11.3: Geometric Sequences and Series

Supplementary Notes

A *geometric* sequence $\{a_n\}$ may be defined *recursively* as

$$\begin{aligned} a_1 &= a \ \text{ for real number } \ a, \\ a_n &= a_{n-1} \cdot r \ \text{ for } \ n \geq 2 \end{aligned}$$

or *explicitly* as

$$a_n = a_1 \cdot r^{n-1} \ \text{ for } \ n \geq 1$$

where $r \neq 0, 1$ is a real number called the *common ratio*.

The sum S_n of the first n terms of a geometric sequence $\{a_n\}$ with common ratio r is

$$S_n = \sum_{k=1}^{n} a_k = a_1 \frac{1 - r^n}{1 - r}.$$

The sum S of all terms of a geometric sequence $\{a_n\}$ with common ratio r is called an *infinite geometric series*. If $|r| < 1$, the sum converges as

$$S = \sum_{k=1}^{\infty} a_k = \frac{a_1}{1 - r}.$$

Exercises

1. If a geometric sequence has $a_3 = 9$ and $a_6 = -\frac{1}{3}$, what is the common ratio?
2. Find the 2^{nd} term of a geometric sequence whose 4^{th} term is -3 and whose 6^{th} term is $-\frac{1}{3}$.
3. Find the n^{th} term of a geometric sequence with first term $a_1 = 6$ and common ratio $r = 4$.
4. If the repeating decimal $1.424242\ldots$ is written as $\frac{m}{n}$ in reduced form where m and n are integers, then $m =$