Joseph Silverstein

HW III

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1. Define the following indicator functions:
   $1_t(E)$ indicates that the agent is employed at time t
   $1_t(w_i)$ indicates that the agent receives wage W_i at time t
   $1^a_t(w_i)$ indicates that the agent accepts an offer of wage w_i at time t

Then $\{\{1_t(w_i)\}_{t=0}^{\infty}\}_{i=1}^{5}$ is a Markov chain defined by initial values $1_0(E)$ and $\{1_0(w_i)\}_{i=1}^{5}$ and transition rules characterized by

$$P(1_t(w_i)=1)=\begin{cases} d_{t-1}^{0.5} & \text{if } 1_{t-1}(w_i)=1 \\ \frac{1}{5}e_{t-1}^{0.5}\cdot 1^a_{t-1}(w_i) & \text{if } 1_{t-1}(E)=0 \end{cases}$$

$1_t(E)$ is then defined by

$$1_t(E)=\sum_{i=1}^{5} 1_t(w_i)$$

Then the sequential problem of the agent is

$$\max_{\{c_t, b_{t+1}, d_t, e_t, \{1^a_t(w_i)\}_{i=1}^{5}\}_{t=0}^{\infty}} E_0\left[\sum_{t=0}^{\infty} 0.96^t \log(c_t-0.8d_t^{1.5}\cdot 1_t(E)-0.9e_t^{1.5}(1-1_t(E))\right]$$

s.t.

$$c_t+\frac{b_{t+1}}{1.03}\leq\sum_{i=1}^{5} w_i\cdot 1_t(w_i)+(1-1_t(E))z+b_t \quad \forall t\geq 0$$

$$d_t, e_t\in[0,1] \quad \forall t\geq 0$$

$$1^a_t(w_i)\in\{0,1\} \quad \forall t\geq 0, i=1,\dots,5$$

$$c_t, b_{t+1}\geq 0 \quad \forall t\geq 0$$

b_0 given

2. Employed agent:

$$W(w,b) = \max_{c,b',d\geq 0} \log(c-0.8d^{1.5}) + 0.95[d^{0.5}W(w,b') + (1-d^{0.5})U(b')]$$

$$\text{s.t.} \quad c+b' = 1.03b + w$$
$$0 \leq d \leq 1$$

Unemployed Agent: Denoting $W := \{w_1, \dots, w_5\} = \{0.2, 0.4, 0.6, 0.8, 1\}$,

$$U(b) = \max_{c,b',e\geq 0} \log(c-0.8e^{1.5}) + 0.95[e^{0.5}\sum_{i=1}^{5}\frac{1}{5}\max\{W(w_i,b'), U(b')\} + (1-e^{0.5})U(b')]$$

$$\text{s.t.} \quad c+b' = 1.03b + 0.1$$
$$0 \leq e \leq 1$$

3. The feasible set of the sequential problem is
$\Gamma(b) = [0, \sum_{i=1}^{5} \mathbb{1}(w_i) \cdot w_i + (1-\mathbb{1}(E))z + 1.03b]$.
Note that $\Gamma(b) \neq 0$.

The return function is
$$F := \log(c - \mathbb{1}(E) \cdot 0.8 d^{1.5} - (1-\mathbb{1}(E)) \cdot 0.9 e^{1.5})$$

WTS: $\lim_{t\to\infty} \sum_{t=0}^{\infty} \beta^t F_t$ exists. $(*)$

If we show $(*)$, then we know that the principle of optimality holds, in which case the sequential and recursive problems are equivalent.

A sufficient condition for $(*)$ to hold is that $\forall x_0$ and for all feasible plans, $\exists \theta \in (0, \frac{1}{0.96})$ and $0 < c < \infty$ s.t. $\forall t$, $F_t \leq c\theta^t$.

Set $c = b_0 + w_5$ and $\theta = 1.03$. Then
WTS: $\log(c_t - \mathbb{1}_t(E) \cdot 0.8 d_t^{1.5} - (1-\mathbb{1}_t(E)) \cdot 0.9 e_t^{1.5}) \leq (b_0 + w_5) \cdot 1.03^t$

The highest possible savings of the agent at period t occurs in the theoretical case where she saves all of her wealth in all previous periods and always worked at wage w_5. Thus we know that
$$c_t \leq b_0 (1.03)^t + w_5 \sum_{\tau=0}^{t-1} (1.03)^\tau$$

Then
$$\sum_{t=0}^{\infty} \beta^t \log(c_t - \mathbb{1}_t(E) \cdot 0.8 d_t^{1.5} - (1-\mathbb{1}_t(E)) \cdot 0.9 e_t^{1.5}) < \sum \beta^t c_t$$
$$= b_0 \sum [\beta(1.03)]^t + \frac{1}{1.03} \sum [\beta(1.03)]^t - \frac{w_5}{1.03} \sum \beta^t$$
$$=: K \in \mathbb{R} \quad \text{because } \beta(1.03) < 1.$$

Thus, we have verified $(*)$. Then we know that the value and policy functions of the two problems are equivalent.

4. Substituting for c and writing separately for each value of $w \in W$, we get 6 value functions:

$$\begin{cases} W_1(b) = \max_{b', d \geq 0} \log(1.03b + w_1 - b' - 0.8d^{1.5}) \\ \qquad + 0.95[d^{0.5} W_1(b') + (1-d^{0.5}) U(b')] \quad \text{s.t. } 0 \leq d \leq 1 \\ \vdots \\ W_5(b) = \max_{b', d \geq 0} \log(1.03b + w_5 - b' - 0.8d^{1.5}) \\ \qquad + 0.95[d^{0.5} W_5(b') + (1-d^{0.5}) U(b')] \quad \text{s.t. } 0 \leq d \leq 1 \\ U(b) = \max_{b', e \geq 0} \log(1.03b + 0.1 - b' - 0.8e^{1.5}) \\ \qquad + 0.95[e^{0.5} \sum_{i=1}^{5} \frac{1}{5} \max\{W_i(b'), U(b')\} + (1-e^{0.5}) U(b')] \\ \qquad \text{s.t. } 0 \leq e \leq 1 \end{cases}$$

From the graph of the value functions (Figure 5), we can see that the agent will always accept a wage of 0.4 or higher. She will accept a wage of 0.2 iff her savings are below 0.14.

