MAC2311: Calculus 1 - Section 1

Final Exam Review — April 20-24, 2015

1. Evaluate the following limits
   (a) $\lim_{u\to -2} \sqrt{u^4+3u+6}$
   (b) $\lim_{h\to 0} \dfrac{\sqrt{9+h}-3}{h}$
   (c) $\lim_{t\to\infty} \dfrac{t-t\sqrt{t}}{2t^{3/2}+3t-5}$
   (d) $\lim_{x\to -\infty} \dfrac{\sqrt{9x^6-x}}{x^3+1}$
   (e) $\lim_{x\to\infty} \arctan(e^x)$
2. If $2x \le g(x) \le x^4 - x^2 + 2$ for all x, evaluate $\lim_{x\to 1} g(x)$.
3. Find an equation of the tangent line to the curve at the given point
   (a) $y = 4x - x^2$; $(1, 3)$
   (b) $y = \dfrac{2x+1}{x+2}$; $(1, 1)$
   (c) $y = x^4 + 2e^x$; $(0, 2)$
4. Find the points on the curve $y = 2x^3 + 3x^2 - 12x + 1$ where the tangent is horizontal.
5. At what point on the curve $y = 1 + 2e^x - 3x$ is the tangent line parallel to the line $3x - y = 5$?
6. Find the n^{th} derivative of x^n.
7. Differentiate.
   (a) $y = \dfrac{x^2+4x+3}{\sqrt{x}}$
   (b) $H(x) = (x + x^{-1})^3$
   (c) $y = e^p(p + p\sqrt{p})$
   (d) $y = \dfrac{t^2+2}{t^4-3t^2+1}$
   (e) $F(t) = (3t-1)^4(2t+1)^{-2}$
   (f) $f(s) = \sqrt{\dfrac{s^2+1}{s^2+4}}$
   (g) $y = \sin^2(e^{\sin^2 t})$
8. Find $\frac{dy}{dx}$ by implicit differentiation.
   (a) $x^4(x+y) = y^2(3x-y)$
   (b) $e^{x/y} = x - y$
9. Find an equation of the tangent line to the curve at the given point
   (a) $x^2 + 2xy - y^2 + x = 2$ (hyperbola); $(1, 2)$
   (b) $x^2 + xy + y^2 = 3$ (ellipse); $(1, 1)$
10. Differentiate $(\ln x)^{\cos x}$ using logarithmic differentiation.
11. (a) Show that of all the rectangles with constant area, the one with the smallest perimeter is a square.
    (b) Show that of all the rectangles with a constant perimeter, the one with the largest area is a square.

12. Differentiate $g(x) = \displaystyle\int_x^{2x} u(u-1)\ du$.

13. Find the general indefinite integrals

    (a) $\displaystyle\int (u+4)(2u+1)\ du$

    (b) $\displaystyle\int (\theta - \csc\theta\cot\theta)\ d\theta$

    (c) $\displaystyle\int \frac{t^2-1}{t^4-1}\ dt$

14. Evaluate the integrals

    (a) $\displaystyle\int_{\pi/4}^{\pi/2} \cot x\ dx$

    (b) $\displaystyle\int_e^{e^4} \frac{dx}{x\sqrt{\ln x}}$

15. Find the area of the region bounded by the curves $y = 6x - x^2$ and $y = x^3$.

16. Find the volume of the solid obtained by rotating the region bounded by $y = x^2$ and $y = \sqrt{x}$ about the y-axis.

17. Use the method of cylindrical shells to find the volume generated by rotating the region bounded by $y = \sqrt[3]{x}$, $y = 0$, and $x = 1$ about the y-axis.

18. Find the average value of $f(x) = 4x - x^2$ on the interval $[0, 4]$.