Tutorial – Mathematics for Political Science

Winter semester 2024/25

Basics and Preliminaries

GitHub: https://github.com/joerdisstrack/tutorial_mathematics_social_science

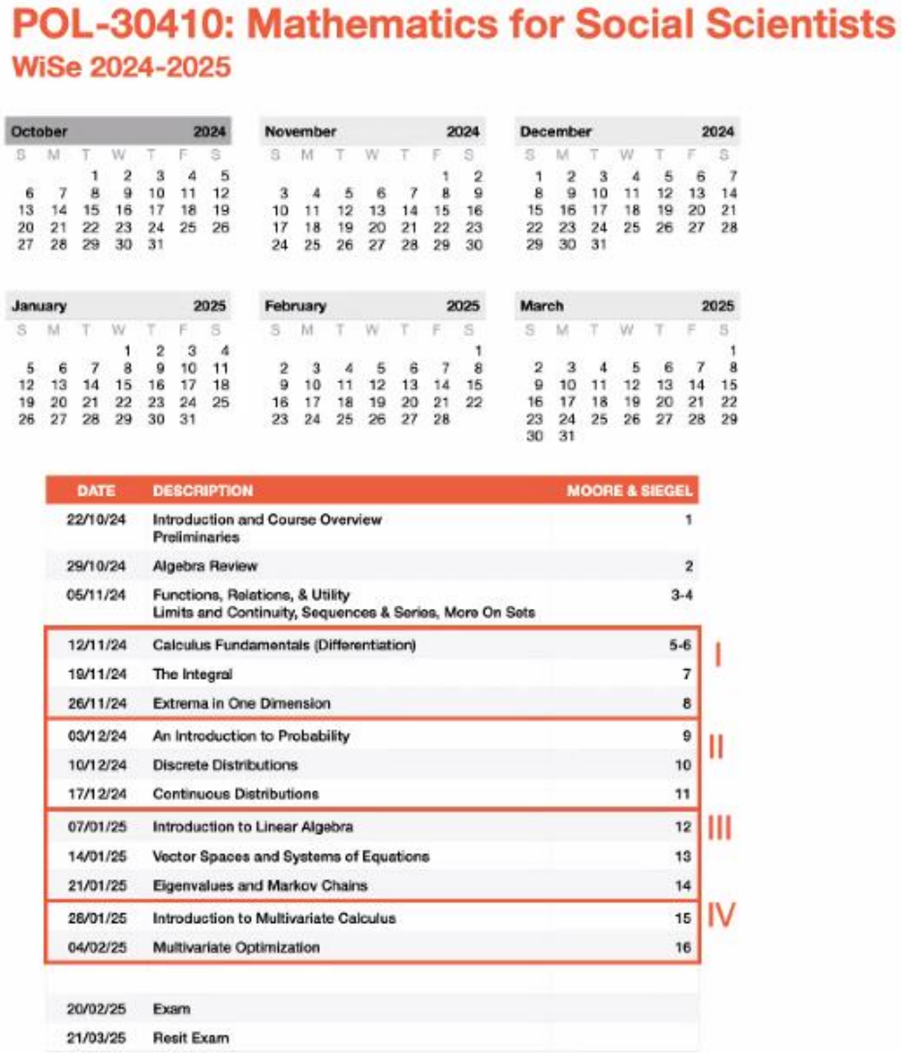
To do

- introduction: problem sets
- weekly recap
  - basics and preliminaries
  - intro to algebra
- hands on practice
- questions

Introduction

Problem Sets:

- There will be four problem sets throughout the semester, each is worth 12.5% of your final grade
- There will be 1 PS for each main block:
  - Algebra – 05/11/2024
  - Calculus ID – 26/11/2024
  - Probability – 07/01/2025
  - Multivariate Calculus – 21/01/2025
- Note that these dates are preliminary and may change throughout the semester



POL-30410: Mathematics for Social Scientists

WiSe 2024-2025

October						2024
S	M	T	W	T	F	S
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30	31		

November						2024
S	M	T	W	T	F	S
					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30

December						2024
S	M	T	W	T	F	S
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31				

January						2025
S	M	T	W	T	F	S
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30	31	

February						2025
S	M	T	W	T	F	S
						1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23	24	25	26	27	28	

