Proyecto

Historia

En el Spider Verse, la discontinuidad de eventos canónicos tiene un impacto significativo en la trama y en la interacción entre los diferentes universos. Cuando se detiene un evento canónico, se altera drásticamente la escala y las perturbaciones que emergen debido a la presencia de los personajes principales, como Spider-Man y sus múltiples versiones. La siguiente ecuación demuestra los efectos que romper un evento canónico tiene sobre el Spider Verse:

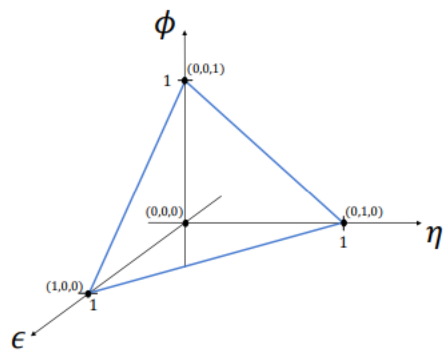
$$\eta^3 \nabla(\eta^3 \nabla * X) = \eta^3 + \epsilon^2$$

Cada versión de Spider-Man representa una escala de longitud, donde cada uno responde de manera única a las perturbaciones del Spider Verse. La presencia de η y ϵ se traduce en la diversidad y singularidad de cada Spider-Man, sus habilidades y personalidades. Al detener un evento canónico, se rompe la continuidad del flujo temporal, y los universos paralelos se ven afectados.

Proceso del MEF

Paso #1: Localización

Según lo establecido en clases, se localiza el problema a un elemento en las dimensiones épsilon, eta y phi, la cual surge de la izoparametrización del elemento en el plano x y z. A partir de este nuevo plano se determinan las funciones de forma:



$$\boxed{\begin{aligned} N_1 &= 1 - \epsilon - \eta - \phi \\ N_2 &= \epsilon \\ N_3 &= \eta \\ N_4 &= \phi \end{aligned}}$$

Paso #2: Interpolación

Se determina una aproximación para la incógnita X haciendo uso de las funciones de forma previamente establecidas:

$$X \approx N_1 * X_1 + N_2 * X_2 + N_3 * X_3 + N_4 * X_4 = \begin{bmatrix} N_1 & N_2 & N_3 & N_4 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix} = \mathbf{NX}$$

$$X \approx \mathbf{NX}$$

Paso #3: Aproximación del modelo

Sustituimos la aproximación del paso anterior en nuestro modelo:

$$\eta^3\nabla(\eta^3\nabla * X) = \eta^3 + \epsilon^2$$

$$\eta^3\nabla(\eta^3\nabla * \mathbf{NX}) \approx \eta^3 + \epsilon^2$$

Paso #3.5: Definición del Residual

Despejamos el modelo aproximado y obtenemos el residual:

$$\Re = (\eta^3 + \epsilon^2) - \eta^3\nabla(\eta^3\nabla * \mathbf{NX})$$

Paso #4: Método de los residuos ponderados

$$\Re = (\eta^3 + \epsilon^2) - \eta^3\nabla(\eta^3\nabla * \mathbf{NX})$$

$$\int_V \mathbf{W}\Re dV = 0$$

$$\int_V \mathbf{W}[(\eta^3 + \epsilon^2) - \eta^3\nabla(\eta^3\nabla * \mathbf{NX})]dV = 0$$

Paso #5: Método de Galerkin

$$\mathbf{W} = \mathbf{N^T}$$

$$\int_V \mathbf{N^T}[(\eta^3 + \epsilon^2) - \eta^3\nabla(\eta^3\nabla * \mathbf{NX})]dV = 0$$

Interludio

Ordenamos la ecuacion:

$$\int_V \mathbf{N^T}[(\eta^3 + \epsilon^2) - \eta^3\nabla(\eta^3\nabla * \mathbf{NX})]dV = 0$$

$$\int_V \mathbf{N^T}(\eta^3 + \epsilon^2) - \mathbf{N^T}[\eta^3\nabla(\eta^3\nabla * \mathbf{NX})]dV = 0$$

$$\int_V \mathbf{N^T}(\eta^3 + \epsilon^2)dV - \int_V \mathbf{N^T}[\eta^3\nabla(\eta^3\nabla * \mathbf{NX})]dV = 0$$

$$\int_V \mathbf{N^T}[\eta^3\nabla(\eta^3\nabla * \mathbf{NX})]dV = \int_V \mathbf{N^T}(\eta^3 + \epsilon^2)dV$$

$$(\int_V \mathbf{N^T}\eta^3\nabla(\eta^3\nabla * \mathbf{N})dV)\mathbf{X} = \int_V \mathbf{N^T}(\eta^3 + \epsilon^2)dV$$

$$\mathbf{KX} = \mathbf{Q}$$

Paso #6: Resolución de integrales

Empezamos resolviendo la integral del lado derecho:

