Type-Level Programming 101

The Subspace of Scala

Joe Barnes

Follow along at http://192.168.183.88:8080

Powered by

Joe Barnes

@joescii

prose :: and :: conz

Primarily Java from 2004-2013

Started Scala late 2012

Started Scala at Mentor Graphics late 2013

Lift committer in July 2014

So... Type programming...

What do *you* know about *that*, Joe??

Not much...

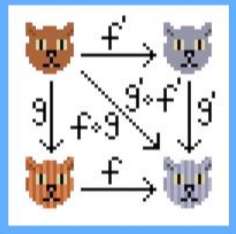
So... Type programming...

What do *you* know about *tha*

I've made zero contributions to shapeless

Not much...

Also no contributions to Cats



Type-Level Programming 101

The Subspace of Scala

Joe Barnes

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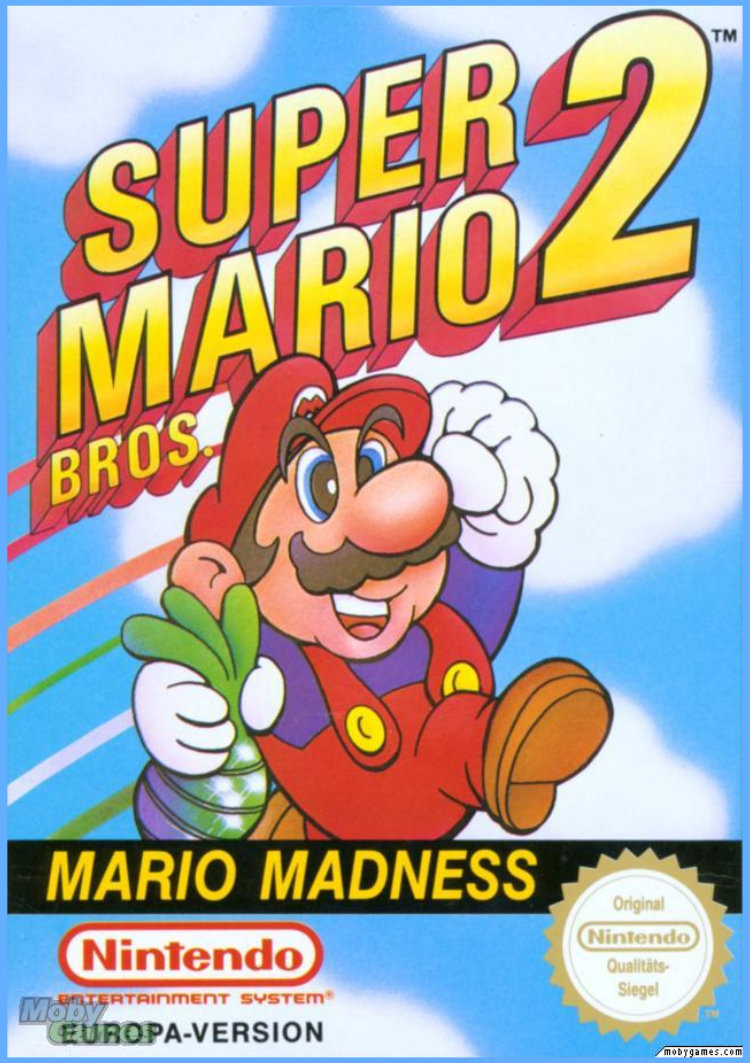
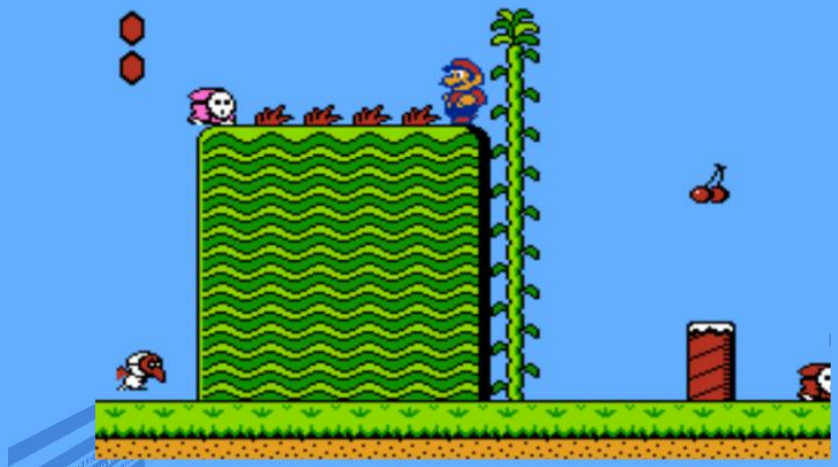
Although I have contributed plenty of cat memes!

I'm not here to share expertise on the subject,
but rather my *Aha!* moment.

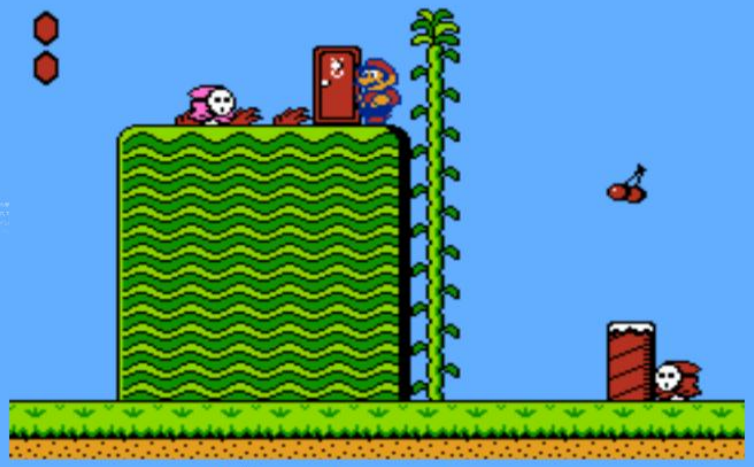
Programming in Scala is like...


SUPER MARIO 2
BROS.
MARIO MADNESS
Nintendo
ENTERTAINMENT SYSTEM
EUROPA-VERSION
Original
Nintendo
Qualitäts-
Siegel
mobygames.com



Normal value programming

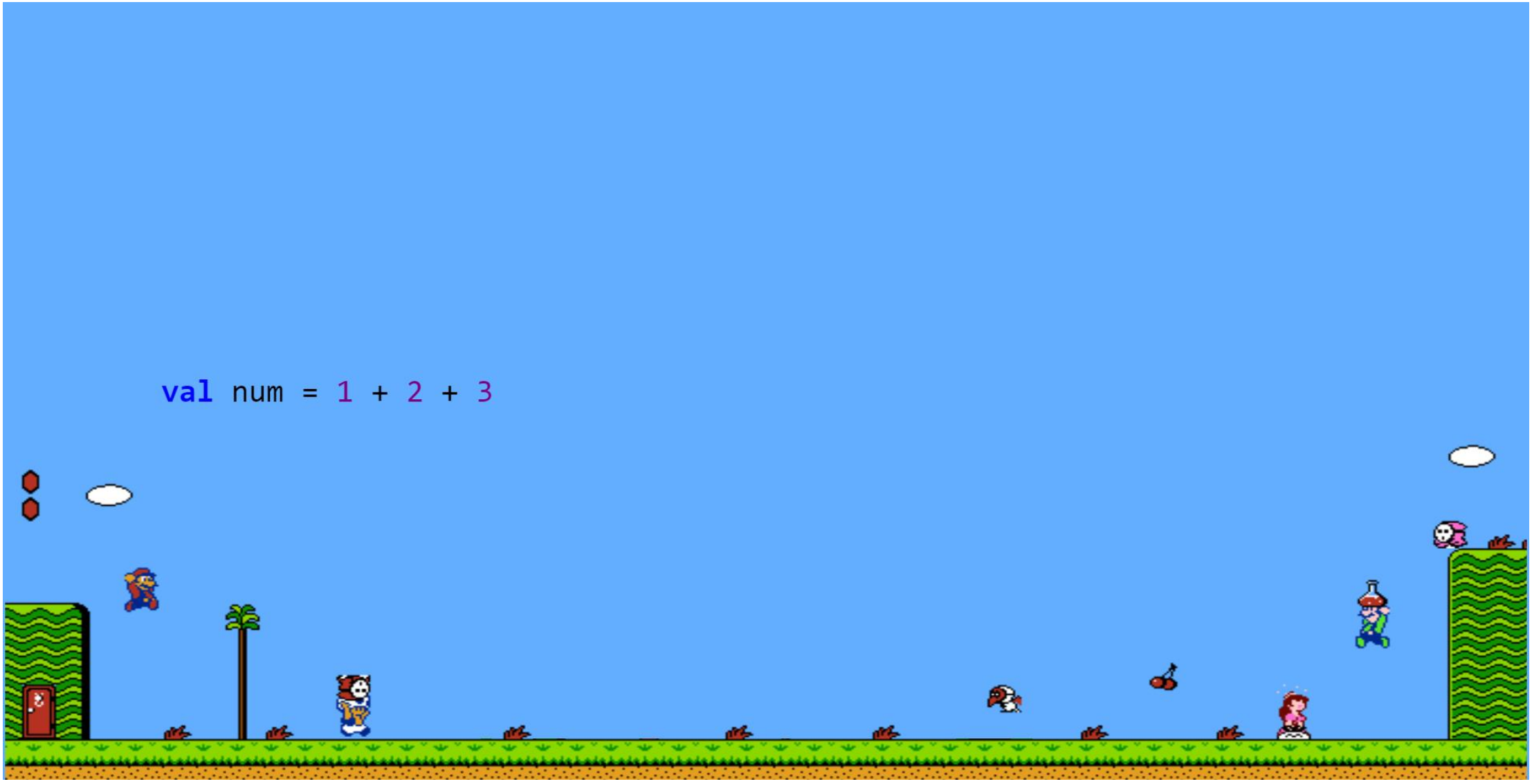
A flask of type programming!



Where does this go?

SUBSPACE!

```
val num = 1 + 2 + 3
```



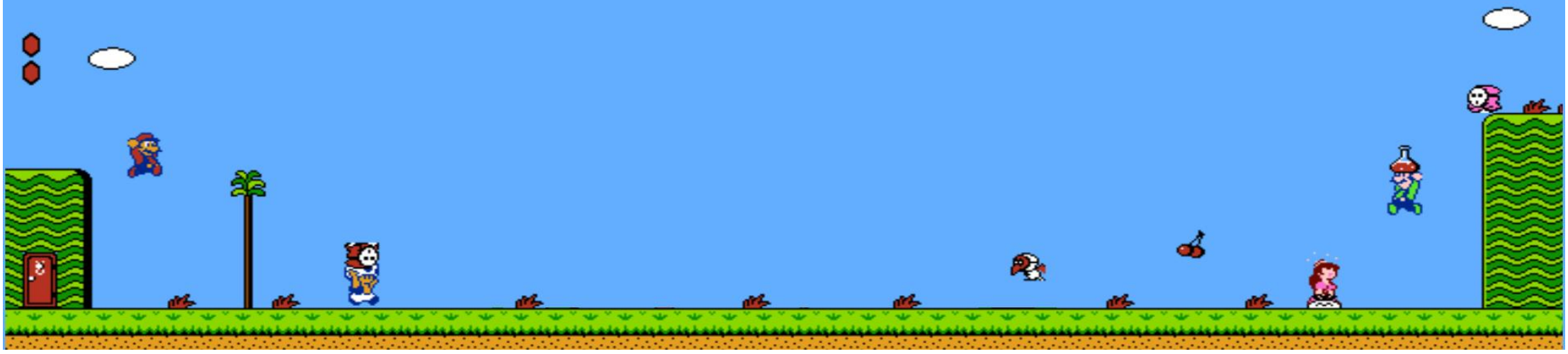
```
val num = 1 + 2 + 3
```

```
lazy val str = "a"+"b"+"c"
```

```
val num = 1 + 2 + 3
```

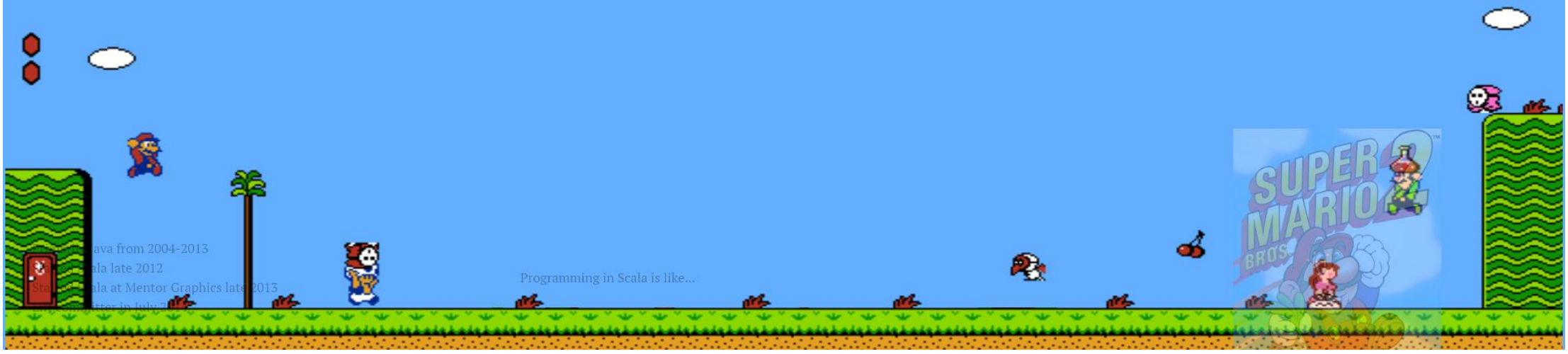
```
lazy val str = "a"+"b"+"c"
```

```
def now = new java.util.Date
```

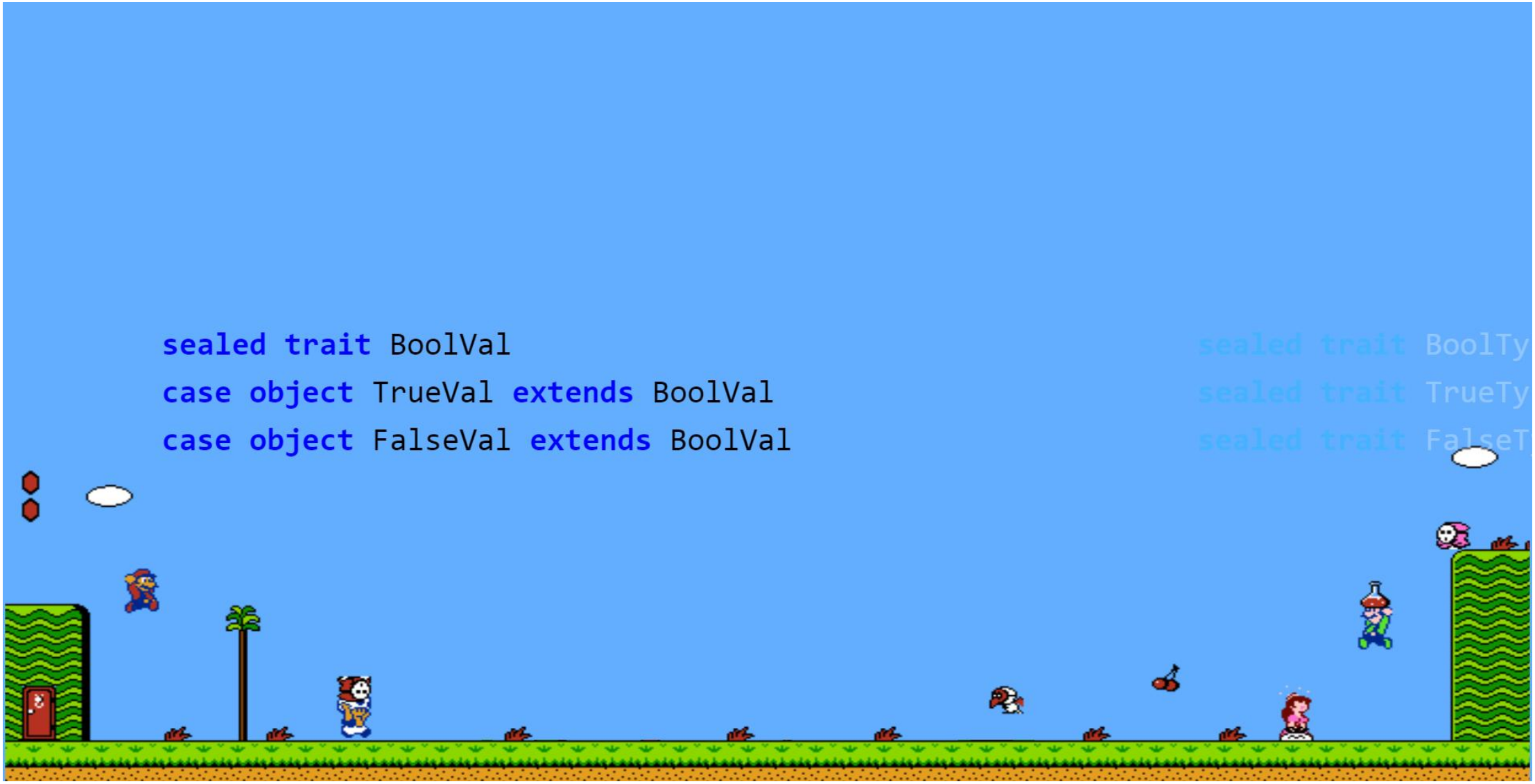


```
type MyMap = Map[Int, String]
```

Let's compare values to types with the simplest domain ever

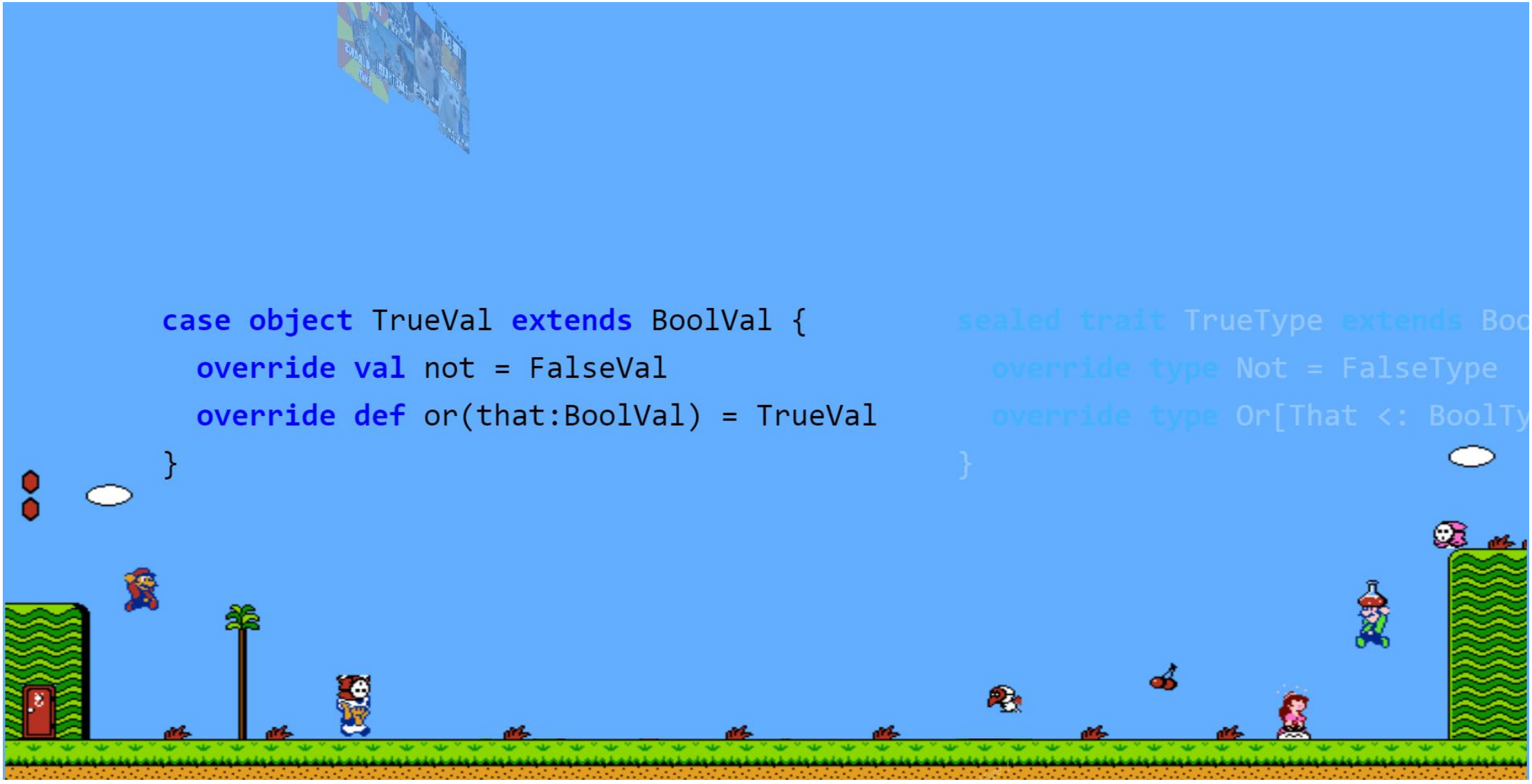


```scala
sealed trait BoolVal
case object TrueVal extends BoolVal
case object FalseVal extends BoolVal
```