5. The distribution of assets is presented in Figure 5 in histogram form.

539 agents are workers and 461 agents are searchers.

The distribution of wages is

Wage	Count	Percent
0.2	9	1.95%
0.4	57	12.36%
0.6	91	19.74%
0.8	100	21.69%
1	204	44.25%

6. a) When unemployment insurance is lowered to 0.05, the agent is willing to accept all wages including the low wage because the opportunity cost of remaining unemployed is now higher.
Running the simulation, unemployment rises to 50.3% and the distribution of wages becomes

Value	Count	Percent
0.2	26	5.17%
0.4	65	12.92%
0.6	92	18.29%
0.8	112	22.27%
1	208	41.35%

b) When unemployment insurance is raised to 0.15, the agent is never willing to accept the low wage because the opportunity cost of remaining unemployed is now lower. Running the simulation, unemployment falls to 42.2% and the distribution of wages becomes

Value	Count	Percent
0.2	2	0.40%
0.4	66	13.12%
0.6	95	18.89%
0.8	126	25.05%
1	214	42.54%

7. Looking at Figure 17 (the distribution of assets), we can see that it is centered far to the right of where it was in the benchmark case. This makes sense because the agents are risk-averse and thus prefer to save more when there is higher turbulence.

8. The wording of the question is ambiguous, so I clearly state my assumptions:

i. Conditional on paying search cost 0.25, agent receives offer for sure.

ii. If agent accepts an offer, she can't lose her job at the beginning of the next period.

The recursive formulation of this problem is:

$$
\begin{cases}
W_i(b) = \max\limits_{b',d\geq 0}\{V_i^{NS}(b), V_i^{S}(b)\}, \quad 0 \leq d \leq 1, \quad \text{for } i=1,\dots,5 \\
\text{where} \\
\quad V_i^{NS}(b) := \log(1.03b + w_i - b' - 0.8d^{1.5}) \\
\qquad\qquad + 0.95[d^{0.5} W_i(b') + (1-d^{0.5})U(b')] \\
\text{and} \\
\quad V_i^{S}(b) := \log(1.03 + w_i - b' - 0.8d^{1.5} - 0.25) \\
\qquad\qquad + 0.95 \sum_{i=1}^{5} \frac{1}{5} \max\{\underbrace{W_i(b')}_{\text{accept offer}}, \underbrace{d^{0.5} W_i(b') + (1-d^{0.5})U(b')}_{\substack{\text{reject offer and remain in current job} \\ \text{(could still lose job during next period}}}\} \\
U(b) = \max\limits_{b',e\geq 0} \log(1.03b + 0.1 - b' - 0.8e^{1.5}) \\
\qquad\qquad + 0.95[e^{0.5} \sum_{i=1}^{5} \frac{1}{5} \max\{W_i(b'), U(b')\} + (1-e^{0.5})U(b')]
\end{cases}
$$

In Figure 19, we can see that on-the-job search causes the agents to exert diligence iff they have the high wage $w_5 = 1$. This makes sense because if they have the option to search for a new job while on the job and they get an offer for sure, maintaining their current wage through diligence is less important to them than before. It is only worth it to them if they have no incentive to search because they have the highest wage already, in which case their problem is equivalent to the benchmark problem without on-the-job search.

The stationary distribution becomes of assets, excluding 0, becomes more spread out as the people who are able to maintain the high wage are able to maintain savings.

1.4

Figure 1:

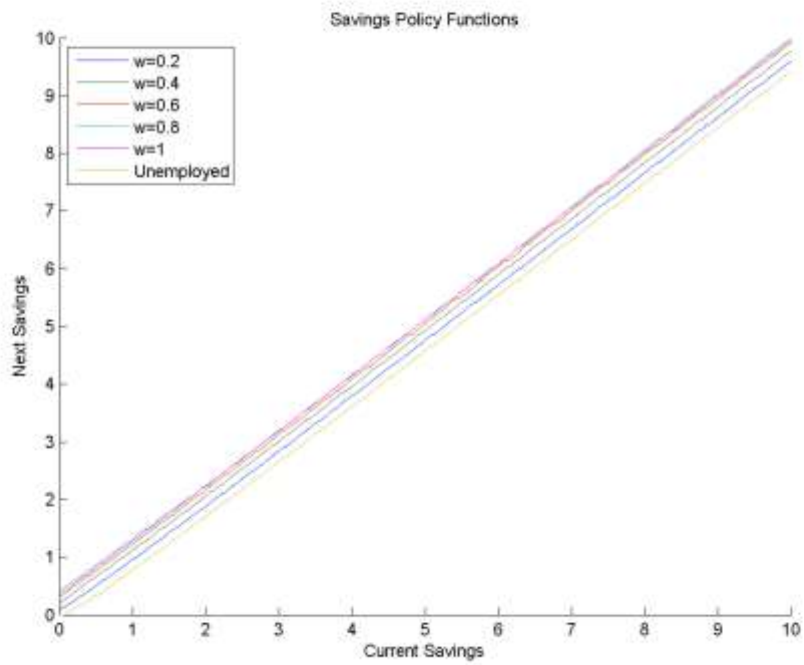
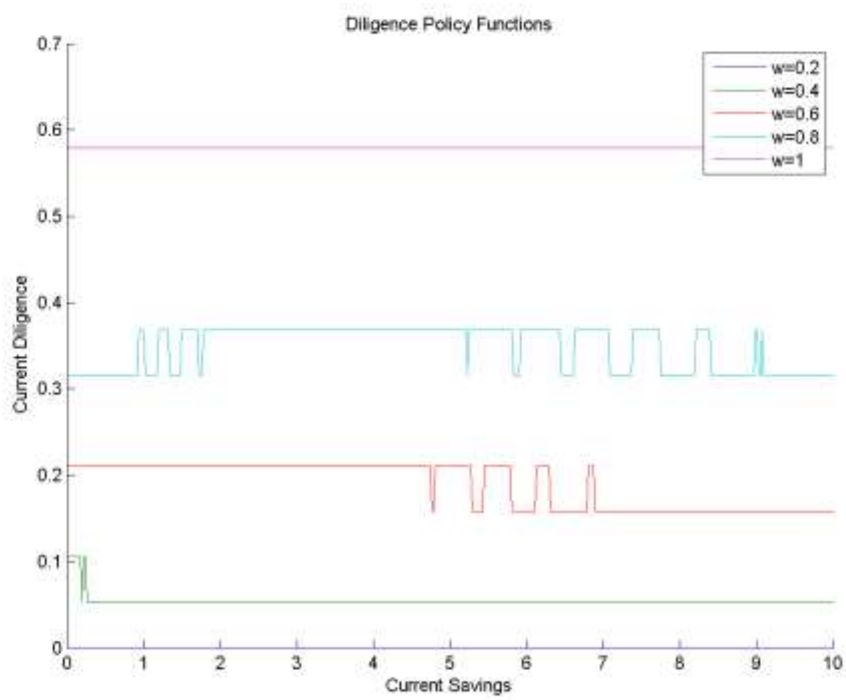