$$\int_V \mathbf{N^T}(\eta^3+\epsilon^2)d_V$$

$$\iiint \mathbf{N^T}(\eta^3+\epsilon^2)d_x d_y d_z ; \mathbf{d_V} \equiv \mathbf{d_x d_y d_z}$$

$$\iiint \begin{bmatrix} N_1 \\ N_2 \\ N_3 \\ N_4 \end{bmatrix} (\eta^3+\epsilon^2)d_x d_y d_z$$

$$\iiint \begin{bmatrix} 1-\epsilon-\eta-\emptyset \\ \epsilon \\ \eta \\ \emptyset \end{bmatrix} (\eta^3+\epsilon^2)d_x d_y d_z$$

$$d_x d_y d_z = \mathbf{J} d_\epsilon d_\eta d_\emptyset$$

$$\mathbf{J} = \begin{bmatrix} \frac{\partial_x}{\partial_\epsilon} & \frac{\partial_x}{\partial_\eta} & \frac{\partial_x}{\partial_\emptyset} \\ \frac{\partial_y}{\partial_\epsilon} & \frac{\partial_y}{\partial_\eta} & \frac{\partial_y}{\partial_\emptyset} \\ \frac{\partial_z}{\partial_\epsilon} & \frac{\partial_z}{\partial_\eta} & \frac{\partial_z}{\partial_\emptyset} \end{bmatrix}$$

$$\int_0^1 \int_0^{1-\emptyset} \int_0^{1-\eta-\emptyset} \epsilon(\eta^3+\epsilon^2)d_\epsilon d_\eta d_\emptyset$$

$$\mathbf{J} \iiint \begin{bmatrix} 1-\epsilon-\eta-\emptyset \\ \epsilon \\ \eta \\ \emptyset \end{bmatrix} (\eta^3+\epsilon^2)d_x d_y d_z$$

Resolviendo:

$$\int_0^{1-\eta-\emptyset} (\eta^3+\epsilon^2)(1-\epsilon-\eta-\emptyset)d_\epsilon$$

$$\int (-\epsilon-\emptyset-\eta-1)(\epsilon^2+\eta^3)d_\epsilon$$

$$\int (-\epsilon^3-(\emptyset+\eta-1)\epsilon^2-\eta^3\epsilon-\eta^3(\emptyset+\eta-1))d_\epsilon$$

$$\int \epsilon^3 d_\epsilon + (-\emptyset-\eta+1)\int \epsilon^2 d_\epsilon - \eta^3 \int \epsilon d_\epsilon - \eta^3(\emptyset+\eta-1)\int 1 d_\epsilon$$

$$\int \epsilon^3 d_\epsilon = \frac{\epsilon^4}{4}$$

$$\int \epsilon^2 d_\epsilon = \frac{\epsilon^3}{3}$$

$$\int \epsilon d_\epsilon = \frac{\epsilon^2}{2}$$

$$\int 1d_\epsilon = \epsilon$$

$$-\frac{\epsilon^4}{4} + \frac{(-\emptyset - \eta + 1)\epsilon^3}{3} - \frac{\eta^3\epsilon^2}{2} - \eta^3(\emptyset + \eta - 1)\epsilon|_0^{1-\eta-\emptyset}$$

$$-\frac{\epsilon \cdot \left(3\epsilon^3 + 4\left(o + \eta - 1\right)\epsilon^2 + 6\eta^3\epsilon + 12\eta^3 o + 12\eta^4 - 12\eta^3\right)}{12}|_0^{1-\eta-\emptyset}$$

$$\frac{\emptyset^4 + (4\eta - 4)\emptyset^3 + (6\eta^3 + 6\eta^2 - 12\eta + 6)\emptyset^2 + (12\eta^4 - 8\eta^3 - 12\eta^2 - 4)\emptyset + 6\eta^5 - 11\eta^4 + 2\eta^3 + 6\eta^2 - 4\eta + 1}{12}$$

$$\frac{(\emptyset + \eta - 1)^2(\emptyset(\emptyset + 2\eta - 2) + 6\eta^3 + n^2 - 2\eta + 1)}{12}$$

$$\int \frac{(\emptyset + \eta - 1)^2(\emptyset(\emptyset + 2\eta - 2) + 6\eta^3 + n^2 - 2\eta + 1)}{12}$$

$$\frac{1}{12}\int (\emptyset + \eta - 1)^2(\emptyset(\emptyset + 2\eta - 2) + 6\eta^3 + n^2 - 2\eta + 1)$$

$$\int (\eta + \emptyset - 1)^2(6\eta^3 + \eta^2 + \emptyset(2\eta + \emptyset - 2) - 2\eta + 1)d_\eta$$