March						2025
S	M	T	W	T	F	S
						1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23	24	25	26	27	28	29
30	31					

DATE	DESCRIPTION	MOORE & SIEGEL	
22/10/24	Introduction and Course Overview Preliminaries	1	
29/10/24	Algebra Review	2	
05/11/24	Functions, Relations, & Utility Limits and Continuity, Sequences & Series, More On Sets	3-4	
12/11/24	Calculus Fundamentals (Differentiation)	5-6	I
19/11/24	The Integral	7	
26/11/24	Extrema in One Dimension	8	
03/12/24	An Introduction to Probability	9	II
10/12/24	Discrete Distributions	10	
17/12/24	Continuous Distributions	11	
07/01/25	Introduction to Linear Algebra	12	III
14/01/25	Vector Spaces and Systems of Equations	13	
21/01/25	Eigenvalues and Markov Chains	14	
28/01/25	Introduction to Multivariate Calculus	15	IV
04/02/25	Multivariate Optimization	16	
20/02/25	Exam		
21/03/25	Resit Exam		

Introduction

Problem Sets:

- You have 1 week to complete each PS and to hand in a **scan** in **pdf format** via Ilias **within** the deadline
- You should name your file → PSX_matrikelnr.pdf e.g. PS1/1234567
- Please write your last name and matrikel nr. /Student ID on the last page of your paper version before you scan it

Introduction

There are a bunch of good helping hands out there to support your learning process:

- Wolfram Alpha
- Symbolab
- R, python, MATLAB
- Chat GPT
- GeoGebra
- etc…

→ Do NOT rely on these too much! You CANNOT use them during the exam

Chapter 1 | Preliminaries

Preliminary vocab

Theory

- a set of statements involving concepts and concern relationships among abstract concepts

Statements

- comprise assumptions, propositions, corollaries, and hypotheses

Assumptions are asserted by us

- **propositions** and corollaries are deduced from these assumptions
  - **hypotheses** are derived from these deductions and then empirically challenged

Preliminary vocab

Concepts

- inventions that human beings create to help them understand the world and may take on different values

Variables

- indicators we develop to measure our concepts
- mathematically they take on different values in given sets

Constants

- concept or a measure that has a single value for a given set

Sets

- describe variables as **discrete** or **continuous**
- **discrete**
  - a variable is discrete if each one of its possible values can be associated with a single integer
- **continuous**:
  - a variable is continuous if its values cannot be assigned a single integer

  → typically assumed to be drawn from subset of real numbers

- sets give the domain – the range of values – a concept may take

Table 1.1: Common Sets

Notation	Meaning
$\mathbb{N}$	Natural numbers
$\mathbb{Z}$	Integers
$\mathbb{Q}$	Rational numbers
$\mathbb{R}$	Real (rational and irrational) numbers
$\mathbb{C}$	Complex numbers
Subscript: $\mathbb{N}_+$	Positive (negative) values of the set
Superscript: $\mathbb{N}^d$	Dimensionality (number of dimensions)

Moore and Siegel, 2013, p. 5

Types of sets

Solution set
- all solutions to a problem

Sample space
- contains all values a variable can take on

Spaces
- sets with some structure – e.g. the difference between elements in $\mathbb{Z}$

Finite sets
- have fixed cardinality – e.g. all integers between 1 and 10

Infinite sets … do not
- all numbers in $\mathbb{Z}$

Uncountable sets
- cannot be classified using cardinality – e.g. all decimal numbers between 1 and 3

Tuple
- an ordered pair

Singleton
- only one element

Empty set
- contains no element

Universal set
- contains ALL elements

Ordered sets
- order of elements must be maintained

Unordered sets
- order does not matter

Operators

The classics:

