

ISLR | Chapter 4 Exercises

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Conceptual

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$$f(\alpha) = \text{Var}(\alpha X + (1 - \alpha)Y)$$

$$f(\alpha) = \text{Var}(\alpha X) + \text{Var}((1 - \alpha)Y) + 2\text{Cov}(\alpha X, (1 - \alpha)Y)$$

$$f(\alpha) = \alpha^2 \text{Var}(X) + (1 - \alpha)^2 \text{Var}(Y) + 2\alpha(1 - \alpha)\text{Cov}(X, Y)$$

$$f(\alpha) = \alpha^2 \text{Var}(X) + (1 - \alpha)^2 \text{Var}(Y) + 2(\alpha - \alpha^2)\text{Cov}(X, Y)$$

$$\frac{\partial f(\alpha)}{\partial \alpha} = 2\alpha\sigma_X^2 + 2(1 - \alpha)(-1)\sigma_Y^2 + 2(1 - 2\alpha)\sigma_{XY}$$

$$\frac{\partial f(\alpha)}{\partial \alpha} = \alpha\sigma_X^2 + (-1 + \alpha)\sigma_Y^2 + (1 - 2\alpha)\sigma_{XY}$$

$$\frac{\partial f(\alpha)}{\partial \alpha} = \alpha\sigma_X^2 + -\sigma_Y^2 + \alpha\sigma_Y^2 + \sigma_{XY} - 2\alpha\sigma_{XY}$$

$$\frac{\partial f(\alpha)}{\partial \alpha} = \alpha(\sigma_X^2 + \sigma_Y^2 - 2\sigma_{XY}) - \sigma_Y^2 + \sigma_{XY}$$

$$\frac{\partial f(\alpha)}{\partial \alpha} = \alpha - \frac{\sigma_Y^2 + \sigma_{XY}}{(\sigma_X^2 + \sigma_Y^2 - 2\sigma_{XY})}$$

$$-\alpha = -\frac{\sigma_Y^2 + \sigma_{XY}}{(\sigma_X^2 + \sigma_Y^2 - 2\sigma_{XY})}$$

$$\alpha = \frac{\sigma_Y^2 - \sigma_{XY}}{\sigma_X^2 + \sigma_Y^2 - 2\sigma_{XY}}$$