Sustituyendo $u = \eta + \emptyset - 1 \to d_u = d_\eta$

$$\int u^2(6(u - \emptyset + 1)^3 + (u - \emptyset + 1)^2 + 2(\emptyset - 1)u - 2\emptyset^2 + (\emptyset - 2)\emptyset + 4\emptyset - 1)d_u$$

$$\int (6u^5 + 18(1 - \emptyset)u^4 + u^4 + 2(\emptyset - 1)u^3 + 18(1 - \emptyset)^2u^3 + (-2\emptyset^2 + (\emptyset - 2)\emptyset + 4\emptyset + 6(1 - \emptyset)^3 + (1 - \emptyset)^2 - 1)d_u$$

$$6\int u^5 d_u + (18(1 - \emptyset) + 1)\int u^4 d_u + (2(\emptyset - 1) + 2(1 - \emptyset) + 2(1 - \emptyset) + 18(1 - \emptyset)^2)\int u^3 d_u + (-2\emptyset^2 + (\emptyset - 2)\emptyset +$$

Resolviendo integrales

$$\int u^5 d_u = \frac{u^6}{6}$$

$$\int u^4 d_u = \frac{u^5}{5}$$

$$\int u^3 d_u = \frac{u^4}{4}$$

$$\int u^2 d_u = \frac{u^3}{3}$$

Reemplazando resultados

$$u^6 + \frac{(18(1 - \emptyset) + 1)(\eta + \emptyset - 1)^5}{5} + \frac{(2(\emptyset - 1) + 2(1 - \emptyset) + 18(1 - \emptyset)^2)u^4}{4} + \frac{(-2\emptyset^2 + (\emptyset - 2)\emptyset + 4\emptyset + 6(1 - \emptyset)^3 +}{3}$$

Recordando $u = \eta + \emptyset - 1$, y sustituyendo

$$(\eta + \emptyset - 1)^6 + \frac{(18(1 - \emptyset) + 1)(\eta + \emptyset - 1)^5}{5} + \frac{(2(\emptyset - 1) + 2(1 - \emptyset) + 18(1 - \emptyset)^2)(\eta + \emptyset - 1)^4}{4} + \frac{(-2\emptyset^2 + (\emptyset - 2)\emptyset}{}$$

$$\frac{1}{12}\int(\eta+\emptyset-1)^2(6\eta^3+\eta^2+\emptyset(2\eta+\emptyset-2)-2\eta+1)d_\eta$$

$$=\frac{(\eta+\emptyset-1)^6}{12}+\frac{(18(1-\emptyset)+1)(\eta+\emptyset-1)^5}{60}+\frac{(2(\emptyset-1)+2(1-\emptyset)+18(1-\emptyset)^2)(\eta+\emptyset-1)^4}{48}+\frac{(-2\emptyset^2+(\emptyset-2}{}$$

$$=\frac{(\eta+\emptyset-1)^3(\eta(2\eta(5\eta-3\emptyset+4)+(\emptyset-1)(3\emptyset+1))-(\emptyset-3)(\emptyset-1)^2}{120}$$

$$\frac{n\cdot\left(10n^5+(24o-22)\,n^4+\left(15o^2-20o+5\right)n^3+\left(20o^2-40o+20\right)n^2+\left(20o^3-60o^2+60o-20\right)n+10o^4-4\right.}{120}$$

$$\frac{\emptyset^6-8\emptyset^5+25\emptyset^4-40\emptyset^3+35\emptyset^2-16\emptyset+3}{120}$$

$$\int_0^1\frac{\emptyset^6-8\emptyset^5+25\emptyset^4-40\emptyset^3+35\emptyset^2-16\emptyset+3}{120}d_\emptyset$$

$$\frac{1}{120}\int\emptyset^6d_\emptyset-\frac{1}{15}\int\emptyset^5d_\emptyset+\frac{5}{24}\int\emptyset^4d_\emptyset-\frac{1}{3}\int\emptyset^3d_\emptyset+\frac{7}{24}\int\emptyset^2d_\emptyset-\frac{2}{15}\int\emptyset d_\emptyset+\frac{1}{40}\int1d_\emptyset$$

$$=\frac{\emptyset^7}{840}-\frac{\emptyset^6}{90}+\frac{\emptyset^5}{24}-\frac{\emptyset^4}{12}+\frac{7\emptyset^3}{72}-\frac{\emptyset^2}{15}+\frac{\emptyset}{40}$$

$$=\frac{\emptyset(3\emptyset^6-28\emptyset^5+105\emptyset^4-210\emptyset^3+245\emptyset^2-168\emptyset+63}{2520}\Big|_0^1$$

$$=\frac{1}{252}$$

$$\int_0^{1-\eta-\emptyset}(\eta^3+\epsilon^2)\epsilon d_\epsilon$$

Sustituyendo $u=\epsilon^2+\eta^3\rightarrow d_u=2\epsilon d_\epsilon$

$$\frac{1}{2}\int ud_u=\frac{1}{2}(\frac{u^2}{2})=\frac{1}{4}u^2$$

Como $\mathbf{u}=\epsilon^{\mathbf{2}}+\eta^{\mathbf{3}}$

$$\frac{1}{4}(\epsilon^2+\eta^3)^2\Big|_0^{1-\eta-\emptyset}$$

$$\frac{o^4+(4n-4)\,o^3+\left(2n^3+6n^2-12n+6\right)o^2+\left(4n^4-12n^2+12n-4\right)o+n^6+2n^5-3n^4-2n^3+6n^2-4n+1}{4}$$

