Q 2-1)

$T(n) = 9\,T(n/3) + n$ Assume $T(1) = 1$

$= 9(9T(\frac{n}{9}) + \frac{n}{3}) + n = 81(T(\frac{n}{9})) + 4n$

$= \cancel{81(9T(\frac{n}{27})}\ 81(9T(\frac{n}{27}) + \frac{n}{9}) + 4n$

$= 729\,T(\frac{n}{27}) + 13n$

$= 9^t \cdot T(\frac{n}{3^t}) + n + 3n + 9n$

$\hookrightarrow n\sum_{x=0}^{x=k-1} (\frac{9^x}{3^x}) \to$ geometric

We want $T(1)$, this occurs when $3^t = n$ or $t = \log_3 n$

$9^{\log_3 n} \cdot T(1) + n\sum_{x=0}^{x=\log_3 n} (\frac{9^x}{3^x})$ ~~common ration~~ $n\sum 3^x$ where $a=1$

$S_n = n\left[\frac{1-3^{\log_3 n}}{1-3}\right]$

$= n\left[\frac{1-n}{-2}\right] \to n^2$

=> The geometric series approximates to n^2 so ~~t~~ lets guess $O(n^2)$

Prove our guess $T(n) \le cn^2$ for $\forall n$ & constant $c > 0$

~~Base Case: $T(1) = 1$ ... $9T(1)+$~~

$T(n) \le c9(\frac{n}{3})^2 + n$

$\le cn^2 + n$

$n^2 \le cn^2 + n$ for all $c \ge 1$

$\therefore O(n^2)$