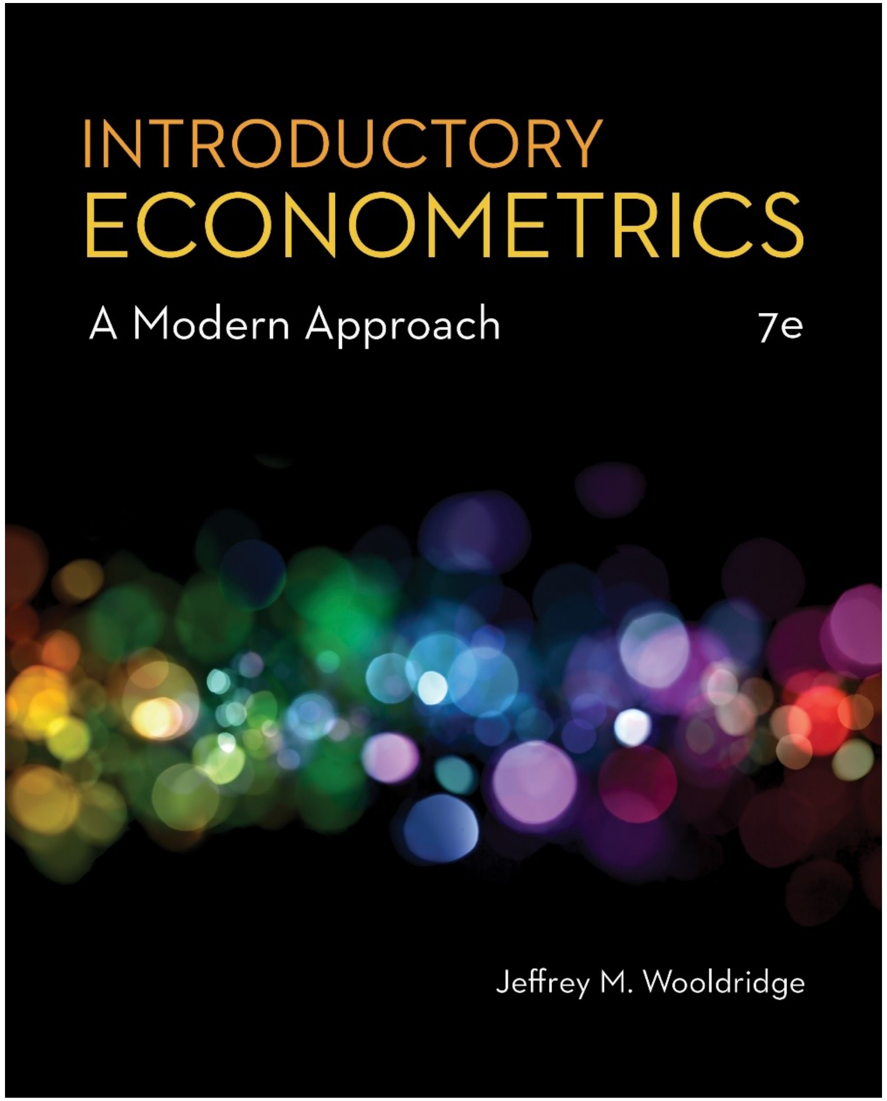


Chapter 10

Basic Regression Analysis with Time Series Data

Basic Regression Analysis with Time Series Data (1 of 25)

- **The nature of time series data**
- Temporal ordering of observations; may not be arbitrarily reordered
- Typical features: serial correlation/nonindependence of observations

- How should we think about the randomness in time series data?
  - The outcome of economic variables (e.g. GNP, Dow Jones) is uncertain; they should therefore be modeled as random variables.
  - Time series are sequences of r.v. (= stochastic processes)
  - Randomness does not come from sampling from a population.
  - "Sample" = the one realized path of the time series out of the many possible paths the stochastic process could have taken.

Basic Regression Analysis with Time Series Data (2 of 25)

- **Example: US inflation and unemployment rates 1948-2003**

TABLE 10.1 Partial Listing of Data on U.S. Inflation and Unemployment Rates, 1948–2003

Year	Inflation	Unemployment
1948	8.1	3.8
1949	−1.2	5.9
1950	1.3	5.3
1951	7.9	3.3
.	.	.
.	.	.
.	.	.
1998	1.6	4.5
1999	2.2	4.2
2000	3.4	4.0
2001	2.8	4.7
2002	1.6	5.8
2003	2.3	6.0

- Here, there are only two time series. There may be many more variables whose paths over time are observed simultaneously.