Figure 2:



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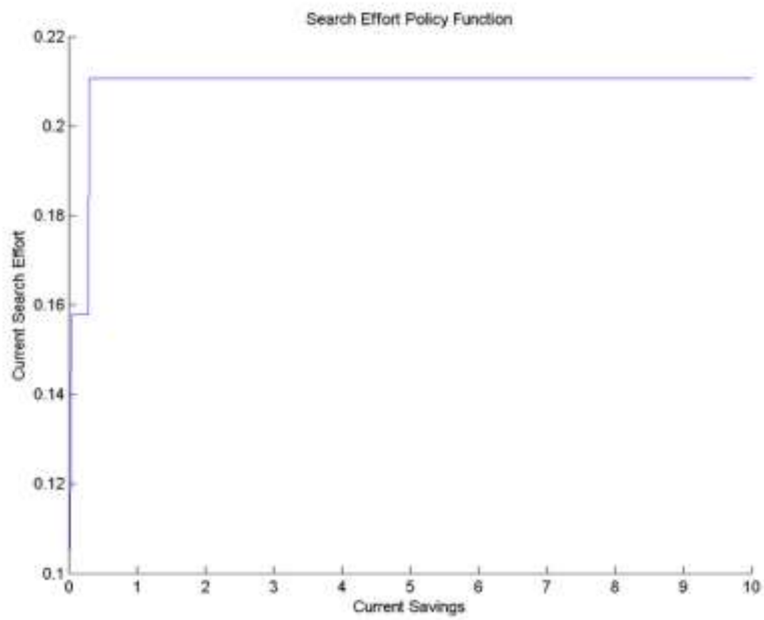

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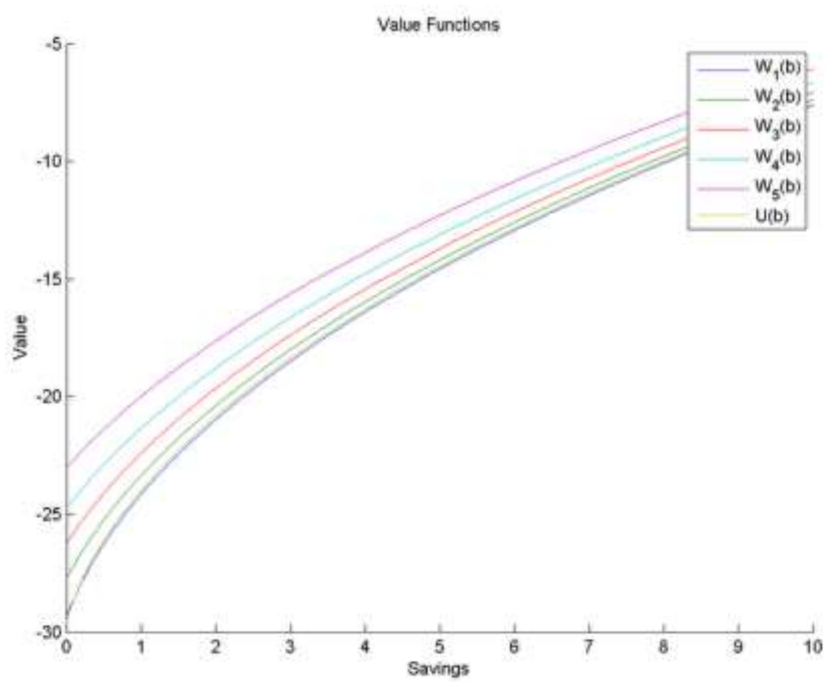
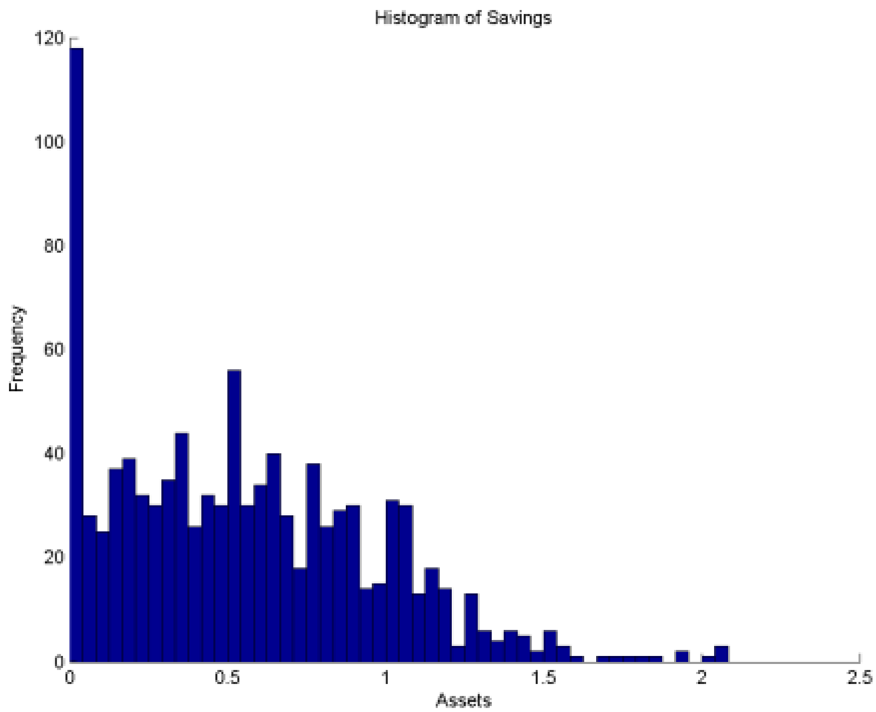


