In [10]:

```
from sympy import *
init_printing()
```

In [11]:

```
q1, q2, q3 = symbols('q_1 q_2 q_3',
                     real=True,
                     positive=True,
                     finite=True)

# Define payoff functions for solvable system
payoff1 = Rational(1,4)*(q1**2) - q1 * q2 + Rational(1,3) * q1 * q3
payoff2 = Rational(1,4) * q1 - Rational(1,4)*q2**2 + q2 * q3
payoff3 = Rational(1,8) * q3 * (q1 + q2) - q3

# Define payoff functions for unsolvable system
# payoff1_ns
# payoff2_ns
# payoff3_ns

payoff1, payoff2, payoff3
```

Out[11]:

$$\left(\frac{q_1^2}{4} - q_1 q_2 + \frac{q_1 q_3}{3}, \quad \frac{q_1}{4} - \frac{q_2^2}{4} + q_2 q_3, \quad \frac{q_3}{8}(q_1 + q_2) - q_3 \right)$$

In [12]:

```
D1 = payoff1.diff(q1)
D2 = payoff2.diff(q2)
D3 = payoff3.diff(q3)
D = [D1, D2, D3]
D1, D2, D3
```

Out[12]:

$$\left(\frac{q_1}{2} - q_2 + \frac{q_3}{3}, \quad -\frac{q_2}{2} + q_3, \quad \frac{q_1}{8} + \frac{q_2}{8} - 1 \right)$$

In [13]:

```
# Solve the FOC system
solve([D1, D2, D3], [q1, q2, q3], dict=True)
```

Out[13]:

$$\left[\left\{ q_1 : 5, \quad q_2 : 3, \quad q_3 : \frac{3}{2} \right\} \right]$$

In [14]:

```
A = Matrix([
    [Rational(1,2), -1, Rational(1,3)],
    [0, -Rational(1,2), 1],
    [Rational(1,8), Rational(1,8), 0]
])
A
b = Matrix([[0],
            [0],
            [1]])

A, b
```

Out[14]:

$$\left(\begin{bmatrix} \frac{1}{2} & -1 & \frac{1}{3} \\ 0 & -\frac{1}{2} & 1 \\ \frac{1}{8} & \frac{1}{8} & 0 \end{bmatrix}, \quad \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right)$$

In [15]:

```
A.rref()
```

Out[15]:

$$\left(\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad [0, \quad 1, \quad 2] \right)$$

In [16]:

```
A.inv()
```

Out[16]:

$$\begin{bmatrix} \frac{3}{4} & -\frac{1}{4} & 5 \\ -\frac{3}{4} & \frac{1}{4} & 3 \\ -\frac{3}{8} & \frac{9}{8} & \frac{3}{2} \end{bmatrix}$$

In [17]:

```
A.det()
```

Out[17]:

$$-\frac{1}{6}$$

In [18]:

```
A.inv() * b
```

Out[18]:

$$\begin{bmatrix} 5 \\ 3 \\ \frac{3}{2} \end{bmatrix}$$