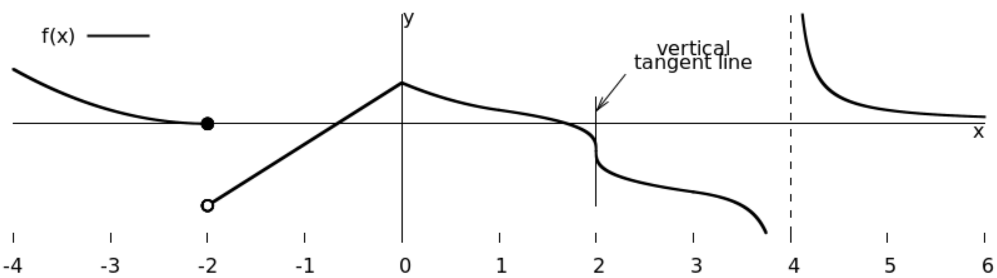
MAC2311: Calculus 1 - Section 1

Quiz 2: Sections 2.8, 3.1-3.3 — February 12, 2015

Name: ______________________________

1. [8 points] Use the following graph of f to answer the questions below it.



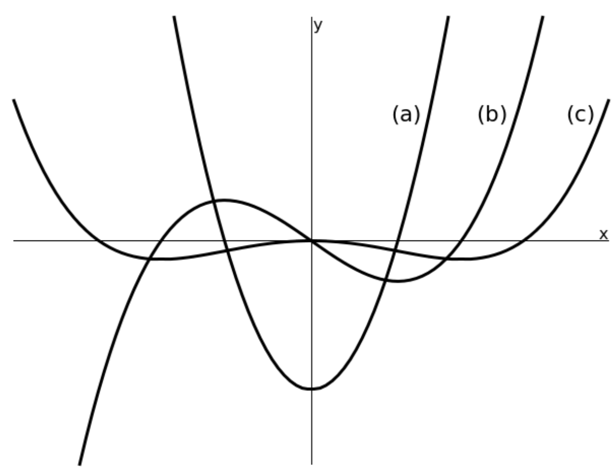
(a) [4 points] List all x-values at which f is not differentiable: $x = -2$, 0, 2, and 4

(b) [4 points] For each x-value that you listed in part (a), state why f is not differentiable.

f is not differentiable at...

- $x = -2$ because f is not continous at $x = -2$.
- $x = 0$ because f has a "corner" or "kink" $x = 0$.
- $x = 2$ because f has a vertical tangent line at $x = 2$.
- $x = 4$ because f is not continous at $x = 4$.

2. [3 points] The following figure shows the graphs of f, f', and f''.



Complete the following statements by filling in one of f, f', or f'' in each blank.

(a) is the graph of f''

(b) is the graph of f'

(c) is the graph of f

3. [4 points] Let $f(x) = (x^2 - x)e^x$.

(a) [3 points] Differentiate $f(x)$.

Using the product rule, power rule, and $\frac{d}{dx}(e^x) = e^x$,

$$\begin{aligned} f'(x) &= (x^2 - x)\frac{d}{dx}(e^x) + e^x\frac{d}{dx}(x^2 - x) \\ &= (x^2 - x)e^x + (2x - 1)e^x \\ &= (x^2 + x - 1)e^x \end{aligned}$$

(b) [1 point] Find the slope of the line tangent to f at $x = 1$.

The slope of the line tangent to f at $x = 1$ is

$$f'(1) = (1^2 + 1 - 1)e^1 = e$$

4. [5 points] Consider taking the derivative of $\sec x$ by first expressing $\frac{d}{dx}(\sec x)$ as a quotient, then using the quotient rule.

(a) [1 point] Select the equation that correctly expresses $\frac{d}{dx}(\sec x)$ as a quotient.

A. $\frac{d}{dx}(\sec x) = \frac{d}{dx}\left(\frac{1}{\sin x}\right)$
B. $\frac{d}{dx}(\sec x) = \frac{d}{dx}\left(\frac{1}{\cos x}\right)$
C. $\frac{d}{dx}(\sec x) = \frac{d}{dx}\left(\frac{\sin x}{\cos x}\right)$
D. $\frac{d}{dx}(\sec x) = \frac{d}{dx}\left(\frac{\cos x}{\sin x}\right)$

(b) [4 points] Prove that $\frac{d}{dx}(\sec x) = \sec x \tan x$ by using the quotient rule to evaluate the righthand side of your answer to part (a).

From part (a), $\frac{d}{dx}(\sec x) = \frac{d}{dx}\left(\frac{1}{\cos x}\right)$. By the quotient rule, and using $\frac{d}{dx}(1) = 0$ and $\frac{d}{dx}(\cos x) = -\sin x$,

$$\begin{aligned} \frac{d}{dx}\left(\frac{1}{\cos x}\right) &= \frac{(\cos x)\frac{d}{dx}(1) - (1)\frac{d}{dx}(\cos x)}{\cos^2 x} \\ &= \frac{(\cos x)(0) - (-\sin x)}{\cos^2 x} \\ &= \frac{\sin x}{\cos^2 x} \\ &= \frac{1}{\cos x} \cdot \frac{\sin x}{\cos x} = \sec x \tan x. \end{aligned}$$