1.5

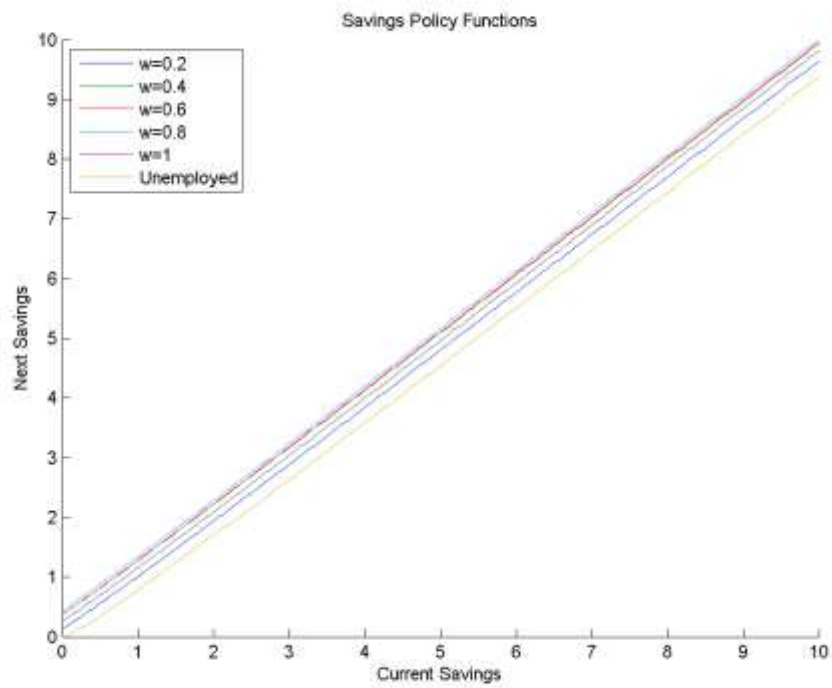
Figure 5:



Histogram of Savings
Frequency
Assets
0
20
40
60
80
100
120
0
0.5
1
1.5
2
2.5

1.6(a)

Figure 6:



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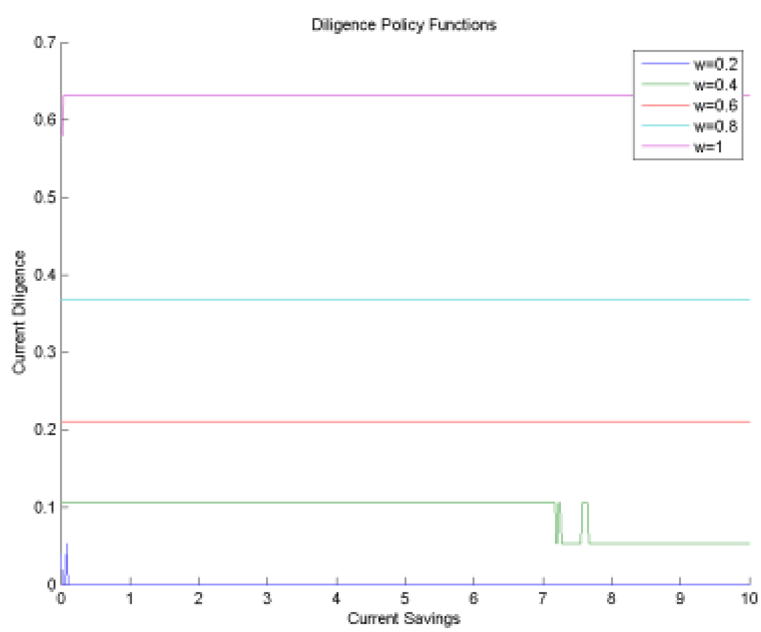

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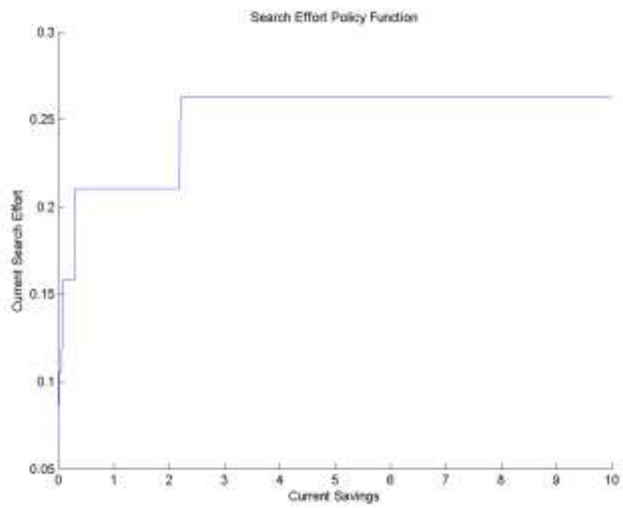

