MAC2311: Calculus 1 - Section 1

Test 3 Review March 18, 2015

1. Find the differential dy of each function
   (a) $y = x^2 \sin 5x$
   (b) $y = \ln(\tan t)$
   (c) $y = \dfrac{s}{1+6s}$

2. Find the critical numbers of each function
   (a) $f(x) = x^{4/5}(x-4)^2$
   (b) $f(x) = 3x - \arcsin x$

3. Find the absolute minimum and maximum values of $f(x)$ on the given interval
   (a) $f(x) = x^3 - 6x^2 + 5$, $[-3, 5]$
   (b) $f(x) = x + \frac{5}{x}$, $[1, 5]$
   (c) $f(x) = x\sqrt{4 - x^2}$, $[-1, 2]$

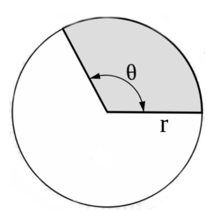
4. Find the linearization $L(x)$ of the function $f(x)$ at a and use it to approximate the given number.
   (a) $f(x) = e^x$, $a = 0$, $e^{-.05}$
   (b) $f(x) = x^4$, $a = 2$, $(1.95)^4$
   (c) $f(x) = \sqrt{5-x}$, $a = 1$, $\sqrt{4.95}$

5. Decide whether each function satisfies the hypotheses of the Mean Value Theorem on the given interval. If so, find all numbers c that satisfy the conclusion of the Mean Value Theorem.
   (a) $f(x) = 2x^2 - 3x + 1$, $[0, 2]$
   (b) $f(x) = e^{-x}$, $[0, 2]$

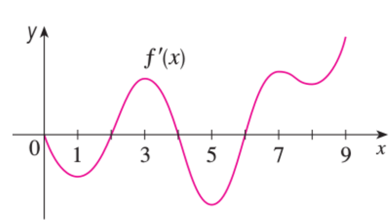
6. The area of a sector with angle θ of a circle of radius r is

$$A = \frac{\theta r^2}{2}$$

   for $0 \le \theta \le 2\pi$. Suppose θ and r are functions of t, and that θ changes at the rate 1 and r changes at the rate 2. At what rate is A changing when $\theta = \pi$ and $r = 2$?



7. Below is the graph of $\frac{dy}{dx} = f'(x)$, the derivative of $y = f(x)$.



(a) On what intervals is f increasing? decreasing?

(b) At what x-values does f have a local minimum or maximum?

(c) On what intervals is f concave upward? concave downward?

(d) At what x-values does f have an inflection point?

8. What is the first derivative test? the second derivative test? When is the second derivative test inconclusive?

9. Find the intervals on which f is increasing or decreasing, any local minima or maxima of f, the intervals on which f is concave upward or downward, any inflection points of f, and any asymptotes of f.

   (a) $f(x) = 2 + 2x^2 - x^4$

   (b) $f(x) = x\sqrt{6 - x}$

10. Evaluate the following limits

    (a) $\lim\limits_{x \to 0^+} \sin x \ln x$

    (b) $\lim\limits_{x \to 0^+} \left(\dfrac{1}{x} - \dfrac{1}{e^x - 1} \right)$

    (c) $\lim\limits_{x \to 0} (1 - 2x)^{1/x}$