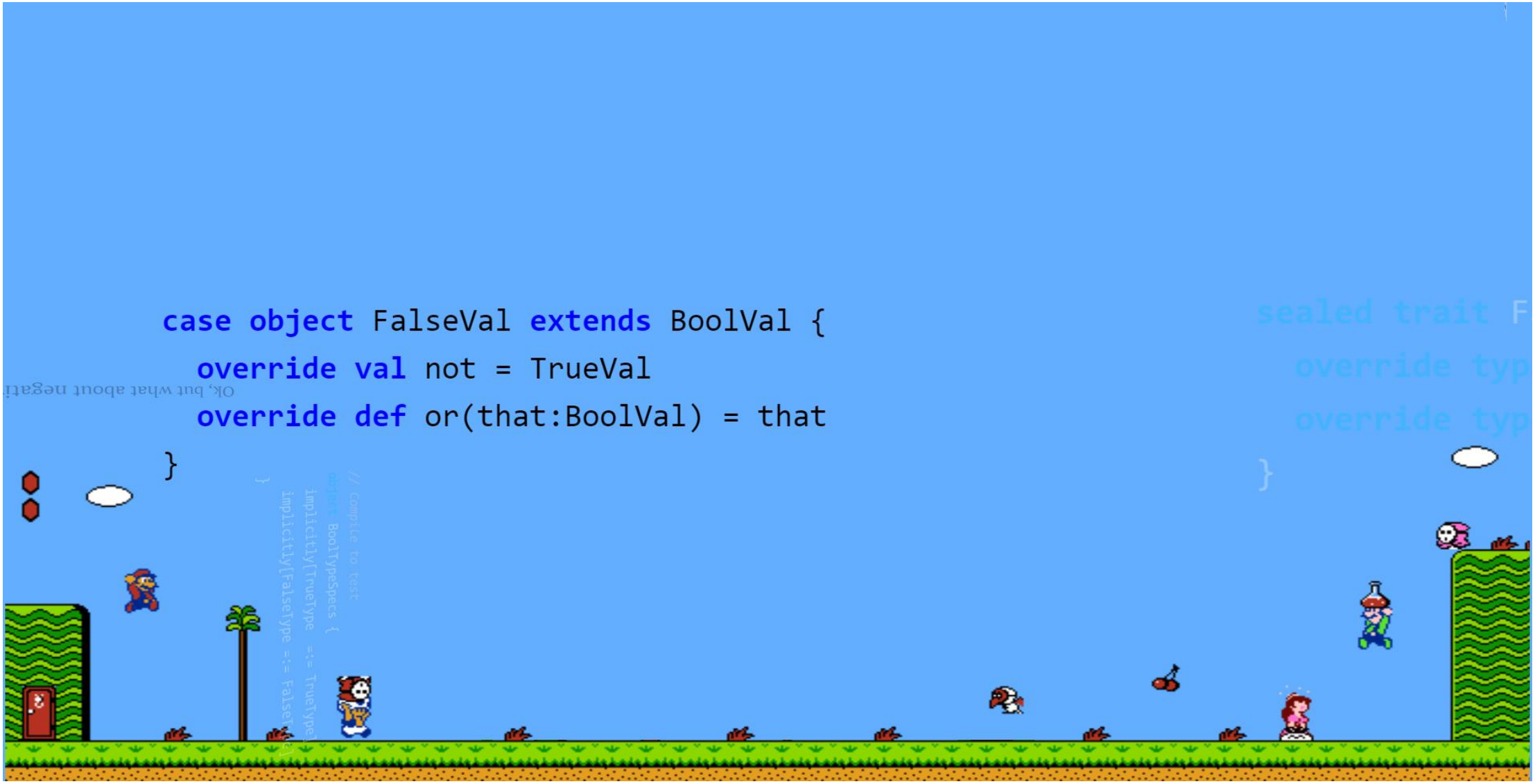


```scala
sealed trait BoolVal {
  def not:BoolVal
  def or(that:BoolVal):BoolVal
}
```

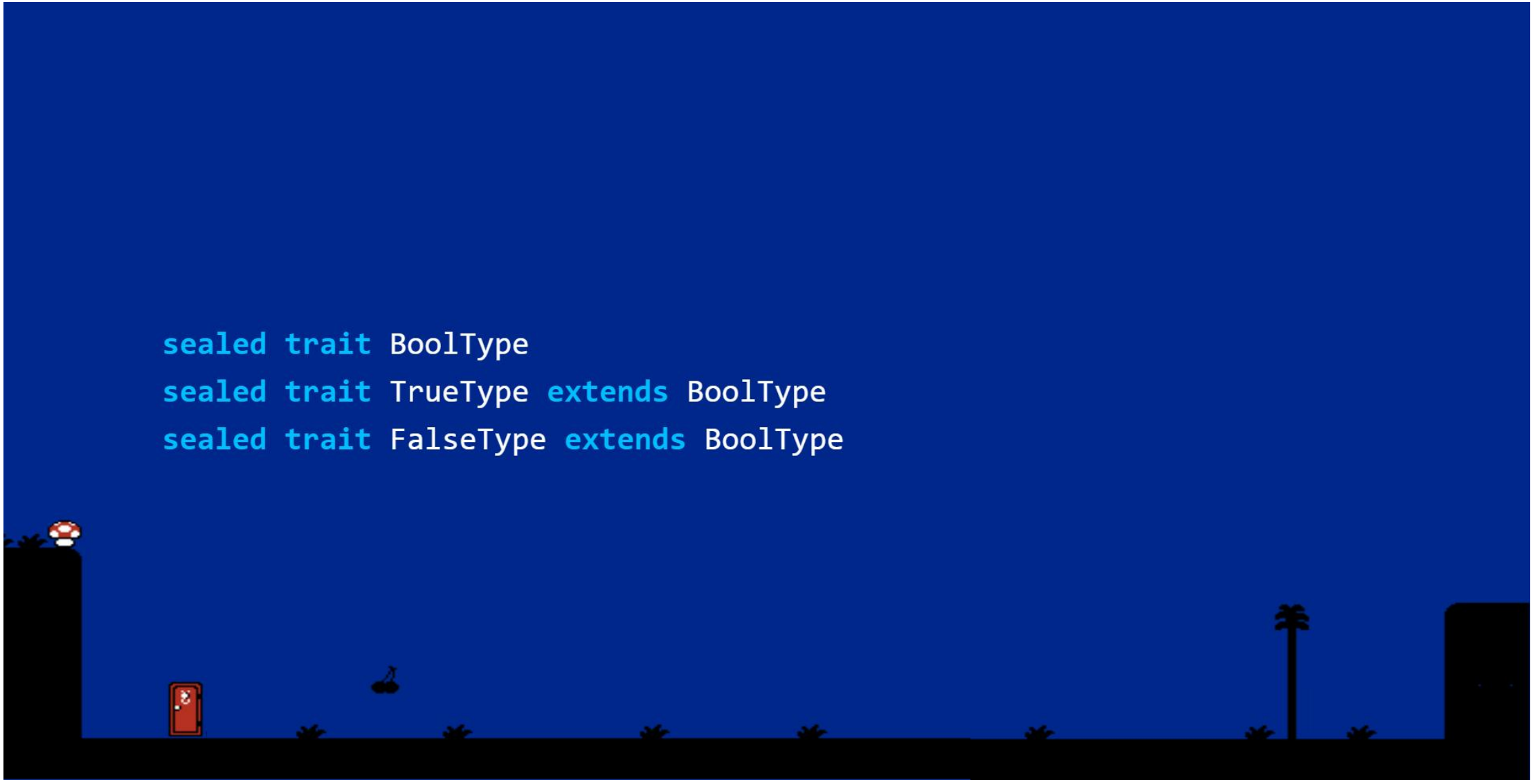
```scala
sealed trait BoolType {
  type Not <: BoolType
  type Or[That <: BoolType] <:
}
```


case object TrueVal extends BoolVal {
override val not = FalseVal
override def or(that:BoolVal) = TrueVal
}
sealed trait TrueType extends Boo
override type Not = FalseType
override type Or[That <: BoolTy
}

```
case object FalseVal extends BoolVal {
  override val not = TrueVal
  override def or(that:BoolVal) = that
}
```



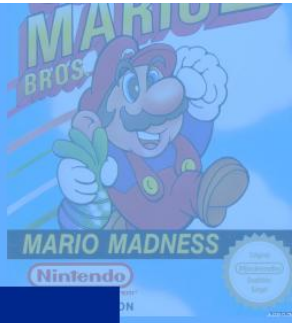
```
sealed trait BoolType
sealed trait TrueType extends BoolType
sealed trait FalseType extends BoolType
```



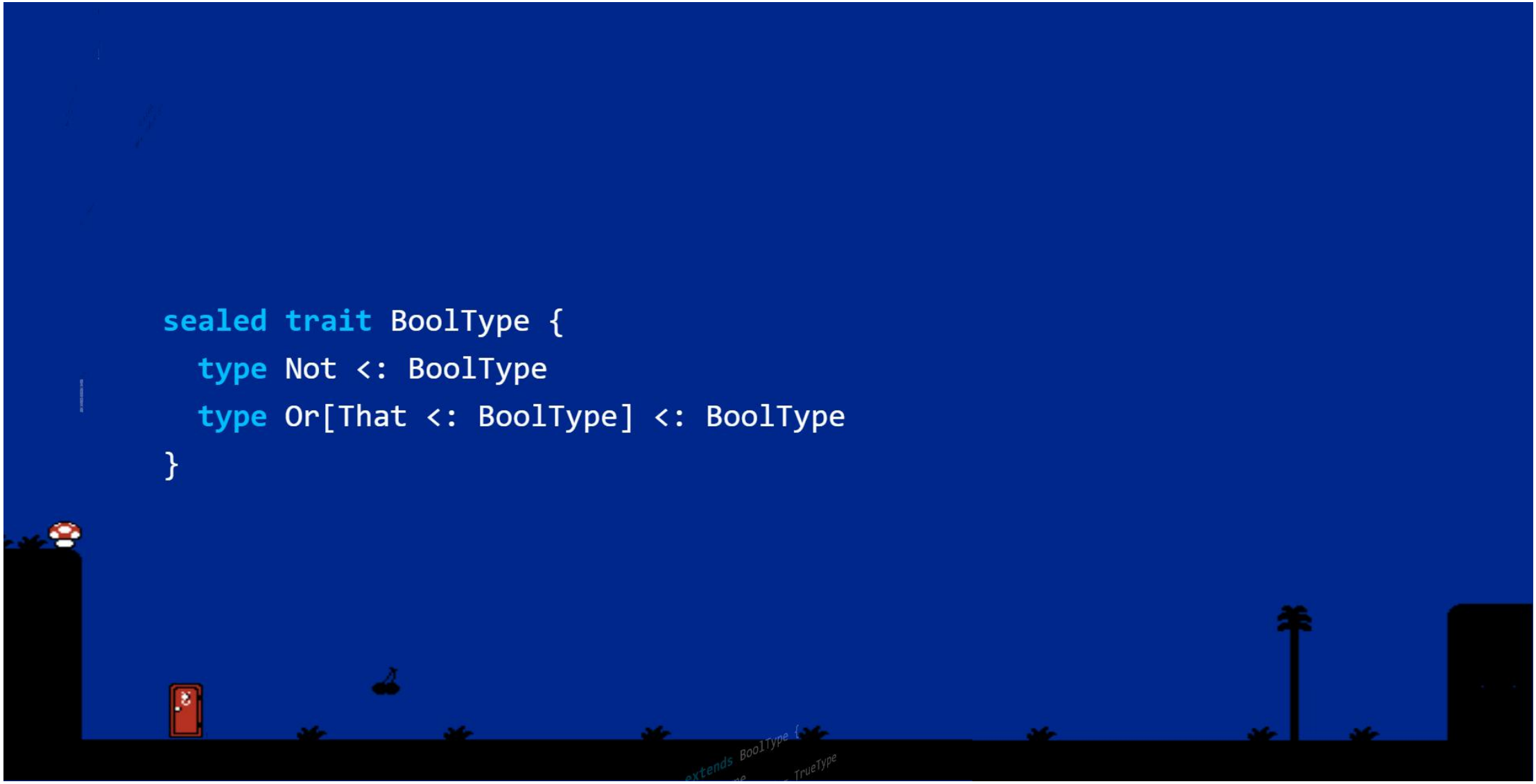
oe Barnes
@joescii
rose :: and :: conz

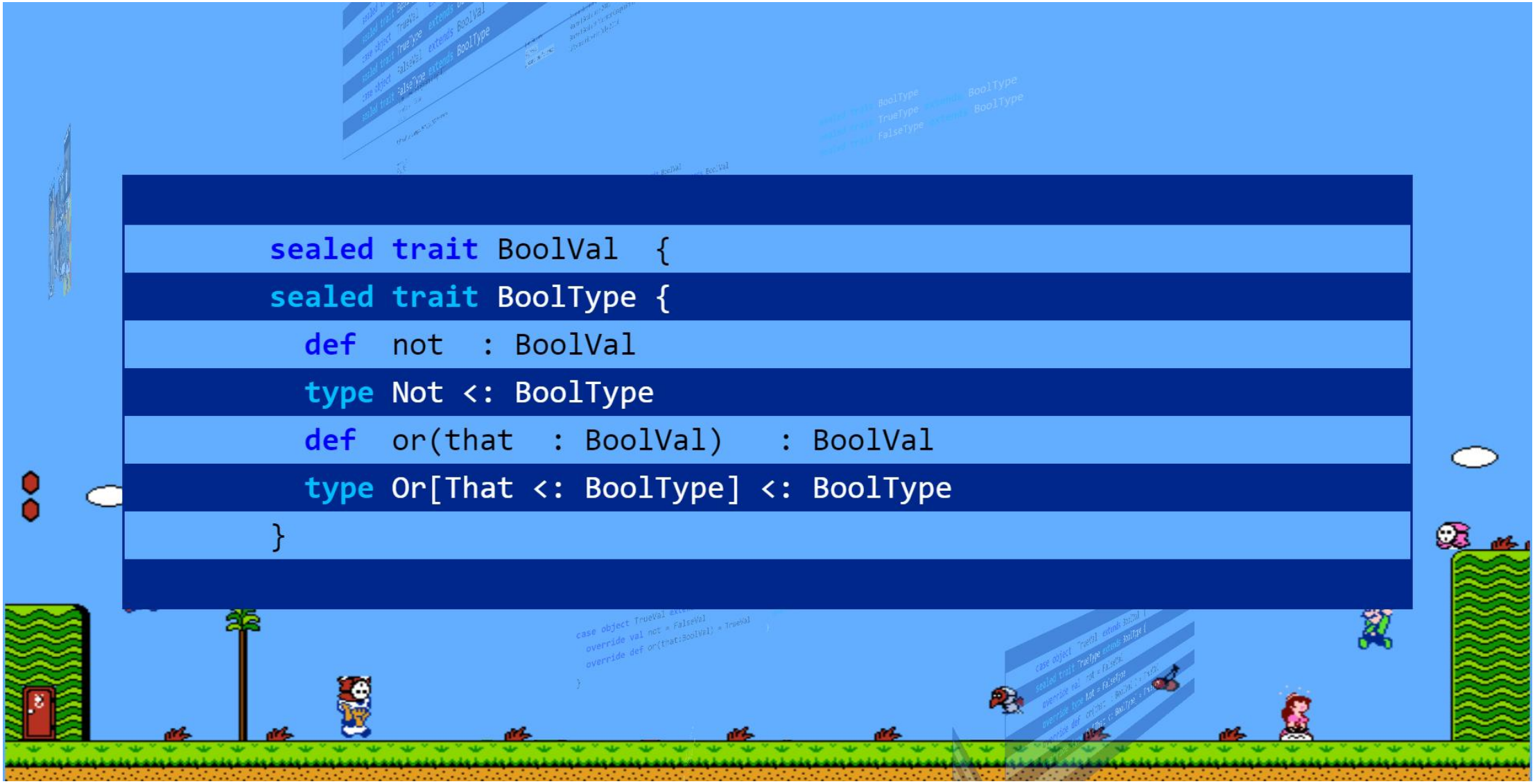
Primarily Java from 2004-2013
Started Scala late 2012
Started Scala at Mentor Graphics late 2013
Lift committer in July 2014

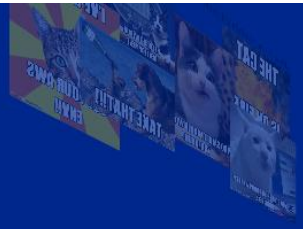
Programming in Scala is like...