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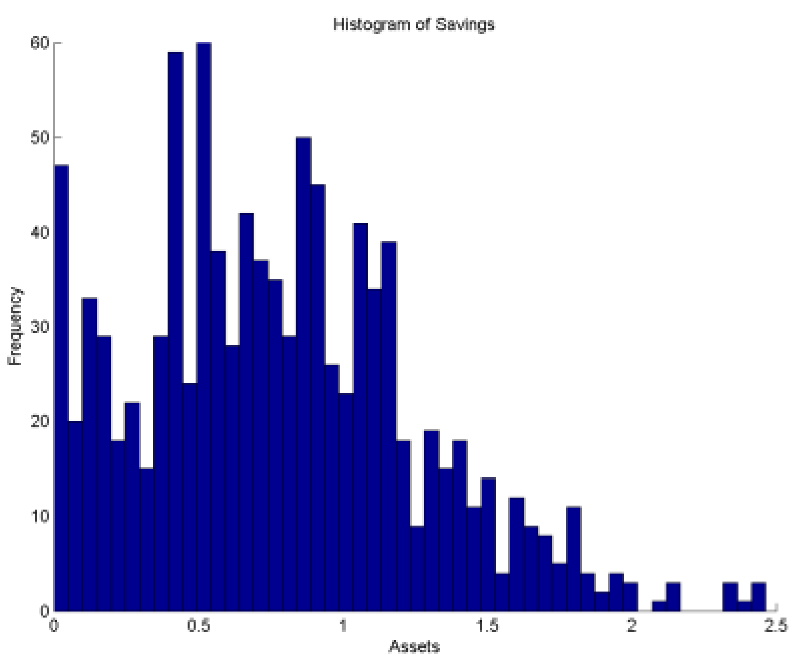
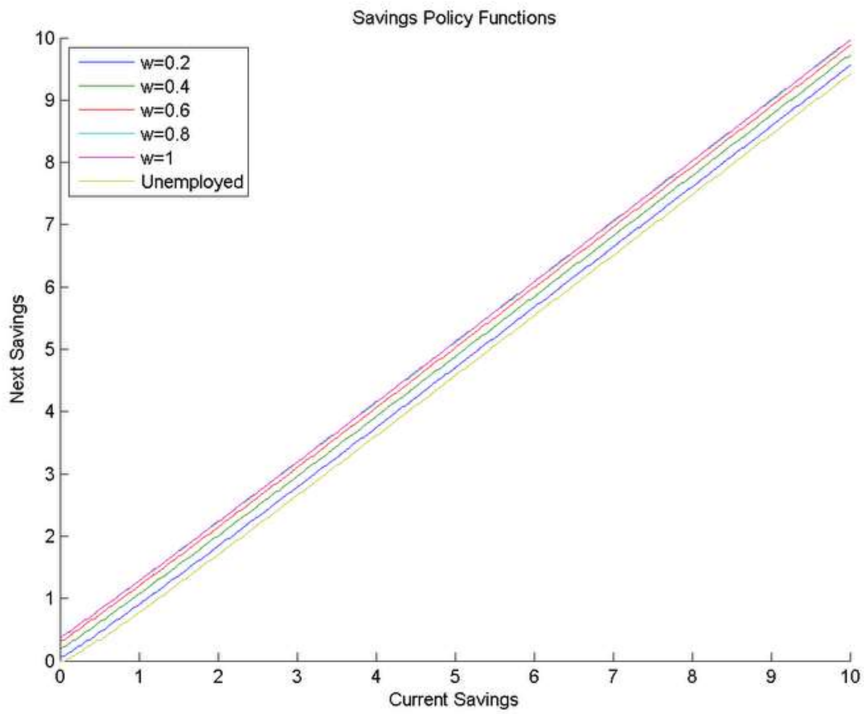


1.6(b)

Figure 10:



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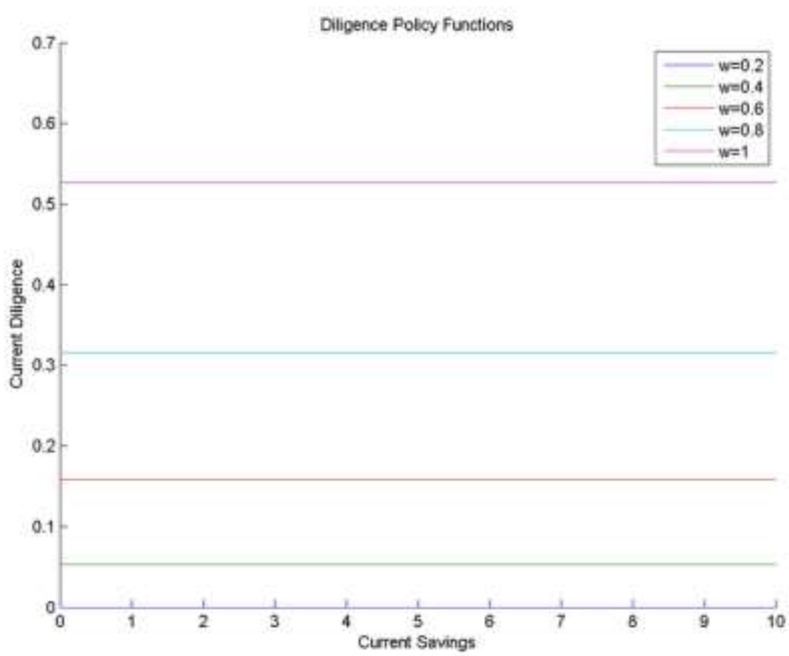

