11.1: Sequences

Supplementary Notes

A *sequence* $\{a_n\}$ is a function whose domain is the set of positive integers, i.e an ordered list of numbers. $\{a_n\}$ may be defined *recursively* as a function of previous terms in the sequence

$$a_1 = a \text{ for real number } a,$$
$$a_n = f(a_{n-1}) \text{ for } n \geq 2$$

or *explicitly* as a function of n

$$a_n = f(n) \text{ for } n \geq 1.$$

The sum of the first n terms of the sequence $\{a_n\}$ is denoted

$$\sum_{k=1}^{n} a_k = a_1 + a_2 + a_3 + \cdots + a_{n-1} + a_n.$$

Exercises

1. The first four terms of the sequence $\{(-1)^{n+1}(1+\frac{1}{n})^n\}$ are

   A. $2, -\frac{9}{4}, \frac{64}{27}, -\frac{625}{256}$
   B. $\frac{625}{256}, \frac{64}{27}, \frac{9}{4}, 2$
   C. $-2, \frac{9}{4}, -\frac{64}{27}, \frac{625}{256}$
   D. $2, -\frac{5}{4}, \frac{28}{27}, -\frac{257}{256}$

2. The n^{th} term of the sequence $-1, \frac{1}{9}, -\frac{1}{125}, \frac{1}{2401}, \ldots$ is

   A. $(-1)^{n+1}(\frac{1}{n})^{n-1}$
   B. $(-1)^{n+1}(\frac{1}{n})^{n+1}$
   C. $(-1)^{n-1}(\frac{1}{2n-1})^{n}$
   D. $(-1)^{n}(\frac{1}{2n-1})^{n}$

3. Choose the sum equivalent to

   $$\sum_{i=1}^{100} f(x_i)\Delta x$$

   where $f(x_i) = 2i$ and $\Delta x = 0.1$

   A. $.2 + .4 + .6 + \cdots + 2$
   B. $2 + 4 + 6 + \cdots + 20$
   C. $.2 + .4 + .6 + \cdots + 200$
   D. $2 + 4 + 6 + \cdots + 200$
   E. $.2 + .4 + .6 + \cdots + 20$

4. Select the statement that is true

   A. $\sum_{k=1}^{9} \frac{k}{(k+1)(k+2)} = \sum_{k=1}^{3} \frac{k}{(k+1)(k+2)} + \frac{2}{15} + \sum_{k=6}^{9} \frac{k}{(k+1)(k+2)}$
   B. $\sum_{k=1}^{10} \frac{k}{(k+1)(k+2)} = \sum_{k=1}^{4} \frac{k}{(k+1)(k+2)} + \frac{5}{42} + \sum_{k=6}^{10} \frac{k}{(k+1)(k+2)}$
   C. $\sum_{k=1}^{10} \frac{k}{(k+1)(k+2)} = \sum_{k=1}^{4} \frac{k}{(k+1)(k+2)} + \frac{5}{42} + \sum_{k=7}^{10} \frac{k}{(k+1)(k+2)}$
   D. $\sum_{k=1}^{9} \frac{k}{(k+1)(k+2)} = \sum_{k=1}^{3} \frac{k}{(k+1)(k+2)} + \frac{2}{15} + \sum_{k=5}^{10} \frac{k}{(k+1)(k+2)}$

5. Select the statement that is true

   A. $\sum_{m=0}^{n} \frac{(m+2)(m+1)}{3^{m+2}} = \sum_{m=-2}^{n-2} \frac{m(m-1)}{3^m}$
   B. $\sum_{m=0}^{n} \frac{(m+2)(m+1)}{3^{m+2}} = \sum_{m=-2}^{n-2} \frac{(m+4)(m+3)}{3^{m+2}}$
   C. $\sum_{m=0}^{n} \frac{(m+2)(m+1)}{3^{m+2}} = \sum_{m=2}^{n+2} \frac{(m+4)(m+3)}{3^{m+2}}$
   D. $\sum_{m=0}^{n} \frac{(m+2)(m+1)}{3^{m+2}} = \sum_{m=2}^{n+2} \frac{m(m-1)}{3^m}$