- Time series analysis focuses on modeling the dependency of a variable on its own past, and on the present and past values of other variables.

Basic Regression Analysis with Time Series Data (3 of 25)

- **Examples of time series regression models**
- Static models
  - In static time series models, the current value of one variable is modeled as the result of the current values of explanatory variables

- Examples for static models

$$inf_t = \beta_0 + \beta_1 unem_t + u_t$$

There is a contemporaneous relationship between unemployment and inflation (= Phillips curve).

$$mrdrte_t = \beta_0 + \beta_1 convrte_t + \beta_2 unem_t + \beta_3 yngmle_t + u_t$$

The current murder rate is determined by the current conviction rate, unemployment rate, and the fraction of young males in the population.

Basic Regression Analysis with Time Series Data (4 of 25)

- **Finite distributed lag models**
  - In finite distributed lag models, the explanatory variables are allowed to influence the dependent variable with a time lag.
- Example for a finite distributed lag model
  - The fertility rate may depend on the tax value of a child, but for biological and behavioral reasons, the effect may have a lag.

$$\underbrace{gfr_t}_{\text{Children born per 1,000 women in year t}} = \alpha_0 + \delta_0 \underbrace{pe_t}_{\text{Tax exemption in year t}} + \delta_1 \underbrace{pe_{t-1}}_{\text{Tax exemption in year t - 1}} + \delta_2 \underbrace{pe_{t-2}}_{\text{Tax exemption in year t - 2}} + u_t$$

Basic Regression Analysis with Time Series Data (5 of 25)

- **Interpretation of the effects in finite distributed lag models**

$$y_t = \alpha_0 + \delta_0 z_t + \delta_1 z_{t-1} + \ldots + \delta_q z_{t-q} + u_t$$

- Effect of a past shock on the current value of the dep. variable

$$\frac{\Delta y_t}{\Delta z_{t-s}} = \delta_s$$

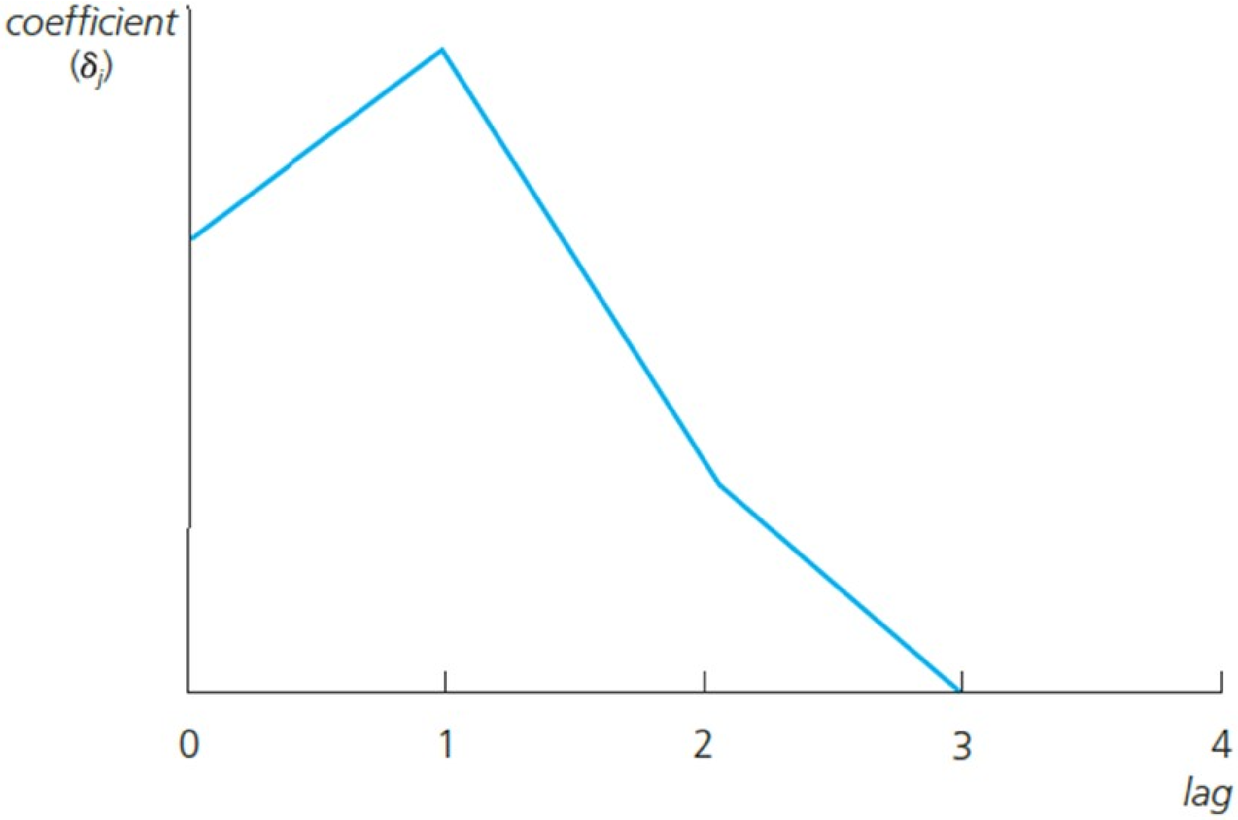
$$\frac{\Delta y_t}{\Delta z_{t-q}} + \ldots + \frac{\Delta y_t}{\Delta z_t} = \delta_1 + \ldots + \delta_q$$

