MAC2312: Calculus 2 - Section 3

Quiz 9: 8.3 Applications to Physics and Engineering — June 17, 2015

1. Find the center of mass (centroid) of the region bounded by $y = x$, $y = 0$, and $x = 1$.

   A. $\left(\frac{1}{2}, \frac{1}{2}\right)$

   B. $\left(\frac{1}{2}, \frac{1}{3}\right)$

   C. $\left(\frac{2}{3}, \frac{1}{2}\right)$

   D. $\left(\frac{2}{3}, \frac{1}{3}\right)$

   $f(x) = x$, $a = 0$, $b = 1$, and the center of mass is given by $(\bar{x}, \bar{y}) = \left(\frac{1}{A}\int_a^b xf(x)\,dx, \frac{1}{A}\int_a^b \frac{1}{2}[f(x)]^2\,dx\right)$.

$$
\begin{aligned}
A = \int_a^b f(x)\,dx &= \int_0^1 x\,dx = \frac{1}{2} \\
\bar{x} &= \frac{1}{A}\int_a^b xf(x)\,dx \\
&= 2\int_0^1 x^2\,dx = \frac{2}{3} \\
\bar{y} &= \frac{1}{A}\int_a^b \frac{1}{2}[f(x)]^2\,dx \\
&= \int_0^1 x^2\,dx = \frac{1}{3}
\end{aligned}
$$

   so the center of mass is $\left(\frac{2}{3}, \frac{1}{3}\right)$.