3.7: Rational Functions

Supplementary Notes

$$f(x) = \frac{g(x)}{h(x)}$$

where g and h are polynomial functions.

The *domain* of $f(x)$ is all real numbers x except the zeros of $h(x)$.

- If $x = z$ is a real zero of $h(x)$ but not $g(x)$, then the graph of $f(x)$ has a *vertical asymptote* at $x = z$.

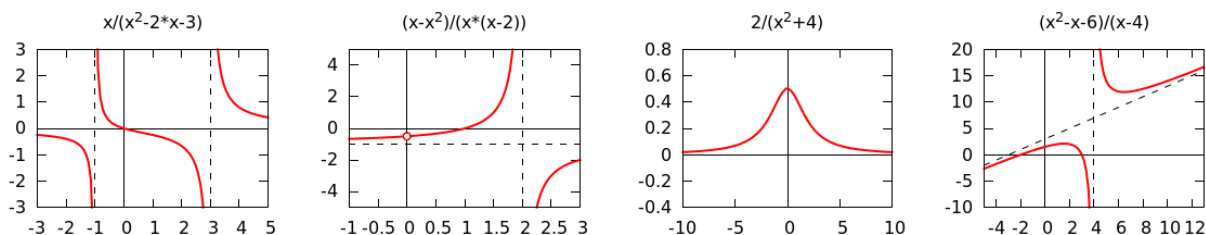- If $x = z$ is a real zero of $h(x)$ and $g(x)$, then the graph of $f(x)$ has a *jump discontinuity* at $x = z$.

For

$$f(x) = \frac{a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0}{b_m x^m + b_{m-1} x^{m-1} + \cdots + b_1 x + b_0},$$

- if $n < m$, the graph of f has a *horizontal asymptote* $y = 0$.

- if $n = m$, the graph of f has a *horizontal asymptote* $y = \frac{a_n}{b_m}$.

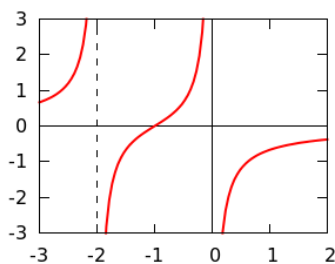- if $n = m+1$, the graph of f has an *oblique asymptote* $q(x)$, the quotient of $g(x) = q(x)h(x) + r(x)$.

Below are the graphs of

$$y = \frac{x}{x^2 - 2x - 3}, \qquad y = \frac{x - x^2}{x(x-2)}, \qquad y = \frac{2}{x^2 + 4}, \text{ and } \qquad y = \frac{x^2 - x - 6}{x - 4}$$
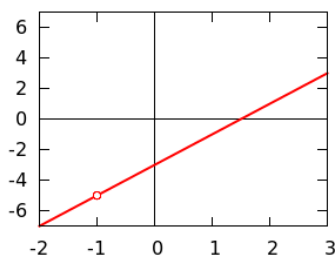
Exercises

1. Select the statement that is false for $f(x) = -\frac{(x+1)(x+2)}{x^2+2x}$

   A. The domain of f is $(-\infty, -2) \cup (-2, 0) \cup (0, \infty)$

   B. The graph of f has one vertical asymptote at $x = 0$

   C. The $x-$intercept of the graph is -1.

   D. The graph of f has one horizontal asymptote at $y = -1$.

   E. The $y-$intercept of the graph is -1

2. Select the equation of the following graph

   A $y = -\frac{x+1}{x(x+2)}$

   B $y = \frac{x+1}{x(x+2)}$

   C $y = \frac{1-x}{x(x-2)}$

   D $y = \frac{x-1}{x(x-2)}$

3. Select the equation of the following graph

   A $y = -\frac{2x^2-x-3}{x+1}$

   B $y = 2x - 3$

   C $y = \frac{2x^2-x-3}{x(x+1)}$

   D $y = \frac{2x^2-x-3}{x-1}$