# Hydromechanics - Principal Stress Output - Tube

## Analytical Solution

To test the correct output of the principal stresses, a tube under plane strain is simulated with high pressure on the inside - see Kolditz et al. [2015] for more details. A 2D axial symmetric model is used for this simulation. The principal stresses in this model are equal to the radial and circumferential stresses with the analytical solution:

$$\sigma_{rr} = \frac{p_i R_i^3 - p_a R_a^3}{R_a^3 - R_i^3} - \frac{R_a^3 R_i^3 (p_i - p_a)}{(R_a^3 - R_i^3) r^3} \tag{1}$$

$$\sigma_{\phi\phi} = \frac{p_i R_i^3 - p_a R_a^3}{R_a^3 - R_i^3} + \frac{R_a^3 R_i^3 (p_i - p_a)}{(R_a^3 - R_i^3) r^3} \tag{2}$$

$$\sigma_{\theta\theta} = \frac{2\nu p_a R_a^2 - p_i R_i^2}{R_i^2 - R_a^2} \tag{3}$$

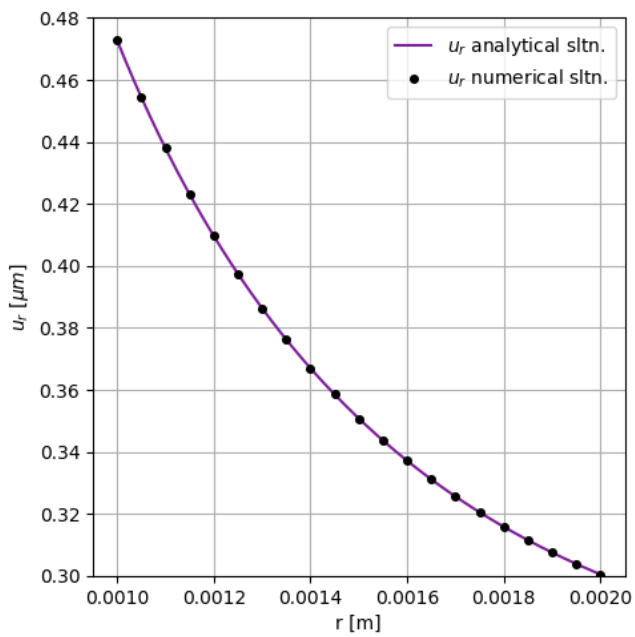
Additionally the displacement result of the simulation is also compared with the analytical solution:

$$u_r = \frac{R_i^2 p_i (1+\nu) r}{E(R_i^2 - R_a^2)} \left[ \left( \frac{p_a}{p_i} \left( \frac{R_a}{R_i} \right)^2 - 1 \right) (1 - 2\nu) + \left( \frac{p_a}{p_i} - 1 \right) \left( \frac{R_a}{r} \right)^2 \right] \tag{4}$$

With the parameters

$$p_i = 52.2 \cdot 10^6\,\mathrm{Pa} \qquad p_a = 0.1 \cdot 10^6\,\mathrm{Pa} \qquad E = 210\,\mathrm{Pa}$$
$$R_i = 0.001\,\mathrm{m} \qquad R_a = 0.002\,\mathrm{m} \qquad \nu = 0.3$$

the simulation was executed with the model having a 400 elements in r-direction and being 1 element high. The results show a good agreement with the analytical solution.



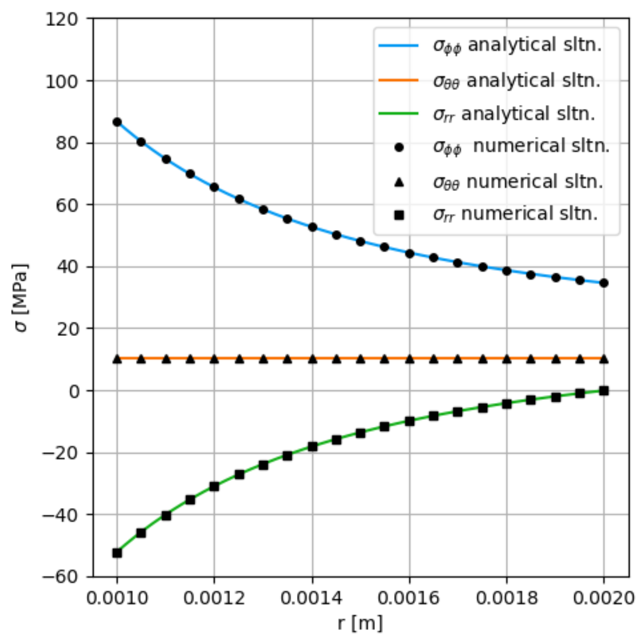




Fig. 1: Radial and circumferential stresses

Fig. 2: Radial displacement

### Relative Error

The following figures show the relative errors between the numerical and the analytical solutions. The error in displacement is near numerical precision and the error of the principal stresses is low enough to be sufficient for a derived quantity.