<u>Effect of a transitory shock:</u>
If there is a one time shock in a past period, the dep. variable will change temporarily by the amount indicated by the coefficient of the corresponding lag.

<u>Effect of permanent shock:</u>
If there is a permanent shock in a past period, i.e. the explanatory variable permanently increases by one unit, the effect on the dep. variable will be the cumulated effect of all relevant lags. This is a long-run effect on the dependent variable.

Basic Regression Analysis with Time Series Data (6 of 25)

- **Graphical illustration of lagged effects**



- The effect is biggest after a lag of one period. After that, the effect vanishes (if the initial shock was transitory).
- The long run effect of a permanent shock is the cumulated effect of all relevant lagged effects. It does not vanish (if the initial shock is a permanent one).

Basic Regression Analysis with Time Series Data (7 of 25)

- **Finite sample properties of OLS under classical assumptions**
- *Assumption TS.1 (Linear in parameters)*

$$y_t = \beta_0 + \beta_1 x_{t1} + \beta_2 x_{t2} + \ldots + \beta_k x_{tk} + u_t$$

The time series involved obey a linear relationship. The stochastic processes y_t, x_{t1},…, x_{tk} are observed, the error process u_t is unobserved. The definition of the explanatory variables is general, e.g. they may be lags or functions of other explanatory variables.

- *Assumption TS.2 (No perfect collinearity)*
  "In the sample (and therefore in the underlying time series process), no independent variable is constant nor a perfect linear combination of the others."

Basic Regression Analysis with Time Series Data (8 of 25)

- Notation

$$\mathbf{X} = \begin{pmatrix} x_{11} & x_{12} & \cdots & x_{1k} \\ \vdots & \vdots & & \vdots \\ x_{t1} & x_{t2} & \cdots & x_{tk} \\ \vdots & \vdots & & \vdots \\ x_{n1} & x_{n2} & \cdots & x_{nk} \end{pmatrix}$$

This matrix collects all the information on the complete time paths of all the explanatory variables

The values of all the explanatory variables in period number t

- *Assumption TS.3 (Zero conditional mean)*

$$E(u_t|\mathrm{X}) = 0, t = 1, 2, \ldots, n$$

The mean value of the unobserved factors is uncorrelated to the values of the explanatory variables in all periods

Basic Regression Analysis with Time Series Data (9 of 25)

- **Discussion of assumption TS.3**

Exogeneity: $E(u_t|\mathbf{x}_t) = 0 \leftarrow$ The mean of the error term is uncorrelated to the explanatory variables of the *same period*

Strict exogeneity: $E(u_t|\mathbf{X}) = 0 \leftarrow$ The mean of the error term is uncorrelated to the values of the explanatory variables of *all periods*

- Strict exogeneity is stronger than contemporaneous exogeneity
  - TS.3 rules out feedback from the dependent variable on future values of the explanatory variables; this is often questionable especially if explanatory variables “adjust” to past changes in the dependent variable.
  - If the error term is related to past values of the explanatory variables, one should include these values as contemporaneous regressors

Basic Regression Analysis with Time Series Data (10 of 25)