Simplificando:

$$\frac{o^4+(4n-4)\,o^3+\left(2n^3+6n^2-12n+6\right)o^2+\left(4n^4-12n^2+12n-4\right)o+2n^5-3n^4-2n^3+6n^2-4n+1}{4}$$

$$\int_0^{1-\emptyset}\frac{o^4+(4n-4)\,o^3+\left(2n^3+6n^2-12n+6\right)o^2+\left(4n^4-12n^2+12n-4\right)o+2n^5-3n^4-2n^3+6n^2-4n+1}{4}d_\eta$$

$$\int \frac{2\eta^5 + \emptyset(4\eta^4 - 12\eta^2 - 4) - 3\eta^4 + \emptyset^2(2\eta^3 + 6\eta^2 - 12\eta + 6) - 2\eta^3 + 6\eta^2 + \emptyset^3(\eta - 4) - 4\eta + \emptyset^4 + 1}{4} d_\eta$$

Reordenando la expresión:

$$\frac{1}{2}\int \eta^5 d_\eta + (\emptyset - \frac{3}{4})\int \eta^4 d_\eta + (\frac{\emptyset^2}{2} - \frac{1}{2})\int \eta^3 d_\eta + (\frac{3\emptyset^2}{2} - 3\eta + \frac{3}{2})\int \eta^2 d_\eta + (\emptyset^3 - 3\emptyset^2 + 3\eta - 1)\int \eta d_\eta + (\frac{\emptyset^4}{4} - \emptyset^3$$

Resolviendo integrales

$$\int \eta^5 d_\eta = \frac{\eta^6}{6}$$

$$\int \eta^4 d_\eta = \frac{\eta^5}{5}$$

$$\int \eta^3 d_\eta = \frac{\eta^4}{4}$$

$$\int \eta^2 d_\eta = \frac{\eta^3}{3}$$

$$\int \eta d_\eta = \frac{\eta^2}{2}$$

$$\int 1 d_\eta = \eta$$

Remplazando resultados obtenidos en la expresión

$$\frac{\eta^6}{12} + \frac{(\emptyset - \frac{3}{4})\eta^5}{5} + \frac{(\frac{\emptyset^2}{2} - \frac{1}{2})\eta^4}{4} + \frac{(\frac{3\emptyset^2}{2} - 3\eta + \frac{3}{2})\eta^3}{3} + \frac{(\emptyset^3 - 3\emptyset^2 + 3\eta - 1)\eta^2}{2} + (\frac{\emptyset^4}{4} - \emptyset^3 + \frac{3\emptyset^2}{2} - \emptyset + \frac{1}{4})\eta$$

Simplificando:

$$\frac{\eta \cdot \left(\eta \cdot \left(\eta \cdot \left(\eta \cdot \left(2\eta \cdot (5\eta + 12\emptyset - 9) + 15\emptyset^2 - 15\right) + 60\,(\emptyset - 1)^2\right) + 60\,(\emptyset - 1)^3\right) + 30\,(\emptyset - 1)^4\right)}{120}\Bigg|_0^{1-\emptyset}$$

$$= \frac{\emptyset^6 - 12\emptyset^5 + 45\emptyset^4 - 80\emptyset^3 + 75\emptyset^2 - 36\emptyset + 7}{120}$$

$$\int_0^1 \frac{\emptyset^6 - 12\emptyset^5 + 45\emptyset^4 - 80\emptyset^3 + 75\emptyset^2 - 36\emptyset + 7}{120}$$

$$\frac{1}{120}\int \emptyset^6 d_\emptyset - \frac{1}{10}\int \emptyset^5 d_\emptyset + \frac{3}{8}\int \emptyset^4 d_\emptyset - \frac{2}{3}\int \emptyset^3 d_\emptyset + \frac{5}{8}\int \emptyset^2 d_\emptyset - \frac{3}{10}\int \emptyset d_\emptyset + \frac{7}{120}\int 1 d_\emptyset$$

$$\int \emptyset^6 d_\emptyset = \frac{\emptyset^7}{7}$$

$$\int \emptyset^5 d_\emptyset = \frac{\emptyset^6}{6}$$

$$\int \emptyset^4 d_\emptyset = \frac{\emptyset^5}{5}$$

$$\int \emptyset^3 d_\emptyset = \frac{\emptyset^4}{4}$$

$$\int \emptyset^2 d_\emptyset = \frac{\emptyset^3}{3}$$

$$\int \emptyset d_\emptyset = \frac{\emptyset^2}{2}$$

$$\int 1 d_\emptyset = \emptyset$$

$$= \frac{\emptyset^7}{840} - \frac{\emptyset^6}{60} + \frac{3\emptyset^5}{40} - \frac{\emptyset^4}{6} + \frac{5\emptyset^3}{24} - \frac{3\emptyset^2}{20} + \frac{7\emptyset}{120}$$

