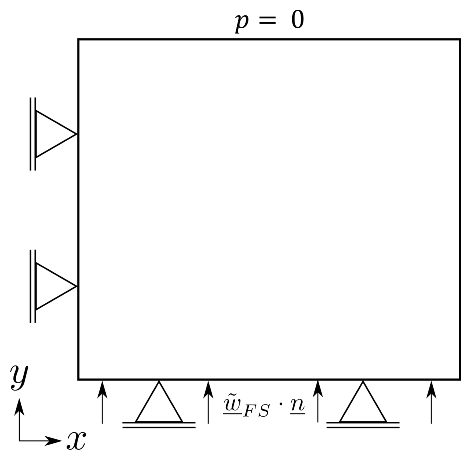
Hydromechanics: Darcy-flow

This benchmark uses a square of 1 m x 1 m with a fluid inflow at the bottom edge and zero pressure at the top edge.



The relevant parameters are:

- fluid-density $\varrho_{\mathrm{FR}} = 1000\,\mathrm{kg\,m^{-3}}$
- fluid-viscosity $\mu = 10^{-3}\,\mathrm{Pa\,s}$
- solid-density $\varrho_{\mathrm{SR}} = 2000\,\mathrm{kg\,m^{-3}}$
- intrinsic permeability $\boldsymbol{K} = 10^{-12}\,\mathrm{m^2}$
- specific storage $S = 0$
- porosity $\phi = 0.1$
- fluid inflow $\underline{\tilde{w}}_{\mathrm{FS}} \cdot \underline{n} = 10^{-5}\,\mathrm{m^3/m^2 s}$
- body force vector
  - with gravity: $\underline{b} = \{0, -10\}\mathrm{m\,s^{-2}}$
  - without gravity: $\underline{b} = \{0, 0\}\mathrm{m\,s^{-2}}$

Analytical solution

By using Darcy's law we can calculate the resulting pressure gradient. Because there is no change of pressure in x-direction, only the y-direction is of interest.

$$
\begin{aligned}
\underline{\tilde{w}}_{\mathrm{FS}} &= -\frac{\boldsymbol{K}}{\mu}\left(\operatorname{grad} p_{\mathrm{FR}} - \varrho_{\mathrm{FR}}\underline{b}\right) \\
\operatorname{grad} p_{\mathrm{FR}} &= -\underline{\tilde{w}}_{\mathrm{FS}}\frac{\mu}{\boldsymbol{K}} + \varrho_{\mathrm{FR}}\underline{b}
\end{aligned}
$$

With gravity

$$
\begin{aligned}
\operatorname{grad} p_{\mathrm{FR}} &= -10^{-5}\,\mathrm{m/s}\frac{10^{-3}\,\mathrm{Pa\,s}}{10^{-12}\,\mathrm{m^2}} + 1000\,\frac{\mathrm{kg}}{\mathrm{m^3}}\left(-10\,\frac{\mathrm{m}}{\mathrm{s^2}}\right) \\
\operatorname{grad} p_{\mathrm{FR}} &= -2 \cdot 10^{-4}\,\frac{\mathrm{Pa}}{\mathrm{m}}
\end{aligned}
$$

Without gravity

$$
\begin{aligned}
\operatorname{grad} p_{\mathrm{FR}} &= -10^{-5}\,\mathrm{m/s}\frac{10^{-3}\,\mathrm{Pa\,s}}{10^{-12}\,\mathrm{m^2}} + 1000\,\frac{\mathrm{kg}}{\mathrm{m^3}}\,0\,\frac{\mathrm{m}}{\mathrm{s^2}} \\
\operatorname{grad} p_{\mathrm{FR}} &= -10^{-4}\,\frac{\mathrm{Pa}}{\mathrm{m}}
\end{aligned}
$$

Simulation result with OGS 6

In both cases the simulation with OGS 6 yields the correct result for the pressure gradient.