- **Theorem 10.1 (Unbiasedness of OLS)**

$$TS.1 - TS.3 \quad \Rightarrow \quad E(\hat{\beta}_j) = \beta_j, \quad j = 0, 1, \ldots, k$$

- *Assumption TS.4 (Homoskedasticity)*

$$Var(u_t|\mathrm{X}) = Var(u_t) = \sigma^2, t = 1, 2, \ldots, n$$

The volatility of the errors must not be related to the explanatory variables in any of the periods

  - A sufficient condition is that the volatility of the error is independent of the explanatory variables and that it is constant over time.
  - In the time series context, homoskedasticity may also be easily violated, e.g. if the volatility of the dep. variable depends on regime changes.

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- *Assumption TS.5 (No serial correlation)*

$$Corr(u_t, u_s|\mathbf{X}) = 0, \;\; t \neq s$$ ← Conditional on the explanatory variables, the unobserved factors must not be correlated over time

- Discussion of assumption TS.5
  - Why was such an assumption not made in the cross-sectional case?
  - The assumption may easily be violated if, conditional on knowing the values of the indep. variables, omitted factors are correlated over time.
  - The assumption may also serve as substitute for the random sampling assumption if sampling a cross-section is not done completely randomly.
  - In this case, given the values of the explanatory variables, errors have to be uncorrelated across cross-sectional units (e.g.

Basic Regression Analysis with Time Series Data (12 of 25)

- **Theorem 10.2 (OLS sampling variances)**
  Under assumptions TS.1 – TS.5:

$$Var(\widehat{\beta}_j|\mathbf{X}) = \frac{\sigma^2}{SST_j(1-R_j^2)}, \quad j=1,\ldots,k$$

The conditioning on the values of the explanatory variables is not easy to understand. It effectively means that, in a finite sample, one ignores the sampling variability coming from the randomness of the regressors. This kind of sampling variability will normally not be large (because of the sums).

- **Theorem 10.3 (Unbiased estimation of the error variance)**

$$TS.1 - TS.5 \quad \Rightarrow \quad E(\widehat{\sigma}^2) = \sigma^2$$

Basic Regression Analysis with Time Series Data (13 of 25)

- **Theorem 10.4 (Gauss-Markov Theorem)**
  - Under assumptions TS.1 – TS.5, the OLS estimators have the minimal variance of all linear unbiased estimators of the regression coefficients.
  - This holds conditional as well as unconditional on the regressors.
- *Assumption TS.6 (Normality)*

$u_t \sim \text{Normal}(0, \sigma^2)$ independently of X ← This assumption implies TS.3 – TS.5

- **Theorem 10.5 (Normal sampling distributions)**
  - Under assumptions TS.1 – TS.6, the OLS estimators have the usual normal distribution (conditional on X). The usual F and t-tests are valid.

Basic Regression Analysis with Time Series Data (14 of 25)

- **Example: Static Phillips curve**

$$\widehat{inf}_t = \underset{(1.72)}{1.42} + \underset{(.289)}{.468}\ unem_t$$

Contrary to theory, the estimated Phillips Curve does not suggest a tradeoff between inflation and unemployment

$$n = 49, R^2 = .053, \bar{R}^2 = .033$$

- Discussion of CLM assumptions
  - TS.1: The error term contains factors such as monetary shocks, income/demand shocks, oil price shocks, supply shocks, or exchange rate shocks.
  - TS.2: A linear relationship might be restrictive, but it should be a good approximation. Perfect collinearity is not a problem as long as unemployment varies over time.

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- **Discussion of CLM assumptions (cont.)**

TS.3: $E(u_t|unem_1, \ldots, unem_n) = 0$ ← Easily violated

$unem_{t-1} \uparrow \; \rightarrow \; u_t \downarrow$ ← For example, past unemployment shocks may lead to future demand shocks which may dampen inflation

$u_{t-1} \uparrow \; \rightarrow \; unem_t \uparrow$ ← For example, an oil price shock means more inflation and may lead to future increases in unemployment

TS.4: $Var(u_t|unem_1, \ldots, unem_n) = \sigma^2$ ← Assumption is violated if monetary policy is more "nervous" in times of high unemployment

TS.5: $Corr(u_t, u_s|unem_1, \ldots, unem_n) = 0$ ← Assumption is violated if exchange rate influences persist over time (they cannot be explained by unemployment)

