Functions: Practice

MATH 1200 Section A Tutorial 02

November 13, 2023

Question 1. Determine whether f is a function from $\mathbb{Z}$ to $\mathbb{R}$ if

(a) $f(n) = \pm n$

(b) $f(n) = \sqrt{n^2 + 1}$

(c) $f(n) = \dfrac{1}{n^2 - 4}$

Question 2. Find the domain and range of these functions. Note that in each case, to find the domain, determine the set of elements assigned values by the function.

(a) The function that assigns to each nonnegative integer its last digit.

(b) The function that assigns the next largest integer to a positive integer.

Question 3. Find the domain and range of these functions:

(a) The function that assigns to each pair of positive integers the first integer of the pair.

(b) The function that assigns each positive integer its largest decimal digit.

(c) The function that assigns to each positive integer the largest integer not exceeding the square root of the integer.

Question 4. Determine whether each of these functions from $\{a, b, c, d\}$ to itself is one-to-one.

(a) $f(a) = b$, $f(b) = a$, $f(c) = c$, $f(d) = d$.

(b) $f(a) = b$, $f(b) = b$, $f(c) = d$, $f(d) = c$.

(c) $f(a) = d$, $f(b) = b$, $f(c) = c$, $f(d) = d$.

Question 5. Determine whether $f : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$ is onto if

(a) $f(m, n) = 2m - n$

(b) $f(m, n) = m^2 - n^2$

(c) $f(m, n) = m + n + 1$

(d) $f(m, n) = |m| - |n|$

(e) $f(m, n) = m^2 - 4$

Question 6. Determine whether each of these functions is a bijection from $\mathbb{R}$ to $\mathbb{R}$.

(a) $f(x) = -3x + 4$

(b) $f(x) = -3x^2 + 7$

(c) $f(x) = \dfrac{x+1}{x+2}$

(d) $f(x) = x^5 + 1$.

Question 7. Show that the function $f(x) = e^x$ from the set of real numbers to the set of real numbers is not invertible, but if the codomain is restricted ot the set of positive real numbers, the resulting function is invertible.

Question 8. Find $f + g$ and fg for $f(x) = x^2 + 1$ and $g(x) = x + 2$.

Question 9. Draw the graphs of each of these functions:

(a) $f(x) = \left\lfloor x + \dfrac{1}{2} \right\rfloor$

(b) $f(x) = \lfloor 2x + 1 \rfloor$

(c) $f(x) = \left\lceil \dfrac{x}{3} \right\rceil$

(d) $f(x) = \left\lceil \dfrac{1}{x} \right\rceil$

(e) $f(x) = \lceil x - 2 \rceil + \lfloor x + 2 \rfloor$