3.1: Derivatives of Polynomial and Exponential Functions

Constant Functions

The derivative of a constant function $f(x) = c$ is

$$\frac{d}{dx}(c) = 0.$$

Example 1. *Prove that $\frac{d}{dx}(c) = 0$ using (a) the definition of f' and (b) the geometric interpretation of f'.*

Power Functions

Power Rule: The derivative of a power function $f(x) = x^n$ is

$$\frac{d}{dx}(x^n) = nx^{n-1}.$$

Example 2.

(a) *Prove that $\frac{d}{dx}(x) = 1$ using the geometric interpretation of f'.*

(b) *Prove that $\frac{d}{dx}(x^n) = nx^{n-1}$ where $n > 1$ is an integer using the definition of f'.*

Example 3. *Complete the following equalities*

(a) *If $f(x) = x^6$, then $f'(x) =$*

(b) *If $y = x^{1000}$, then $y' =$*

(c) *If $y = t^4$, then $\frac{dy}{dt} =$*

(d) $\frac{d}{dr}(r^3) =$

Example 4. *Differentiate*

(a) $f(x) = \frac{1}{x^2}$

(b) $y = \sqrt[3]{x^2}$

Example 5. *Find equations of the tangent and normal line to the curve $y = x\sqrt{x}$ at the point $(1, 1)$.*

Differentiation Rules

If f and g are differentiable functions and c is a constant, then the following are true

- $\frac{d}{dx}[cf(x)] = c\frac{d}{dx}f(x)$
- $\frac{d}{dx}[f(x) + g(x)] = \frac{d}{dx}f(x) + \frac{d}{dx}g(x)$
- $\frac{d}{dx}[f(x) - g(x)] = \frac{d}{dx}f(x) - \frac{d}{dx}g(x)$

Note that, in general, $\frac{d}{dx}[f(x)g(x)] \neq \frac{d}{dx}f(x) \cdot \frac{d}{dx}g(x)$ and $\frac{d}{dx}\frac{f(x)}{g(x)} \neq \frac{\frac{d}{dx}f(x)}{\frac{d}{dx}g(x)}$. We will see how to differentiate products $f(x)g(x)$ and quotients $\frac{f(x)}{g(x)}$ in the next section.

Example 6. *Prove the first two differentiation rules above using the definition of the derivative.*

Example 7. *Differentiate* $x^8 + 12x^5 - 4x^4 + 10x^3 - 6x + 5$.

Example 8. *Find the points on the curve* $y = x^4 - 6x^2 + 4$ *where the tangent line is horizontal.*

Example 9. *The equation of motion of a particle is* $s = 2t^3 - 5t^2 + 3t + 4$, *where* s *is measured in centimeters and* t *in seconds. Find the acceleration as a function of time. What is the acceleration after 2 seconds?*

Exponential Functions

Example 10. *Show that the derivative of the exponential function* $f(x) = a^x$ *is* $f'(x) = f'(0)a^x$.

e is the number such that

$$\lim_{h\to 0} \frac{e^h - 1}{h} = 1 \quad \text{or} \quad e = \lim_{h\to 0}(1+h)^{1/h}.$$

The derivative of the natural exponential function $f(x) = e^x$ is

$$\frac{d}{dx}(e^x) = e^x.$$

This means that the slope of the line tangent to each point (x, e^x) on the graph $y = e^x$ is the y-coordinate of the point.

Example 11. *If* $f(x) = e^x - x$, *find* f' *and* f''.

Example 12. *At what point on the curve* $y = e^x$ *is the tangent line parallel to the line* $y = 2x$*?*