MAC2311: Calculus 1 - Section 1

Test 2 February 19, 2015

Name: _____

| Answer each question in the space provided on the question sheets. If you run out of space for an answer, continue on the back of the page. Credit will only be given if you clearly show all of your work. Calculators may be used for this test. |
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Question	Points	Score
1	8	
2	3	
3	5	
4	5	
5	5	
6	4	
7	4	
8	5	
9	13	
Extra Credit	–	
Total:	52	

1. [8 points] Use the following graph of f to answer the questions below it.

f(x) ————

vertical
tangent line

-4   -3   -2   -1   0   1   2   3   4   5   6
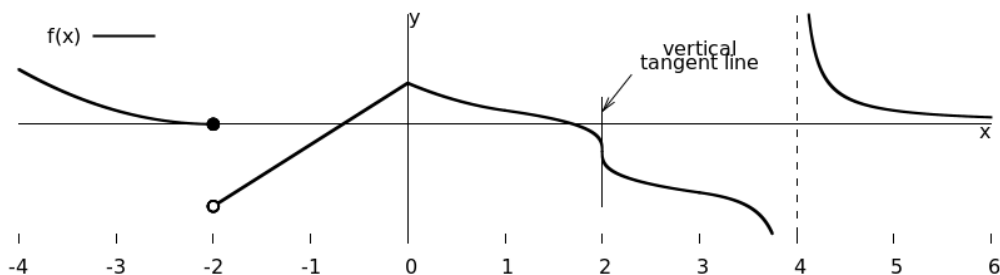
(a) [4 points] List all x-values at which f is not differentiable: _____

(b) [4 points] For each x-value that you listed in part (a), state why f is not differentiable.

2. [3 points] The following figure shows the graphs of f, f', and f''.

(b)

(a)

(c)
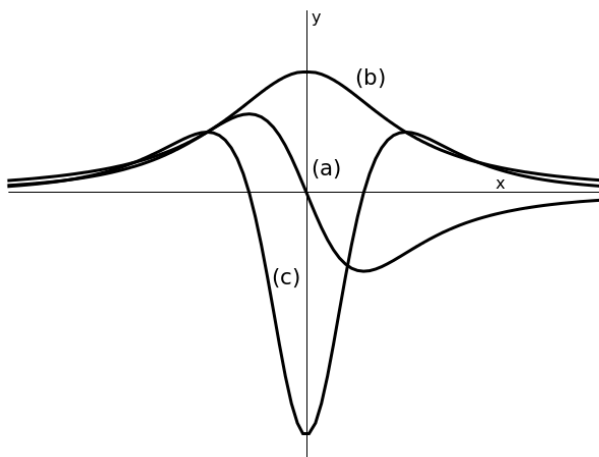
Complete the following statements by filling in one of f, f', or f'' in each blank.

(a) is the graph of _____

(b) is the graph of _____

(c) is the graph of _____

3. [5 points] Let f and g be differentiable functions. Complete each of the following equations using differentiation rules.

(a) $\frac{d}{dx}[f(x)g(x)] =$

(b) $\frac{d}{dx}[f(g(x))] =$

(c) $\frac{d}{dx}(2^x) =$

(d) $\frac{d}{dx}(\cos x) =$

(e) $\frac{d}{dx}(\log_2 x) =$

4. [5 points] Let $f(x) = (x^2 + 1)e^x$

(a) [3 points] Differentiate $f(x)$.

(b) [2 points] Find the x-value(s) at which f has a horizontal tangent line.

5. [5 points] Consider taking the derivative of $\cot x$ by first expressing $\frac{d}{dx}(\cot x)$ as the derivative of a quotient of trigonometric functions, and then using the quotient rule.

(a) [1 point] Select the equation that correctly expresses $\frac{d}{dx}(\cot x)$ as the derivative of a quotient of trigonometric functions.

A. $\frac{d}{dx}(\cot x) = \frac{d}{dx}\left(\frac{1}{\sin x}\right)$

B. $\frac{d}{dx}(\cot x) = \frac{d}{dx}\left(\frac{1}{\cos x}\right)$

C. $\frac{d}{dx}(\cot x) = \frac{d}{dx}\left(\frac{\cos x}{\sin x}\right)$

D. $\frac{d}{dx}(\cot x) = \frac{d}{dx}\left(\frac{\sec x}{\csc x}\right)$

(b) [4 points] Prove that $\frac{d}{dx}(\cot x) = -\csc^2 x$ by using the quotient rule to evaluate the righthand side of your answer to part (a).

6. [4 points] Differentiate $e^{\cot \sqrt{x}}$.

7. [4 points] Use implicit differentiation to find y' if $x^2 + y^2 + xy = 1$.

8. [5 points] Use logarithmic differentiation to find a formula for y' in terms of x if $y = x^x$.

9. [13 points] For each part of the following question, use correct units in your final answer.

The position of a particle is given by the equation

$$s(t) = t^3 - 9t^2 + 15t$$

where s is measured in meters and $t \geq 0$ is measured in seconds.

(a) [4 points] Find the velocity at time t.

(b) [3 points] When is the particle at rest?

(c) [3 points] When is the particle moving forward?

(d) [3 points] Find the acceleration at time t.

Extra Credit [2 points]: Let $f(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3$.

(a) Show that $f'(x) = ADX$ where

$$A = \begin{pmatrix} a_0 & a_1 & a_2 & a_3 \end{pmatrix}, \quad D = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \end{pmatrix}, \quad \text{and } X = \begin{pmatrix} 1 \\ x \\ x^2 \\ x^3 \end{pmatrix}.$$

(b) What is $AD^k X$ for $k > 3$?