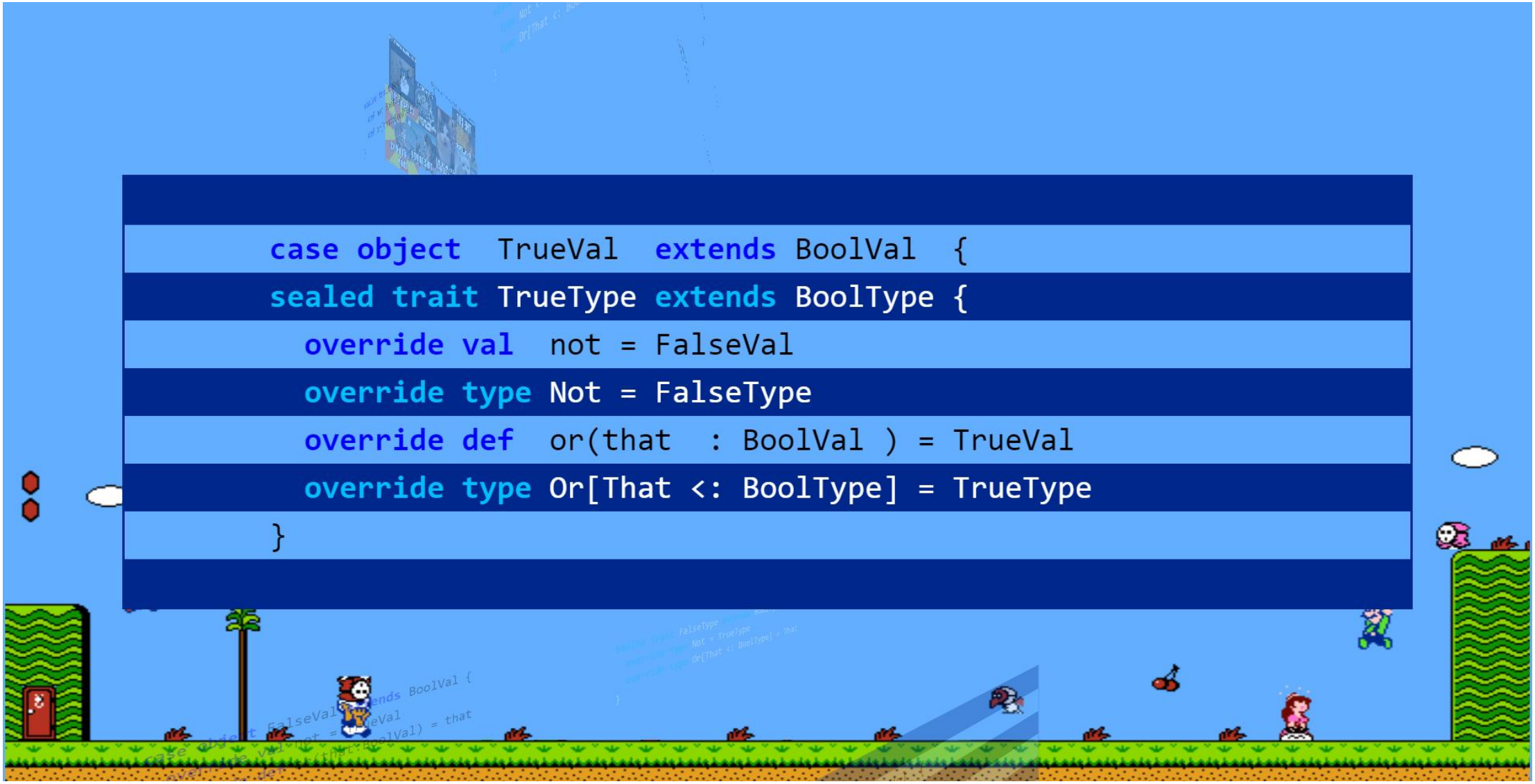

```
sealed trait BoolVal
sealed trait BoolType
case object  TrueVal   extends BoolVal
sealed trait TrueType  extends BoolType
case object  FalseVal  extends BoolVal
sealed trait FalseType extends BoolType
```


sealed trait BoolType {
type Not <: BoolType
type Or[That <: BoolType] <: BoolType
}

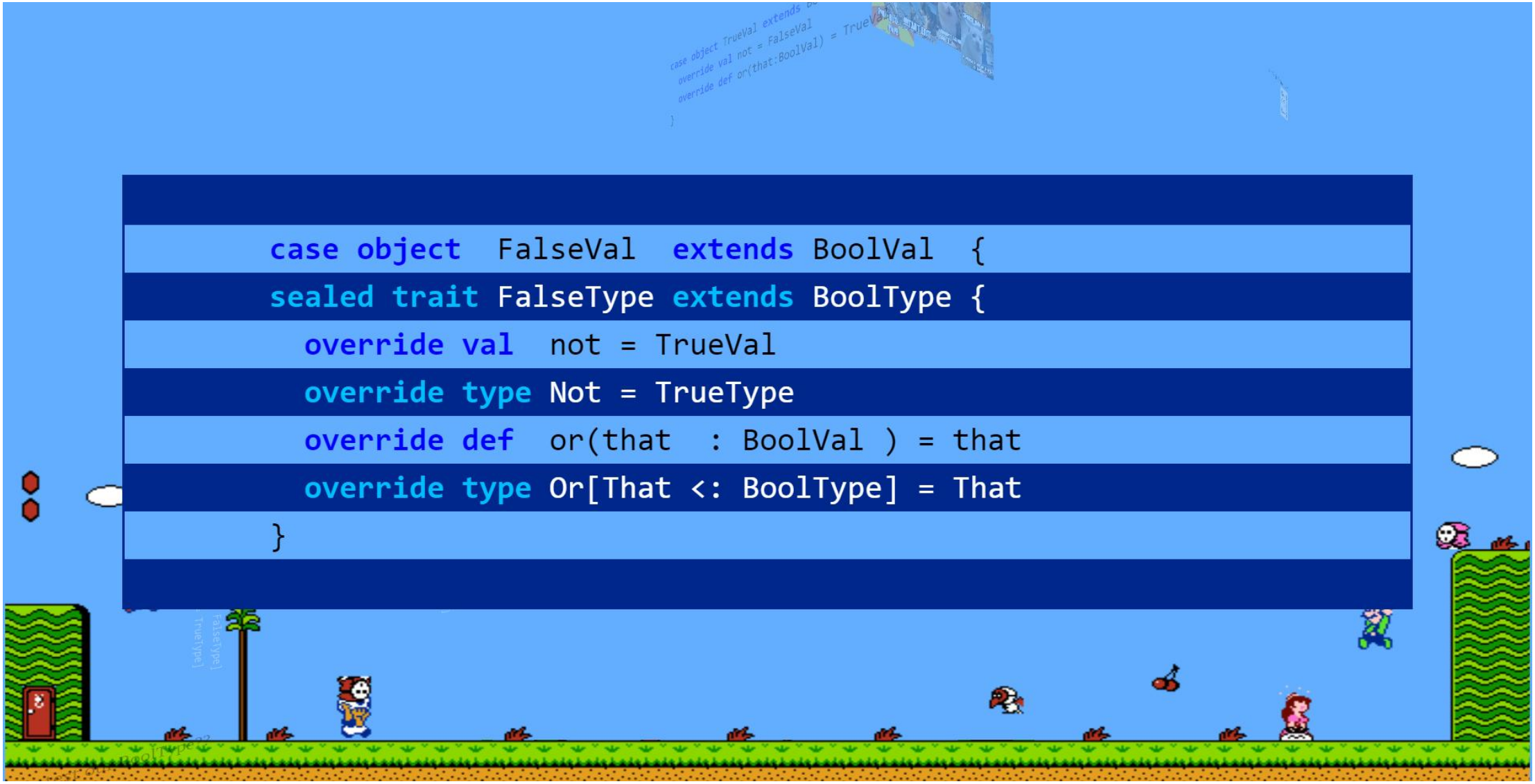

sealed trait BoolVal {
sealed trait BoolType {
def not : BoolVal
type Not <: BoolType
def or(that : BoolVal)   : BoolVal
type Or[That <: BoolType] <: BoolType
}



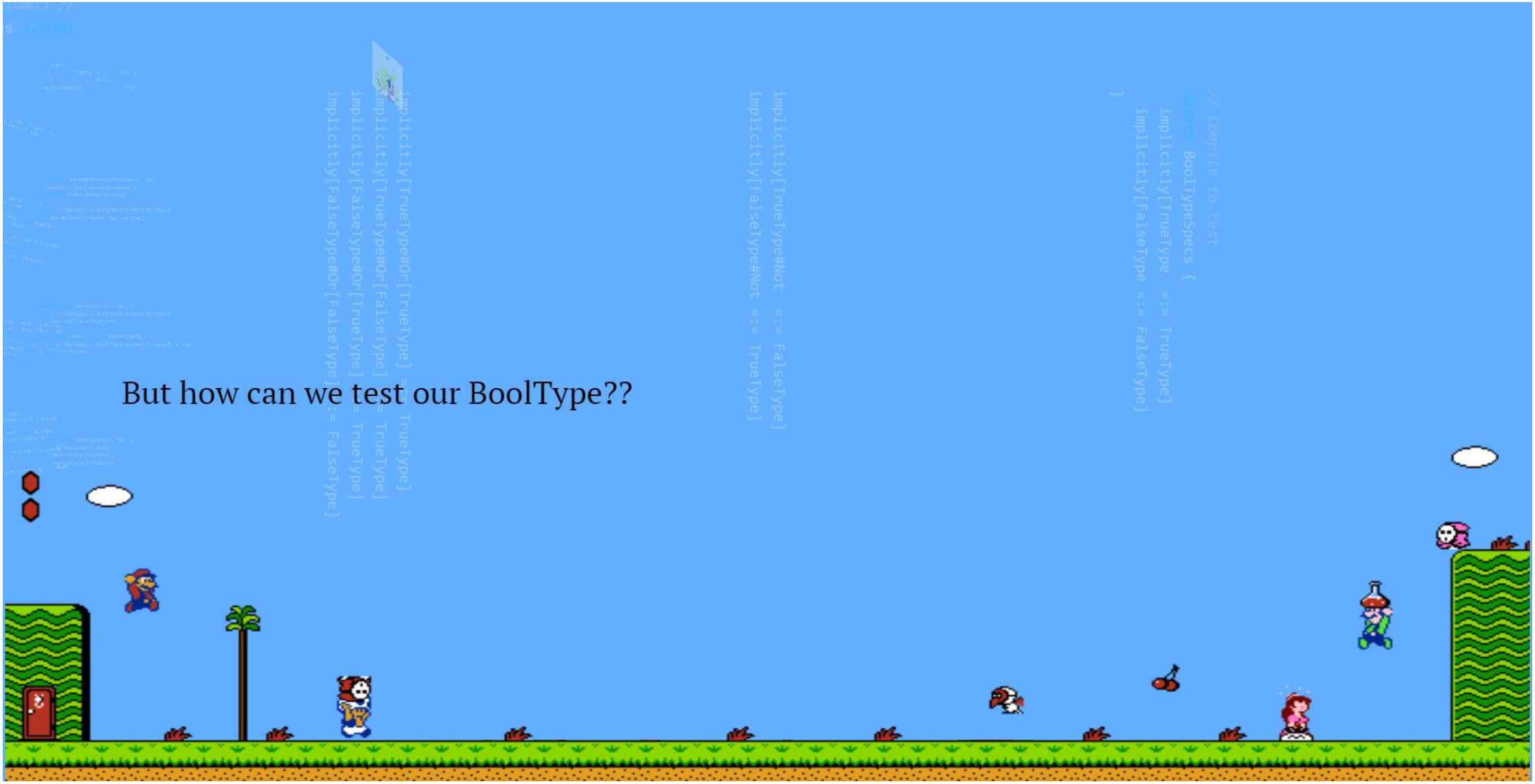
```
sealed trait TrueType extends BoolType {
  override type Not = FalseType
  override type Or[That <: BoolType] = TrueType
}
```


case object TrueVal extends BoolVal {
sealed trait TrueType extends BoolType {
override val not = FalseVal
override type Not = FalseType
override def or(that : BoolVal) = TrueVal
override type Or[That <: BoolType] = TrueType
}

```scala
sealed trait FalseType extends BoolType {
  override type Not = TrueType
  override type Or[That <: BoolType] = That
}
```

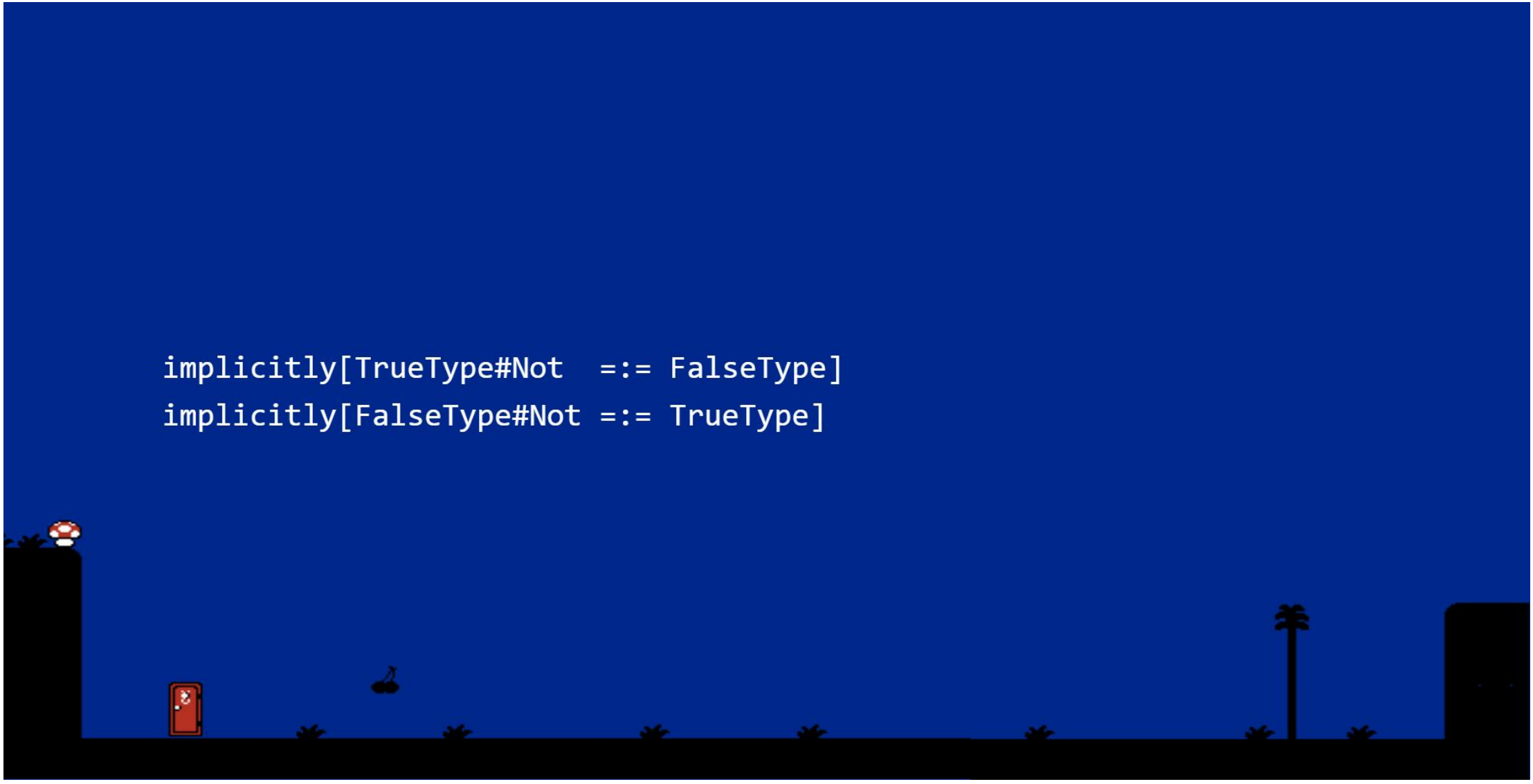

case object FalseVal extends BoolVal {
sealed trait FalseType extends BoolType {
override val not = TrueVal
override type Not = TrueType
override def or(that : BoolVal) = that
override type Or[That <: BoolType] = That
}

But how can we test our BoolType??

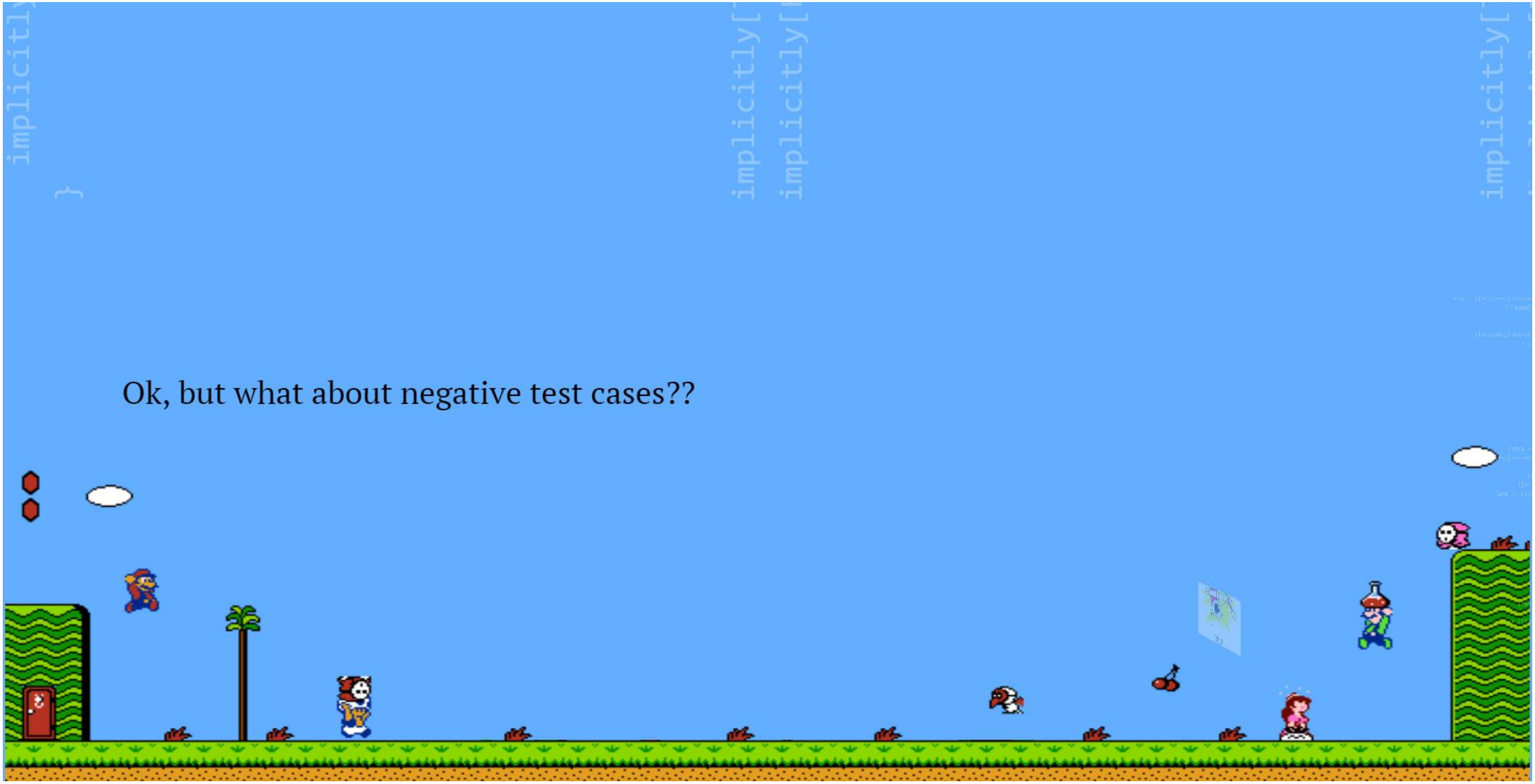


```scala
// Compile to test
object BoolTypeSpecs {
  implicitly[TrueType  =:= TrueType]
  implicitly[FalseType =:= FalseType]
}
```

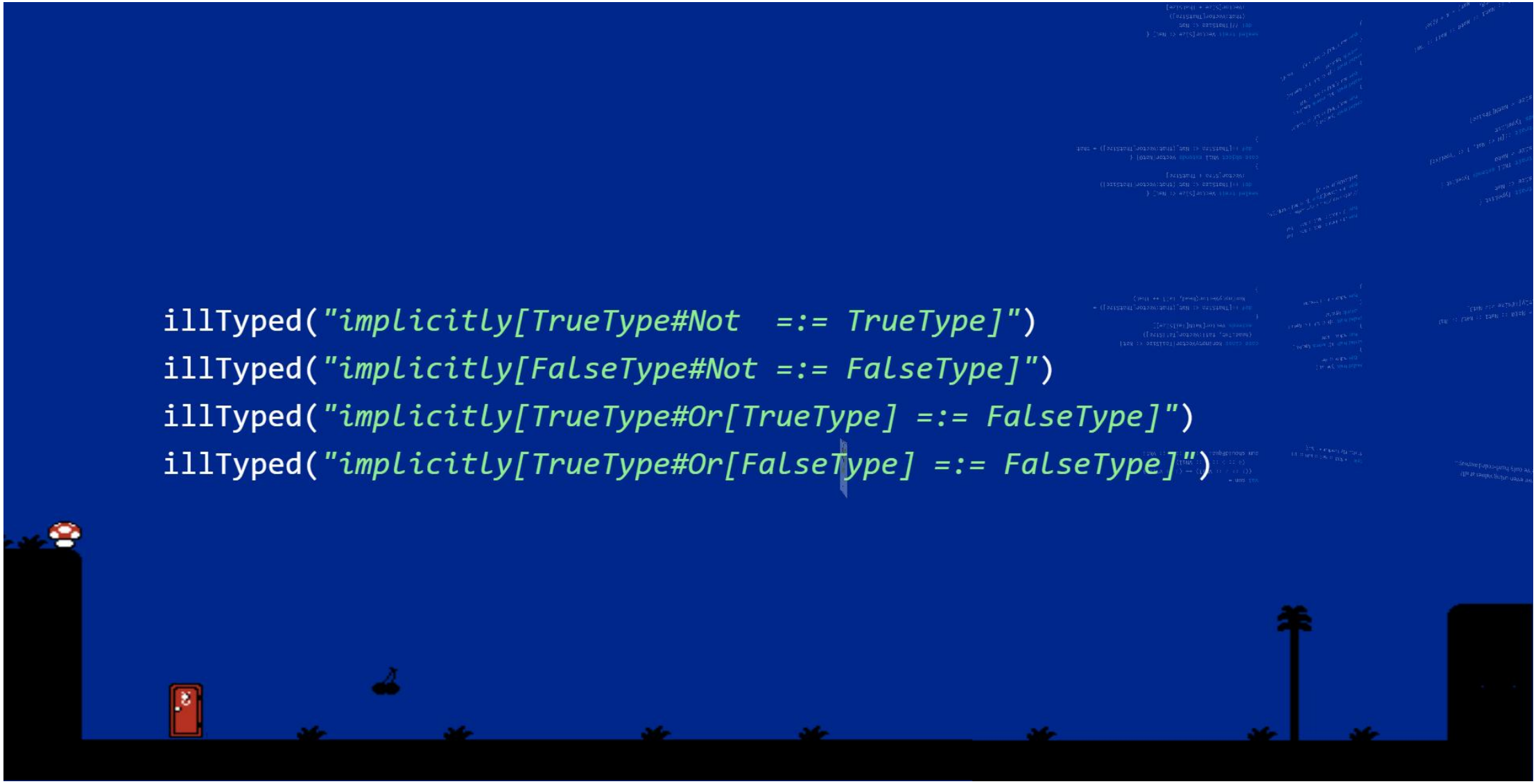
```
implicitly[TrueType#Not  =:= FalseType]
implicitly[FalseType#Not =:= TrueType]
```



```
implicitly[TrueType#Or[TrueType] =:= TrueType]
implicitly[TrueType#Or[FalseType] =:= TrueType]
implicitly[FalseType#Or[TrueType] =:= TrueType]
implicitly[FalseType#Or[FalseType] =:= FalseType]
```


