10.3: Linear Systems of Equations: Matrices

Supplementary Notes

A *matrix* is a rectangular array of numbers

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1j} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2j} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots & & \vdots \\ a_{i1} & a_{i2} & \cdots & a_{ij} & \cdots & a_{in} \\ \vdots & \vdots & & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mj} & \cdots & a_{mn} \end{bmatrix}.$$

Notice the above matrix is $m \times n$, that is, has m *rows* and n *columns*, and that a_{ij} is the number in row i, column j.

A system of linear equations can be represented as an *augmented matrix*, and can be solved by reducing the matrix using the following row operations

- interchange two rows,
- multiply a row by a nonzero constant, or
- replace a row by a nontrivial linear combination of that and another row, e.g. replace row n by ($a\times$ row m) + ($b\times$ row n) where $a, b \neq 0$.

A matrix is in *row echelon form* when

- the leading nonzero entry in each row is 1 and, in the same column, only 0's are below it,
- the leading 1 of a row is to the right of any leading 1 in the rows above it, and
- any rows of all 0's are at the bottom.

A matrix is in *reduced row echelon form* if in addition to being in row echelon form

- only 0's are above, in the same column, the leading 1 in each row.

Exercises

1. Select the matrix that is in reduced row echelon form

   A. $\begin{bmatrix} 1 & 0 & 0 & 2 \\ 1 & 0 & 0 & 3 \\ 0 & 0 & 1 & 1 \end{bmatrix}$

   B. $\begin{bmatrix} 1 & 1 & 0 & 5 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 1 \end{bmatrix}$

   C. $\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$

   D. $\begin{bmatrix} 1 & 2 & -5 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

   E. $\begin{bmatrix} 1 & 0 & 1 & -1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

2. Find the reduced row echelon form of the matrix $\begin{bmatrix} -3 & 2 & -3 \\ 12 & -8 & -12 \end{bmatrix}$.

3. Find the reduced row echelon form of the matrix $\begin{bmatrix} 1 & -1 & 1 & -4 \\ 2 & -3 & 4 & -15 \\ 5 & 1 & -2 & 12 \end{bmatrix}$.