Homework 5 Foundations of Computational Math 2 Spring 2012

Solutions will be posted Monday, 2/13/12

Problem 5.1

Recall that we have derived different sets of linear equations for the coefficients of an interpolating cubic spline.

Assume that $f(x) = x^3$ and analyze the equations and boundary conditions that define $s(x)$ in the forms below and determine what can be said about the relationship between $s(x)$ and $f(x)$.

5.1.a. $s(x)$ is determined by $Ts'' = d$ where s'' is a vector containing s''_i $1 \leq i \leq n-1$ and boundary conditions $s''_0 = f''(x_0)$ and $s''_n = f''(x_n)$.

5.1.b. $s(x)$ is determined by $\tilde{T}s' = \tilde{d}$ where s' is a vector containing s'_i $1 \leq i \leq n-1$ and boundary conditions $s'_0 = f'(x_0)$ and $s'_n = f'(x_n)$.

Problem 5.2

Assuming that the nodes are uniformly spaced, we have derived the form of the cubic B-spline $B_{3,i}(t)$ and determined its values and the values of $B'_{3,i}(t)$ and $B''_{3,i}(t)$ at the nodes t_{i-2}, t_{i-1}, t_{i-1}, t_i, t_{i+1}, and t_{i+2}. We also derived $B_{1,i}(t)$ and saw that it was the familiar hat function.

5.2.a. Derive the formula of the quadratic B-spline $B_{2,i}(t)$ and determine its values and the values of $B'_{2,i}(t)$ and $B''_{2,i}(t)$ at the appropriate nodes.

5.2.b. Derive the formula of the quintic B-spline $B_{5,i}(t)$ and determine its values and the values of $B'_{5,i}(t)$ and $B''_{5,i}(t)$ at the appropriate nodes.

Problem 5.3

Consider a set of equidistant mesh points, $x_k = x_0 + kh$, $0 \leq k \leq m$.

5.3.a. Determine a cubic spline $b_i(x)$ that satisfies the following conditions:

$$b_i(x_j) = \begin{cases} 0 & \text{if } j < i-1 \text{ or } j > i+1 \\ 1 & \text{if } j = i \end{cases}$$

$$b'_i(x) = b''_i(x) = 0 \quad \text{for } x = x_{i-2} \text{ and } x = x_{i+2}$$

(For simplicity, you may assume that $2 < i < m-2$.)

5.3.b. Show that $b_i(x) = 0$ when $|x - x_i| \geq 2h$.

5.3.c. Show that $b_i(x) > 0$ when $|x - x_i| < 2h$.

5.3.d. What is the relationship between the spline, $b_i(x)$, and a B-spline?