- addition, subtraction, multiplication, division

Sum operator

$$\sum_{i}^{n} x_i$$

- the sum of x_i over the range from $i = 1$ through $i = 4$

$$\sum_{i=1}^{4} x_{i=1+2+3+4=10}$$

Multiplication operator

$$\prod_{i}^{n} x_i$$

- the product of x_i over the range from $i = 1$ through $i = 4$

$$\prod_{i=1}^{4} x_i = 1 \cdot 2 \cdot 3 \cdot 4 = 24$$

Set operators

Union

- $A \cup B$

Intersection

- $A \cap B$

Difference

- $A \setminus B$

Complement

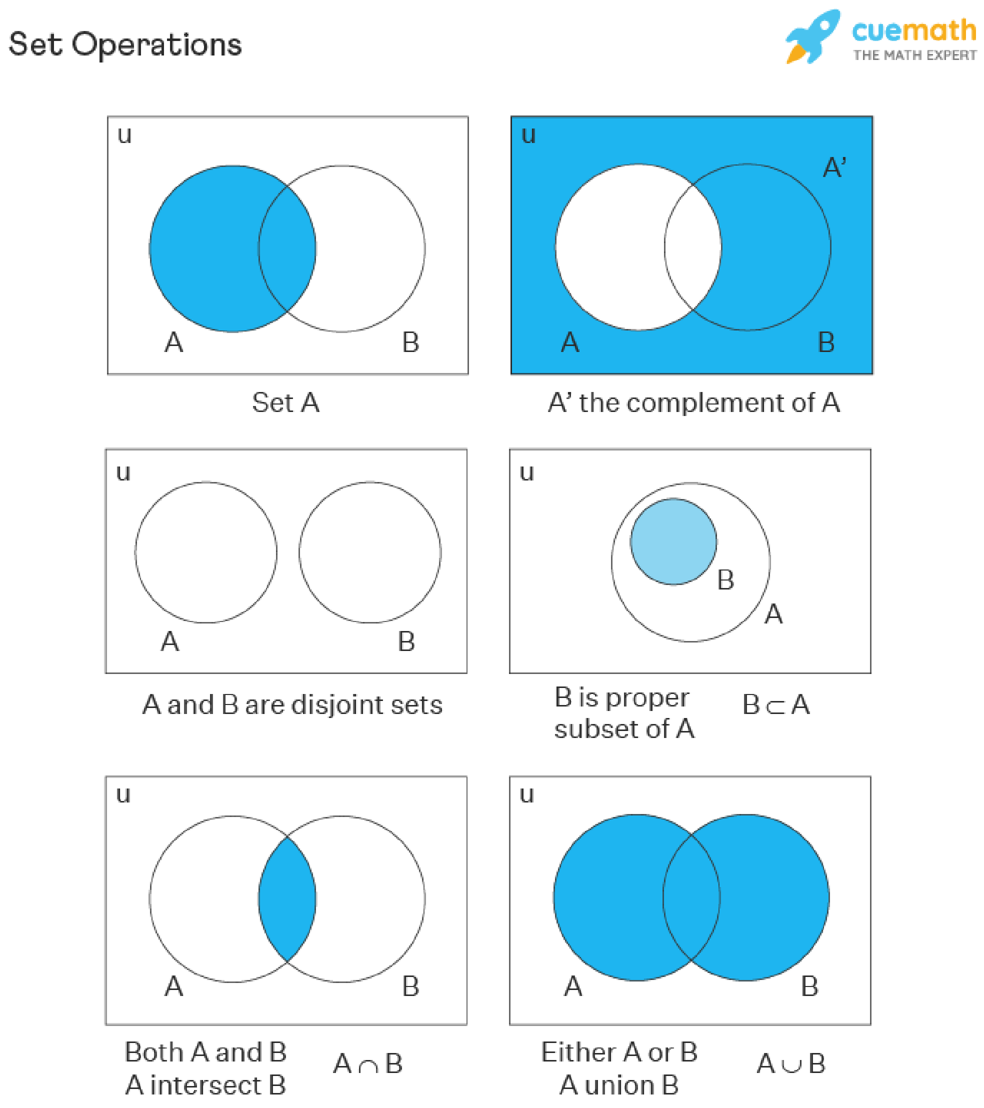
- $\neg B$

Partition P of M

- $P = \{\{blue\}, \{green\}\}$ *and* $M = \{blue, green\}$

Cartesian Product

- $A \times B$



Hands on – Set operators

Task: Let $A = \{1, 3, 5, 7, 9\}$, $B = \{2, 4, 6, 8, 10\}$, $C = \{2, 5, 8, 9\}$ from the universal set U= $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$. Assuming that we do not include the same number as duplicates, find the following:

