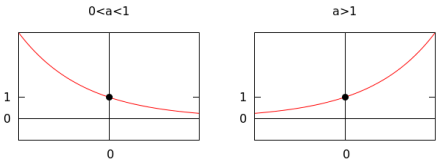
Unit II

Supplementary Notes

4.2 Exponential Functions

$$f(x) = a^x \qquad (a > 0, a \neq 1)$$

- domain: $(-\infty, \infty)$
- range: $(0, \infty)$
- $y-$intercept: 1
- horiz. asymp.: $y = 0$ ($x-$axis)
- $\begin{cases} \text{decreasing} & \text{if } 0 < a < 1 \\ \text{increasing} & \text{if } a > 1 \end{cases}$



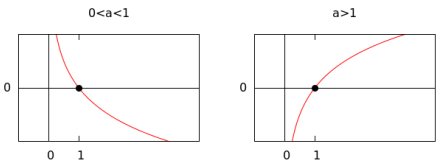
Laws of Exponents	
$a^s \cdot a^t = a^{s+t}$	$(a^s)^t = a^{s \cdot t}$
$(a \cdot b)^s = a^s \cdot b^s$	$a^{-s} = \left(\frac{1}{a}\right)^s = \frac{1}{a^s}$
$1^s = 1$	$a^0 = 1$

Reflection and Translation ($h, k > 0$), to obtain the graph of		
a^{-x-h}	translate the graph of a^x then reflect the graph of a^{x-h}	rightward h units about the $y-axis$
a^{-x+h}	translate the graph of a^x then reflect the graph of a^{x+h}	leftward h units about the $y-axis$
$-a^x + k$	reflect the graph of a^x then translate the graph of $-a^x$	about the $x-$axis upward k units
$-a^x - k$	reflect the graph of a^x then translate the graph of $-a^x$	about the $x-$axis downward k units

4.3 Logarithmic Functions

$$f(x) = \log_a x \qquad (a > 0, a \neq 1)$$

- domain: $(0, \infty)$
- range: $(-\infty, \infty)$
- $x-$intercept: 1
- vert. asymp.: $x = 0$ ($y-$axis)
- $\begin{cases} \text{decreasing} & \text{if } 0 < a < 1 \\ \text{increasing} & \text{if } a > 1 \end{cases}$



Special Logarithms	
$\log x = \log_{10} x$	$\ln x = \log_e x$

4.4 Properties of Logarithms ($a, b, m, n > 0$, $a, b \neq 1$, real number p)

- $\log_a 1 = 0$
- $\log_a a = 1$
- $\log_a(mn) = \log_a m + \log_a n$
- $\log_a\left(\frac{m}{n}\right) = \log_a m - \log_a n$
- $\log_a n^p = p \log_a n$
- $\log_a n = \frac{\log_b n}{\log_b a}$

4.5 Log. and Exponential Equations ($a > 0$, $a \neq 1$, real numbers $s, t > 0$)

- $x = \log_a y \Leftrightarrow y = a^x$
- $a^s = a^t \Leftrightarrow s = t$
- $\log_a s = \log_a t \Leftrightarrow s = t$

4.6 Compound Interest

- Periodic: $A = P(1 + \frac{r}{n})^{nt}$
- Continuous: $A = Pe^{rt}$
- Effective interest rate: $r_e = \left(1 + \frac{r}{n}\right)^n - 1$

A: future amount (\$)
P: initial amount a.k.a. principal (\$)
r: annual interest rate (%)
n: periods per year
t: time (years)

4.7 Exponential Growth and Decay

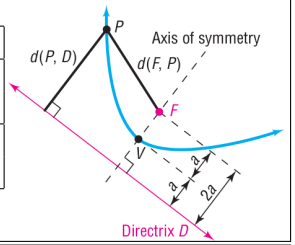
Exponential law a.k.a Law of uninhibited growth ($k > 0$) or decay ($k < 0$):

- $A(t) = A_0 e^{kt}$ where $A_0 = A(0)$ $\qquad (k \neq 0)$

9.2 The Parabola

- V: (h, k)
- $d(V, F) = d(V, D) = a$

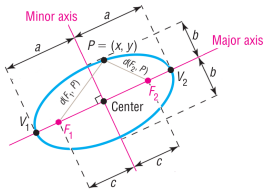
Equation	Opens	F	D
$(x-h)^2 = 4a(y-k)$	up	$(h, k+a)$	$y = k - a$
$(x-h)^2 = -4a(y-k)$	down	$(h, k-a)$	$y = k + a$
$(y-k)^2 = 4a(x-h)$	right	$(h+a, k)$	$x = h - a$
$(y-k)^2 = -4a(x-h)$	left	$(h-a, k)$	$x = h + a$



