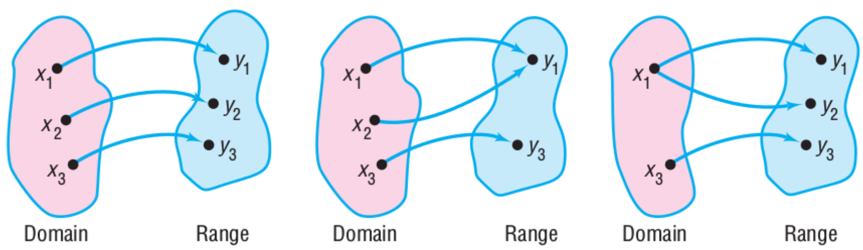
4.1: One-to-one and Inverse Functions

Supplementary Notes

A function f is *one-to-one* if $f(x_1) \neq f(x_2)$ for any distinct x_1 and x_2 in the domain of f.



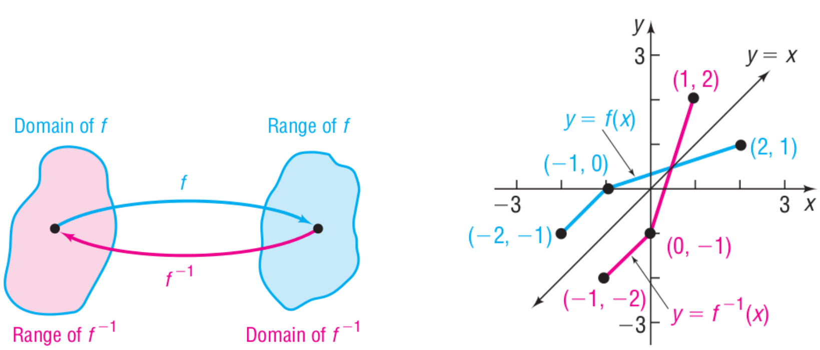
(a) One-to-one function: Each x in the domain has one and only one image in the range.
(b) Not a one-to-one function: y_1 is the image of both x_1 and x_2.
(c) Not a function: x_1 has two images, y_1 and y_2.

<u>Vertical Line Test</u>: If every *vertical* line intersects the graph of f at most once, then f is a *function*

<u>Horizontal Line Test</u>: If every *horizontal* line intersects the graph of f at most once, then f is *one-to-one*

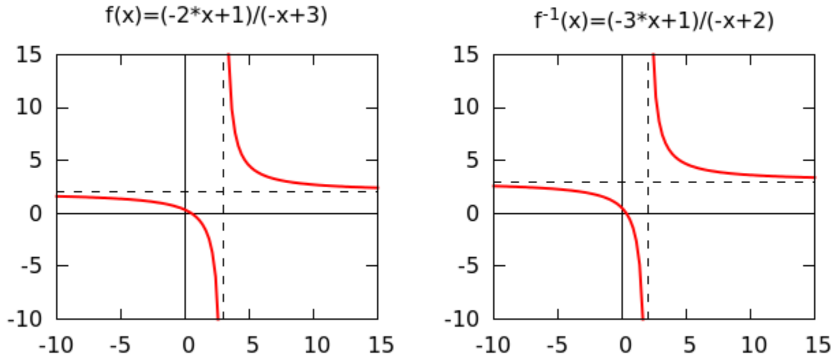
Any one-to-one function f has an *inverse* f^{-1} with the following properties

- $x \xrightarrow{f} y$ if and only if $x \xleftarrow{f^{-1}} y$
- (domain of f) = (range of f^{-1}) AND (range of f) = (domain of f^{-1})
- The graphs of f and f^{-1} are symmetric about the line $y = x$



Exercises

1. Find the formula for the inverse of the function $f(x) = \frac{-2x+1}{-x+3}$
2. Find the inverse of $f(x) = -4x + 2$ on $[-2, 2]$.



3. If $f(x) = a(x-3)^2 + 2$ on $(3, \infty)$ and $a < 0$, then the inverse function is

   A. $f^{-1}(x) = \sqrt{\frac{x-2}{a}} + 3$ on $(2, \infty)$

   B. $f^{-1}(x) = \sqrt{\frac{x-2}{a}} + 3$ on $(-\infty, 2)$

   C. $f^{-1}(x) = \frac{1}{a(x-3)^2+2}$ on $(3, \infty)$

   D. $f^{-1}(x) = -\sqrt{\frac{x-2}{a}} + 3$ on $(3, \infty)$

   E. $f^{-1}(x) = -\sqrt{\frac{x-2}{a}} + 3$ on $(-\infty, 2)$