Simplificando:

$$= \frac{o^6 - 12o^5 + 45o^4 - 80o^3 + 75o^2 - 36o + 7}{120}|_0^1$$

$$= \frac{1}{105}$$

$$\int_0^{1-\eta-\emptyset} (\eta^3 + \epsilon^2)\eta d_\epsilon$$

$$\eta \int \epsilon^2 d_\epsilon + \eta^4 \int 1 d_\epsilon$$

$$\int \epsilon^2 d_\epsilon = \frac{\epsilon^3}{3}$$

$$\int 1 d_\epsilon = \epsilon$$

$$= \frac{\eta\epsilon^3}{3} + \eta^4 \epsilon|_0^{1-\eta-\emptyset}$$

$$-\frac{\eta \cdot \left(\emptyset^3 + (3\eta - 3)\emptyset^2 + \left(3\eta^3 + 3\eta^2 - 6\eta + 3\right)\emptyset + 3\eta^4 - 2\eta^3 - 3\eta^2 + 3\eta - 1\right)}{3}$$

$$\int_0^{1-\emptyset} -\frac{\eta \cdot \left(\emptyset^3 + (3\eta - 3)\emptyset^2 + \left(3\eta^3 + 3\eta^2 - 6\eta + 3\right)\emptyset + 3\eta^4 - 2\eta^3 - 3\eta^2 + 3\eta - 1\right)}{3} d_\eta$$

$$\int -\frac{\eta(3\eta^4 + \emptyset(3\eta^3 + 3\eta^2 - 6\eta + 3) - 2\eta^3 - 3\eta^2 + \emptyset^2(3\eta - 3) + 3\eta + \emptyset^3 - 1)}{3} d_\eta$$

$$\int (-\eta^5 - \emptyset\eta^4 + \frac{2\eta^4}{3} - \emptyset\eta^3 + \eta^3 - \emptyset^2\eta^2 + 2\emptyset\eta^2 - \eta^2 - \frac{(\emptyset^3 - 3\emptyset^2 + 3\emptyset - 1)\eta}{3}) d_\eta$$

$$-\int \eta^5 d_\eta - \frac{3\emptyset - 2}{3} \int \eta^4 d_\eta - \frac{3\emptyset - 3}{3} \int \eta^3 d_\eta - \frac{3\emptyset^2 - 6\emptyset + 3}{3} \int \eta^2 d_\eta - \frac{\emptyset^3 - 3\emptyset^2 + 3\emptyset - 1}{3} \int \eta d_\eta$$

$$\int \eta^5 d_\eta = \frac{\eta^6}{6}$$

$$\int \eta^4 d_\eta = \frac{\eta^5}{5}$$

$$\int \eta^3 d_\eta = \frac{\eta^4}{4}$$

$$\int \eta^2 d_\eta = \frac{\eta^3}{3}$$

$$\int \eta d_\eta = \frac{\eta^2}{2}$$

$$= -\frac{\eta^6}{6} - \frac{(3\emptyset - 2)\eta^5}{15} - \frac{(3\emptyset - 3)\eta^4}{12} - \frac{(3\emptyset^2 - 6\emptyset + 3)\eta^3}{9} - \frac{(\emptyset^3 - 3\emptyset^2 + 3\emptyset - 1)\eta^2}{6}$$

Simplificando:

$$-\frac{\eta^2 \cdot \left(\eta \cdot \left(\eta \cdot (2\eta \cdot (5\eta + 6\emptyset - 4) + 15\emptyset - 15) + 20\,(\emptyset - 1)^2\right) + 10\,(\emptyset - 1)^3\right)}{60}\Big|_0^{1-\emptyset}$$

$$= \frac{2\emptyset^6 - 13\emptyset^5 + 35\emptyset^4 - 50\emptyset^3 + 40\emptyset^2 - 17\emptyset + 3}{60}$$

$$\int_0^1 \frac{2\emptyset^6 - 13\emptyset^5 + 35\emptyset^4 - 50\emptyset^3 + 40\emptyset^2 - 17\emptyset + 3}{60} d_\emptyset$$

$$= \frac{1}{30}\int \emptyset^6 d_\emptyset - \frac{13}{60}\int \emptyset^5 d_\emptyset + \frac{7}{12}\int \emptyset^4 d_\emptyset - \frac{5}{6}\int \emptyset^3 d_\emptyset + \frac{2}{3}\int \emptyset^2 d_\emptyset - \frac{17}{60}\int \emptyset d_\emptyset + \frac{1}{20}\int 1 d_\emptyset$$

$$\int \emptyset^6 d_\emptyset = \frac{\emptyset^7}{7}$$

$$\int \emptyset^5 d_\emptyset = \frac{\emptyset^6}{6}$$

$$\int \emptyset^4 d_\emptyset = \frac{\emptyset^5}{5}$$

$$\int \emptyset^3 d_\emptyset = \frac{\emptyset^4}{4}$$

$$\int \emptyset^2 d_\emptyset = \frac{\emptyset^3}{3}$$

$$\int \emptyset d_\emptyset = \frac{\emptyset^2}{2}$$

$$\int 1 d_\emptyset = \emptyset$$

$$= \frac{\emptyset^7}{210} - \frac{13\emptyset^6}{360} + \frac{7\emptyset^5}{60} - \frac{5\emptyset^4}{24} + \frac{2\emptyset^3}{9} - \frac{17\emptyset^2}{120} + \frac{\emptyset}{20}$$