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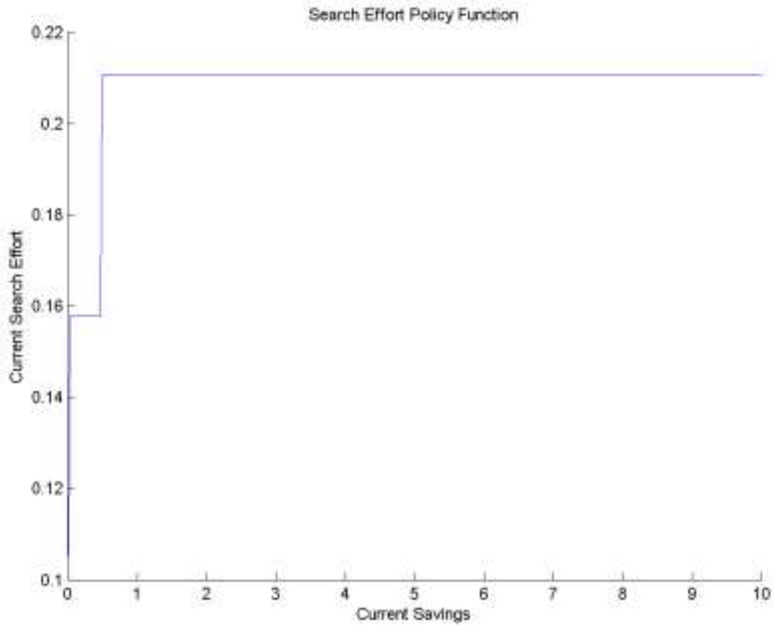

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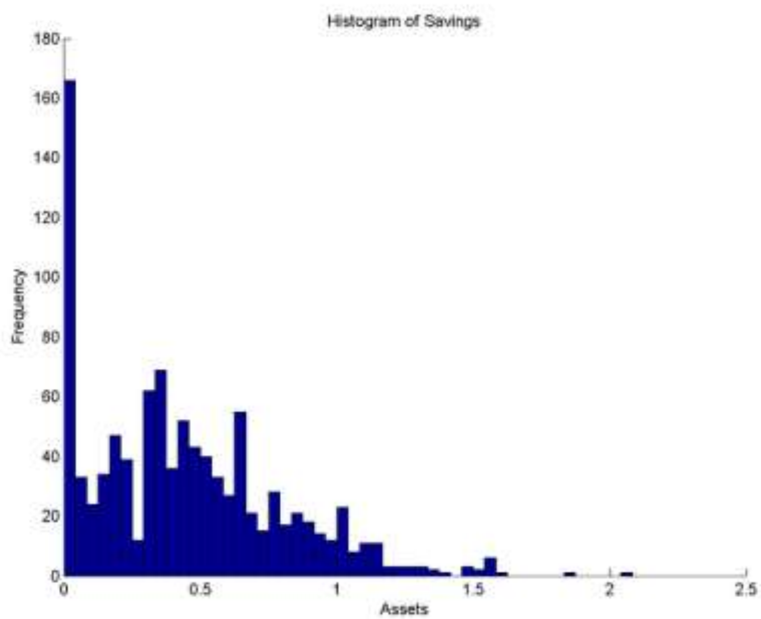
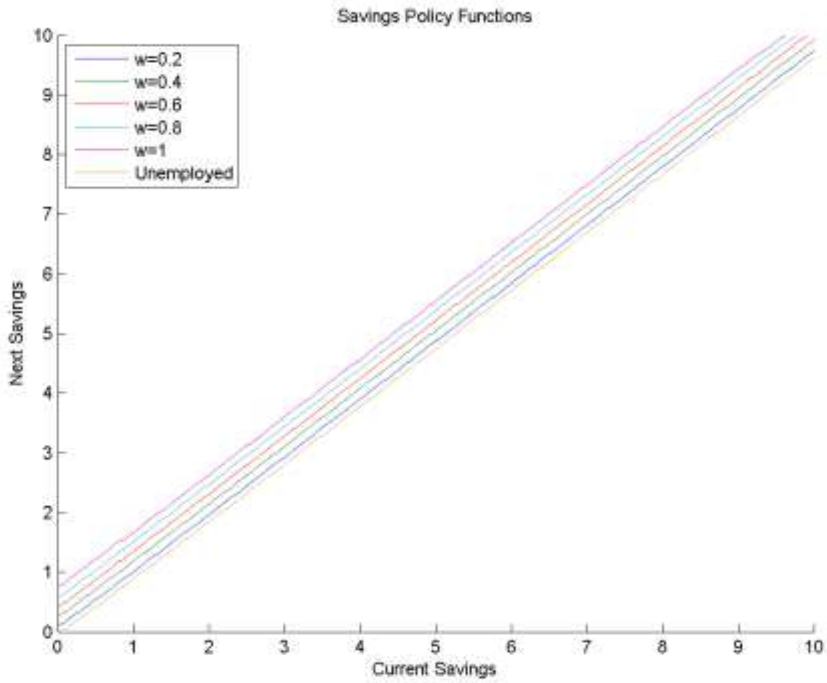


1.7

Figure 14:



