2.6: Limits at Infinity: Horizontal Asymptotes

Horizontal Asymptotes

If $f(x)$ approaches L as x approaches ∞, then

$$\lim_{x\to\infty} f(x) = L.$$

If $f(x)$ approaches L as x approaches $-\infty$, then

$$\lim_{x\to-\infty} f(x) = L.$$

$f(x)$ has a **horizontal asymptote** $y = L$ if one of the following is true

- $\lim_{x\to\infty} f(x) = L$
- $\lim_{x\to-\infty} f(x) = L$

Example 1. *Find the horizontal asymptotes of each function*

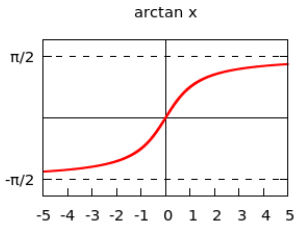
(a) e^x

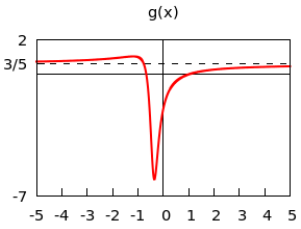
(b) $f(x) = \frac{1}{(x-c)^r} \quad (r > 0)$

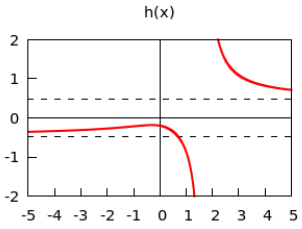
(c) $\arctan x$

(d) $g(x) = \frac{3x^2-x-2}{5x^2+4x+1}$

(e) $h(x) = \frac{\sqrt{2x^2+1}}{3x-5}$







Example 2. *Evaluate each infinite limit*

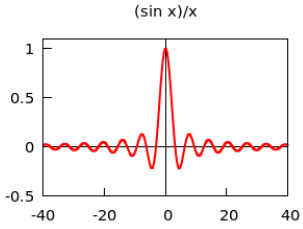
(a) $\lim_{x\to\infty} \sin x$

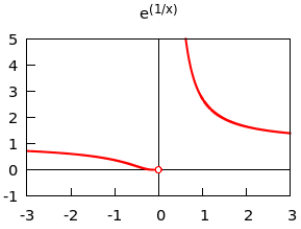
(b) $\lim_{x\to\infty} \frac{\sin x}{x}$

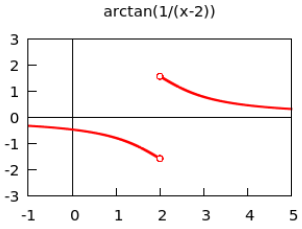
Example 3. *Evaluate each limit by transforming it to an infinite limit*

(a) $\lim_{x\to0^-} e^{\frac{1}{x}}$

(b) $\lim_{x\to2^+} \arctan\left(\frac{1}{x-2}\right)$







Infinite Limits at Infinity

If $f(x)$ approaches ∞ as x approaches ∞, then

$$\lim_{x\to\infty} f(x) = \infty.$$

Similar notation is used for the other three cases: (i) $f(x)$ approaches ∞ as x approaches $-\infty$, (ii) $f(x)$ approaches $-\infty$ as x approaches ∞, and (iii) $f(x)$ approaches $-\infty$ as x approaches $-\infty$.

Example 4. *Find the limit as x approaches $\pm\infty$ for each function*

(a) e^x *(b)* $f(x) = x^3$ *(c)* $g(x) = \frac{x^2+x}{3-x}$