Quotient Set Notation There are many ways denote the quotient set (check the *Book of Proof* for one way). A common method is to get a notation for an individual equivalence class first. Say $[x] = \{y \in C : x \sim y\}$ which is the set of all bundles in C that are indifferent to x. Then we might create a class of sets (set of sets) using the indexed collection of sets notation you covered in the prerequisite readings. For example $\mathcal{I} = \{[x] : x \in C\}$ where $[x] = \{y \in C : x \sim y\}$. Remember that sets only contain distinct elements so even though we "loop" through all values x any two values x, y such that $x \sim y$ will lead to $[x] = [y]$ and the distinct equivalence class will only show up once in the collection $\mathcal{I}$.

Problem 10 Let $\succsim$ be rational preference relation and define $\sim$ as the indifference relation where $x \sim y$ iff $[x \succsim y \wedge y \succsim x]$. Because $\succsim$ is rational we know that it is complete, reflexive and transitive. To show $(x, x) \in \sim$, $\forall x$ we can refer to completeness to know that either $x \succsim x$ or $x \succsim x$ and by reflexiveness we know both statements are true so $x \sim x$.

Symmetry would imply that for any $x, y \in C$ if $(x, y) \in \sim$ then $(y, x) \in \sim$. Again we now know that $\sim$ is reflexive and we still know $\succsim$ is complete, reflexive and transitive and we should use these facts to establish symmetry and transitivity of $\sim$. Let $x, y \in C$ and suppose $x \sim y$. Then we know $[x \succsim y \wedge y \succsim x]$ is true by definition of $\sim$. Because the order of arguments in the conjunction don't alter its truth value we could say $[y \succsim x \wedge x \succsim y]$ which is the definition of $y \sim x$.

For transitivity, suppose $x, y, z \in C$ and $[x \sim y \wedge y \sim z]$. Then by definition $[x \succsim y \wedge y \succsim x]$ and $[y \succsim z \wedge z \succsim y]$. Hence we have $x \succsim y \succsim z$ which by transitivity of $\succsim$ implies $x \succsim z$ and we also have $z \succsim y \succsim x$ which by transitivity of $\succsim$ implies $z \succsim x$. Since we have $[x \succsim z \wedge z \succsim x]$ we see that $x \sim z$ and therefore $\sim$ is transitive.