Homework 1 Foundations of Computational Math 2 Spring 2012

Solutions will be posted Friday, 1/20/12

Problem 1.1

Consider the data points

$$(x, y) = \{(0, 2),\ \ (0.5, 5),\ \ (1, 8)\}$$

Write the interpolating polynomial in both Lagrange and Newton form for the given data.

Problem 1.2

Use this divided difference table for this problem. Justify all of your answers.

i	0		1		2		3		4		5
x_i	-1		0		2		4		5		6
f_i	13		2		-14		18		67		91
$f[-,-]$		-11		-8		16		49		24	
$f[-,-,-]$			1		6		11		$-25/2$		
$f[-,-,-,-]$				1		1		$-47/8$			
$f[-,-,-,-,-]$					0		$-55/48$				
$f[-,-,-,-,-,-]$						$-55/336$					

1.2.a

Use the divided difference information about the unknown function $f(x)$ and consider the unique polynomial, denoted $p_{1,5}(x)$, that interpolates the data given by pairs (x_1, f_1), (x_2, f_2), (x_3, f_3), (x_4, f_4) , and (x_5, f_5). Use two different sets of divided differences to express $p_{1,5}(x)$ in two distinct forms.

1.2.b

What is the significance of the value of 0 for $f[x_0, x_1, x_2, x_3, x_4]$?

1.2.c

Denote by $p_{0,4}(x)$, the unique polynomial, that interpolates the data given by pairs (x_0, f_0), (x_1, f_1), (x_2, f_2), (x_3, f_3), and (x_4, f_4) and recall the definition of $p_{1,5}(x)$ from part (a). Use the divided difference information about the unknown function $f(x)$ to derive error estimates for $f(x) - p_{1,5}(x)$ and $f(x) - p_{0,4}(x)$ for any $x_0 \le x \le x_5$.

Problem 1.3

Assume you are given distinct points $x_0, \ldots, x_n$ and, $p_n(x)$, the interpolating polynomial defined by those points for a function f.

1.3.a. If $p_n(x) = \sum_{i=0}^{n} f(x_i)\ell_i(x)$ is the Lagrange form show that

$$\sum_{i=0}^{n} \ell_i(x) = 1$$

1.3.b. Assume $x \neq x_i$ for $0 \le i \le n$ and show that the divided difference $f[x_0, \ldots, x_n, x]$ satisfies

$$f[x_0, \ldots, x_n, x] = \sum_{i=0}^{n} \frac{f[x, x_i]}{\prod_{j=0, j\neq i}^{n} (x_i - x_j)}$$

Problem 1.4

Text exercise 8.10.1 on page 375

Problem 1.5

Text exercise 8.10.3 on page 376

Problem 1.6

Text exercise 8.10.4 on page 376