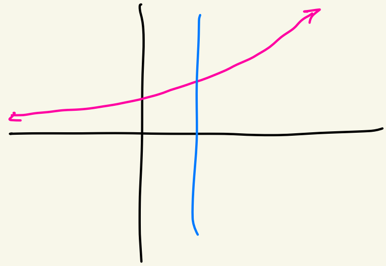
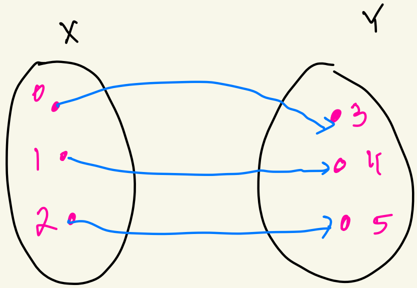
Function

$f(x) = x^2 \quad f(x) = e^{\wedge}$



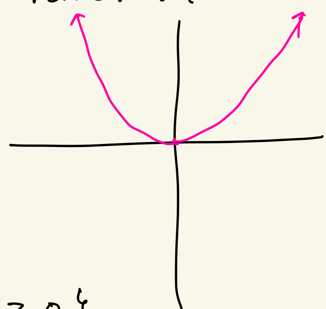


$f: X \to Y$

Def: Let $X, Y \neq \emptyset$, a function from X to Y is an assignment of exactly one element of Y to each element in X. $f: X \to Y$, if $x \in X$, then $f(x) = y$.

- We call X the **domain** of the function
- we call Y the **codomain** of the function.

$f: \mathbb{R} \to \mathbb{R} \quad f(x) = x^2$



- The **image** (or **range**) is the set

$f(X) = \{ f(x) : x \in X \}$.

$f: \mathbb{R} \to f(\mathbb{R}) \quad f(x) = x^2$

$f(\mathbb{R}) = \{ x^2 : x \geq 0 \} . = \{ y \in \mathbb{R} : y \geq 0 \}$

One-to-One, Onto, Bijections

Def: Let $f: X \to Y$ be a function

- f is said to be **one-to-one** if $f(x) \neq f(y)$, then $x \neq y$ for all $x, y \in X$. Note: could also say, for all $x, y \in X$ if $x = y$, then $f(x) = f(y)$.
- f is said to be **onto** if for all $y \in Y$, there exists an $x \in X$ such that $f(x) = y$

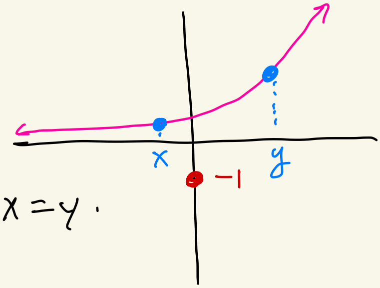
- If f is both one-to-one and onto, then f is a bijection.

Example: Let $f: \mathbb{R} \to \mathbb{R}$ be given by $f(x) = e^x$.

(a) Is it one-to-one

(b) Is it onto

(c) Is it a bijection.



Solution: (a) f is one-to-one, let $x = y$.

Then $e^x = e^y \Rightarrow f(x) = f(y)$

Alternatively, if $f(x) \neq f(y) \Rightarrow e^x \neq e^y \Rightarrow x \neq y$

domain

(b) f is not onto. Take $-1 \in \mathbb{R}$, then no such $x \in \mathbb{R}$

codomain

such that $e^x = -1$.

f is onto if $f: X \to f(X)$.

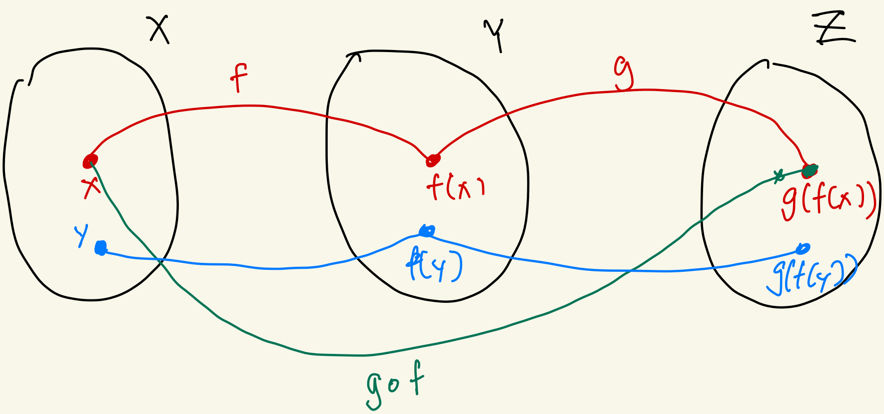
$f: \mathbb{R} \to (0, \infty)$

$f(x) = e^x$

(c) f is not a bijection.

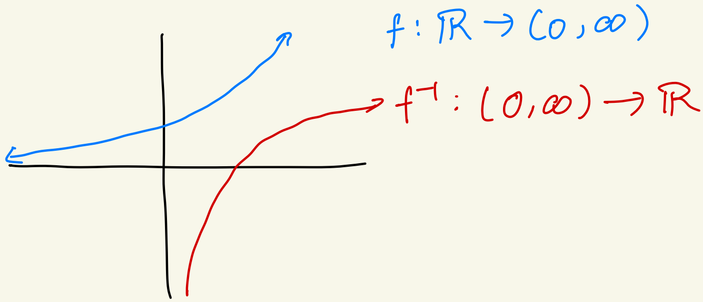
Def: Let $X, Y, Z \neq \emptyset$, let $f: X \to Y$, and $g: Y \to Z$, then

Then the composition is $g \circ f : X \to Z$



Prop: If $f: X \to Y$ and $g: Y \to Z$ are bijections, then $g \circ f$ is also a bijection.

Def: If $f: X \to Y$ is a bijection, its inverse map $f^{-1}: Y \to X$ is called the inverse.



Prop: Every bijection has an inverse and is also a bijection.