Math 1200 Fall 2023 Section A Tutorial 02
Equivalence Relations and Classes (Practice)

Feel free to reach out to me by email if you have any questions about the problems below.

(1) Which of these relations on $\{0, 1, 2, 3\}$ are equivalence relations? Determine their properties of an equivalence relation that the others lack.
   (a) $\{(0, 0), (1, 1), (2, 2), (3, 3)\}$
   (b) $\{(0, 0), (0, 2), (2, 0), (2, 2), (2, 3), (3, 2), (3, 3)\}$
   (c) $\{(0, 0), (1, 1), (1, 2), (2, 1), (2, 2), (3, 3)\}$
   (d) $\{(0, 0), (1, 1), (1, 3), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)\}$
   (e) $\{(0, 0), (0, 1), (0, 2), (1, 0), (1, 1), (1, 2), (2, 0), (2, 2), (3, 3)\}$

(2) Which of these relations on the set of all functions from $\mathbb{Z}$ to $\mathbb{Z}$ are equivalence relations? Determine the properties of an equivalence relation that the others lack.
   (a) $\{(f, g) : f(1) = g(1)\}$
   (b) $\{(f, g) : f(0) = g(0) \text{ or } f(1) = g(1)\}$
   (c) $\{(f, g) : f(x) - g(x) = 1 \text{ for all } x \in \mathbb{Z}\}$
   (d) $\{(f, g) : \text{for some } c \in \mathbb{Z}, \text{ for all } x \in \mathbb{Z}, \ f(x) - g(x) = c\}$
   (e) $\{(f, g) : f(0) = g(1) \text{ and } f(1) = g(0)\}$

(3) Let R be the relation on the set of ordered pairs of positive integers such that $((a, b), (c, d)) \in R$ if and only if $a + d = b + c$. Show that R is an equivalence relation.

(4) Give a description of each of the congruence classes modulo 6.

(5) Which of these collections of subsets are partitions of $\{1, 2, 3, 4, 5, 6\}$?
   (a) $\{1, 2\}, \{2, 3, 4\}, \{4, 5, 6\}$
   (b) $\{1\}, \{2, 3, 6\}, \{4\}, \{5\}$.
   (c) $\{2, 4, 6\}, \{1, 3, 5\}$
   (d) $\{1, 4, 5\}, \{2, 6\}$

(6) Which of these collections of subsets are partitions of $\{-3, -2, -1, 0, 1, 2, 3\}$?
   (a) $\{-3, 1, 1, 3\}, \{-2, 0, 2\}$
   (b) $\{-3, -2, -1, 0\}, \{0, 1, 2, 3\}$
   (c) $\{-3, 3\}, \{-2, 2\}, \{-1, 1\}, \{0\}$
   (d) $\{-3, -2, 2, 3\}, \{-1, 1\}$

(7) List the ordered pairs in the equivalence relations produced by these partitions of $\{0, 1, 2, 3, 4, 5\}$
   (a) $\{0\}, \{1, 2\}, \{3, 4, 5\}$
   (b) $\{0, 1\}, \{2, 3\}, \{4, 5\}$
   (c) $\{0, 1, 2\}, \{3, 4, 5\}$
   (d) $\{0\}, \{1\}, \{2\}, \{3\}, \{4\}, \{5\}$