TS.6: $u_t \sim \text{Normal}(0, \sigma^2)$ ← Questionable

Basic Regression Analysis with Time Series Data (16 of 25)

- **Example: Effects of inflation and deficits on interest rates**

Interest rate on 3-months T-bill

Government deficit as percentage of GDP

$$\widehat{i3}_t = \underset{(0.43)}{1.73} + \underset{(.082)}{.606}\ inf_t + \underset{(.118)}{.513}\ def_t$$

$$n = 56, R^2 = .602, \bar{R}^2 = .587$$

- Discussion of CLM assumptions
  - TS.1: The error term represents other factors that determine interest rates in general, e.g. business cycle effects.
  - TS.2: A linear relationship might be restrictive, but it should be a good approximation. Perfect collinearity will seldomly be a problem in practice.

Basic Regression Analysis with Time Series Data (17 of 25)

- **Discussion of CLM assumptions (cont.)**

TS.3: $E(u_t|inf_1, \ldots, inf_n, def_1, \ldots, def_n) = 0$ ← Easily violated

$def_{t-1} \uparrow \rightarrow \; u_t \uparrow$ ← For example, past deficit spending may boost economic activity, which in turn may lead to general interest rate rises

$u_{t-1} \uparrow \rightarrow \; inf_t \uparrow$ ← For example, unobserved demand shocks may increase interest rates and lead to higher inflation in future periods

TS.4: $Var(u_t|inf_1, \ldots, def_n) = \sigma^2$ ← Assumption is violated if higher deficits lead to more uncertainty about state finances and possibly more abrupt rate changes

TS.5: $Corr(u_t, u_s|inf_1, \ldots, def_n) = 0$ ← Assumption is violated if business cycle effects persist across years (and they cannot be completely accounted for by inflation and the evolution of deficits)

TS.6: $u_t \sim \text{Normal}(0, \sigma^2)$ ← Questionable

Basic Regression Analysis with Time Series Data (18 of 25)

- **Using dummy explanatory variables in time series**

Children born per 1,000 women in year t

Tax exemption in year t

Dummy for World War II years (1941-45)

Dummy for availabity of contraceptive pill (1963-present)

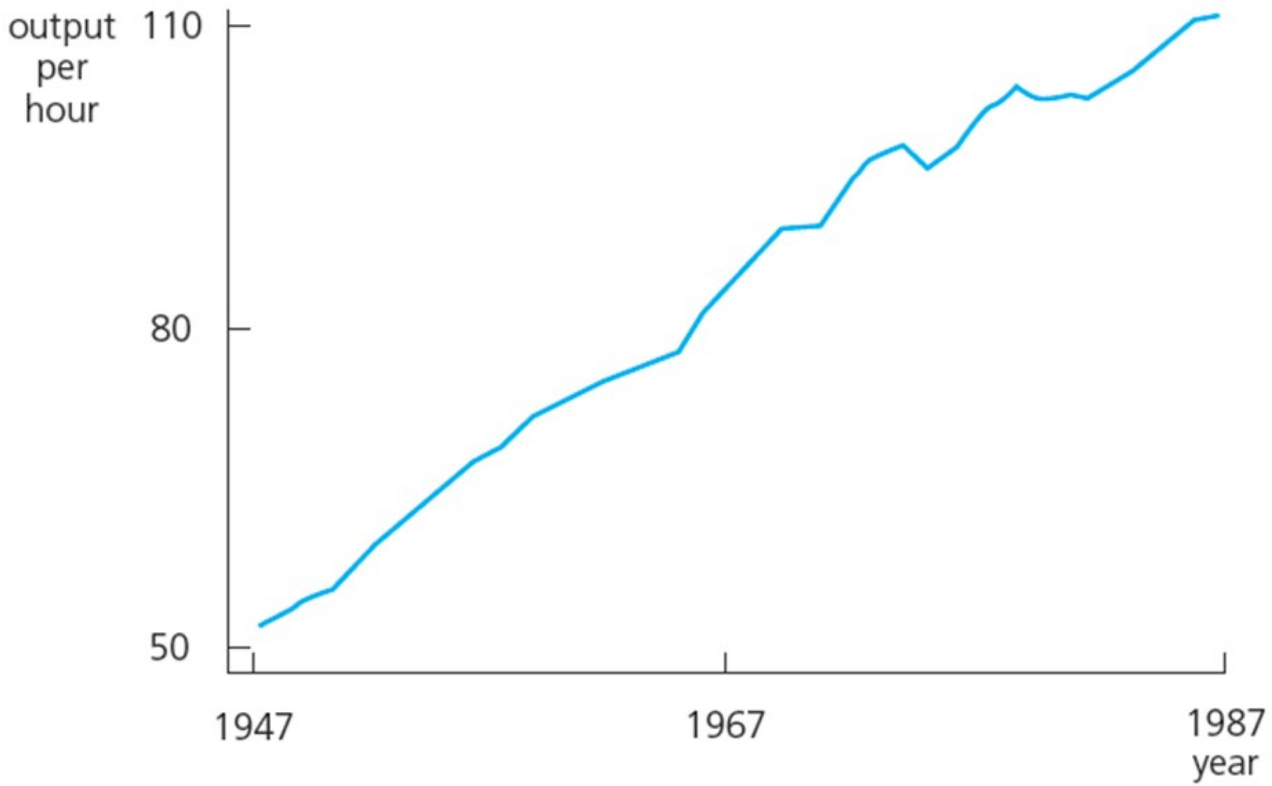
$$\widehat{gfr}_t = \underset{(3.21)}{98.68} + \underset{(.030)}{.083}\, pe_t - \underset{(7.46)}{24.24} ww2_t - \underset{(4.08)}{31.59} pill_t$$