- $A \cap C$
- $A \cup C$
- $B \cap C$
- $A - C$
- $\neg B$
- $(A \cup C)\setminus \mathrm{B}$
- $(A \cap B)\setminus C$
- $\neg(A \cup C)$

Hands on – Set operators

Solution:

- $A \cap C = \{5, 9\}$
- $A \cup C = \{1, 2, 3, 5, 7, 8, 9\}$
- $B \cap C = \{2, 8\}$
- $A \backslash C = \{1, 3, 7\}$
- $\neg B = A$
- $(A \cup C) \backslash \mathrm{B} = A$
- $(A \cap B) \backslash C = \{\quad\}$
- $\neg(A \cup C) = \{4, 6, 10\}$

Hands on – Partitions

Task: Find all partitions of $M = \{1, 3, 5\}$

Hands on – Partitions

Solution: M has five partitions:

- $P_1 = \{\{1, 3, 5\}\}$
- $P_2 = \{\{1\}, \{3, 5\}\}$
- $P_3 = \{\{3\}, \{1, 5\}\}$
- $P_4 = \{\{5\}, \{1, 3\}\}$
- $P_5 = \{\{1\},\ \{3\},\ \{5\}\}$

Set operators

Mutually exclusive

- intersection equal to the empty set, i.e., sets with no elements in their intersection

Collectively exhaustive

- a group of sets is collectively exhaustive if together the sets constitute the universal set

Relations

- **used to compare** variables, constants and concepts via >, ≥, ≤, <, =, ≠
- **binary relation**
  - ordered by size (a, b) or a > b

- **functions** are relations, too!
- **consider a function f(x)**
  - **domain**

    → The domain consists of all possible values that x can take on
  - **range**

    → The range consists of all possible values y takes on given x

Level of measurement

Difference of kind

- nominal – distinction by name, type [Greens, SPD, CDU, …]

Difference of degree

- ordinal – distinction by order, size [language ability on your CV]
- interval – same difference between each element [$\mathbb{Z}$ - set of all integers, temperature]
- ratio – ‚meaningful' or true 0 as starting point [length in metres]

	Distinct categories	Meaningful order	Equal spacing	True zero
Nominal	✓			
Ordinal	✓	✓		
Interval	✓	✓	✓	
Ratio	✓	✓	✓	✓

Proofs

Axioms and assumptions

- stated to begin and assumed as true

Proposition

- considered as true based on prior assumptions

Theorem

- a proven proposition

Lemma

- a theorem of ‚little interest‘ used as a prior step to solve another problem

Corollary

- proposition following from the proof of a 2nd proposition which requires no further proof

Proofs

Direct proofs

- proof by deduction
- proof by exhaustion
- proof by construction
- proof by induction

Indirect proofs

- counterexample
- contradiction

Proof by induction

Initial step

- provide base case for assumption $A(1)$
- necessary to show validity often for n = 1

Inductive hypothesis

- assume that $A(n)$ for $n \in \mathbb{N}$ is true
- this step requires no computation, it can be a sentence you learn by heart ☺

Inductive step

- increment n by one and prove that $A(n+1)$ is true

→ if case is true for both n and $n+1$ we know our case is true for $\mathrm{n} \in \mathbb{N}$

Proof by induction

Let‘s look at Gauss $\sum_{k=0}^{n} k = \frac{n \cdot (n+1)}{2}$ holds for $\forall\, n \in \mathbb{N}$

Initial step for $n = 1$

$$\sum_{k=0}^{1} 0 + 1 = \frac{1 \cdot (1+1)}{2} = 1$$

Inductive hypothesis

→ statement A(n) holds for any $n \in \mathbb{N}$

Inductive step for $n + 1$

$$\sum_{k=0}^{n+1} k = (n+1) + \sum_{k=0}^{n} k = (n+1) + \frac{n \cdot (n+1)}{2}$$

$$= \frac{2\,(n+1)}{2} + \frac{n\,(n+1)}{2} = \frac{2\,(n+1) + n(n+1)}{2} = \frac{(n+2) + (n+1)}{2}$$

Time for your questions

- Any questions during the week?
  - → joerdis.strack@uni-konstanz.de