11.4: Mathematical Induction

Supplementary Notes

Mathematical Induction is a method for proving a proposition $P(n)$ is true for all positive integers n. $P(n)$ is true for all positive integers n if the following conditions hold

- $P(1)$ is true, and (base case)
- if $P(k)$ is true, then $P(k+1)$ is true for some positive integer $k \geq 1$. (induction step)

Exercises

1. To prove by induction that $7 + 12 + 17 + \cdots + (5n + 2) = \frac{1}{2}(5n^2 + 9n)$ is true for all positive integers n, we assume $7 + 12 + 17 + \cdots + (5k + 2) = \frac{1}{2}(5k^2 + 9k)$ is true for some positive integer k, and show that $7 + 12 + 17 + \cdots + (5k + 2) + (5(k + 1) + 2) = A$ where A is

2. To prove by induction that $5 + 9 + 13 + \cdots + (4n + 1) = 2n^2 + 3n$ is true for all positive integers n, we assume $5 + 9 + 13 + \cdots + (4k + 1) = 2k^2 + 3k$ is true for some positive integer k, and show that $5 + 9 + 13 + \cdots + (4k + 1) + A = 2(k + 1)^2 + 3(k + 1)$ where A is

3. To prove by induction that $n^2 - 5n - 2$ is divisible by 2 is true for all positive integers n, we assume $k^2 - 5k - 2$ is divisible by 2 is true for some positive integer k, and we show that A is divisible by 2, where A is

4. To prove by induction that $n^2 - 3n + 2$ is divisible by 2 is true for all positive integers n, we assume $k^2 - 3k + 2$ is divisible by 2 is true for some positive integer k, and we show that $k^2 - 3k + 2 + A$ is divisible by 2, where A is

5. Find a_2 and a_3 such that $1 + a_2 + a_3 + \cdots + a_n = 2^n - 1$ for all n.