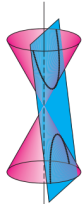


9.4: The Hyperbola

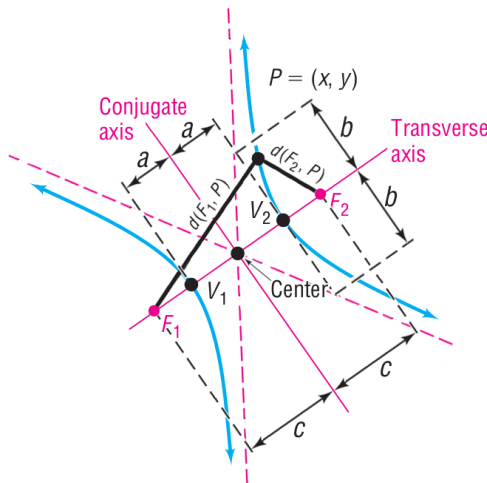
Supplementary Notes

A *hyperbola* is an example of a *conic* since it is the cross-section of a cone with a plane parallel to the axis of the cone.

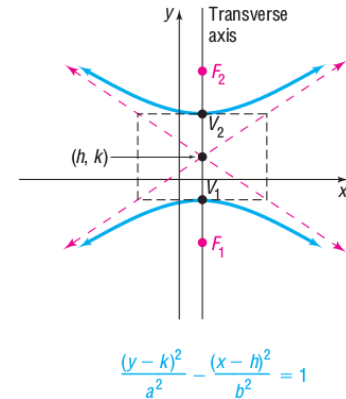
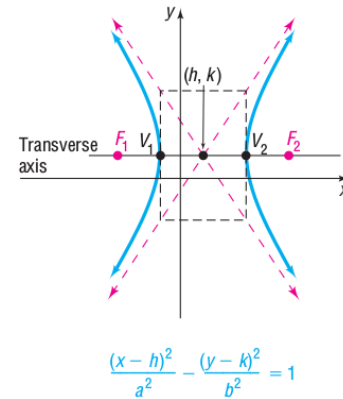


A *hyperbola* is the collection of points in the xy -plane, the difference of whose distance from two fixed points F_1 and F_2 , is constant. F_1 and F_2 are the *foci*. The midpoint of the line segment joining the foci is the *center*. The line containing both foci is the *transverse axis*, and the two points where the transverse axis intersects the hyperbola are the *vertices*. The line perpendicular to the major axis that contains the center is the *conjugate axis*. The branches of a hyperbola approach two *asymptotes* as $|x| \rightarrow \infty$.

- (distance from center to vertex) = a
- (distance from vertex to asymptote along line parallel to conjugate axis) = b
- (distance from center to focus) = c



Equations of hyperbolas with center (h, k) and transverse axis parallel to a coordinate axis				
Equation	Trans. Axis	Foci	Vertices	Asymptotes
$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$	horizontal	$(h \pm c, k)$	$(h \pm a, k)$	$y - k = \pm \frac{b}{a}(x - h)$
$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$	vertical	$(h, k \pm c)$	$(h, k \pm a)$	$y - k = \pm \frac{a}{b}(x - h)$



The distances a , b , and c defined for a hyperbola as above are related by

- $c^2 = a^2 + b^2$

The general formula for a conic is

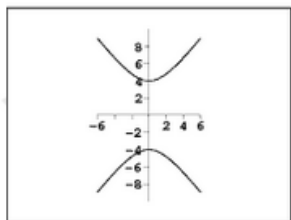
$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0.$$

This formula describes

- a parabola if $AC = 0$
- an ellipse if $AC > 0$, or
- a hyperbola if $AC < 0$.

Exercises

1. Sketch the graph of $\frac{x^2}{9} - \frac{y^2}{16} = 1$.
2. Write (using lowercase x and y) the equation of the following graph.



3. Find the vertices of the hyperbola given by $\frac{(y-5)^2}{36} - \frac{(x-3)^2}{13} = 1$.
4. Find the foci of the hyperbola given by $\frac{y^2}{25} - \frac{x^2}{39} = 1$.
5. Find the foci of the hyperbola given by $\frac{(x-6)^2}{4} - \frac{(y+4)^2}{12} = 1$.
6. Write (using lowercase x and y) the equation of the hyperbola with center $(0, 0)$, focus at $(-7, 0)$, and vertex $(-4, 0)$.
7. Write (using lowercase x) the formula for y^2 in the hyperbola with center $(0, 0)$, focus at $(0, -4)$, and vertex at $(0, -2)$.
8. Select the asymptotes of the hyperbola given by $\frac{x^2}{16} - \frac{y^2}{9} = 1$.
9. Describe the graph of $2y^2 - 4y = x - 2$.
10. Describe the graph of $4x^2 + 9y^2 - 16x - 18y = 11$.
11. Describe the graph of $y^2 - 4y - 4x^2 + 8x = 4$.