Ok, but what about negative test cases??


import shapeless.test.illTyped
// Compiles only if string DOESN'T compile
illTyped("implicitly[TrueType =:= FalseType]")
illTyped("implicitly[FalseType =:= TrueType]")

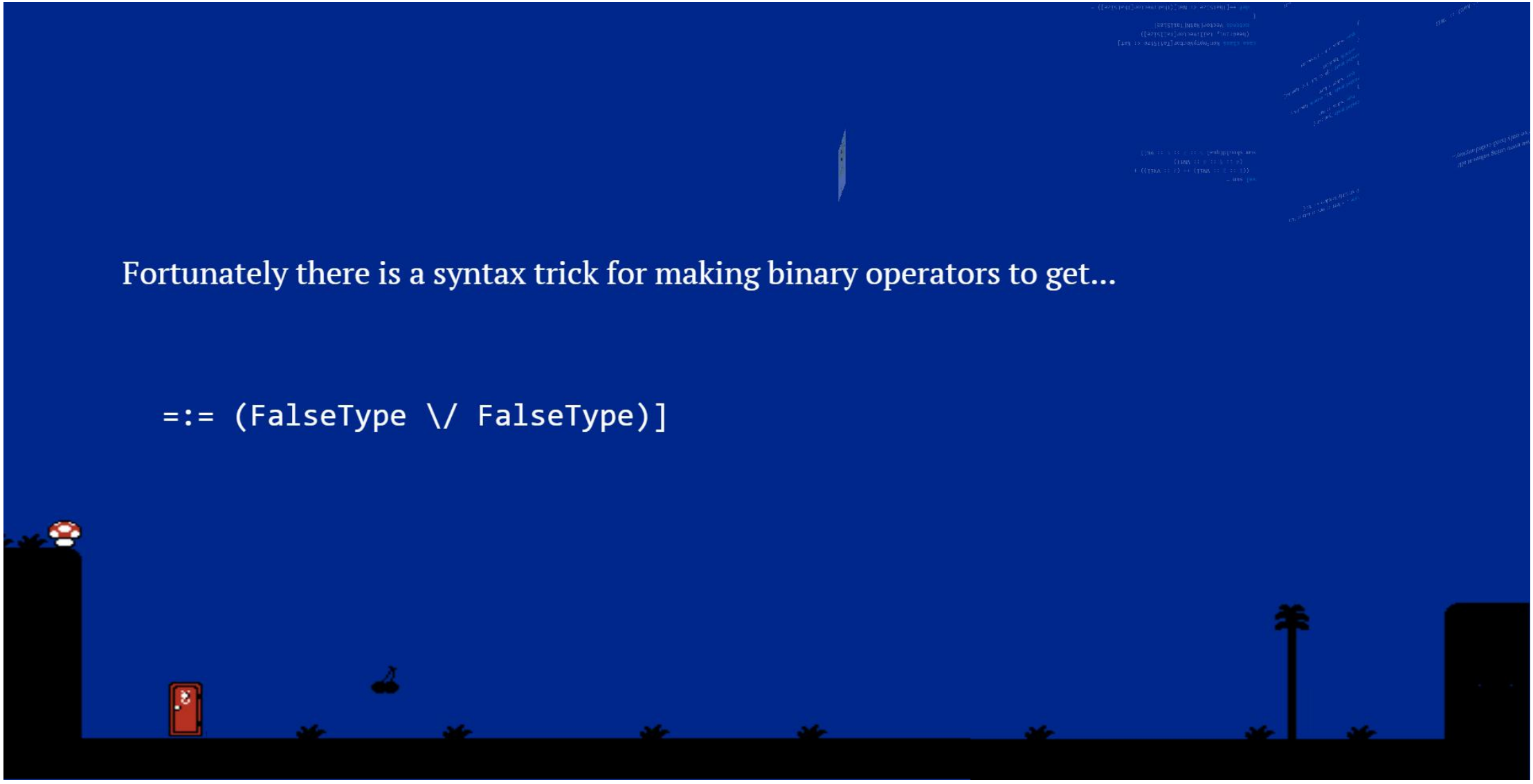

illTyped("implicitly[TrueType#Not =:= TrueType]")
illTyped("implicitly[FalseType#Not =:= FalseType]")
illTyped("implicitly[TrueType#Or[TrueType] =:= FalseType]")
illTyped("implicitly[TrueType#Or[FalseType] =:= FalseType]")

That's cool and everything, but this is kinda ugly for Scala...

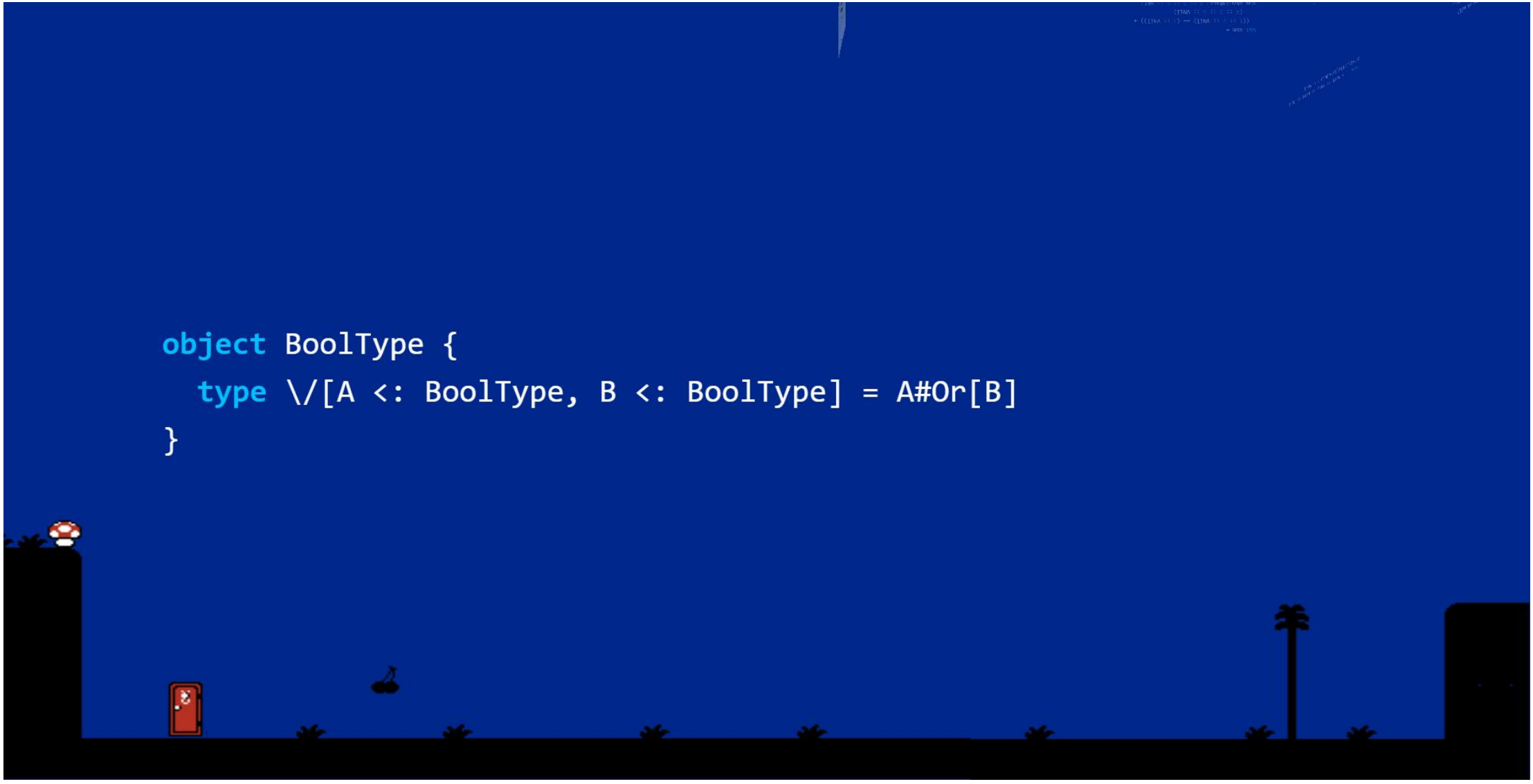
```
implicitly[TrueType#Or[FalseType] =:= TrueType]
```

Fortunately there is a syntax trick for making binary operators to get...

```
=:= (FalseType \/ FalseType)]
```



```
object BoolType {
  type \/[A <: BoolType, B <: BoolType] = A#Or[B]
}
```

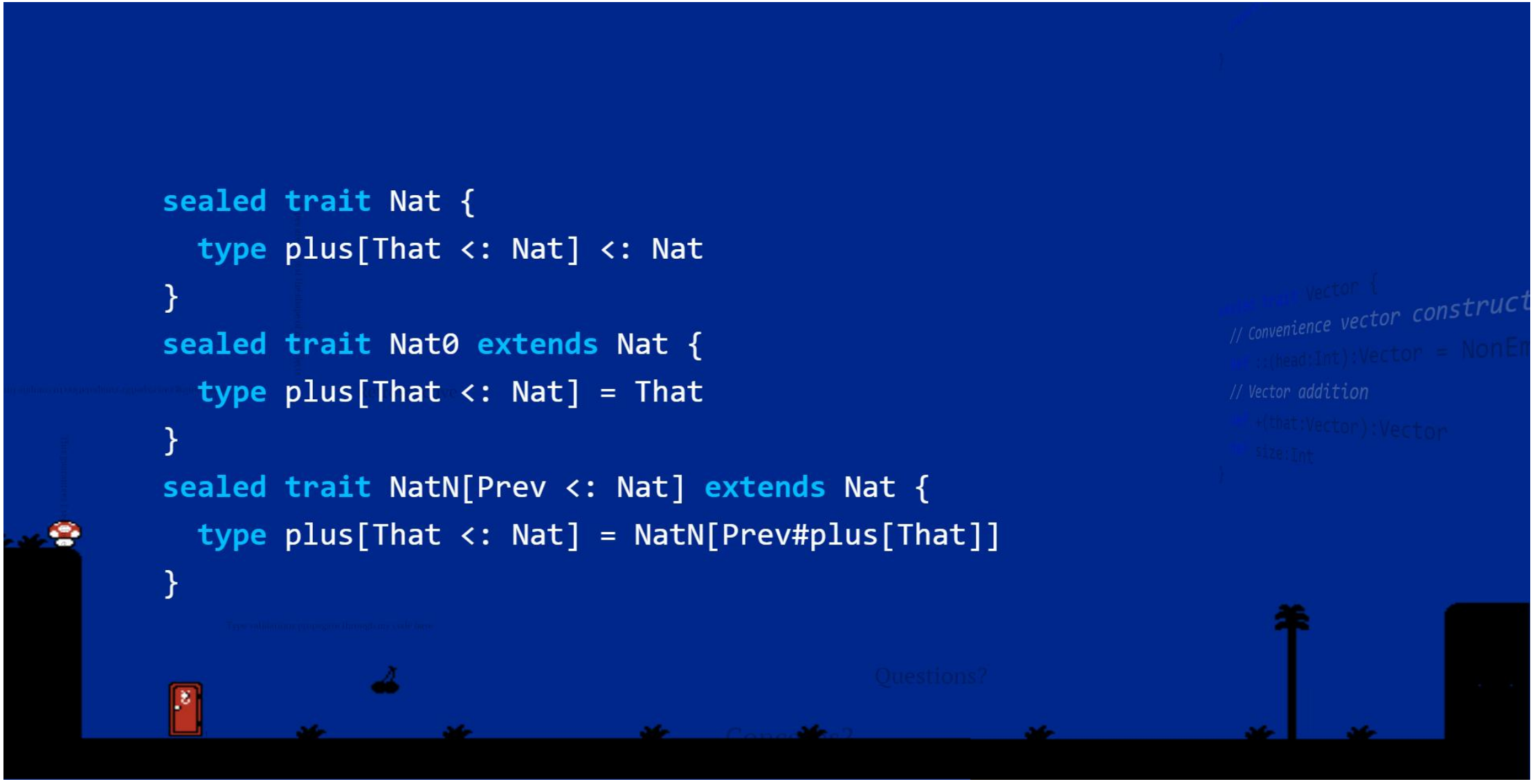



// A type with two parameters
// can be written in infix notation:
implicitly[\/[FalseType, FalseType]
=:= (FalseType \/ FalseType)]

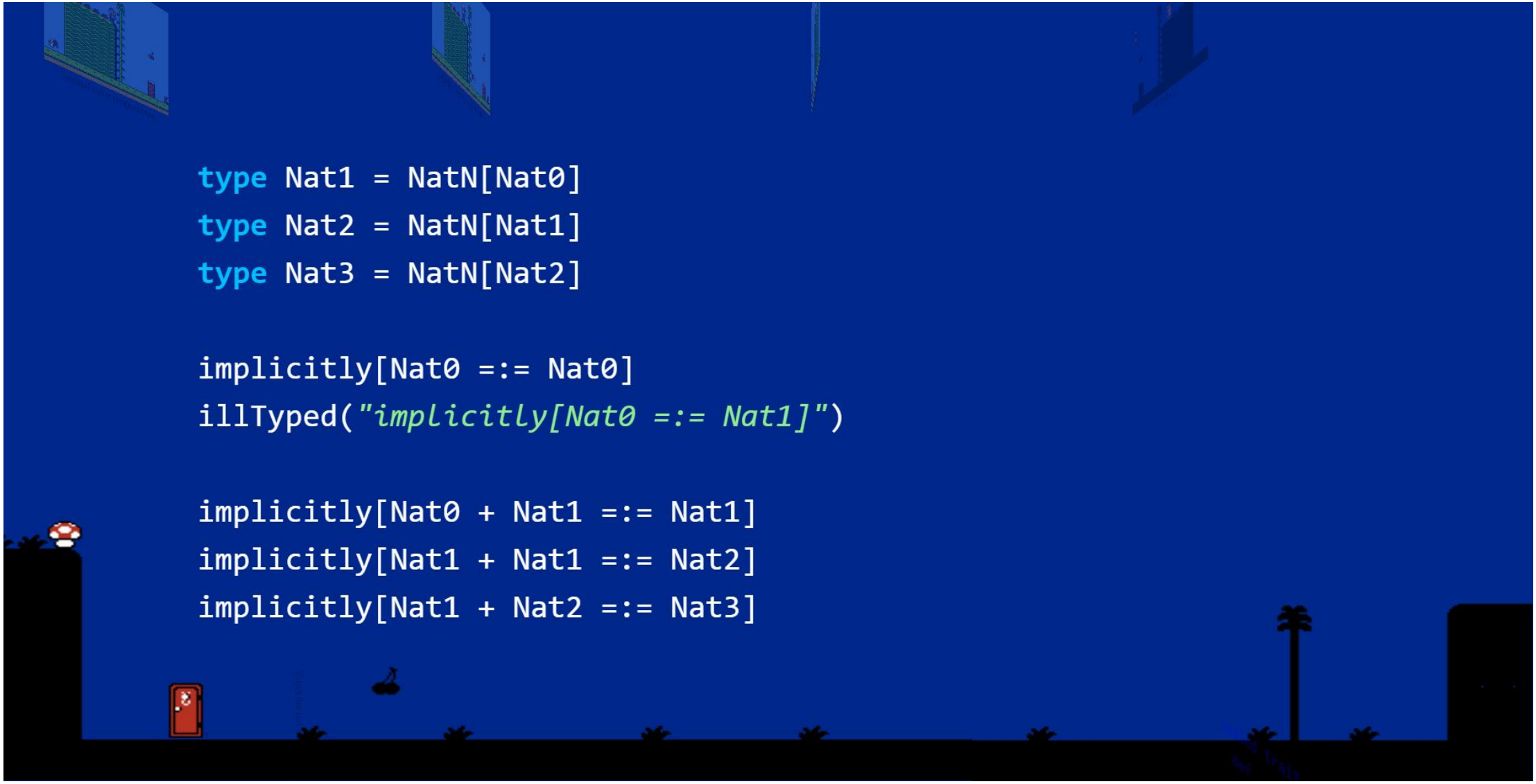

MARIO MADNESS
Nintendo
type Nat
type Nat
type Nat
implicit
illTyped
implicit
Well that was easy... It's just two types, really...


sealed trait Nat {
def +(that:Nat):Nat
}
case object Nat0 extends Nat {
override def +(that:Nat) = that
}
case class NatN(prev:Nat) extends Nat {
override def +(that:Nat) =
NatN(prev + that)
}


val nat1 = NatN(Nat0)
val nat2 = NatN(nat1)
val nat3 = NatN(nat2)
Nat0 + nat1 shouldEqual nat1
nat1 + nat1 shouldEqual nat2
nat1 + nat2 shouldEqual nat3


sealed trait Nat {
type plus[That <: Nat] <: Nat
}
sealed trait Nat0 extends Nat {
type plus[That <: Nat] = That
}
sealed trait NatN[Prev <: Nat] extends Nat {
type plus[That <: Nat] = NatN[Prev#plus[That]]
}

