Tutorial 7 (Happy Halloween)

MATH 1200A02: Problems, Conjectures, and Proofs

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Set Theory

Definition (Set)

A set is a collection of unordered objects.

Definition (Members of a Set)

We say that x is a member of a set A, and write $x \in A$. If x is not a member of A, then write $x \notin A$.

Definition (Union of Two Sets)

Let A and B be two sets on some universal set X. The union of A and B, written $A \cup B$ is the set of all elements that are in A or in B.

$$A \cup B = \{x \in X : x \in A \text{ or } x \in B\}$$

Definition (Intersection of Two Sets)

Let A and B be two sets on some universal set X. The intersection of A and B, written $A \cap B$, is the set of all elements that are in A and B.

$$A \cap B = \{x \in X : x \in A \text{ and } x \in B\}$$

Definition

Let A and B be two sets on some universal set X. The difference $A \setminus B$ (or $A - B$) is the set of all elements that are in A but not in B.

$$A \setminus B = \{x \in X : x \in A \text{ and } x \notin B\}$$

Definition

Let A be a universal set on some universal set X. The complement of A, written as A^c is the set of all elements in X that are not in A.

$$A^c = \{x \in X : x \notin A\}$$

Example

Let $X = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, $A = \{1, 3, 4, 5, 8\}$ and $B = \{0, 2, 6, 7, 9\}$. Find the following:

1. $A \cup B =$
2. $A \cap B =$
3. $A \setminus B =$
4. $B \setminus A =$
5. $A^c =$
6. $B^c =$
7. $(A \setminus B) \cap (B \setminus A) =$
8. $A^c \cup B^c =$
9. $A^c \cap B^c =$
10. $A^c \setminus B^c =$

Definition

Let A and B be sets on some universal set X. Then we say that A is a subset of B, and write $A \subset B$, if every element in A is inside B. In other words, for every $x \in A$, then $x \in B$.

Definition

Let A and B be sets on some universal set X. Then we say that the set A is equal to the set B, and write $A = B$ if and only if $A \subset B$ and $B \subset A$.

Example

$$\mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R} \subset \mathbb{C}$$

Example

Let $X = \mathbb{R}$, let $A = (0, 1)$, $B = [0, 1]$, and $C = [0, 6]$. Determine whether the following statements are true or false:

1. $A \subset B$
2. $B \subset A$
3. $A \subset C$
4. $C \subset A$
5. $B \subset C$
6. $C \subset B$

Definition

Let A be a set of some universal set X. Then the cardinality of A is the number of elements in A, and denote it by $|A|$.

Example

If $A = \{0, 2\}$ then $|A| = 2$, but if $B = [0, 1]$, then $|B| = \infty$.

Definition

Let A be a set of some universal set X. Then the power set of A is the set of all subsets of A, and denote it by $\mathcal{P}(A)$.[a]

[a] In LaTeX to use $\mathcal{P}$, you type \mathcal{P}

Example

If $A = \{a, b, c\}$, then

$$\mathcal{P}(A) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}$$

Question 1

Let $C = \{x \in \mathbb{Z} : x \equiv 7 \pmod{9}\}$ and $D = \{x \in \mathbb{Z} : x \equiv 1 \pmod{3}\}$

(a) List at least five different elements of the set A and at least five elements of the set B.

(b) Is $A \subset B$?

(c) Is $B \subset A$?

Question 2 (Exercise)

We can extend the idea of consecutive integers to represent four consecutive integers as m, $m+1$, $m+2$ and $m+3$, where $m \in \mathbb{Z}$. There are other ways to represent four consecutive integers. For example if $k \in \mathbb{Z}$, then $k-1$, k, $k+1$, and $k+2$ are four consecutive integers.

(a) Prove that for each $n \in \mathbb{Z}$, n is the sum of four consecutive integers if and only if $n \equiv 2 \pmod 4$.

(b) Use set builder notation or the roster method to specify the set of integers that are the sum of four consecutive integers.

(c) Specify the set of all natural numbers that can be written as the sum of four consecutive natural numbers.

(d) Prove that for each $n \in \mathbb{Z}$, n is the sum of eight consecutive integers if and only if $n \equiv 4 \pmod 8$

(e) Use set builder notation or the roster method to specify the set of integers that are the sum of eight consecutive integers.

(f) Specify the set of all natural numbers can be written as the sum of eight consecutive natural numbers.

Question 3

Completely factor $x^4 + 16$ as a product of linear factors.