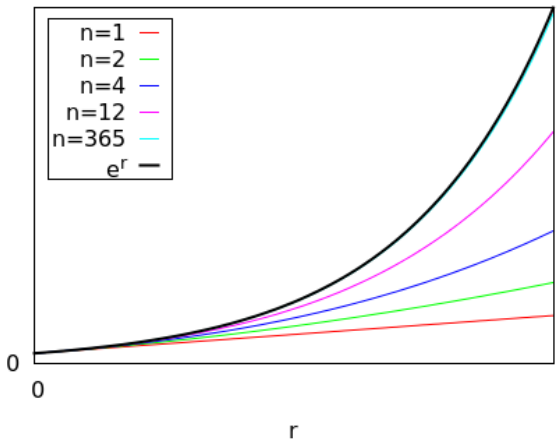
| | $n$ |
|---|---|
| annually | 1 |
| semi-annually | 2 |
| quarterly | 4 |
| monthly | 12 |
| daily | 365 |

or compounded *continuously*.

Periodic Compounding: The amount $A$ after $t$ years due to a principal $P$ invested at an annual interest rate $r$ compounded $n$ times per year is

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

Below are the graphs of $\left(1 + \frac{r}{n}\right)^n$ for increasing $n$ and the graph of $e^r$. This illustrates that $\left(1 + \frac{r}{n}\right)^n \to e^r$ as $n \to \infty$.




Continuous Compounding: The amount $A$ after $t$ years due to a principal $P$ invested at an annual interest rate $r$ compounded continuously is

$$A = Pe^{rt}$$

Effective interest rate: The *effective interest rate* $r_e$ is the interest rate of an annually compounded investment that yields the same return as a periodically compounded investment with interest rate $r$.

$$r_e = \left(1 + \frac{r}{n}\right)^n - 1$$

## Exercises

1. If an investment pays 4.5% compounded quarterly, how much should you deposit now to have $5,000 in four years?

2. What interest rate will take an initial investment of $20,000 to $30,000 in 6 years with annual compounding?

3. How long will it take money to double if it is invested at 6% compounded monthly?

4. What interest rate will triple an initial investment in 4 years if compounded annually?

5. A note will pay $11,857 at maturity in 6 years. How much is the note worth now, assuming continuous compounding at 5.8%?

6. How many years will it take for an investment of $30,000 to grow to $80,000? Assume a rate of interest of 4% compounded continuously.

7. What interest rate, compounded continuously, will take an initial investment of $15,000 to $25,000 in 3 years?

8. How long will it take money to double if its invested at 11% compounded continuously?

9. Find the effective rate of interest for 9% compounded monthly.