Math 1200 Fall 2023 Section A Tutorial 02
Cardinality of Sets (Practice)

Feel free to reach out to me by email if you have any questions about the problems below.

(1) Determine whether each of these sets is finite, countably infinite, or uncountable. For those that are infinite, exhibit a bijection between the set of positive integers and that set.
   (a) The integers greater than 10.
   (b) The odd negative integers.
   (c) The integers with absolute value less than 1,000,000.
   (d) The set $A \times \mathbb{Z}^+$ where $A = \{2, 3\}$.
   (e) The integers that are multiples of 10.

(2) Determine whether each of these sets is countable, or uncountable. For those that are countably infinite, exhibit a bijection between the set of positive integers and that set.
   (a) All positive rational numbers that cannot be written with denominators less than 4.
   (b) The real numbers not containing 0 in their decimal representation.
   (c) The real numbers containing only a finite number of 1s in their decimal representation.

(3) Give an example of two uncountable sets A and B such that $A \setminus B$ is
   (a) Finite
   (b) Countably infinite
   (c) Uncountable.

(4) Give an example of two uncountable sets A and B such that $A \cap B$ is
   (a) Finite
   (b) Countably infinite
   (c) Uncountable.

(5) Show that if A and B are sets and $A \subset B$, then $|A| \leq |B|$.

(6) Show that a subset of a countable set is countable.

(7) Show that if A and B are sets such that $|A| = |B|$, then $|\mathcal{P}(A)| = |\mathcal{P}(B)|$.

(8) Show that if A, B, C, D are sets with $|A| = |B|$ and $|C| = |D|$, then $|A \times C| = |B \times D|$.

(9) Show that if $|A| = |B|$ and $|B| = |C|$, then $|A| = |C|$.

(10) Show that $(0, 1)$ and $[0, 1]$ have the same cardinality.