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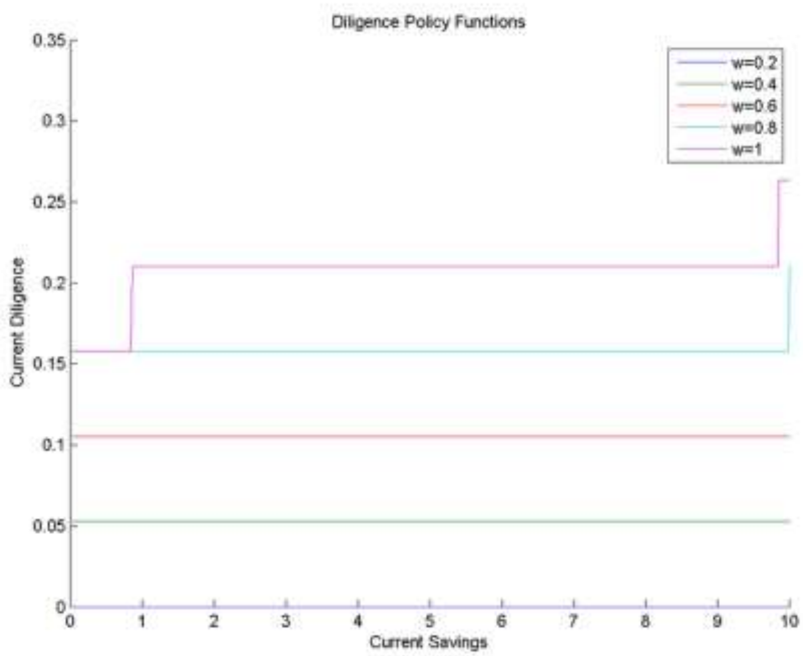

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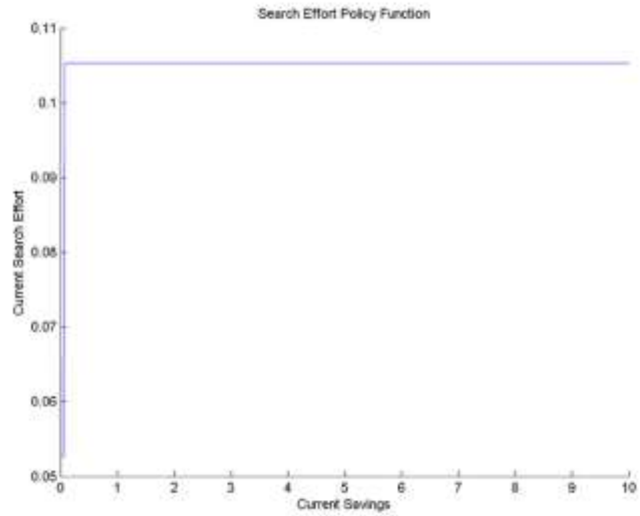
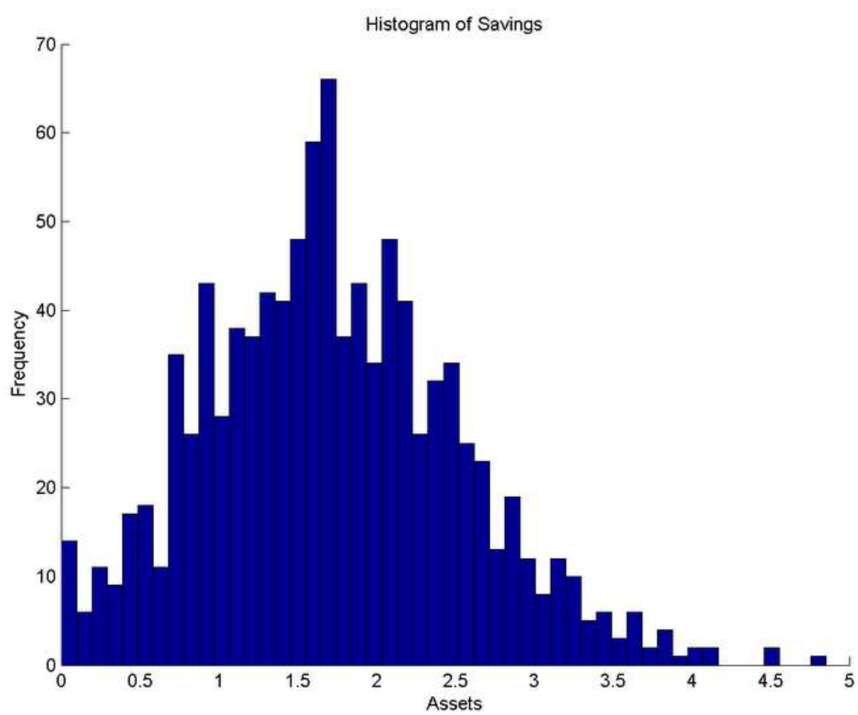
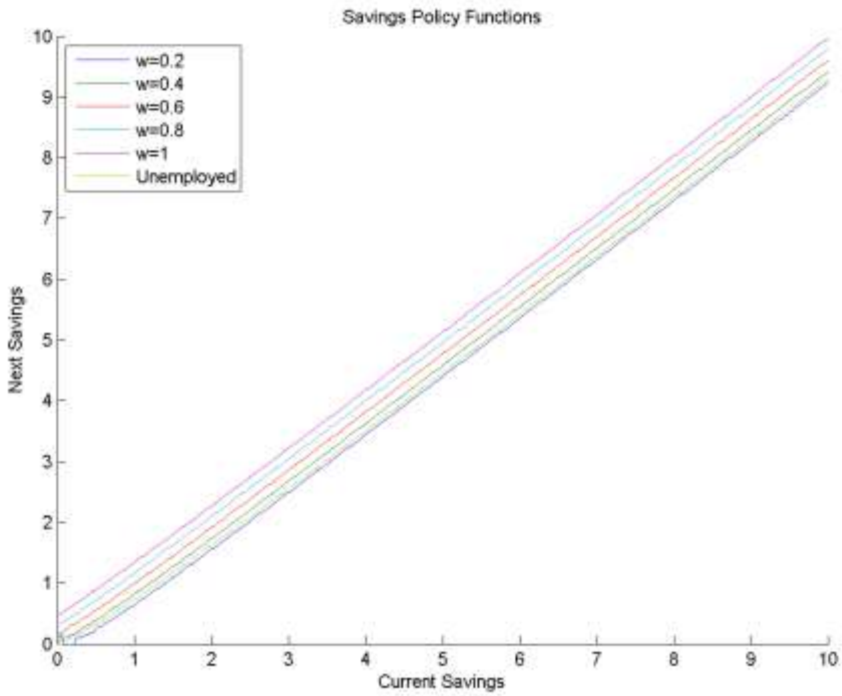