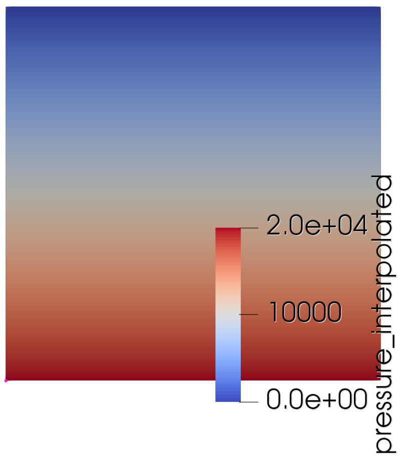


Fig. 1: Solution with gravity

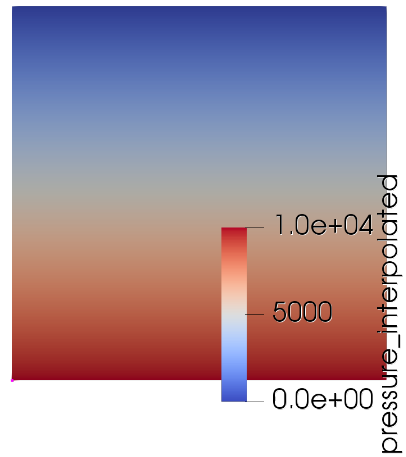


Fig. 2: Solution without gravity

When checking the velocity of the fluid in the case with gravity, the results do not agree with the boundary conditions of the Benchmark. Calculating the velocity from the pressure gradient should naturally result in $10^{-5}\,\mathrm{m/s}$.

$$\underline{\tilde{w}}_{\mathrm{FS}} = -\frac{\boldsymbol{K}}{\mu}(\operatorname{grad} p_{\mathrm{FR}} - \varrho_{\mathrm{FR}}\underline{b})$$
$$\tilde{w}_{\mathrm{FS}} = -\frac{10^{-12}\,\mathrm{m}^2}{10^{-3}\,\mathrm{Pa\,s}}\left(-2\cdot 10^4\,\frac{\mathrm{Pa}}{\mathrm{m}} - 1000\,\frac{\mathrm{kg}}{\mathrm{m}^3}\left(-10\,\frac{\mathrm{m}}{\mathrm{s}^2}\right)\right) = 10^{-5}\,\mathrm{m/s}$$

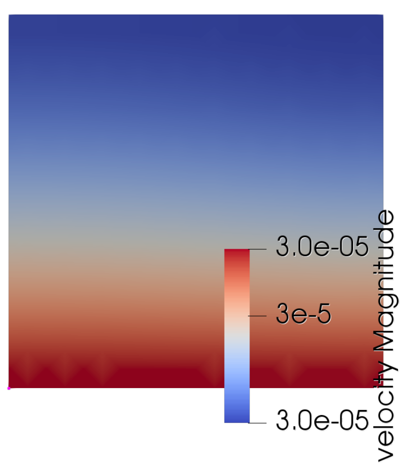


Fig. 3: Wrong velocity output

Therefore the code has been adjusted to solve this error as shown below:

Previous Implementation

```
auto const& dNdx_p = _ip_data[ip].dNdx_p;
    cache_matrix.col(ip).noalias() =
        -K_over_mu * dNdx_p * p - K_over_mu * rho_fr * b;
```

New Implementation

```
auto const& dNdx_p = _ip_data[ip].dNdx_p;
    cache_matrix.col(ip).noalias() =
        -K_over_mu * dNdx_p * p + K_over_mu * rho_fr * b;
```

With that single change the simulation now gives the correct output for the fluid velocity.

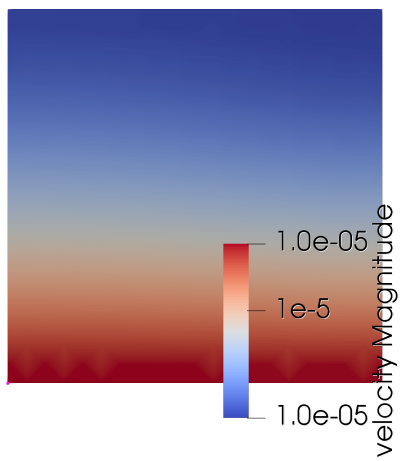


Fig. 4: Correct velocity output with new implementation