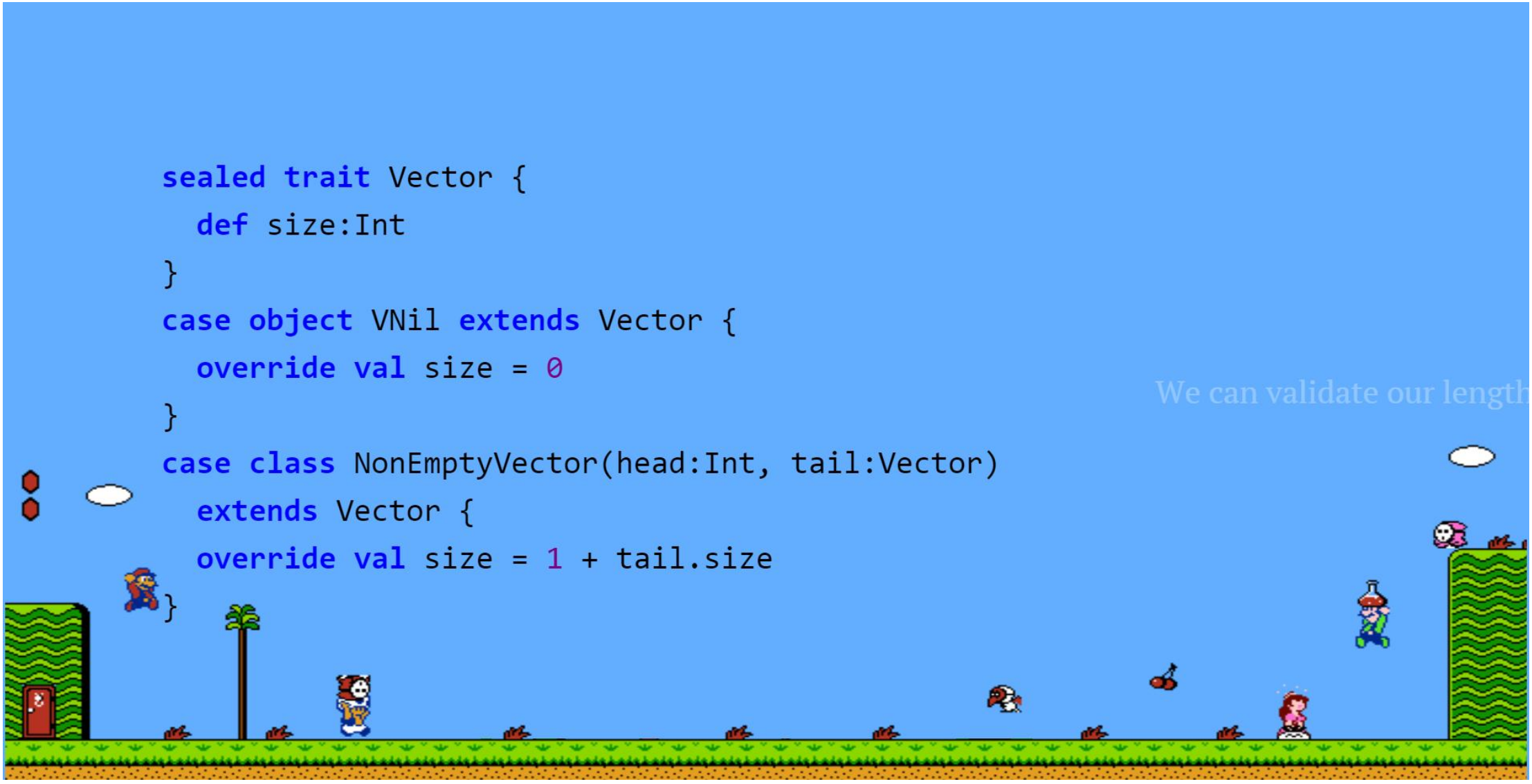

object Nat {
type +[A <: Nat, B <: Nat] = A#plus[B]
}


type Nat1 = NatN[Nat0]
type Nat2 = NatN[Nat1]
type Nat3 = NatN[Nat2]
implicitly[Nat0 =:= Nat0]
illTyped("implicitly[Nat0 =:= Nat1]")
implicitly[Nat0 + Nat1 =:= Nat1]
implicitly[Nat1 + Nat1 =:= Nat2]
implicitly[Nat1 + Nat2 =:= Nat3]


Types, types, and more types, blah, blah, blah. What good is it??

```scala
sealed trait Vector {
  def size:Int
}
case object VNil extends Vector {
  override val size = 0
}
case class NonEmptyVector(head:Int, tail:Vector)
  extends Vector {
  override val size = 1 + tail.size
}
```

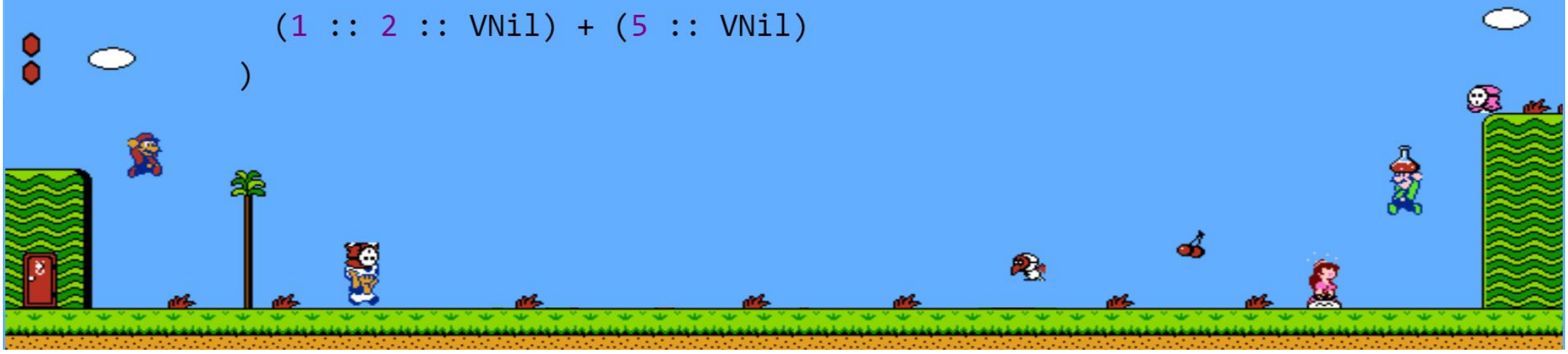
We can validate our length



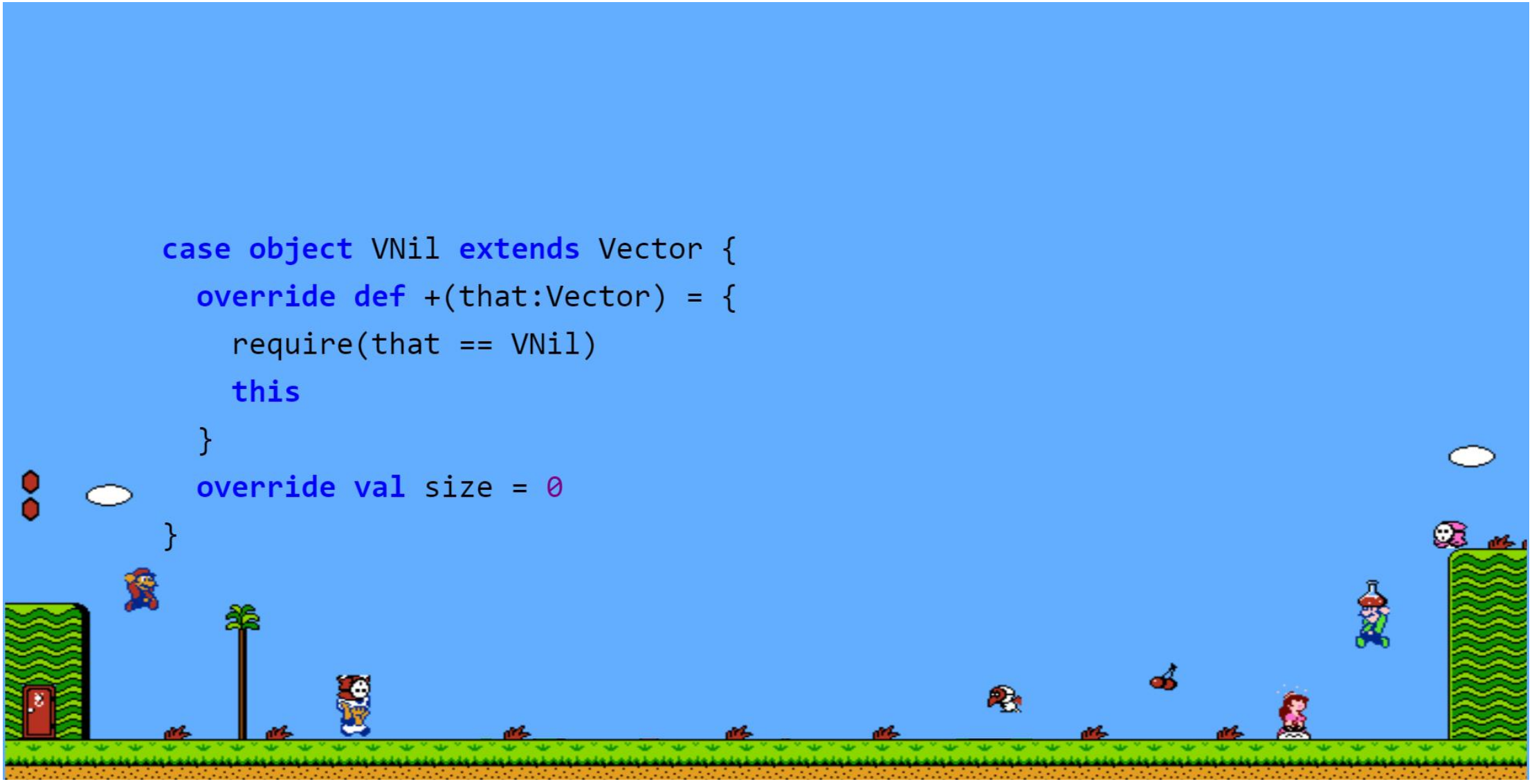
```scala
sealed trait Vector {
  // Convenience vector constructor
  def ::(head:Int):Vector = NonEmptyVector(head, this)
  // Vector addition
  def +(that:Vector):Vector
  def size:Int
}
```

```scala
val sum = (1 ::
sum shouldEqual

illTyped("(1 ::
```

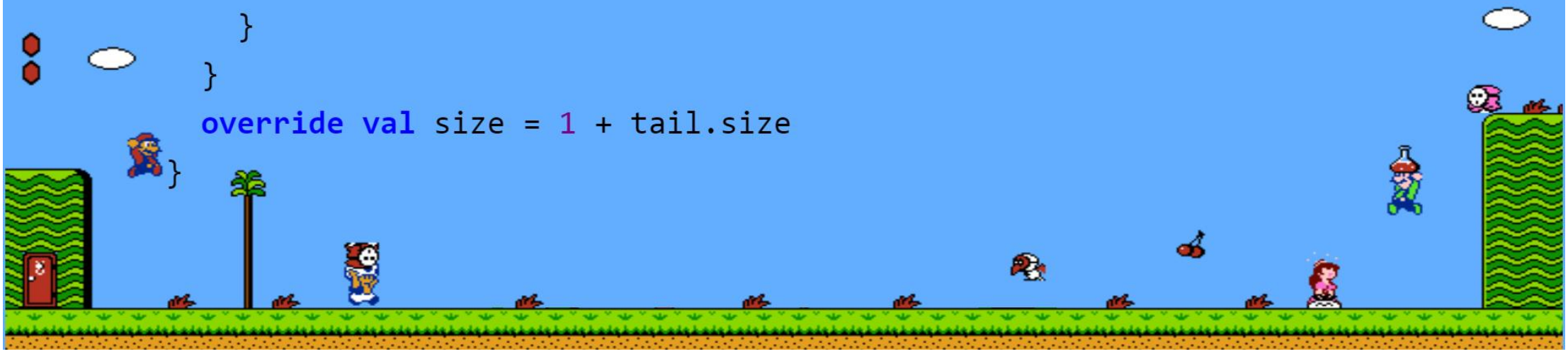
```
val sum = (1 :: 2 :: VNil) + (3 :: 4 :: VNil)
sum shouldEqual 4 :: 6 :: VNil

intercept[IllegalArgumentException](
  (1 :: 2 :: VNil) + (5 :: VNil)
)
```



```
case object VNil extends Vector {
  override def +(that:Vector) = {
    require(that == VNil)
    this
  }
  override val size = 0
}
```

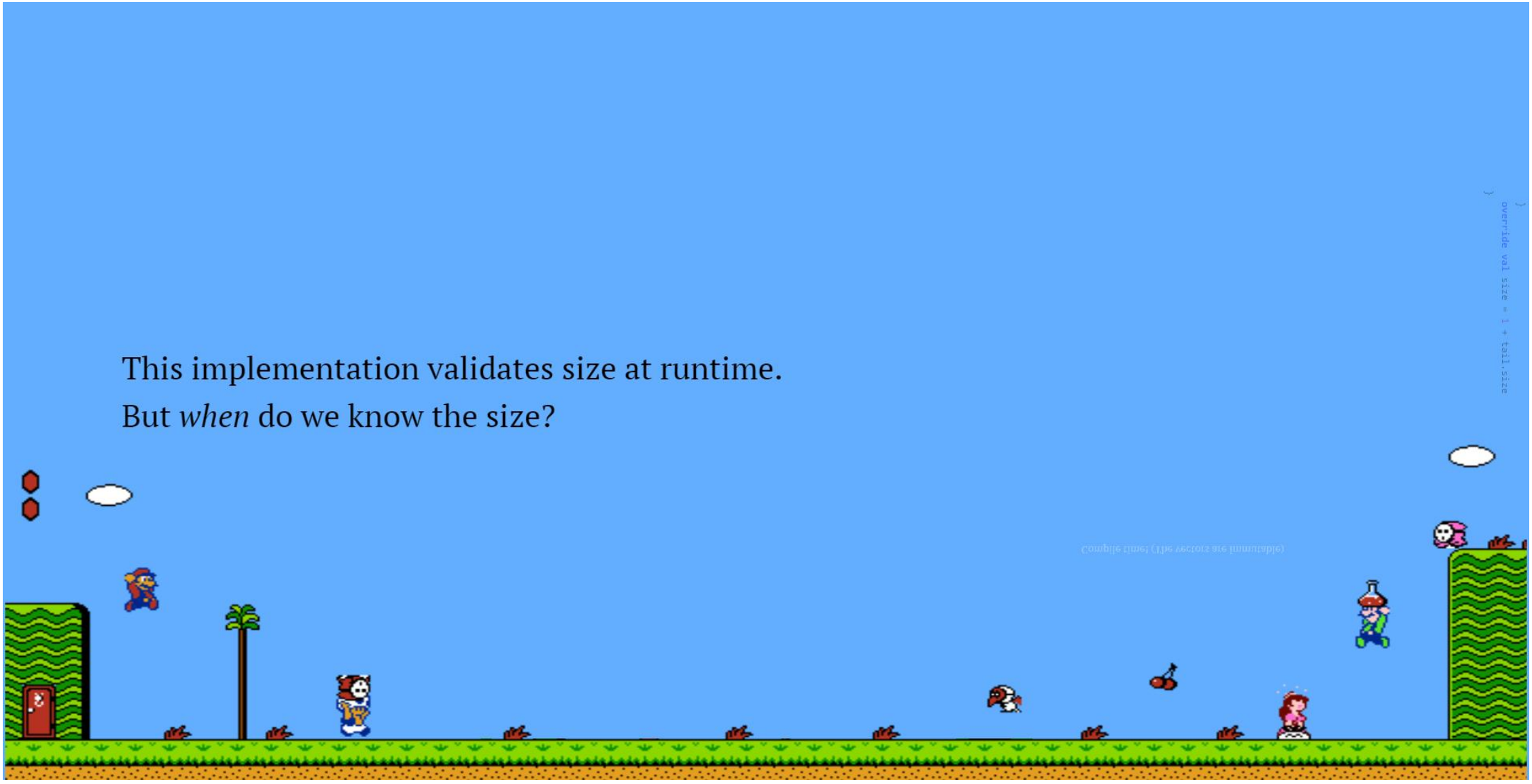


```
case class NonEmptyVector(head:Int, tail:Vector)
  extends Vector {
  override def +(that:Vector) = {
    require(that.size == size)
    that match {
      case NonEmptyVector(h, t) => (head + h) :: (tail + t)
    }
  }
  override val size = 1 + tail.size
}
```

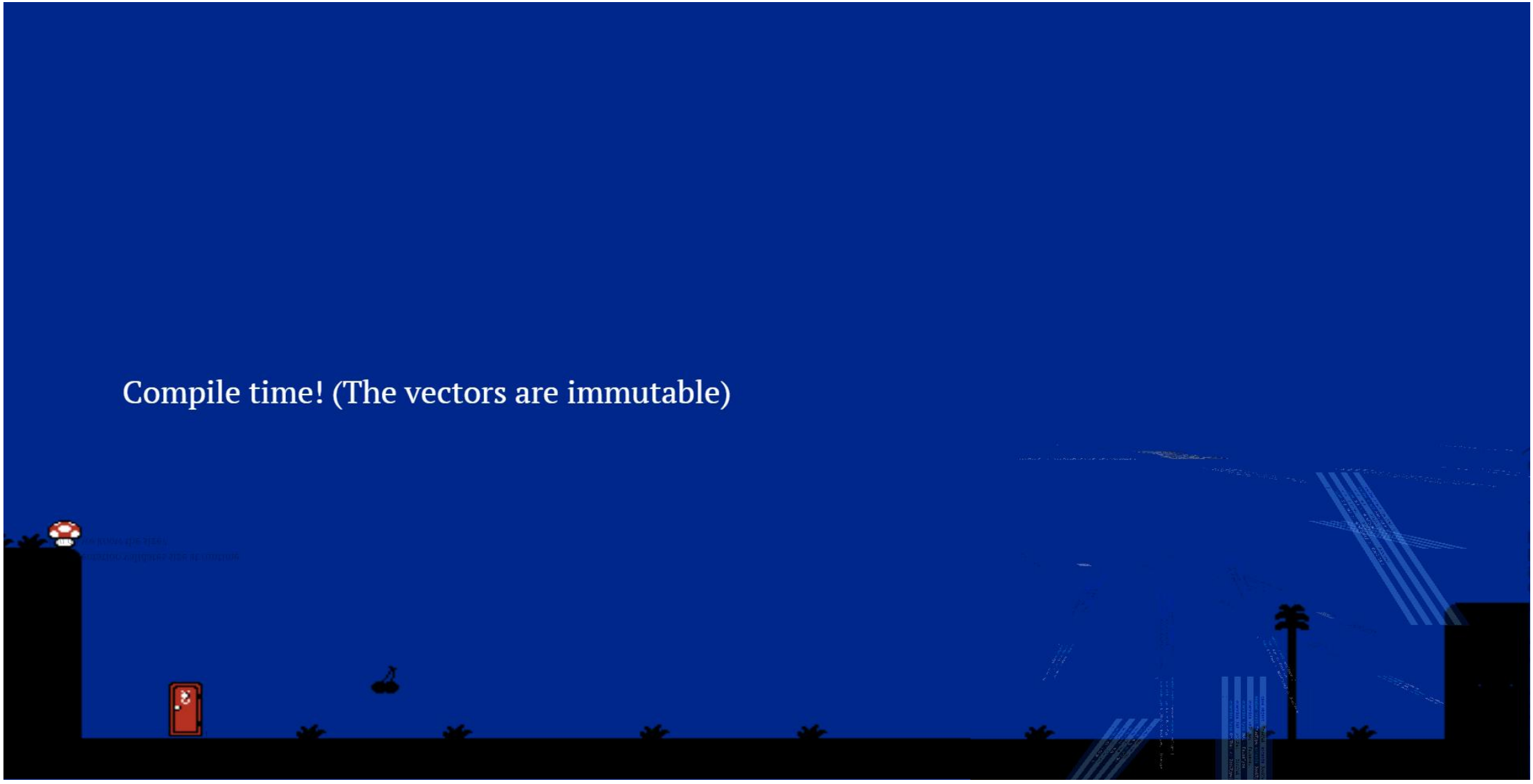


This implementation validates size at runtime.

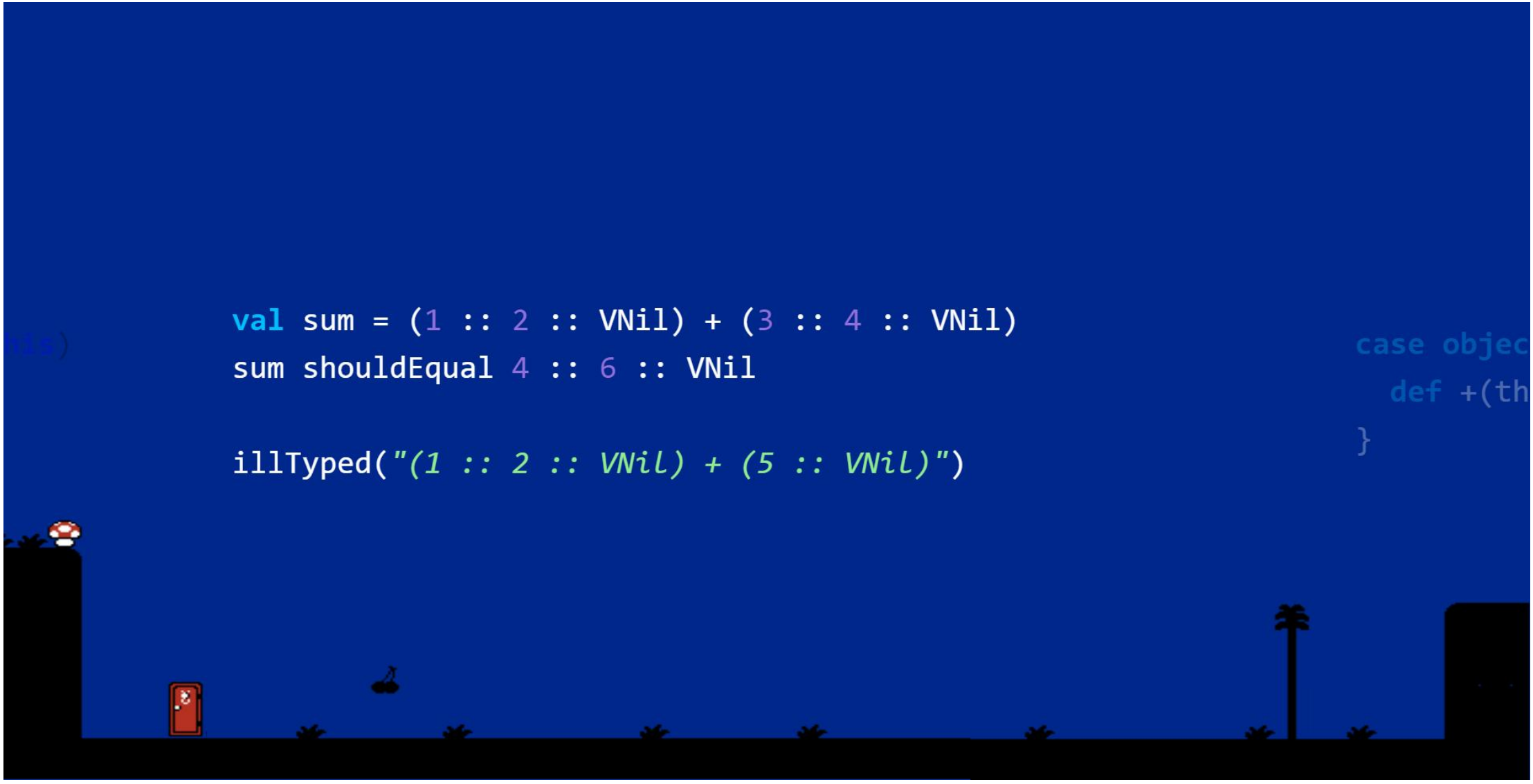
But *when* do we know the size?