Figure 17:



1.8

Figure 18:



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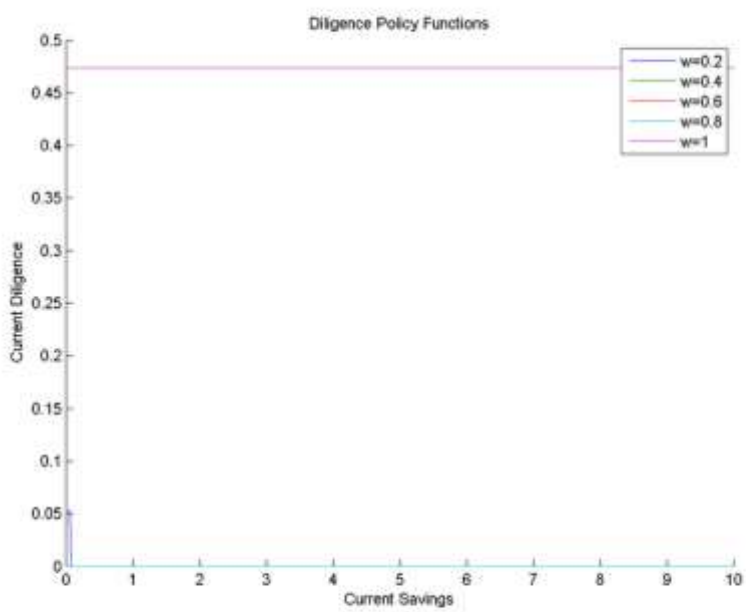

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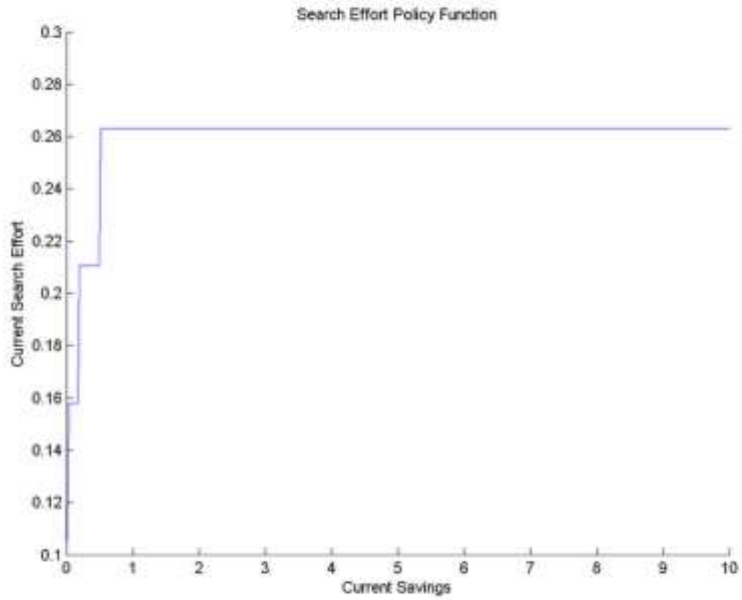
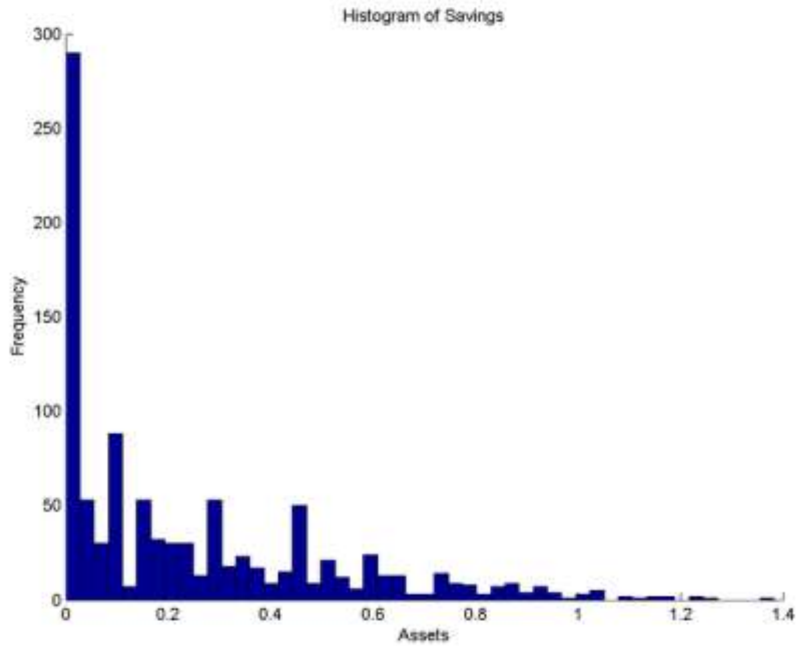


Search Effort Policy Function
Current Search Effort
Current Savings
0.3
0.28
0.26
0.24
0.22
0.2
0.18
0.16
0.14
0.12
0.1
0
1
2
3
4
5
6
7
8
9
10

Figure 21:


Histogram of Savings
Frequency
Assets
300
250
200
150
100
50
0
0
0.2
0.4
0.6
0.8
1
1.2
1.4