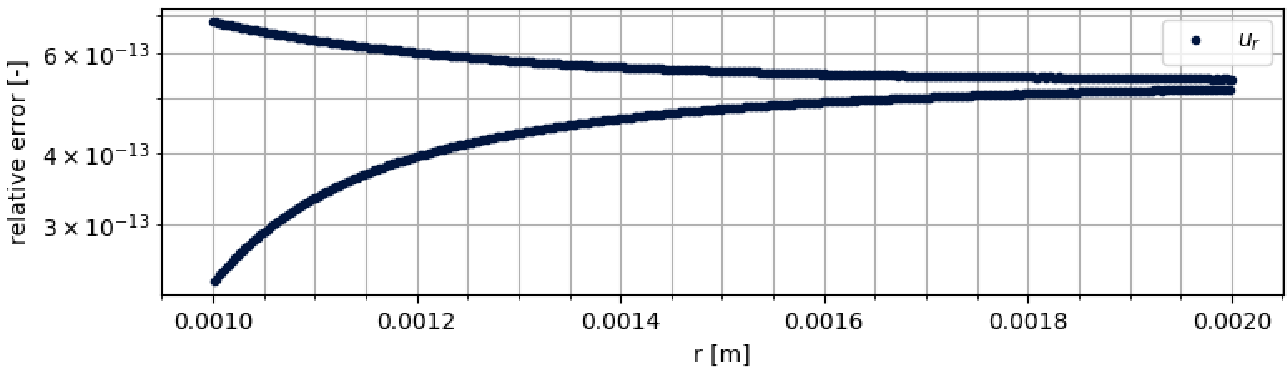



Fig. 3: Relative error for the radial displacement

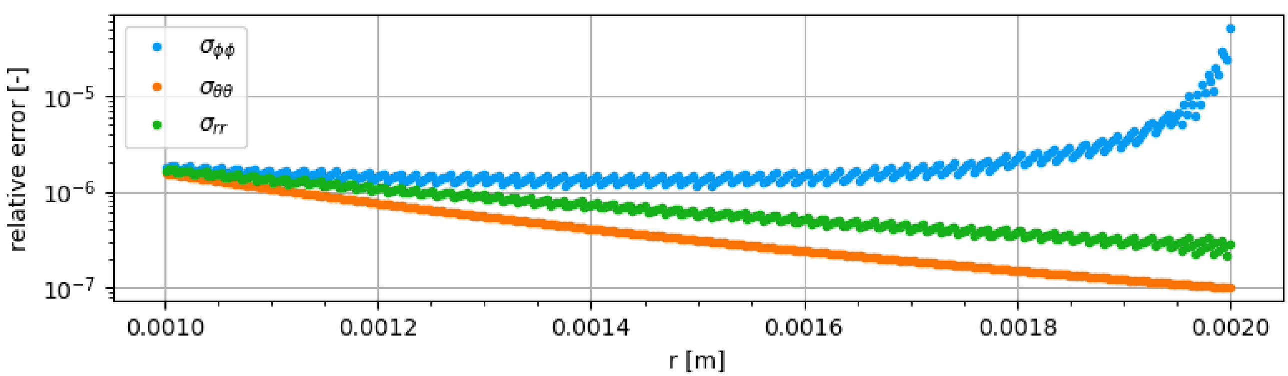



Fig. 4: Relative error for the principal stresses

# References


Olaf Kolditz, Hua Shao, Wenqing Wang, and Sebastian Bauer. *Thermo-Hydro-Mechanical-Chemical Processes in Fractured Porous Media: Modelling and Benchmarking.* Springer International Publishing, Cham, 2015. ISBN 978-3-319-11893-2. doi: 10.1007/978-3-319-11894-9.