Simplificando:

$$\frac{\emptyset \cdot \left(12\emptyset^6 - 91\emptyset^5 + 294\emptyset^4 - 525\emptyset^3 + 560\emptyset^2 - 357\emptyset + 126\right)}{2520}\Big|_0^1$$

$$= \frac{19}{2520}$$

$$\int_0^{1-\eta-\emptyset} (\eta^3 + \epsilon^2)\emptyset d_\epsilon$$

$$\emptyset \int \epsilon^2 d_\epsilon + \eta^3 \emptyset \int 1 d_\epsilon$$

$$\int \epsilon^2 d_\epsilon = \frac{\epsilon^3}{3}$$

$$\int 1 d_\epsilon = \epsilon$$

$$= \frac{\emptyset \epsilon^3}{3} + \emptyset \eta^3 \epsilon |_0^{1-\eta-\emptyset}$$

$$= -\frac{\emptyset \cdot \left(\emptyset^3 + (3\eta - 3)\,\emptyset^2 + \left(3\eta^3 + 3\eta^2 - 6\eta + 3\right)\emptyset + 3\eta^4 - 2\eta^3 - 3\eta^2 + 3\eta - 1\right)}{3}$$

$$\int_0^{1-\emptyset} -\frac{\emptyset \cdot \left(\emptyset^3 + (3\eta - 3)\,\emptyset^2 + \left(3\eta^3 + 3\eta^2 - 6\eta + 3\right)\emptyset + 3\eta^4 - 2\eta^3 - 3\eta^2 + 3\eta - 1\right)}{3} d_\eta$$

$$\int -\frac{\emptyset(3\eta^4 + \emptyset(3\eta^3 + 3\eta^2 - 6\eta + 3) - 2\eta^3 - 3\eta^2 + \emptyset^2(3\eta - 3) + 3\eta + \emptyset^3 - 1)}{3} d_\eta$$

Expandiendo:

$$-\emptyset \int \eta^4 d_\eta + (\frac{2\emptyset}{3} - \emptyset^2) \int \eta^3 d_\eta + (\emptyset - \emptyset^2) \int \eta^2 d_\eta + (-\emptyset^3 + 2\emptyset^2 - \emptyset) \int \eta d_\eta + (-\frac{\emptyset^4}{3} + \emptyset^3 - \emptyset^2 + \frac{\emptyset}{3}) \int d_\eta$$

$$\int \eta^4 d_\eta = \frac{\eta^5}{5}$$

$$\int \eta^3 d_\eta = \frac{\eta^4}{4}$$

$$\int \eta^2 d_\eta = \frac{\eta^3}{3}$$

$$\int \eta d_\eta = \frac{\eta^2}{2}$$

$$\int 1 d_\eta = \eta$$

Usando integrales resueltas:

$$= -\frac{\emptyset \eta^5}{5} + \frac{(\frac{2\emptyset}{3} - \emptyset^2)\eta^4}{4} + \frac{(\emptyset - \emptyset^2)\eta^3}{3} + \frac{(-\emptyset^3 + 2\emptyset^2 - \emptyset)\eta^2}{2} + (-\frac{\emptyset^4}{3} + \emptyset^3 - \emptyset^2 + \frac{\emptyset}{3}\eta)$$

Simplificando:

$$= -\frac{\emptyset \eta \cdot \left(\eta \cdot \left(\eta \cdot (\eta \cdot (12\eta + 15\emptyset - 10) + 20\emptyset - 20) + 30\,(\emptyset - 1)^2\right) + 20\,(\emptyset - 1)^3\right)}{60} |_0^{1-\emptyset}$$

Evaluando Expresion

$$= -\frac{\emptyset \cdot \left(3\emptyset^5 - 20\emptyset^4 + 50\emptyset^3 - 60\emptyset^2 + 35\emptyset - 8\right)}{60}$$

$$\int_0^1 -\frac{\emptyset \cdot \left(3\emptyset^5 - 20\emptyset^4 + 50\emptyset^3 - 60\emptyset^2 + 35\emptyset - 8\right)}{60} d_\emptyset$$

$$= -\frac{1}{60}\int \emptyset(3\emptyset^5 - 20\emptyset^4 + 50\emptyset^3 - 60\emptyset^2 + 35\emptyset - 8)d_\emptyset$$

$$= -\frac{1}{60}(3\emptyset^6 - 20\emptyset^5 + 50\emptyset^4 - 60\emptyset^3 + 35\emptyset^2 - 8\emptyset)d_\emptyset$$

