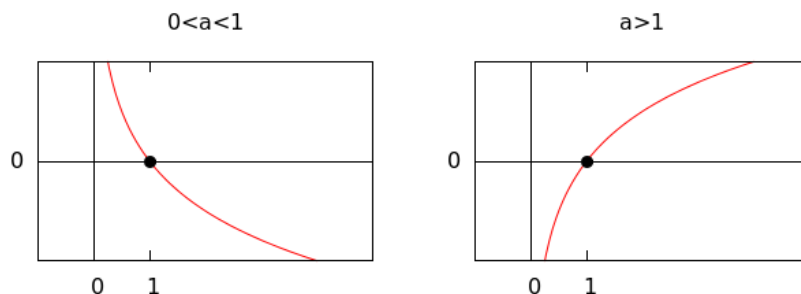# 4.3: Logarithmic Functions

Supplementary Notes

The logarithmic function

$$f(x) = \log_a x \qquad (a > 0, a \neq 1)$$

is defined as the inverse of the exponential function $f^{-1}(x) = a^x$, where a is a real number. Below are the graphs of f both for $0 < a < 1$ and $a > 1$.

The graph of f has the following properties

- domain: $(0, \infty)$

- range: $(-\infty, \infty)$

- $x-$intercept: 1

- vertical asymptote: $x = 0$ $(y-$axis$)$

- $\begin{cases} \text{increasing} & \text{if } a > 1 \\ \text{decreasing} & \text{if } 0 < a < 1 \end{cases}$

Important examples of logarithmic functions:

- The logarithmic function with base 10, $f(x) = \log_{10} x$, is written without the base, $f(x) = \log x$.

- The logarithmic function with base $e \approx 2.72$, $f(x) = \log_e x$, is called the *natural logarithmic function* and is written $f(x) = \ln x$.

Exercises

1. Select ALL of the correct equations for the given graph
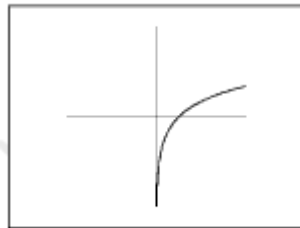
    A. $y = -\log_a(-x)$, $a > 1$

    B. None of these

    C. $y = \log_a(x)$, $0 < a < 1$

    D. $y = -\log_a(x)$, $0 < a < 1$

    E. $y = \log_a(-x)$, $0 < a < 1$