$$n = 72, R^2 = .473, \bar{R}^2 = .450$$

- Interpretation
  - During World War II, the fertility rate was temporarily lower.
  - It has been permanently lower since the introduction of the pill in 1963.

Basic Regression Analysis with Time Series Data (19 of 25)

- **Time series with trends**
  - Example for a time series with a linear upward trend:



Basic Regression Analysis with Time Series Data (20 of 25)

- **Modelling a linear time trend**

$$y_t = \alpha_0 + \alpha_1 t + e_t \quad \Leftrightarrow \quad E(\Delta y_t) = E(y_t - y_{t-1}) = \alpha_1$$

$\Delta y_t / \Delta t = \alpha_1$ ← Abstracting from random deviations, the dependent variable increases by a constant amount per time unit

$E(y_t) = \alpha_0 + \alpha_1 t$ ← Alternatively, the expected value of the dependent variable is a linear function of time

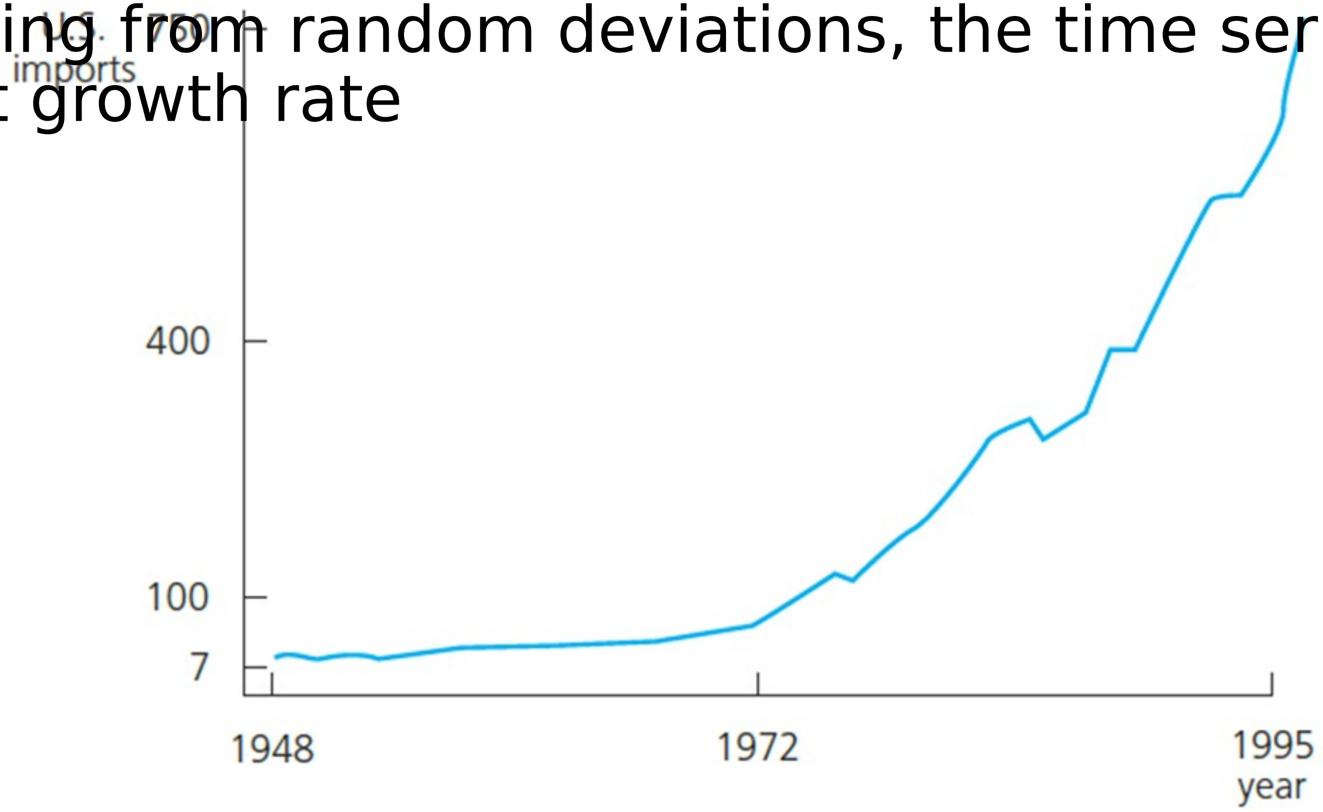
- **Modelling an exponential time trend**

