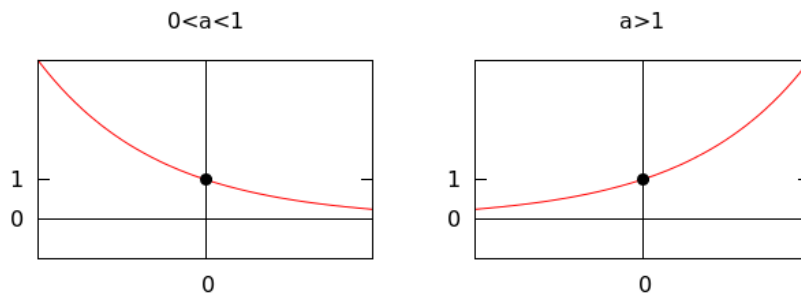


## 4.2: Exponential Functions

### Supplementary Notes

$$f(x) = a^x \quad (a > 0, a \neq 1)$$

where  $a$  is a real number. Below are the graphs of  $f$  both for  $0 < a < 1$  and  $a > 1$ .



The graph of  $f$  has the following properties

- domain:  $(-\infty, \infty)$
- range:  $(0, \infty)$
- $y$ -intercept: 1
- horizontal asymptote:  $y = 0$  ( $x$ -axis)
- $\begin{cases} \text{increasing} & \text{if } a > 1 \\ \text{decreasing} & \text{if } 0 < a < 1 \end{cases}$

Below are laws of exponents and rules for transforming graphs of exponential functions, some of which are review from Section 3.2

Laws of Exponents		
$a^s \cdot a^t = a^{s+t}$	$(a^s)^t = a^{s \cdot t}$	$(a \cdot b)^s = a^s \cdot b^s$
$a^{-s} = \frac{1}{a^s} = \left(\frac{1}{a}\right)^s$	$1^s = 1$	$a^0 = 1$

Reflection		
To obtain the graph of		
$-a^x$	reflect the graph of $a^x$ about the	$x$ -axis
$a^{-x}$	reflect the graph of $a^x$ about the	$y$ -axis

Translation		
For $h, k > 0$ , to obtain the graph of		
$a^x + k$	translate the graph of $a^x$	upward $k$ units
$a^x - k$	translate the graph of $a^x$	downward $k$ units
$a^{x-h}$	translate the graph of $a^x$	rightward $h$ units
$a^{x+h}$	translate the graph of $a^x$	leftward $h$ units

<b>Reflection and Translation</b>		
For $h, k > 0$ , to obtain the graph of		
$-a^x + k$	reflect the graph of $a^x$ then translate the graph of $-a^x$	about the $x$ -axis upward $k$ units
$-a^x - k$	reflect the graph of $a^x$ then translate the graph of $-a^x$	about the $x$ -axis downward $k$ units
$a^{-x-h}$	translate the graph of $a^x$ then reflect the graph of $a^{x-h}$	rightward $h$ units about the $y$ -axis
$a^{-x+h}$	translate the graph of $a^x$ then reflect the graph of $a^{x+h}$	leftward $h$ units about the $y$ -axis

## Exercises

1. Select the graph of  $y = -b^x$ ,  $b > 1$ .



2. Select the graph of  $y = a^{-(1+x)}$ ,  $1 < a$ .



3. Select the graph of  $y = 1 - a^x$ ,  $0 < a < 1$ .



4. Select the graph of  $y = -(\frac{1}{a})^x$ ,  $0 < a < 1$ .

