Math 1200 Fall 2023 Section A Tutorial 02
The Integers (Practice)

Feel free to reach out to me by email if you have any questions about the problems below.

(1) Find the prime factorization of each of the following integers.

(a) 126   (b) 729   (c) 1001

(2) Show that $a^m + 1$ is composite if a and m are integers greater than 1 and m is odd. [Hint: Show that $x + 1$ is a factor of the polynomial $x^m + 1$ if m is odd.]

(3) What are the greatest common divisors of these pairs of integers?
   (a) $2^2 \cdot 3^3 \cdot 5^5, 2^5 \cdot 3^3 \cdot 5^2$
   (b) $2 \cdot 3 \cdot 5 \cdot 7 \cdot 11 \cdot 13, 2^{11} \cdot 3^9 \cdot 11 \cdot 17^{14}$
   (c) $17, 17^{17}$
   (d) $2^2 \cdot 7, 5^3 \cdot 13$
   (e) $0, 5$
   (f) $2 \cdot 3 \cdot 5 \cdot 7, 2 \cdot 3 \cdot 5 \cdot 7$

(4) Use the Euclidean algorithm to find

(a) $\gcd(1, 5)$ (d) $\gcd(1529, 14039)$
(b) $\gcd(100, 101)$ (e) $\gcd(1529, 14038)$
(c) $\gcd(123, 277)$ (f) $\gcd(11111, 111111)$

(5) Use Euclidean algorithm forward, then backward, and use Bèzout's Theorem to express the greatest common divisor of each of these pairs of integers as a linear combination of these integers.

(a) 9, 11   (d) 21, 55   (g) 2002, 2339
(b) 33, 44   (e) 101, 203   (h) 3457, 4669
(c) 35, 78   (f) 124, 323   (i) 10001, 13422