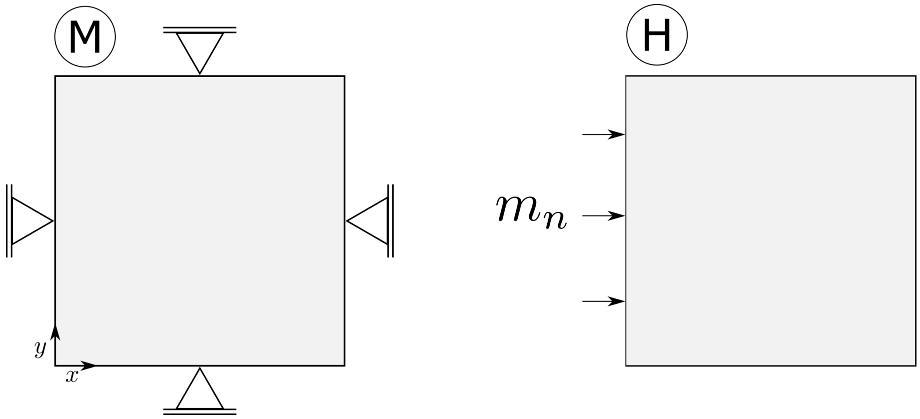
Hydromechanics: flow, no strain



This test uses a square of 1 m x 1 m with a fluid influx from the left edge and no outflow. All displacements are zero. With the mass balance we can derive an analytical solution for the pressure solution.

$$0 = (\varrho_\mathrm{F})'_\mathrm{S} + \mathrm{div}(\varrho_\mathrm{FR}\underline{\tilde{w}}_\mathrm{FS}) \tag{1}$$

The mass balance in turn can be written in integral form as:

$$0 = \int\limits_\Omega \left[(\varrho_\mathrm{F})'_\mathrm{S} + \mathrm{div}(\varrho_\mathrm{FR}\underline{\tilde{w}}_\mathrm{FS})\right]\,\mathrm{d}\Omega = \int\limits_\Omega (\varrho_\mathrm{F})'_\mathrm{S}\,\mathrm{d}\Omega + \int\limits_{\partial\Omega} \varrho_\mathrm{FR}\underline{\tilde{w}}_\mathrm{FS}\cdot\underline{n}\,\mathrm{d}\Gamma \tag{2}$$

For the homogeneous case considered and the boundary conditions given above this translates into the simple form

$$(\varrho_\mathrm{F})'_\mathrm{S}\,\Omega = m_n\Gamma \tag{3}$$

Furthermore we assume material compressibility of both the solid and fluid

$$(\varrho_\mathrm{FR})'_\mathrm{S} = \frac{\partial\varrho_\mathrm{FR}}{\partial p_\mathrm{FR}}\,(p_\mathrm{FR})'_\mathrm{S} = \varrho_\mathrm{FR}\beta_p\,(p_\mathrm{FR})'_\mathrm{S} \tag{4}$$

the following relation can be obtained

$$(\varrho_\mathrm{F})'_\mathrm{S} = \varrho_\mathrm{FR}\beta_p\,(p_\mathrm{FR})'_\mathrm{S}\,\phi + \varrho_\mathrm{FR}(\alpha_\mathrm{B}-\phi)K_\mathrm{SR}^{-1}\,(p_\mathrm{FR})'_\mathrm{S} \tag{5}$$

$$(\varrho_\mathrm{F})'_\mathrm{S} = \varrho_\mathrm{FR}\,(p_\mathrm{FR})'_\mathrm{S}\left[\beta_p\phi + \frac{\alpha_\mathrm{B}-\phi}{K_\mathrm{SR}}\right] \tag{6}$$

For ideal gases as the fluid we can further find that

$$\varrho_\mathrm{FR} = \frac{p_\mathrm{FR}}{R_s T} \qquad \text{and} \qquad \beta_p = \frac{1}{p_\mathrm{FR}} \tag{7}$$