$$\int \emptyset^6 d_\emptyset = \frac{\emptyset^7}{7}$$

$$\int \emptyset^5 d_\emptyset = \frac{\emptyset^6}{6}$$

$$\int \emptyset^4 d_\emptyset = \frac{\emptyset^5}{5}$$

$$\int \emptyset^3 d_\emptyset = \frac{\emptyset^4}{4}$$

$$\int \emptyset^2 d_\emptyset = \frac{\emptyset^3}{3}$$

$$\int \emptyset d_\emptyset = \frac{\emptyset^2}{2}$$

Usando resultados obtenidos y simplificando la expresion

$$= -\frac{\emptyset^7}{140} + \frac{\emptyset^6}{18} - \frac{\emptyset^5}{6} + \frac{\emptyset^4}{4} - \frac{7\emptyset^3}{36} + \frac{\emptyset^2}{15}$$

$$-\frac{\emptyset^7}{140} + \frac{\emptyset^6}{18} - \frac{\emptyset^5}{6} + \frac{\emptyset^4}{4} - \frac{7\emptyset^3}{36} + \frac{\emptyset^2}{15}\Big|_0^1$$

$$= \frac{1}{252}$$

Los resultados de estas integrales conforman la matriz $\mathbf{b}$ local, la cual tiene esta forma:

$$\mathbf{b} = J\begin{bmatrix} \frac{1}{252} \\ \frac{1}{105} \\ \frac{19}{2520} \\ \frac{1}{252} \end{bmatrix}$$

Ahora procedemos a resolver la integral del lado izquierdo:

$$\int_V \mathbf{N^T}\eta^3\nabla(\eta^3\nabla * \mathbf{N})dV$$

$$\iiint \mathbf{N^T}\eta^3\nabla(\eta^3\nabla * \mathbf{N})dxdydz$$

Según lo demostrado en clase, se establece que:

$$dxdydz = Jd\epsilon d\eta d\emptyset$$

Entonces

$$\int_0^1 \int_0^{1-\emptyset} \int_0^{1-\eta-\emptyset} \mathbf{N^T}\eta^3\nabla(\eta^3\nabla * \mathbf{N})Jd\epsilon d\eta d\emptyset$$

Para resolver la integral se hace uso de la integración por partes:

$$\int UdV = UV - \int dUV$$

$$U = \mathbf{N^T}\eta^3$$

$$dV = \nabla\eta^3\nabla\mathbf{N}$$

$$dU = \nabla\mathbf{N^T}\eta^3$$

$$V = \eta^3\nabla\mathbf{N}$$

$$[\mathbf{N^T}\eta^3\eta^3\nabla\mathbf{N}]|_V - \int_0^1 \int_0^{1-\emptyset} \int_0^{1-\eta-\emptyset} \nabla\mathbf{N^T}\eta^3\eta^3\nabla\mathbf{N}Jd\epsilon d\eta d\emptyset$$

Resolvemos la nueva integral:

$$-\int_0^1 \int_0^{1-\emptyset} \int_0^{1-\eta-\emptyset} \nabla\mathbf{N^T}\eta^3\eta^3\nabla\mathbf{N}Jd\epsilon d\eta d\emptyset$$

$$-\int_0^1 \int_0^{1-\emptyset} \int_0^{1-\eta-\emptyset} \nabla\mathbf{N^T}\eta^6\nabla\mathbf{N}Jd\epsilon d\eta d\emptyset$$

Según lo demostrado en clase, se establece que:

$$\nabla\mathbf{N} = \frac{\mathbf{AB}}{J}$$

$$\nabla\mathbf{N^T} = \frac{\mathbf{B^T A^T}}{J}$$

Sustituyendo en la integral:

$$-\int_0^1 \int_0^{1-\emptyset} \int_0^{1-\eta-\emptyset} \frac{\mathbf{B^T A^T}}{J}\eta^6\frac{\mathbf{AB}}{J}Jd\epsilon d\eta d\emptyset$$

Se sacan de la integral todos los valores que no dependen de épsilon, eta o phi:

$$-\frac{\mathbf{B^T A^T}}{J} * \frac{\mathbf{AB}}{J} * J\int_0^1 \int_0^{1-\emptyset} \int_0^{1-\eta-\emptyset} \eta^6 d\epsilon d\eta d\emptyset$$

Se resuelve lo que queda dentro de la integral:

$$\int_0^1 \int_0^{1-\emptyset} \int_0^{1-\eta-\emptyset} \eta^6 d\epsilon d\eta d\emptyset$$

$$= \int_0^1 \int_0^{1-\emptyset} [\eta^6\epsilon]|_0^{1-\eta-\emptyset} d\eta d\emptyset$$

$$= \int_0^1 \int_0^{1-\emptyset} \eta^6(1-\eta-\emptyset)d\eta d\emptyset$$

$$= \int_0^1 \int_0^{1-\emptyset} \eta^6 - \eta^7 - \emptyset\eta^6 d\eta d\emptyset$$

$$= \int_0^1 \int_0^{1-\emptyset} \eta^6(1-\emptyset) - \eta^7 d\eta d\emptyset$$