Compile time! (The vectors are immutable)



We can validate our lengths without a runtime IllegalArgumentException!

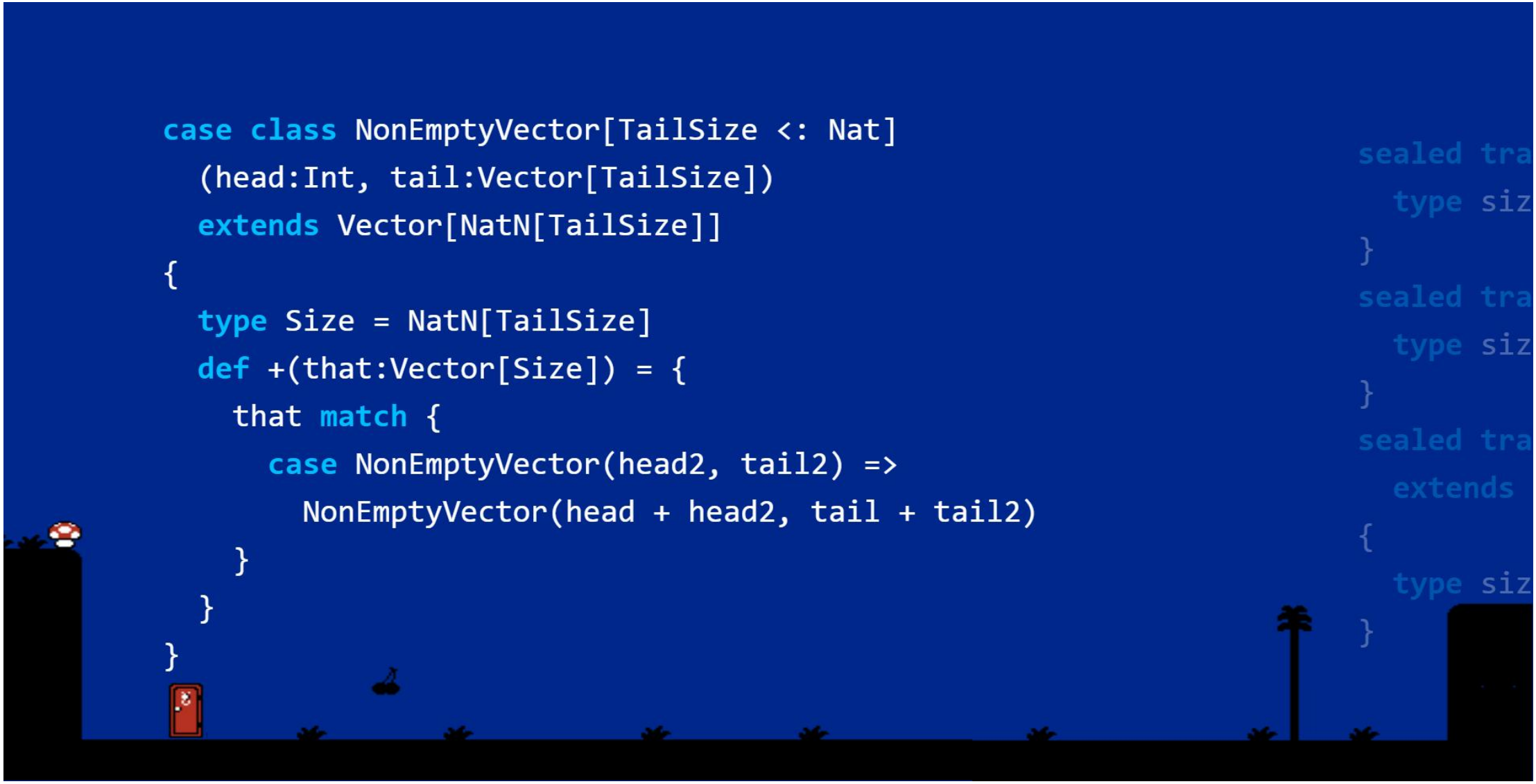

val sum = (1 :: 2 :: VNil) + (3 :: 4 :: VNil)
sum shouldEqual 4 :: 6 :: VNil
illTyped("(1 :: 2 :: VNil) + (5 :: VNil)")

```
sealed trait Vector[Size <: Nat] {
  def ::(head:Int):Vector[NatN[Size]] =
    NonEmptyVector(head, this)
  def +(that:Vector[Size]):Vector[Size]
}
```

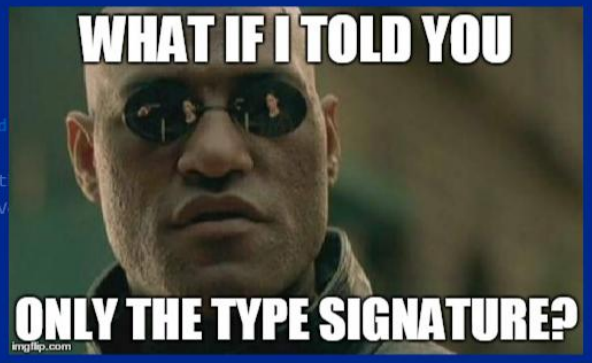
So why are we e
Thus far we've

```scala
case object VNil extends Vector[Nat0] {
  def +(that:Vector[Nat0]) = this
}
```

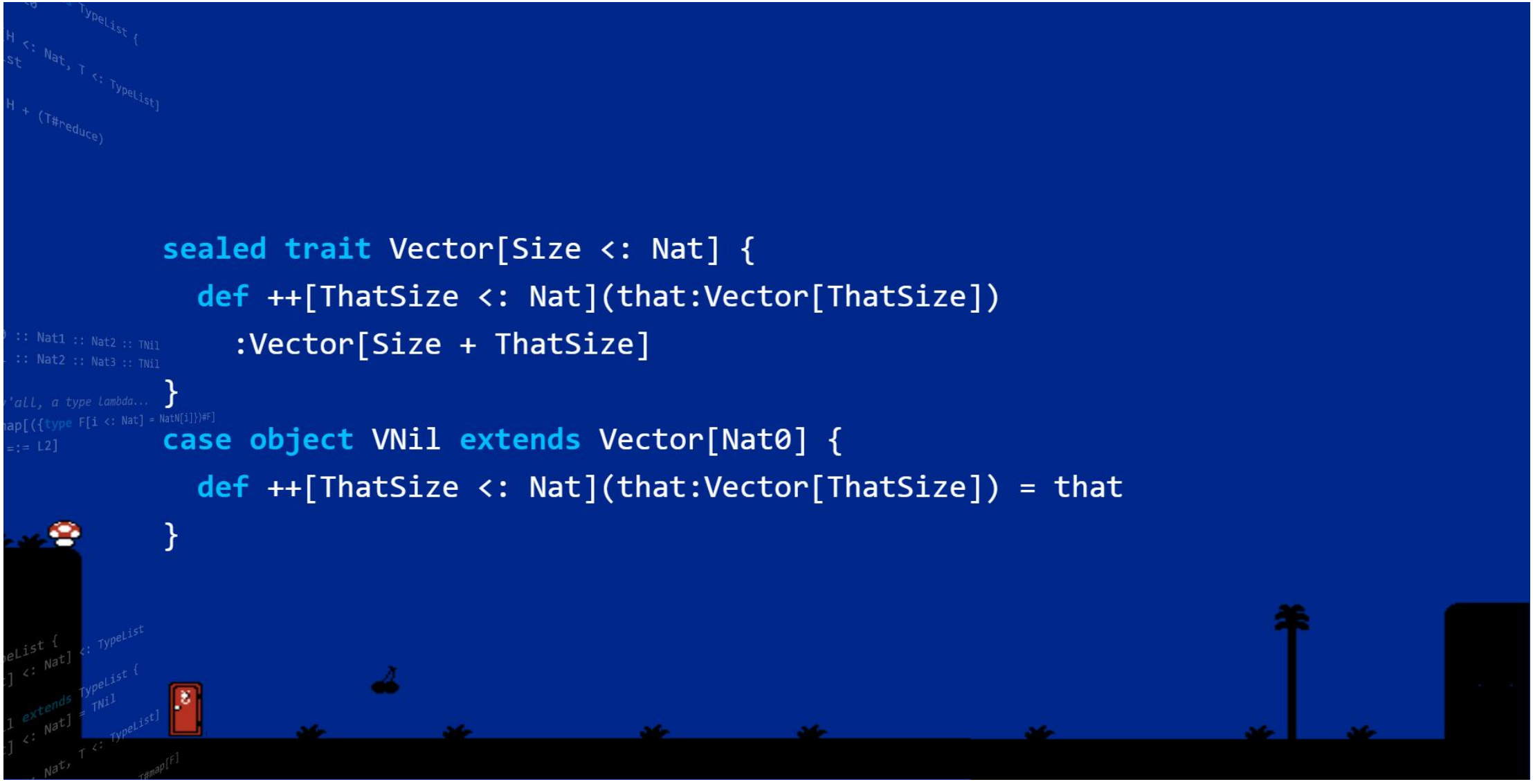
```scala
type L = N
implicitly
```


case class NonEmptyVector[TailSize <: Nat]
(head:Int, tail:Vector[TailSize])
extends Vector[NatN[TailSize]]
{
type Size = NatN[TailSize]
def +(that:Vector[Size]) = {
that match {
case NonEmptyVector(head2, tail2) =>
NonEmptyVector(head + head2, tail + tail2)
}
}
}
sealed tra
type siz
}
sealed tra
type siz
}
sealed tra
extends
{
type siz
}

```
sealed trait Vector[Size <: Nat] {
  def ++[ThatSize <: Nat](that:Vector[ThatSize])
    :Vector[Size + ThatSize]
}
case object VNil extends Vector[Nat0] {
  def ++[ThatSize <: Nat](that:Vector[ThatSize]) = that
}
```




sealed trait Vector[Size <: Nat] {
def ??[ThatSize <: Nat]
(that:Vector[ThatSize])
:Vector[Size + ThatSize]
}


sealed trait Vector[Size <: Nat] {
def ++[ThatSize <: Nat](that:Vector[ThatSize])
:Vector[Size + ThatSize]
}
case object VNil extends Vector[Nat0] {
def ++[ThatSize <: Nat](that:Vector[ThatSize]) = that
}

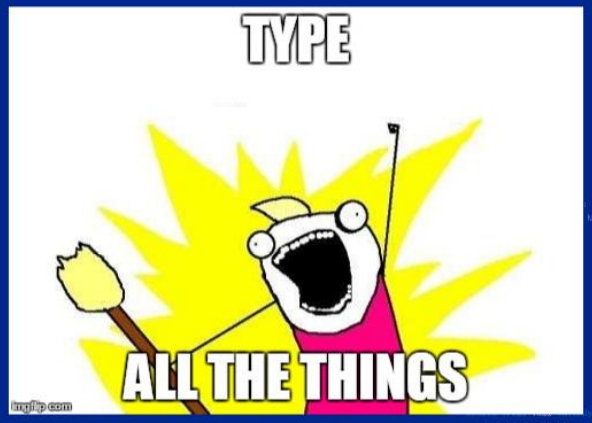

case class NonEmptyVector[TailSize <: Nat]
(head:Int, tail:Vector[TailSize])
extends Vector[NatN[TailSize]]
{
def ++[ThatSize <: Nat](that:Vector[ThatSize]) =
NonEmptyVector(head, tail ++ that)
}

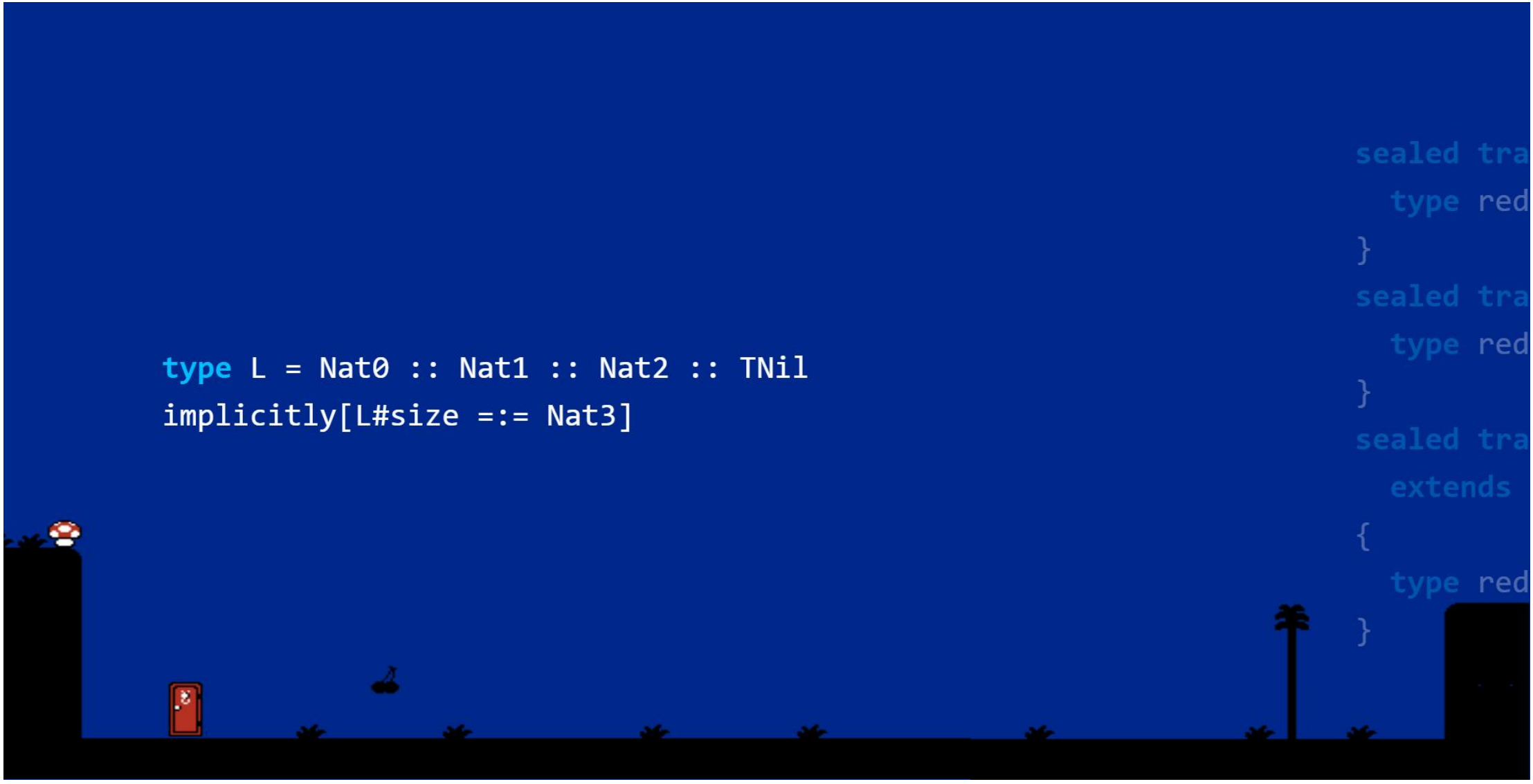

val sum =
((1 :: 2 :: VNil) ++ (3 :: VNil)) +
(4 :: 5 :: 6 :: VNil)
sum shouldEqual 5 :: 7 :: 9 :: VNil

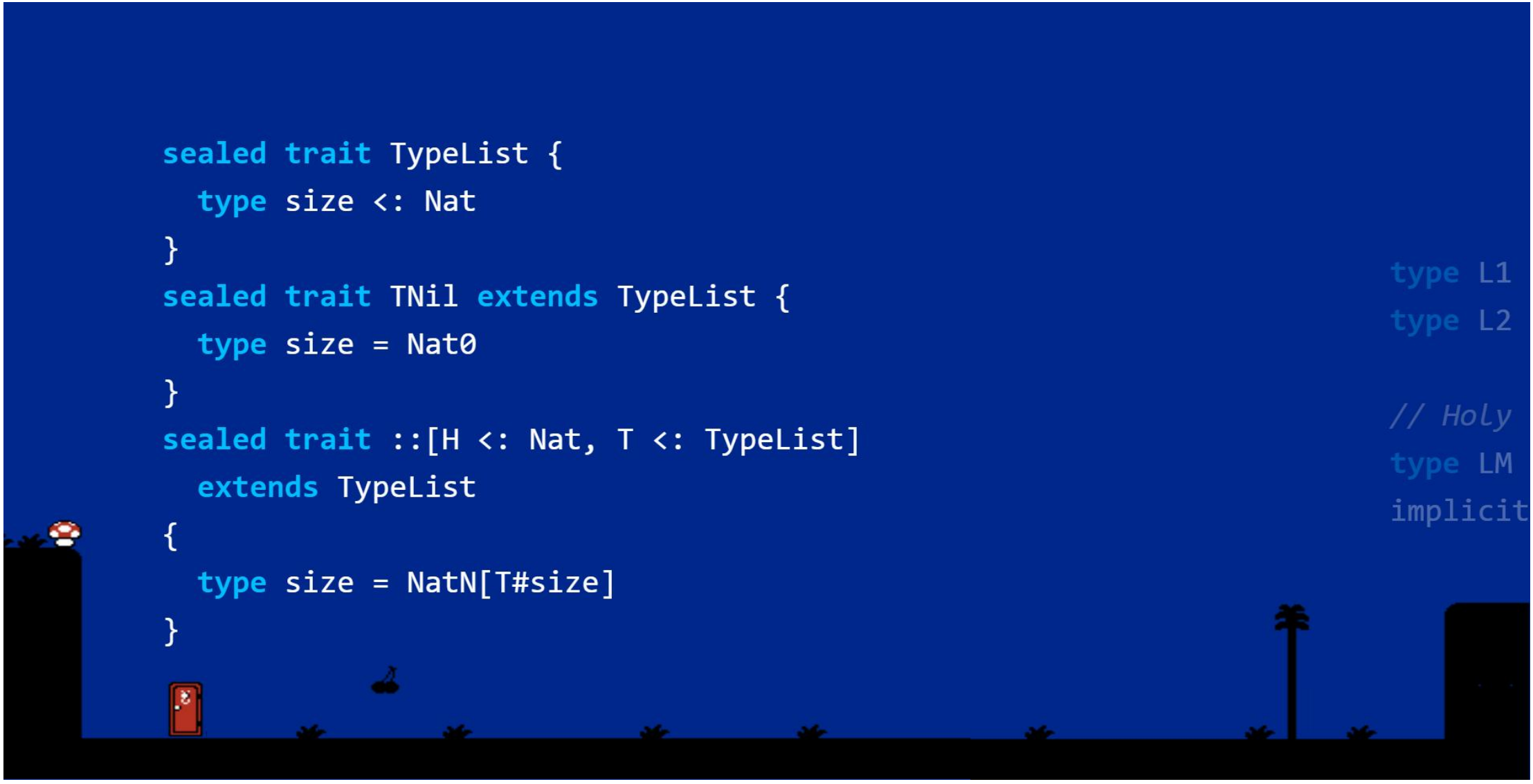
So why are we even using values at all?

