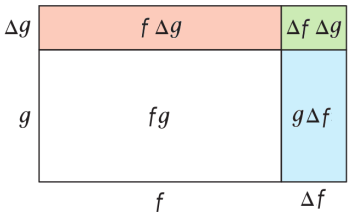
3.2: The Product and Quotient Rules

The Product Rule: If f and g are differentiable, then

$$\frac{d}{dx}[f(x)g(x)] = f(x)\frac{d}{dx}[g(x)] + g(x)\frac{d}{dx}[f(x)].$$

In another notation, the product rule is $(fg)' = fg' + gf'$.



The geometry of the Product Rule

Example 1.

(a) If $f(x) = xe^x$, find $f'(x)$.

(b) Find the n^{th} derivative, $f^{(n)}(x)$.

Example 2. *Differentiate the function $f(t) = \sqrt{t}(a + bt)$.*

Example 3. *If $f(x) = \sqrt{x} \cdot g(x)$, where $g(4) = 2$, and $g'(4) = 3$, find $f'(4)$.*

The Quotient Rule: If f and g are differentiable, then

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)\frac{d}{dx}[f(x)] - f(x)\frac{d}{dx}[g(x)]}{[g(x)]^2}.$$

In another notation, the quotient rule is $\left(\frac{f}{g}\right)' = \frac{gf' - fg'}{g^2}$.

Example 4. *Differentiate $y = \frac{x^2+x-2}{x^3+6}$.*

Example 5. *Find an equation of the tangent line to the curve $y = \frac{e^x}{1+x^2}$ at the point $(1, \frac{1}{2}e)$.*

Example 6.

(a) Differentiate $f(x) = \sqrt{x}e^x$.

(b) Find an equation of the line tangent to $y = \frac{x^2-1}{x^2+x+1}$ at the point $(1, 0)$.

Example 7. *Differentiate $y = \frac{1+xe^x}{e^x}$.*

So far, we have learned the following differentiation formulas,

$\frac{d}{dx}(c) = 0$

$\frac{d}{dx}(x^n) = nx^{n-1}$

$\frac{d}{dx}(e^x) = e^x$

$(cf)' = cf'$

$(f+g)' = f' + g'$

$(f-g)' = f' - g'$

$(fg)' = fg' + gf'$

$\left(\frac{f}{g}\right) = \frac{gf' - fg'}{g^2}$