

Appendix 16A A Short Table of DTFT Pairs

DTFT Pairs	$-\pi \leq \omega \leq \pi$	$-0.5 \leq f \leq 0.5$
$x(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(\omega) e^{j\omega n} d\omega \iff X(\omega) = \sum_{-\infty}^{\infty} x(n) e^{-j\omega n}$ $= \int_{-0.5}^{0.5} X(f) e^{j2\pi f n} df \iff X(f) = \sum_{-\infty}^{\infty} x(n) e^{-j2\pi f n}$		
1, all n	$\iff 2\pi \delta(\omega)$	$\delta(f)$
$d(n)$	$\iff 1, \text{ all } \omega$	1, all f
$d(n - n_0)$	$\iff e^{-jn_0\omega}$	$e^{-j2\pi n_0 f}$
$\begin{cases} 1, & 0 \leq n < N \\ 0, & \text{elsewhere} \end{cases}$	$\iff \frac{\sin \frac{N\omega}{2}}{\sin \frac{\omega}{2}} e^{-j(\frac{N-1}{2})\omega}$	$\frac{\sin(N\pi f)}{\sin(\pi f)} e^{-j(N-1)\pi f}$
$\begin{cases} 1, & -M \leq n \leq M \\ 0, & \text{elsewhere} \end{cases}$	$\iff \frac{\sin(M + \frac{1}{2})\omega}{\sin(\omega/2)}$	$\frac{\sin(2M + 1)\pi f}{\sin(\pi f)}$
$\frac{\sin(\omega_0 n)}{\pi n}$	$\iff \begin{cases} 1, & -\omega_0 \leq \omega \leq \omega_0 \\ 0, & \text{elsewhere} \end{cases}$	$\begin{cases} 1, & -f_0 \leq f \leq f_0, \quad f_0 = \omega_0/(2\pi) \\ 0, & \text{elsewhere} \end{cases}$
$u(n)$	$\iff \frac{1}{1 - e^{-j\omega}} + \pi \delta(\omega)$	$\frac{1}{1 - e^{-j2\pi f}} + \frac{1}{2} \delta(f)$
$u(-n)$	$\iff \frac{1}{1 - e^{j\omega}} + \pi \delta(\omega)$	$\frac{1}{1 - e^{j2\pi f}} + \frac{1}{2} \delta(f)$
$u(n) - u(-n)$	$\iff -\frac{j \sin \omega}{1 - \cos \omega}$	$-\frac{j \sin(2\pi f)}{1 - \cos(2\pi f)}$
$u(n) - u(-n) + d(n)$	$\iff \frac{1 - \cos \omega - j \sin \omega}{1 - \cos \omega}$	$\frac{1 - \cos(2\pi f) - j \sin(2\pi f)}{1 - \cos(2\pi f)}$
$u(n) - u(-n) - d(n)$	$\iff \frac{\cos \omega - j \sin \omega - 1}{1 - \cos \omega}$	$\frac{\cos(2\pi f) - j \sin(2\pi f) - 1}{1 - \cos(2\pi f)}$
$\cos(\omega_0 n)$	$\iff \pi [\delta(\omega + \omega_0) + \delta(\omega - \omega_0)]$	$\frac{1}{2} [\delta(f + f_0) + \delta(f - f_0)], \quad f_0 = \omega_0/(2\pi)$
$\sin(\omega_0 n)$	$\iff -j\pi [\delta(\omega + \omega_0) - \delta(\omega - \omega_0)]$	$\frac{1}{2j} [\delta(f + f_0) - \delta(f - f_0)], \quad f_0 = \omega_0/(2\pi)$
$a^n u(n), \quad a < 1$	$\iff \frac{1}{1 - ae^{-j\omega}}$	$\frac{1}{1 - ae^{-j2\pi f}}$
$a^{-n} u(-n), \quad a < 1$	$\iff \frac{1}{1 - ae^{j\omega}}$	$\frac{1}{1 - ae^{j2\pi f}}$
$a^{ n }, \quad a < 1$	$\iff \frac{1 - a^2}{1 + a^2 - 2a \cos \omega}$	$\frac{1 - a^2}{1 + a^2 - 2a \cos(2\pi f)}$
$na^n u(n), \quad a < 1$	$\iff \frac{ae^{-j\omega}}{(1 - ae^{-j\omega})^2}$	$\frac{ae^{-j2\pi f}}{(1 - ae^{-j2\pi f})^2}$