Unit IV

Supplementary Notes

11.4 Mathematical Induction

- To prove by induction that $P(n)$ is true for all positive integers n, we assume $P(k)$ is true for some positive integer k and show that $P(k+1)$ is true.

11.5 The Binomial Theorem

- factorial function:

$$0! = 1$$
$$n! = n(n-1)(n-2)\cdots 3\cdot 2\cdot 1 \quad \text{for } n \geq 1$$

- binomial coefficient:

$$\binom{n}{k} = \frac{n!}{k!(n-k)!} \quad \text{for } 0 \leq k \leq n$$

- Pascal's triangle:

Row 0:	$\binom{0}{0}$	1
Row 1:	$\binom{1}{0}\ \binom{1}{1}$	1 1
Row 2:	$\binom{2}{0}\ \binom{2}{1}\ \binom{2}{2}$	1 2 1
Row 3:	$\binom{3}{0}\ \binom{3}{1}\ \binom{3}{2}\ \binom{3}{3}$	1 3 3 1
Row 4:	$\binom{4}{0}\ \binom{4}{1}\ \binom{4}{2}\ \binom{4}{3}\ \binom{4}{4}$	1 4 6 4 1
Row 5:	$\binom{5}{0}\ \binom{5}{1}\ \binom{5}{2}\ \binom{5}{3}\ \binom{5}{4}\ \binom{5}{5}$	1 5 10 10 5 1
$\vdots$	$\vdots$	$\vdots$

- binomial theorem:

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k \quad (n \geq 0)$$
$$= \binom{n}{0} a^n + \binom{n}{1} a^{n-1} b^1 + \cdots + \binom{n}{k} a^{n-k} b^k + \cdots + \binom{n}{n-1} a b^{n-1} + \binom{n}{n} b^n$$

Exercises

1. To prove by induction that $n^2 - 7n - 4$ is divisible by 2 is true for all positive integers n, we assume $k^2 - 7k - 4$ is divisible by 2 is true for some positive integer k and we show that $k^2 - 7k - 4 + A$ is divisible by 2, where A is
2. Find a_2 and a_3 such that $-4 + a_2 + a_3 + \cdots + a_n = \frac{n(n-9)}{2}$ for all n.
3. Evaluate the binomial coefficient $\binom{n}{2}$.
4. Find the sixth term of the expansion of $(\frac{3}{c} + \frac{c^2}{4})^7$ if the terms are arranged in decreasing powers of the first term.
5. Find the term that does not contain y in the expansion of $(xy - 3y^{-3})^8$.