9.3 The Ellipse

- center: (h, k)
- $c^2 = a^2 - b^2$
- $d(\text{center}, V) = a$
- $d(\text{center}, F) = c$

Equation	M. Ax.	V	F
$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$	horiz.	$(h \pm a, k)$	$(h \pm c, k)$
$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$	vert.	$(h, k \pm a)$	$(h, k \pm c)$

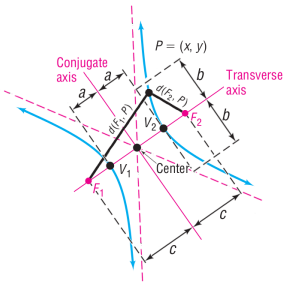


9.4 The Hyperbola

- center: (h, k)
- $c^2 = a^2 + b^2$
- $d(\text{center}, V) = a$
- $d(\text{center}, F) = c$

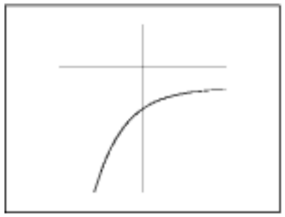
Equation	Tr. Ax.	V	F
$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$	horiz.	$(h \pm a, k)$	$(h \pm c, k)$
$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$	vert.	$(h, k \pm a)$	$(h, k \pm c)$

- asymptotes: set Equation $= 0$, solve for y



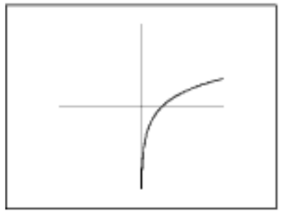
Exercises

1. Select the equation for the following graph.



A. $y = a^{-(1+x)}$, $1 < a$
B. $y = -(1 + a^x)$, $0 < a < 1$
C. $y = -(1 + a^x)$, $1 < a$
D. $y = a^{-(1+x)}$, $0 < a < 1$

2. Select ALL the correct equations for the given graph.



A. $y = -\log_a(x)$, $a > 1$
B. $y = -\log_a(x)$, $0 < a < 1$
C. None of these
D. $y = \log_a(-x)$, $0 < a < 1$
E. $y = -\log_a(-x)$, $a > 1$

3. Find the domain of $\ln(-3x - 1)$.

4. Select ALL of the correct formulas if $x > 0$, $y > 0$, $x \neq 1$, $x \neq 1$.

A. $\log_3 3^{-5}$ is not defined
B. $\log_7 10 = \frac{10}{7}$
C. $\log_{\frac{1}{4}} \frac{1}{4} = 4$
D. $\log xy = \log x + \log y$
E. $\log_x 1 = 1$
F. $\frac{\log x}{\log y} = \log \frac{x}{y}$
G. $\log_y y = 1$
H. $\log_x 1 = 0$
I. $\log(x - y) = \log x - \log y$
J. $\log xy = \log x \log y$
K. $\log_y x = \frac{\log x}{\log y}$
L. $\log_5 \frac{1}{5} = -1$

5. If $\log_4 2 = a$ and $\log_4 7 = b$, then $\log_4 \frac{49}{8} =$

6. Find all x such that $4^{(2x)} = 3$.

7. Expand $\ln\left(\frac{(x-4)^{-2}y^3}{\sqrt{z}}\right)$.

8. Find all x such that $\log_9(x^2 - 4) - \log_3(x - 2) = 1$.

9. If an investment pays 6% compounded daily, how much should you deposit now to have \$500 in three years?

10. What interest rate, compounded continuously, will triple an investment in 5 years?

11. Find the effective rate of interest for 15% compunded semi-annually.

12. The population of a country is known to follow the exponential law. If the initial population is P_0 and after two years the population tripled, what will the population be in 5 years?

13. Find the equation of the parabola with focus at $(2, \frac{5}{2})$ and directrtix $y = \frac{7}{2}$.

14. Find the equation of the ellipse with center at $(-1, \pi)$, focus at $(7, \pi)$, and vertex at $(-11, \pi)$.

15. Find the asymptotes of the hyperbola given by $\frac{y^2}{16} - \frac{x^2}{36} = 1$.

16. Select the best description for the graph of $12x^2 = y$.

A. parabola with focus $(0, 12)$
B. parabola with focus $(0, \frac{1}{3})$
C. parabola with focus $(0, \frac{1}{48})$
D. parabola with focus $(3, 0)$
E. None of these