Thus far we've only hard-coded anyway...

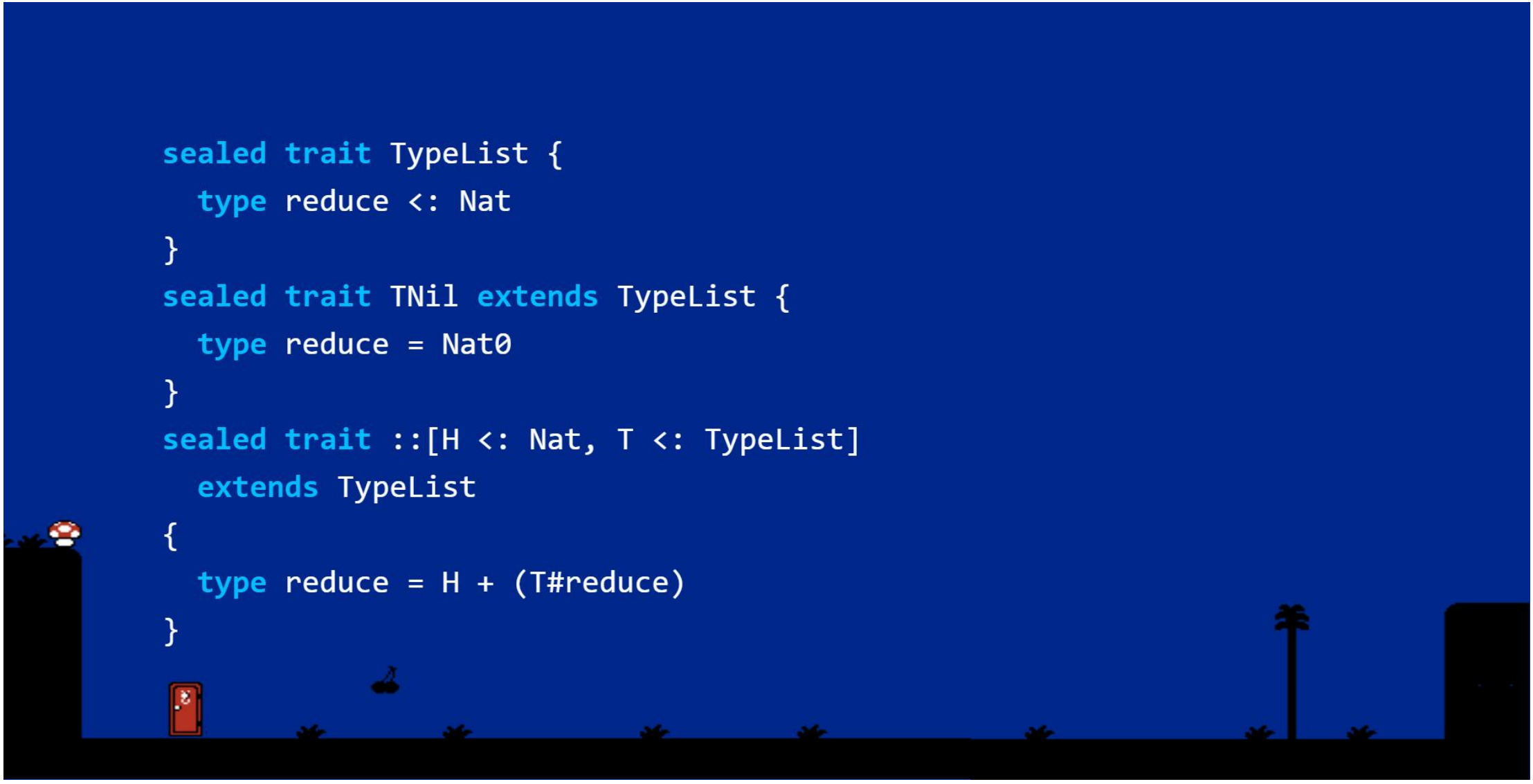
```
type L = N
implicitly
```

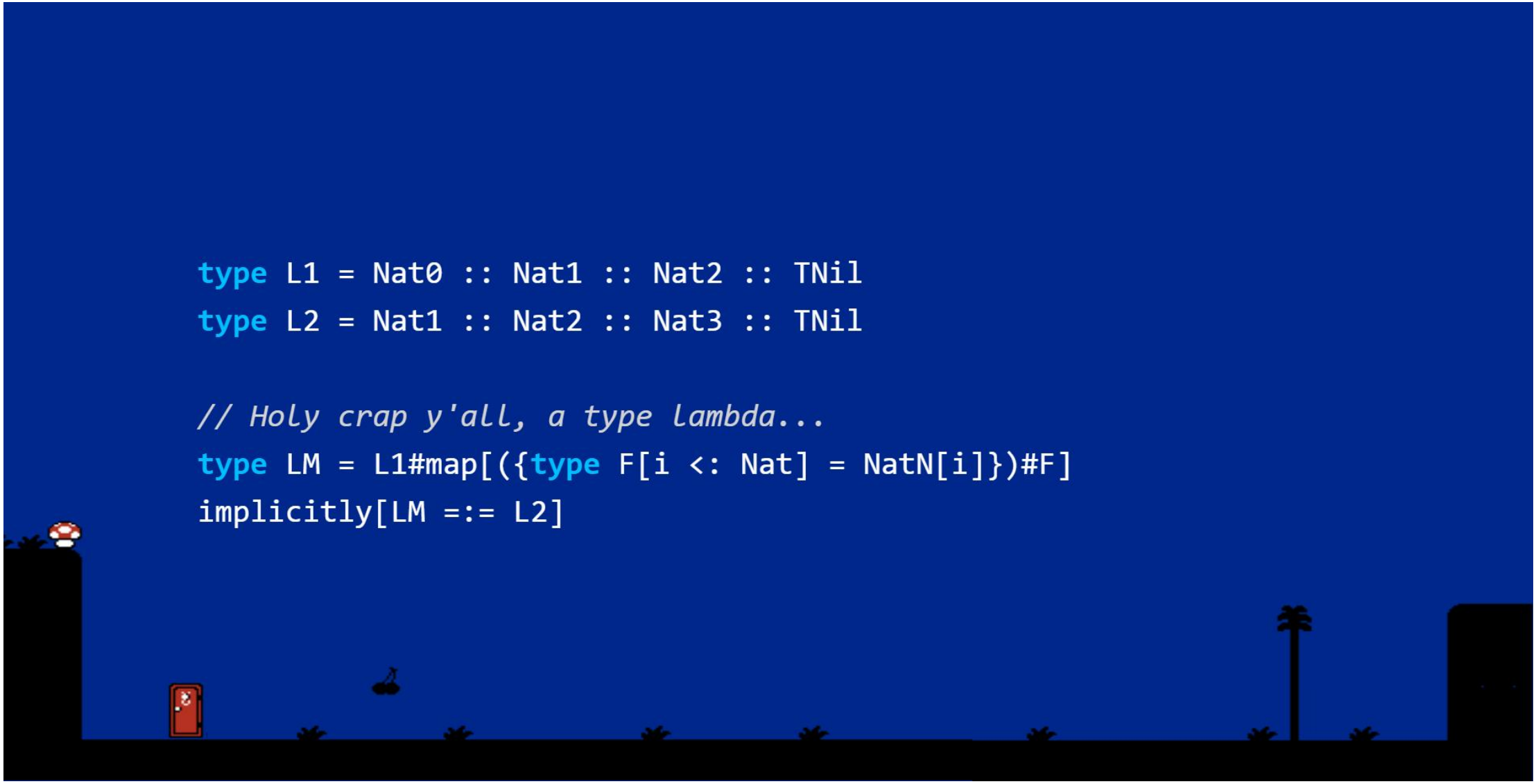

TYPE
ALL THE THINGS
imgflip.com

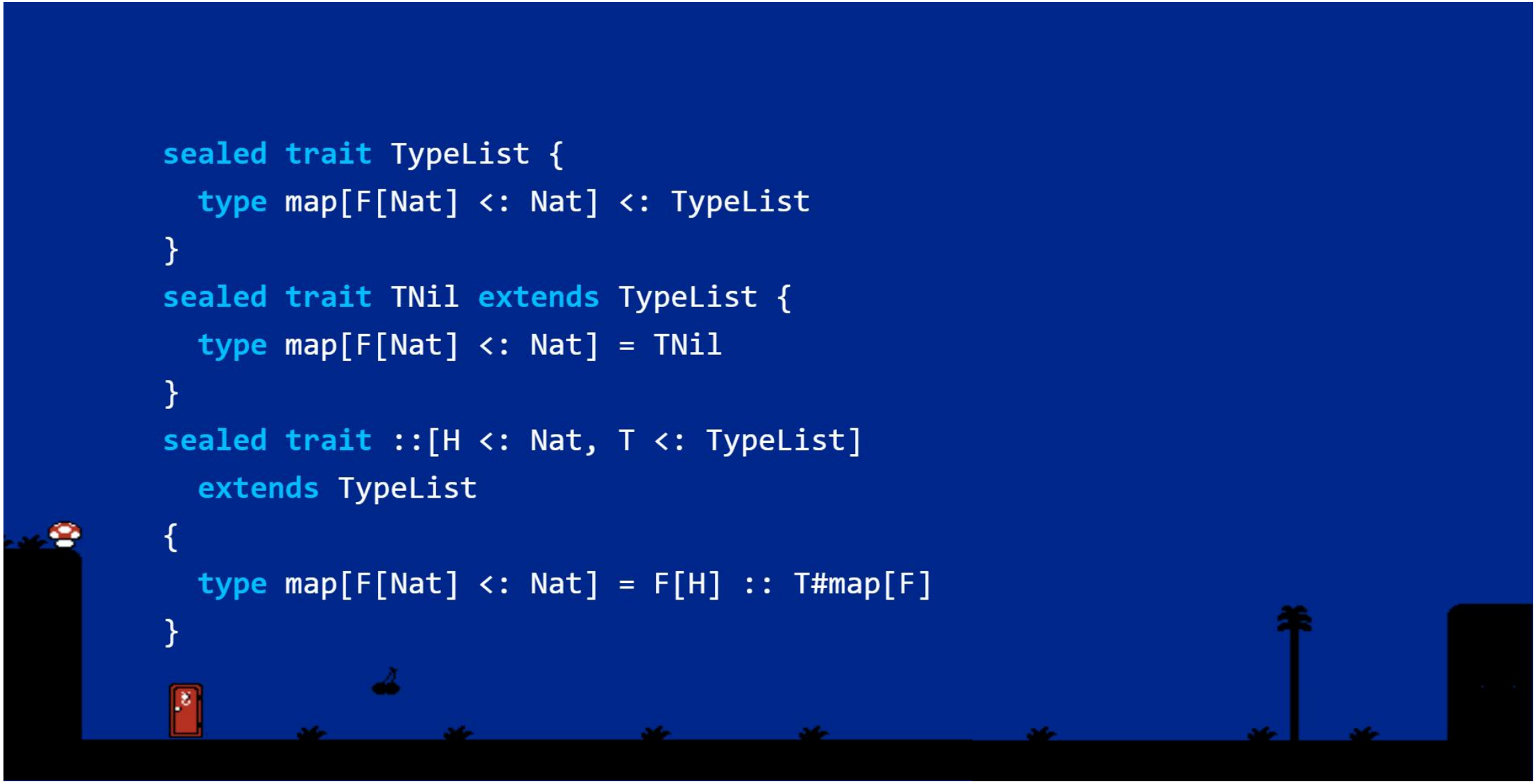

type L = Nat0 :: Nat1 :: Nat2 :: TNil
implicitly[L#size =:= Nat3]
sealed tra
type red
}
sealed tra
type red
}
sealed tra
extends
{
type red
}


sealed trait TypeList {
type size <: Nat
}
sealed trait TNil extends TypeList {
type size = Nat0
}
sealed trait ::[H <: Nat, T <: TypeList]
extends TypeList
{
type size = NatN[T#size]
}
type L1
type L2
// Holy
type LM
implicit

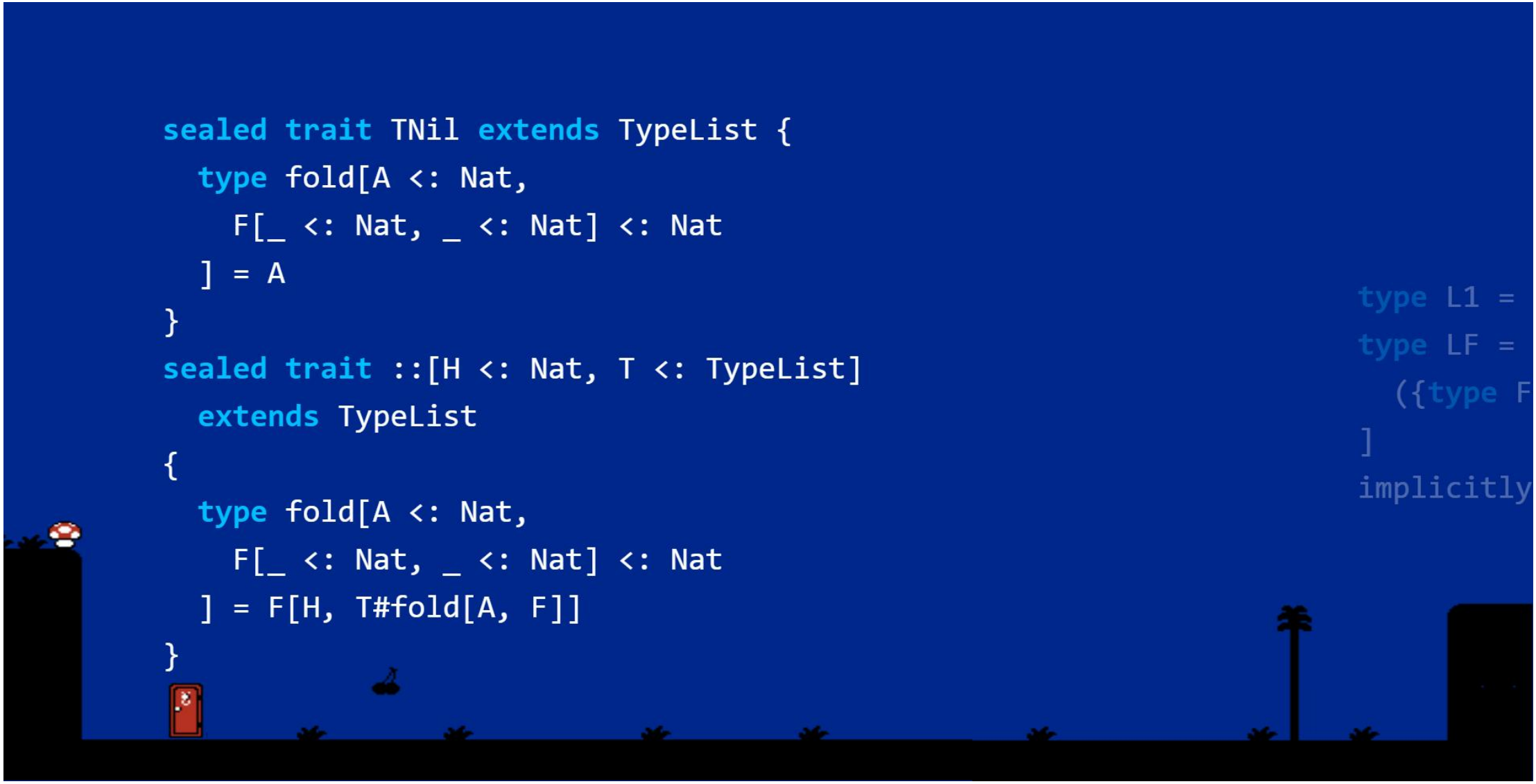
```
type L = Nat1 :: Nat2 :: Nat3 :: TNil
implicitly[L#reduce =:= Nat6]
```


sealed trait TypeList {
type reduce <: Nat
}
sealed trait TNil extends TypeList {
type reduce = Nat0
}
sealed trait ::[H <: Nat, T <: TypeList]
extends TypeList
{
type reduce = H + (T#reduce)
}

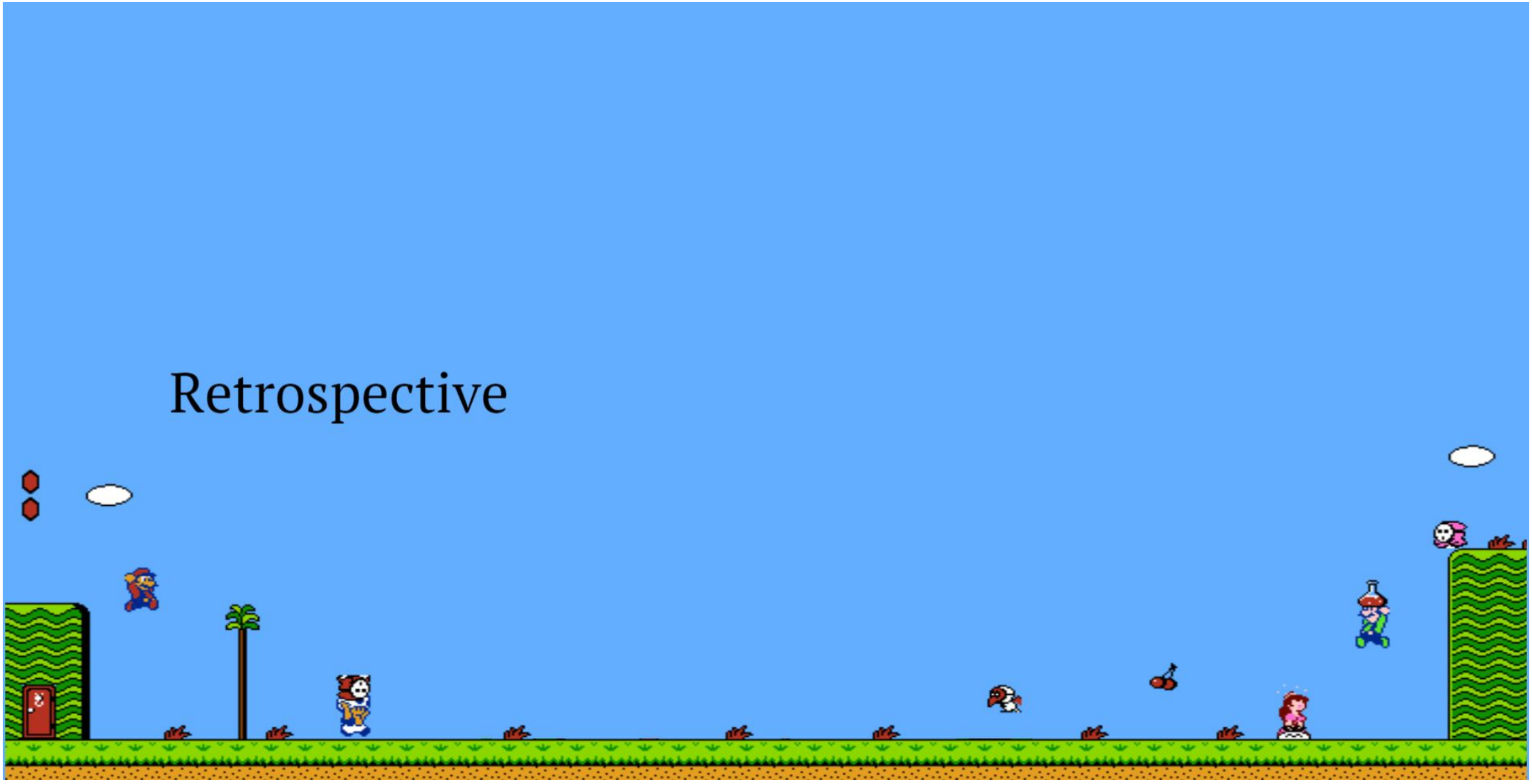

type L1 = Nat0 :: Nat1 :: Nat2 :: TNil
type L2 = Nat1 :: Nat2 :: Nat3 :: TNil
// Holy crap y'all, a type lambda...
type LM = L1#map[({type F[i <: Nat] = NatN[i]})#F]
implicitly[LM =:= L2]


sealed trait TypeList {
type map[F[Nat] <: Nat] <: TypeList
}
sealed trait TNil extends TypeList {
type map[F[Nat] <: Nat] = TNil
}
sealed trait ::[H <: Nat, T <: TypeList]
extends TypeList
{
type map[F[Nat] <: Nat] = F[H] :: T#map[F]
}


type L1 = Nat2 :: Nat1 :: Nat0 :: Nat1 :: TNil
type LF = L1#fold[Nat0,
({type F[A <: Nat, B <: Nat] = A + B})#F
]
implicitly[LF =:= Nat4]
sealed tra
type map
}
sealed tra
type map
}
sealed tra
extends
{
type map
}


sealed trait TNil extends TypeList {
type fold[A <: Nat,
F[_ <: Nat, _ <: Nat] <: Nat
] = A
}
sealed trait ::[H <: Nat, T <: TypeList]
extends TypeList
{
type fold[A <: Nat,
F[_ <: Nat, _ <: Nat] <: Nat
] = F[H, T#fold[A, F]]
}
type L1 =
type LF =
({type F
]
implicitly