$$\log(y_t) = \alpha_0 + \alpha_1 t + e_t \quad \Leftrightarrow \quad E(\Delta \log(y_t)) = \alpha_1$$

$(\Delta y_t / y_t)/\Delta t = \alpha_1$ ← Abstracting from random deviations, the dependent variable increases by a constant percentage per time unit

Basic Regression Analysis with Time Series Data (21 of 25)

- **Example for a time series with an exponential trend**
  - Abstracting from random deviations, the time series has a constant growth rate



Basic Regression Analysis with Time Series Data (22 of 25)

- **Using trending variables in regression analysis**
  - If trending variables are regressed on each other, a spurious relationship may arise if the variables are driven by a common trend.
  - In this case, it is important to include a trend in the regression.
- Example: Housing investment and prices

Per capita housing investment

Housing price index

$$\widehat{\log}(invpc) = -\underset{(.043)}{.550} + \underset{(.382)}{1.241}\,\log(price)$$

$$n = 42, R^2 = .208, \bar{R}^2 = .189$$

It looks as if investment and prices are positively related

Basic Regression Analysis with Time Series Data (23 of 25)

- **Example: Housing investment and prices (cont.)**

$$\widehat{\log}(invpc) = -.913 - .381 \log(price) + .0098\, t$$
$$(1.36) \quad (.679) \qquad\qquad (.0035)$$

$$n = 42, R^2 = .341, \bar{R}^2 = .307$$

There is no significant relationship between price and investment anymore

- When should a trend be included?
  - If the dependent variable displays an obvious trending behaviour
  - If both the dependent and some independent variables have trends
  - If only some of the independent variables have trends; their effect on the dependent variable may only be visible after a trend has been substracted

Basic Regression Analysis with Time Series Data (24 of 25)

- **A detrending interpretation of regressions with a time trend**
  - It turns out that the OLS coefficients in a regression including a trend are the same as the coefficients in a regression without a trend but where all the variables have been detrended before the regression.
  - This follows from the general interpretation of multiple regressions.

- **Computing R-squared when the dependent variable is trending**
  - Due to the trend, the variance of the dependent variable will be overstated.
  - It is better to first detrend the dependent variable and then run the regression on all the independent variables (plus a trend if they are trending as well).

Basic Regression Analysis with Time Series Data (25 of 25)

- **Modelling seasonality in time series**
- A simple method is to include a set of seasonal dummies:

$$y_t = \beta_0 + \delta_1 feb_t + \delta_2 mar_t + \delta_3 apr_t + \ldots + \delta_{11} dec_t$$

$$+ \beta_1 x_{t1} + \beta_2 x_{t2} + \ldots + \beta_k x_{tk} + u_t$$

= 1 if obs. from december
= 0 otherwise

- Similar remarks apply as in the case of deterministic time trends
  - The regression coefficients on the explanatory variables can be seen as the result of first deseasonalizing the dep. and the explanatory variables.
  - An R-squared that is based on first deseasonalizing the dependent variable may better reflect the explanatory power of