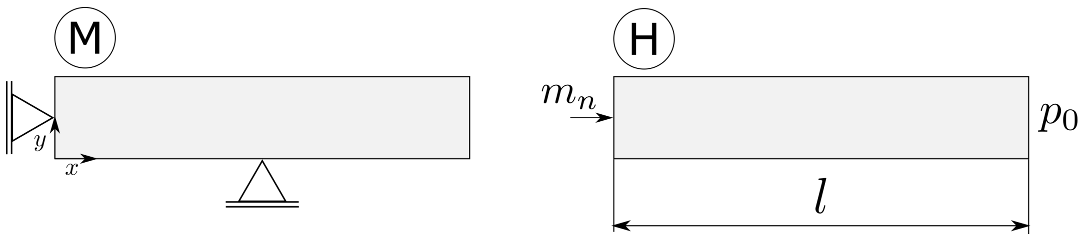
Hydromechanics: flow, pressure boundary



Assuming the fluid is an ideal gas and using Darcy's law (body force is zero) we can derive an analytical solution for the pressure at steady state:

$$\underline{\tilde{w}}_{\mathrm{FS}} = -\frac{\boldsymbol{K}}{\mu}(\operatorname{grad} p - \varrho_{\mathrm{FR}}\underline{b}) \tag{34}$$

$$\underline{\tilde{w}}_{\mathrm{FS}} \cdot \underline{n} = -\frac{\boldsymbol{K}}{\mu}\operatorname{grad}(p) \cdot \underline{n} \tag{35}$$

$$\underline{\tilde{w}}_{\mathrm{FS}} \cdot \underline{n} = -\frac{K}{\mu}\frac{\mathrm{d}p}{\mathrm{d}x} \tag{36}$$

$$m_n = -\frac{K}{\mu}\frac{\mathrm{d}p}{\mathrm{d}x}\varrho_{\mathrm{FR}} = -\frac{K}{\mu}\frac{\mathrm{d}p}{\mathrm{d}x}\frac{p}{R_{\mathrm{s}}T} \tag{37}$$

$$p\,\mathrm{d}p = -m_n R_{\mathrm{s}} T \mu K^{-1}\,\mathrm{d}x \tag{38}$$

$$\int_{p(x)}^{p_0} \bar{p}\,\mathrm{d}\bar{p} = -\int_{x}^{l} m_n R_{\mathrm{s}} T \mu K^{-1}\,\mathrm{d}\bar{x} \tag{39}$$

$$\frac{1}{2}\left(p_0^2 - p(x)^2\right) = -m_n R_{\mathrm{s}} T \mu K^{-1}(l - x) \tag{40}$$

$$p(x) = \sqrt{p_0^2 + 2m_n R_{\mathrm{s}} T \mu K^{-1}(l - x)} \tag{41}$$

$$\tag{42}$$

Given the following parameters

$$R = 287.058\,\mathrm{J/(kgK)} \qquad T = 293.15\,\mathrm{K}$$
$$p_{\mathrm{FR0}} = 0.1\,\mathrm{MPa} \qquad m_n = 10^{-2}\,\mathrm{kg/(m^2s)}$$
$$K = 10^{-12}\,\mathrm{m} \qquad \mu = 10^{-5}\,\mathrm{MPa\,s}$$
$$l = 10\,\mathrm{m}$$

we get a pressure solution which is shown in the figure below. The relative error of the simulation result is about 10^{-13} at the corner nodes of the elements and ranges from 10^{-4} to 10^{-2} at the quadratic nodes which is due to the interpolation of the neighbour values.

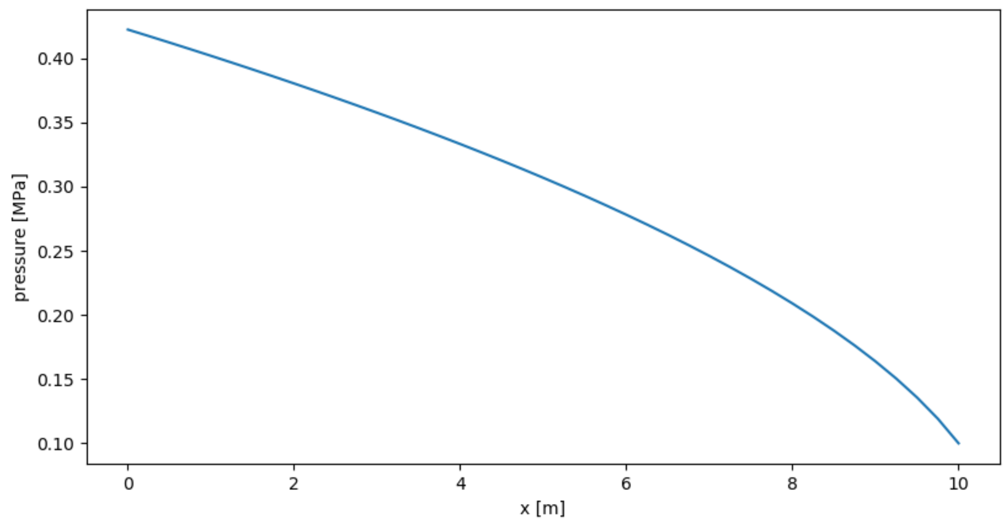


Fig. 3: Analytical solution

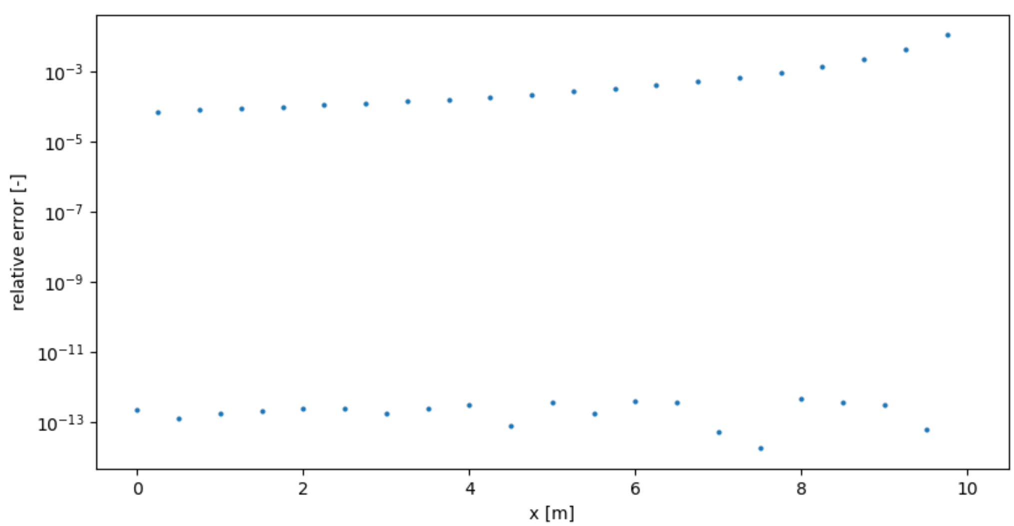


Fig. 4: Relative error