Question 1: Describe the following sets as simply as possible, prove that the description works.

(a) $S = \{x \in \mathbb{Z} : \text{there exists } k \in \mathbb{Z} \text{ such that } x = 2k\}$

S is the set of all integers $x \in \mathbb{Z}$ such that there exists an integer $k \in \mathbb{Z}$ such that $x = 2k$, in other words S is the set of all even integers.

(b) $T = \{x \in \mathbb{R} : \text{there exists } k \in \mathbb{Z} \text{ such that } x = 2k\}$

$\mathbb{Z} \subset \mathbb{R}$

T is the set of all real numbers $x \in \mathbb{R}$ such that there exists an integer $k \in \mathbb{Z}$ such that $x = 2k$. In other words T is the set of all even integers,

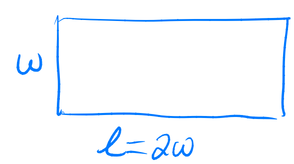
(c) $U = \{x \in \mathbb{Z} : \text{there exists } k \in \mathbb{R} \text{ such that } x = 2k\}$

U is the set of all integers $x \in \mathbb{Z}$ such that there exists a real number $k \in \mathbb{R}$ such that $x = 2k$.

Ex. $k = \frac{1}{2} \Rightarrow x = 2 \times \frac{1}{2} = 1 \in \mathbb{Z}$

$w = \frac{1}{2}$ cm

(d) $V = \{x \in \mathbb{R} : \text{there exists } k \in \mathbb{R} \text{ such that } x = 2k\}$



V is the set of real numbers $x \in \mathbb{R}$ such that there exists a real number $k \in \mathbb{R}$ such that $x = 2k$. In other words, x is twice the value of k.

- The length is twice the width
- The length is two times the width.

(e) $W = \{x \in \mathbb{Z} \setminus\{0\} : x \mid n \text{ and } x \mid n+1 \text{ for some } n \in \mathbb{N}\}$

W is the set of all integers $x \in \mathbb{Z}$ such that $x \mid n$ and $x \mid n+1$ for some $n \in \mathbb{N}$.

$x = 1$

$\bigcup_{i \in I} S_i = \{$ there exists an $i \in I$ such that $x \in S_i \}$.

$\bigcap_{i \in I} S_i = \{$ for every $i \in I$ $x \in S_i \}$.

$\bigcup_{i=1}^{n} A_i = A_1 \cup A_2 \cup \cdots \cup A_n$ $\qquad$ $\bigcap_{i=1}^{n} A_i = A_1 \cap A_2 \cap \cdots \cap A_n$

$x \notin \bigcup_{i=1}^{n} S_i = x \notin (S_1 \cup S_2 \cup S_3 \cup \cdots \cup S_n)$

Question 2: Prove De Morgan's Laws for sets. That is, if $\{S_i\}_{i\in I}$ is a family of sets, with $S_i := \{x : P_i(x)\}$, then prove

(a) $\left(\bigcup_{i\in I} S_i\right)^c = \bigcap_{i\in I} S_i^c.$

Let $x \in \left(\bigcup_{i\in I} S_i\right)^c$ be arbitrary.

Then $x \notin \bigcup_{i\in I} S_i$

$x \notin \{$ there exists an $i \in I$ such that $x \in S_i \}$.

$\neg(\exists i \in I$ s.t $x \in S_i)$

$\forall i \in I$ such that $x \notin S_i$

$\forall i \in I$ s.t $x \in S_i^c$

$\to x \in \{$ for every $i \in I$, $x \in S_i^c \}$

$\Leftrightarrow x \in \bigcap_{i\in I} S_i^c$

(b) $\left(\bigcap_{i\in I} S_i\right)^c = \bigcup_{i\in I} S_i^c$

$(A \cup B)^c = A^c \cap B^c$

To prove the sets are equal

① $A \subset B$ and $B \subset A$

② If and only if.

Let $x \in (A \cup B)^c$ be arbitrary.

Then $x \notin (A \cup B)$ if and only if $x \notin A \wedge x \notin B$ if and only if $x \in A^c \wedge x \in B^c$ if and only if $x \in (A^c \cap B^c)$.

Therefore as $x \in (A \cup B)^c$ was arbitrary, $(A \cup B)^c = A^c \cap B^c$.