BØA 203 – Fasit matterepetisjon

Oppgave 1:

a) $X^2 \cdot X^3 = X^{2+3} = \underline{\underline{X^5}}$

b) $X \cdot X^4 = X^1 \cdot X^4 = X^{1+4} = \underline{\underline{X^5}}$

c) $y + y = \underline{\underline{2y}}$

d) $y^2 + y = y \cdot y + y = \underline{\underline{y(y+1)}}$

e) $(X_1 + X_2)\, X_1^{0.5} = X_1^1 X_1^{0.5} + X_2^1 X_1^{0.5} = \underline{\underline{X_1^{1,5} + X_2 X_1^{0,5}}}$

f) $\frac{1}{X_1} \cdot X_1^3 = \frac{X_1^3}{X_1} = \underline{\underline{X_1^2}}$

g) $\frac{X_1^3}{X_2^3} = \underline{\underline{\left(\frac{X_1}{X_2}\right)^3}}$ (husk: $\frac{a^n}{b^n} = \left(\frac{a}{b}\right)^n$)

Oppgave 2:

a) $y^2 = ab$ (husk: $\sqrt{y^2} = y$)

$\sqrt{y^2} = \sqrt{ab}$

$y = \underline{\underline{\sqrt{ab}}}$

b) $y^{0.5} = a + b$ (husk: $(a^n)^m = a^{n \cdot m}$)

$(y^{0.5})^2 = (a+b)^2$

$y = \underline{\underline{(a+b)^2}}$

Oppgave 3:

a) $f(x) = 4x^3$

$f'(x) = \underline{\underline{12x^2}}$

b) $f(x) = 4x^5 + 2x^{-3}$

$f'(x) = \underline{\underline{20x^4 - 6x^{-4}}}$ eller alternativt $f'(x) = \underline{\underline{20x^4 - \frac{6}{x^4}}}$

c) $f(x) = \frac{1}{6}x^6 a$

$f'(x) = \underline{\underline{x^5 a}}$

d) $f(x) = \frac{1}{6}x^6 + a$

$f'(x) = \underline{\underline{x^5}}$

(husk at den derivert av en konstant $a = 0$)

e) $f(x) = \frac{5x}{2x^2}$

husk: $f(x) = \frac{u}{v} \rightarrow f'(x) = \frac{u'v - uv'}{v^2}$

$f'(x) = \frac{5 \cdot 2x^2 - 5x \cdot 4x}{(2x^2)^2} = \frac{10x^2 - 20x^2}{4x^4} = \frac{-10x^2}{4x^4} = \underline{\underline{-\frac{5}{2x^2}}}$

f) $f(x,y) = 5x^4 + 3y^{0.5}$

$f_x'(x,y) = \underline{\underline{20x^3}}$, $f_y'(x,y) = \underline{\underline{1{,}5y^{-0.5}}}$

g) $f(x,y) = \frac{1}{2}x^2 + xy^{0.5}$

$f_x'(x,y) = \underline{\underline{x + y^{0.5}}}$ $f_y'(x,y) = \underline{\underline{0{,}5xy^{-0.5}}}$

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h) $f(x,y) = 3x^{\frac{1}{6}}y^2 + y^{\frac{1}{3}}x^3$

$f'_x(x,y) = \frac{3}{6}x^{-\frac{5}{6}}y^2 + y^{\frac{1}{3}}3x^2 = \underline{\underline{\frac{1}{2}x^{-\frac{5}{6}}y^2 + 3y^{\frac{1}{3}}x^2}}$

$f'_y(x,y) = \underline{\underline{6x^{\frac{1}{6}}y + \frac{1}{3}y^{-\frac{2}{3}}x^3}}$

i) $f(x_1, x_2) = 3x_1^{1/6} + x_2^{1/3}$

$f'_{x_1}(x_1,x_2) = \underline{\underline{\frac{1}{2}x_1^{-\frac{5}{6}}}}$ $\qquad f'_{x_2}(x_1,x_2) = \underline{\underline{\frac{1}{3}x_2^{-\frac{2}{3}}}}$

Oppgave 4:

a) $\frac{1}{3} + \frac{5}{9} = \frac{3}{9} + \frac{5}{9} = \frac{3+5}{9} = \underline{\underline{\frac{8}{9}}}$

b) $\frac{13}{x} - \frac{2}{x} = \frac{13-2}{x} = \underline{\underline{\frac{11}{x}}}$

c) $\frac{2y}{x^2} + \frac{(x+1)}{x} = \frac{2y}{x^2} + \frac{(x+1)x}{x^2} = \underline{\underline{\frac{2y + x^2 + x}{x^2}}}$

d) $\frac{2y}{x^2} \cdot \frac{(x+1)}{x} = \underline{\underline{\frac{2yx + 2y}{x^3}}}$

e) $\frac{5}{x}$, når $x\uparrow$ vil brøkens verdi bli lavere

f) $\frac{x}{7}$, når $x\uparrow$ ——— ——————— høyere

g) $\frac{x}{2x}$, forholdet mellom teller og nevner er konstant, brøkens verdi er konstant

Oppgave 5:

a)

$$F(x,y) = -4x^2 - 2xy - 3{,}5y^2 + 140x + 100y - 1100 - \lambda(4x + y - 50)$$

$$= -4x^2 - 2xy - 3{,}5y^2 + 140x + 100y - 1100 - 4x\lambda + y\lambda + 50\lambda$$

finner så $F_x'(x,y)$ og $F_y'(x,y)$ og sette disse til 0

$$F_x'(x,y) = -8x - 2y + 140 - 4\lambda = 0$$

$$F_y'(x,y) = -2x - 7y + 100 - \lambda = 0$$

Addisjonsmetoden gir:

$$-8x - 2y + 140 - 4\lambda = 0$$

$$-2x - 7y + 100 - \lambda = 0 \qquad (-4)$$

$$-8x + 2y + 140 - 4\lambda = 0$$

$$8x + 28y - 400 + 4\lambda = 0$$

$$26y - 260 = 0$$

$$\underline{\underline{y = 10}}$$

Vi har da:

$$4x + y = 50$$

$$4x + 10 = 50$$

$$\underline{\underline{x = 10}}$$

b) $F(x,y) = -2x^2 - 2xy - 2y^2 + 280x + 260y - 10.000 - \lambda(x + 2y - 110)$

$F'_x(x,y) = -4x - 2y + 280 - \lambda = 0$

$F'_y(x,y) = -2x - 4y + 260 - 2\lambda = 0$

Addisjonsmetoden:

$$-4x - 2y + 280 - \lambda = 0 \quad | \cdot (-2)$$
$$-2x - 4y + 260 - 2\lambda = 0$$

$$8x + 4y - 560 + 2\lambda = 0$$
$$-2x - 4y + 260 - 2\lambda = 0$$

$$6x - 300 = 0$$

$$\underline{\underline{x = 50}}$$

$$x + 2y = 110$$
$$50 + 2y = 110$$
$$\underline{\underline{y = 30}}$$