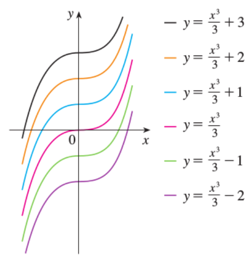
4.9: Antiderivatives

A physicist who knows the velocity of a particle might wish to know its position at a given time. An engineer who can measure the variable rate at which water is leaking from a tank wants to know the amount leaked over a certain time period. A biologist who knows the rate at which a bacteria population is increasing might want to deduce what the size of the population will be at some future time. In each case, the problem is to find a function F whose derivative is a known function f. If such a function F exists, it is called an **antiderivative** of f.

A function F is called an **antiderivative** of f on an interval I if $F'(x) = f(x)$ for all x in I.

Theorem 1. *If F is an antiderivative of f on an interval I, then the most general antiderivative of f on I is $F(x) + C$ where C is an arbitrary constant.*



Example 1. *Find the most general antiderivative of each of the following functions*

(a) $f(x) = \sin x$

(b) $f(x) = \frac{1}{x}$

(c) $f(x) = x^n$, $n \neq -1$

Below is a table of particular antiderivatives for a number of common functions.

Function	Particular antiderivative	Function	Particular antiderivative
$cf(x)$	$cF(x)$	$\sec^2 x$	$\tan x$
$f(x) + g(x)$	$F(x) + G(x)$	$\sec x \tan x$	$\sec x$
$x^n \quad (n \neq -1)$	$\frac{x^{n+1}}{n+1}$	$\frac{1}{\sqrt{1-x^2}}$	$\sin^{-1} x$
$\frac{1}{x}$	$\ln \lvert x \rvert$	$\frac{1}{1+x^2}$	$\tan^{-1} x$
e^x	e^x	$\cosh x$	$\sinh x$
$\cos x$	$\sin x$	$\sinh x$	$\cosh x$
$\sin x$	$-\cos x$		

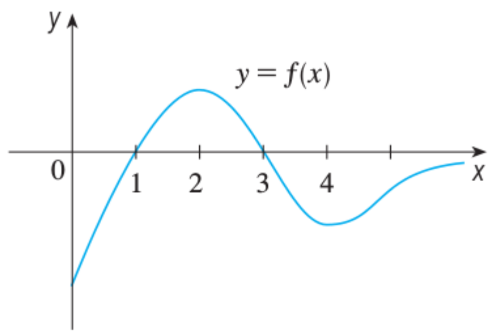
Example 2. *Find all functions such that*

$$g'(x) = 4\sin x + \frac{2x^5 - \sqrt{x}}{x}.$$

Example 3. *Find f is $f'(x) = e^x + 20(1+x^2)^{-1}$ and $f(0) = -2$.*

Example 4. *Find f if $f''(x) = 12x^2 + 6x - 4$, $f(0) = 4$, and $f(1) = 1$.*

Example 5. *The graph of a function f is given below. Make a rough sketch of an antiderivative F, given that $F(0) = 2$.*



Example 6. *A particle moves in a straight line and has acceleration given by $a(t) = 6t + 4$. Its initial velocity is $v(0) = -6$ cm/s and its initial displacement is $s(0) = 9$ cm. Find its position function $s(t)$.*

An object near the surface of the earth is subject to a gravitational force that produces a downward acceleration denoted by g. For motion close to the ground we may assume that g is constant, its value being about 9.8 m/s^2 (or 32 ft/s^2).

Example 7. *A ball is thrown upward with a speed of 48 ft/s from the edge of a cliff 432 ft above the ground. Find its height above the ground t seconds later. When does it reach its maximum height? When does it hit the ground?*