A combination of the above leads to the pressure solution

$$m_n \frac{\Gamma}{\Omega} R_s T = \left(\phi + \frac{\alpha_\mathrm{B} - \phi}{K_\mathrm{SR}} p_\mathrm{FR} \right) (p_\mathrm{FR})'_\mathrm{S} \tag{8}$$

$$m_n \frac{\Gamma}{\Omega} R_s T t = \left(p_\mathrm{FR}^2 - p_\mathrm{FR0}^2 \right) \frac{\alpha_\mathrm{B} - \phi}{2K_\mathrm{SR}} + \left(p_\mathrm{FR} - p_\mathrm{FR0} \right) \phi \tag{9}$$

$$p_\mathrm{FR} = -a \pm \sqrt{\left(a + p_\mathrm{FR0} \right)^2 + b} \tag{10}$$

$$\text{with} \qquad a = K_\mathrm{SR} \frac{\phi}{\alpha_\mathrm{B} - \phi} \tag{11}$$

$$\text{and} \qquad b = p_\mathrm{FR0}^2 + a p_\mathrm{FR0} + m_n \frac{\Gamma}{\Omega} R_s T \frac{a}{\phi} t \tag{12}$$

Given the following parameters

$$\begin{aligned} \Gamma &= 1\,\mathrm{m}^2 & \Omega &= 1\,\mathrm{m}^3 \\ R &= 287.058\,\mathrm{J/(kgK)} & T &= 293.15\,\mathrm{K} \\ E &= 1000\,\mathrm{MPa} & \nu &= 0.3 \\ p_\mathrm{FR0} &= 0.1\,\mathrm{MPa} & m_n &= 10^{-3}\,\mathrm{kg/(m^2 s)} \\ \phi &= 0.03 & \alpha_\mathrm{B} &= 0.6 \\ K_\mathrm{SR} &= \frac{K_\mathrm{S}}{1 - \alpha_\mathrm{B}} & K_\mathrm{S} &= \frac{E}{3(1 - 2\nu)} \end{aligned}$$

we get the following pressure solution for a time of 100 s. The simulation result is reasonable close to the analytical solution with the relative error being below 10^{-6}.

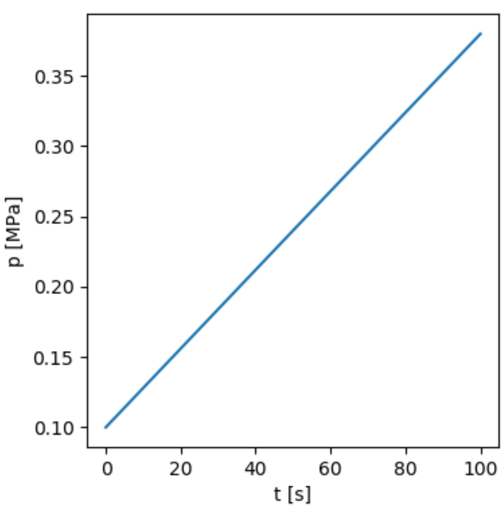


Fig. 1: pressure solution

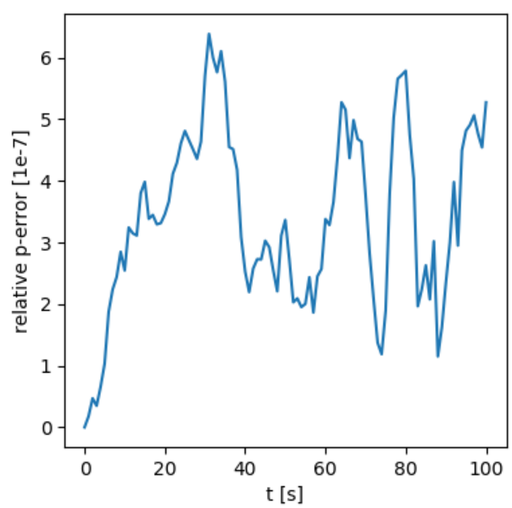


Fig. 2: relative error for pressure