$$= \int_0^1 \int_0^{1-\emptyset} \eta^6(1-\emptyset)d\eta d\emptyset - \int_0^1 \int_0^{1-\emptyset} \eta^7 d\eta d\emptyset$$

$$= \int_0^1 [\frac{\eta^7(1-\emptyset)}{7}]|_0^{1-\emptyset} d\emptyset - \int_0^1 [\frac{\eta^8}{8}]|_0^{1-\emptyset} d\emptyset$$

$$= \int_0^1 \frac{(1-\emptyset)^7(1-\emptyset)}{7} d\emptyset - \int_0^1 \frac{(1-\emptyset)^8}{8} d\emptyset$$

$$= \int_0^1 \frac{(1-\emptyset)^7(1-\emptyset)}{7} - \frac{(1-\emptyset)^8}{8} d\emptyset$$

$$= \int_0^1 \frac{(1-\emptyset)^8}{7} - \frac{(1-\emptyset)^8}{8} d\emptyset$$

$$= \int_0^1 \frac{8(1-\emptyset)^8 - 7(1-\emptyset)^8}{56} d\emptyset$$

$$= \int_0^1 \frac{(1-\emptyset)^8}{56} d\emptyset$$

$$= \int_0^1 \frac{1 - 8\emptyset + 28\emptyset^2 - 56\emptyset^3 + 70\emptyset^4 - 56\emptyset^5 + 28\emptyset^6 - 8\emptyset^7 + \emptyset^8}{56} d\emptyset$$

$$= \int_0^1 \frac{1}{56} - \frac{\emptyset}{7} + \frac{\emptyset^2}{2} - \emptyset^3 + \frac{5\emptyset^4}{4} - \emptyset^5 + \frac{\emptyset^6}{2} - \frac{\emptyset^7}{7} - \frac{\emptyset^8}{56} d\emptyset$$

$$= \int_0^1 \frac{1}{56} d\emptyset - \int_0^1 \frac{\emptyset}{7} d\emptyset + \int_0^1 \frac{\emptyset^2}{2} d\emptyset - \int_0^1 \emptyset^3 d\emptyset + \int_0^1 \frac{5\emptyset^4}{4} d\emptyset - \int_0^1 \emptyset^5 d\emptyset + \int_0^1 \frac{\emptyset^6}{2} d\emptyset - \int_0^1 \frac{\emptyset^7}{7} d\emptyset - \int_0^1 \frac{\emptyset^8}{56} d\emptyset$$

$$= [\frac{\emptyset}{56}]|_0^1 - [\frac{\emptyset^2}{14}]|_0^1 + [\frac{\emptyset^3}{6}]|_0^1 - [\frac{\emptyset^4}{4}]|_0^1 + [\frac{\emptyset^5}{4}]|_0^1 - [\frac{\emptyset^5}{6}]|_0^1 + [\frac{\emptyset^7}{14}]|_0^1 - [\frac{\emptyset^8}{56}]|_0^1 - [\frac{\emptyset^8}{504}]|_0^1$$

$$= \frac{1}{56} - \frac{1}{14} + \frac{1}{6} - \frac{1}{4} + \frac{1}{4} - \frac{1}{6} + \frac{1}{14} - \frac{1}{56} - \frac{1}{504}$$

$$= \frac{1}{504}$$

Con este resultado obtenemos la forma de la matriz $\mathbf{K}$ local:

$$\mathbf{K} = -\frac{\mathbf{B^T A^T}}{J} * \frac{\mathbf{AB}}{J} * \frac{J}{504}$$

Y con esto ultimo finalizamos el paso 6 del MEF y obtenemos nuestro sistema de ecuaciones local:

$$\mathbf{KX} = \mathbf{b}$$

$$[\mathbf{N^T}\eta^3\eta^3\nabla\mathbf{N}]|_V - (\frac{\mathbf{B^T A^T}}{J} * \frac{\mathbf{AB}}{J} * \frac{J}{504})\mathbf{X} = J\begin{bmatrix} \frac{1}{252} \\ \\ \frac{1}{105} \\ \\ \frac{19}{2520} \\ \\ \frac{1}{252} \end{bmatrix}$$

$$-(\frac{\mathbf{B^T A^T}}{J} * \frac{\mathbf{AB}}{J} * \frac{J}{504})\mathbf{X} = J\begin{bmatrix} \frac{1}{252} \\ \\ \frac{1}{105} \\ \\ \frac{19}{2520} \\ \\ \frac{1}{252} \end{bmatrix} - [\mathbf{N^T}\eta^3\eta^3\nabla\mathbf{N}]|_V$$

$$-(\frac{\mathbf{B^T A^T}}{J} * \frac{\mathbf{AB}}{J} * \frac{J}{504})\mathbf{X} = J\begin{bmatrix} \frac{1}{252} \\ \\ \frac{1}{105} \\ \\ \frac{19}{2520} \\ \\ \frac{1}{252} \end{bmatrix}$$