Retrospective



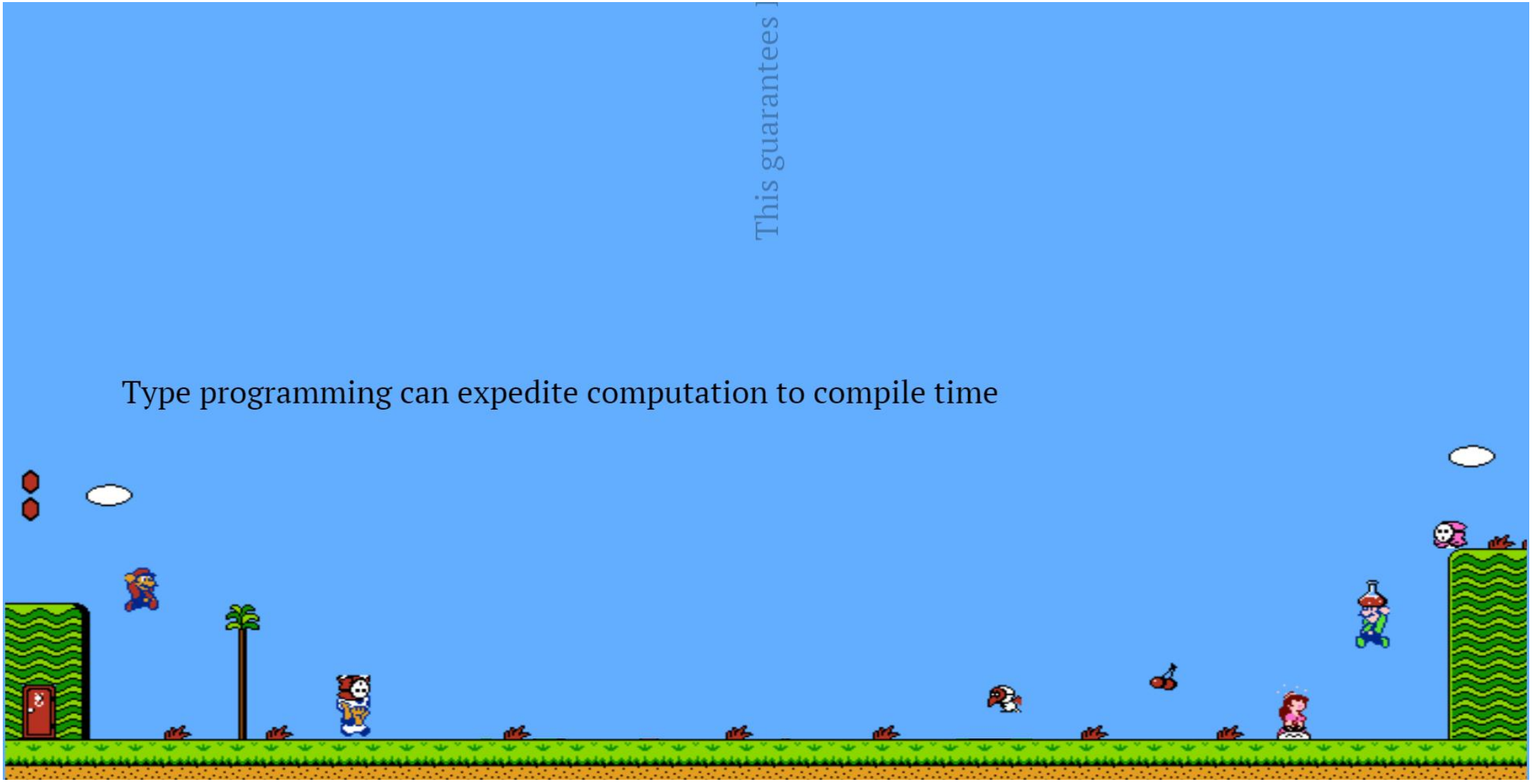
Retrosp

Types are not just the shape of my objects

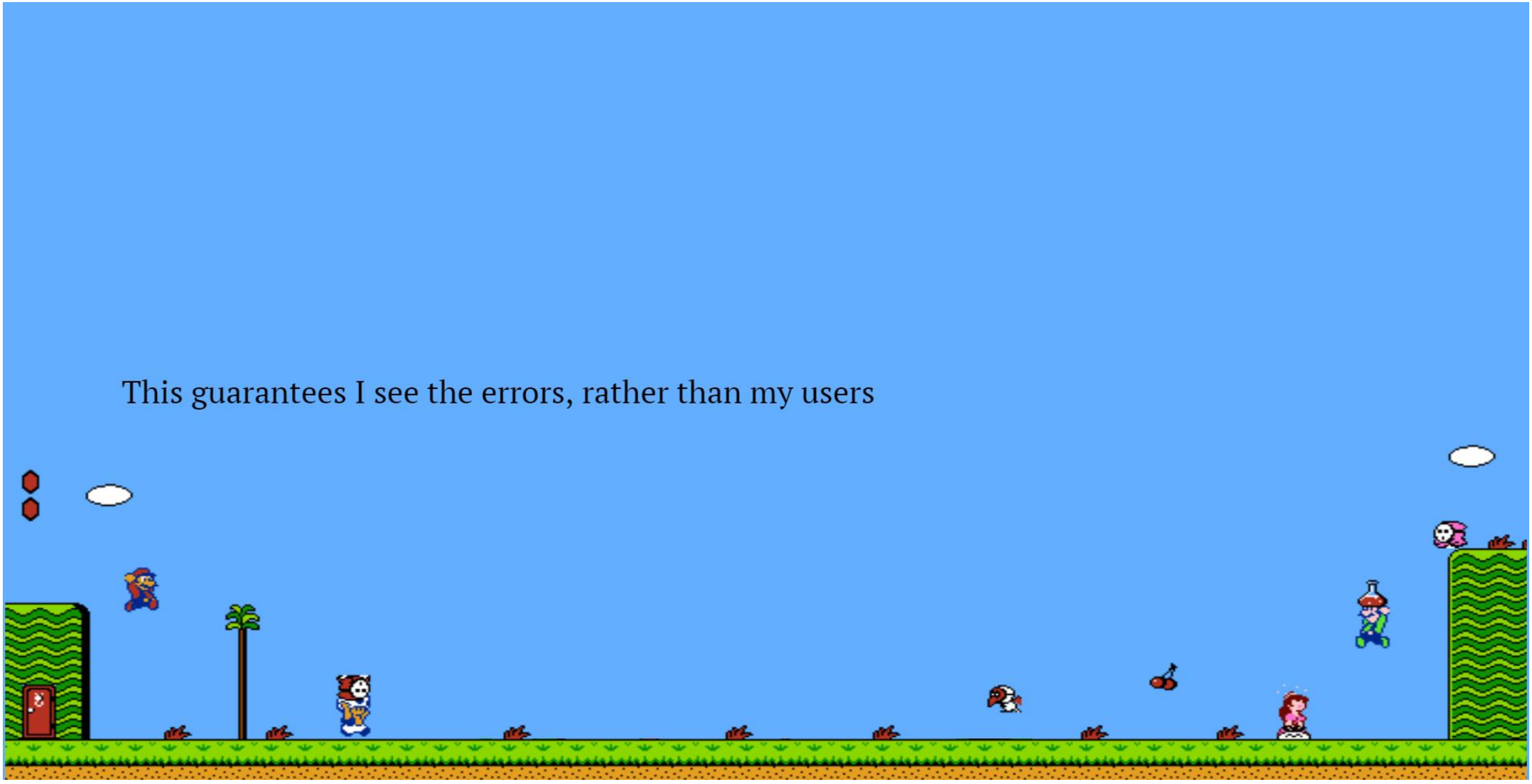
Type programming

This guarantees

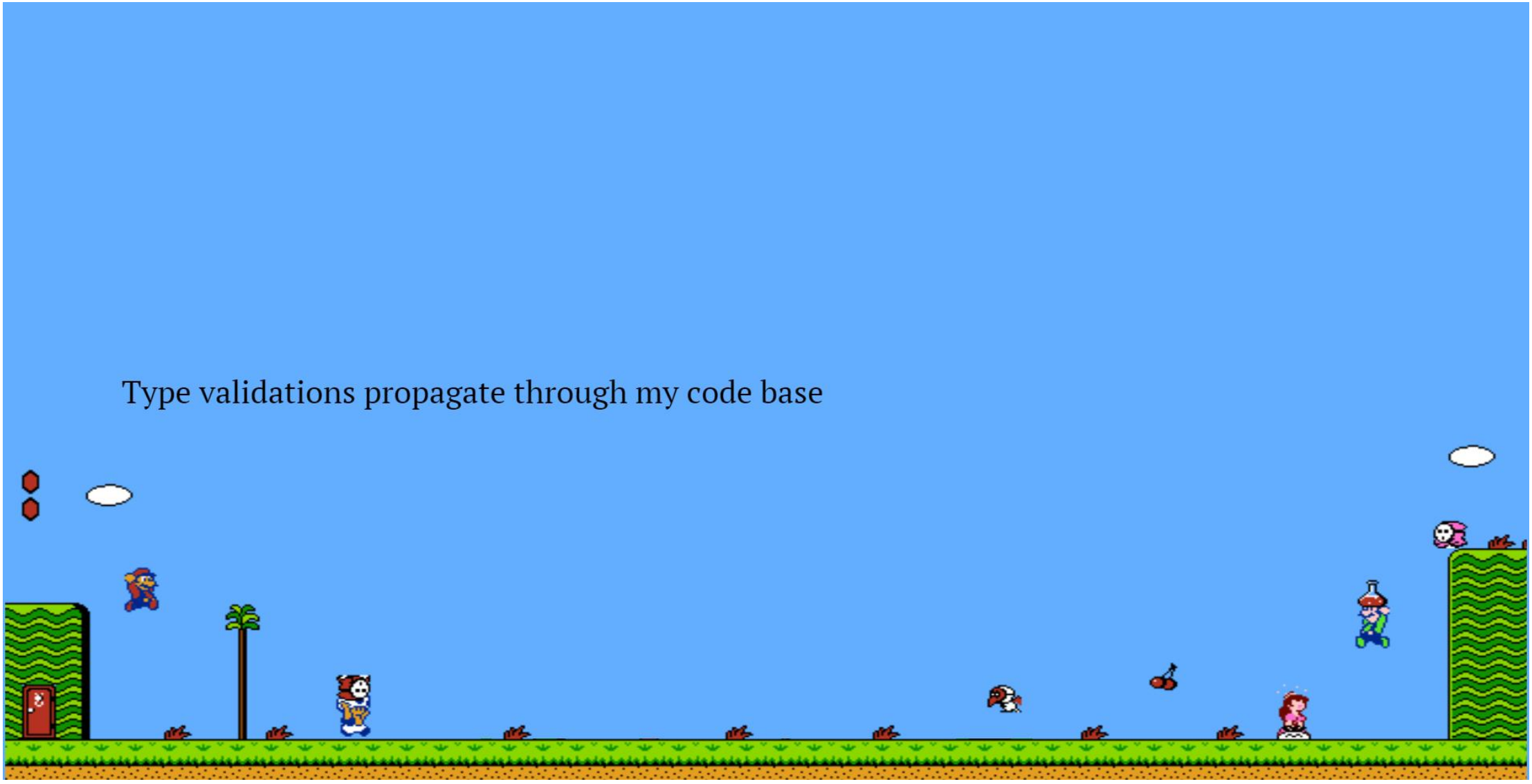
Type programming can expedite computation to compile time



This guarantees I see the errors, rather than my users

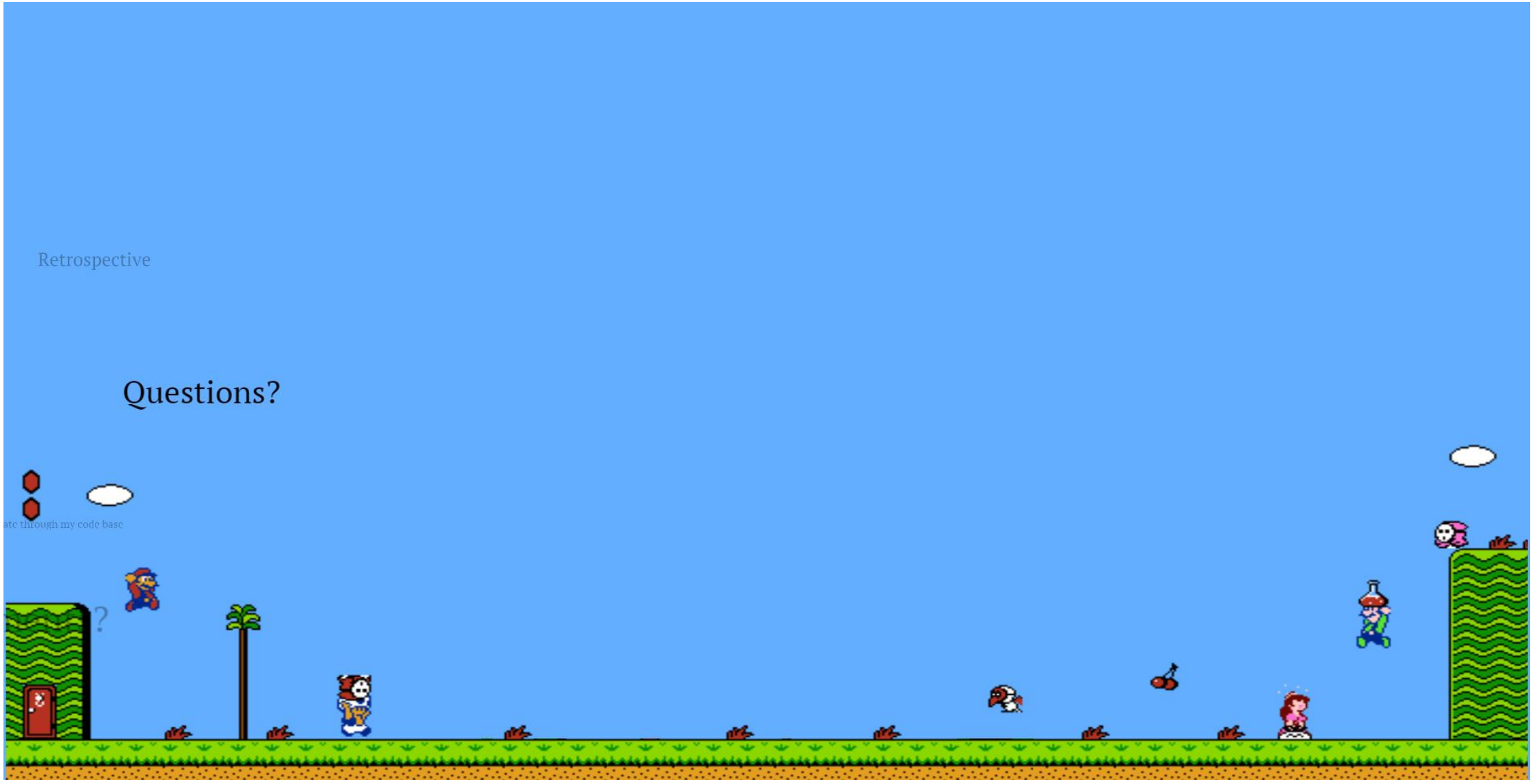


Type validations propagate through my code base



Retrospective

Questions?

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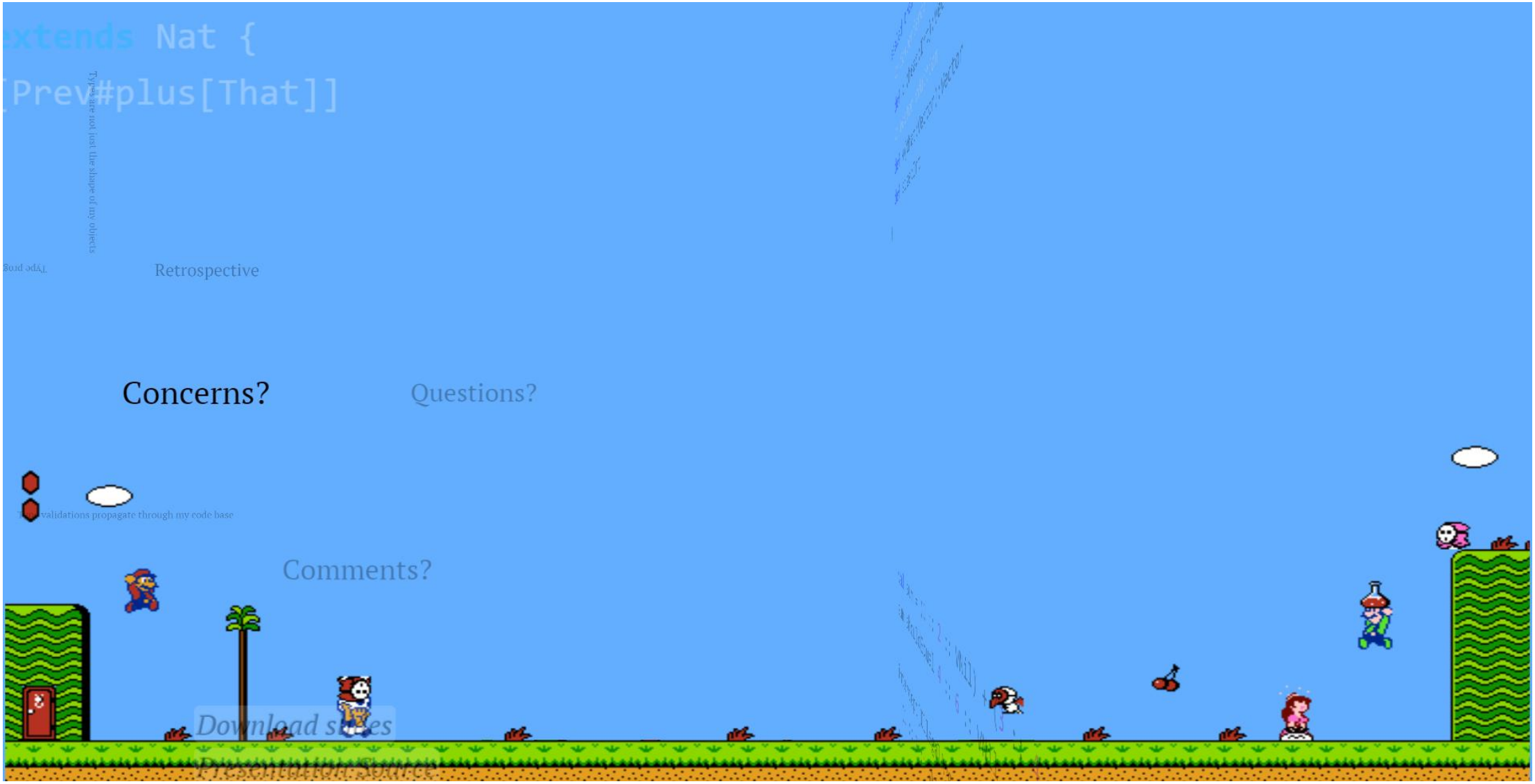
extends Nat {
[Prev#plus[That]]
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