Tutorial 4: Summary of Proof Methods

MATH 1200A02: Problems, Conjectures, and Proofs

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Summary of Proof Methods

To prove an assertion with propositions P and Q that translates to $P \to Q$. We can either proceed by directly proving the assertion or by taking an indirect approach, including contradiction or contrapositive.

Direct Proof

Assume P holds true, then proceed to show Q holds true. It is because for the conditional $\to$, with P true, only the truth value true for Q makes the conditional valid.

Proof by Contraposition

As the word contraposition conveys "taking a contrary position", we assume the "contrary of Q" to be true, i.e. assume $\neg Q$ true, and proceed to show that the "contrary of P", i.e. $\neg P$ is true, we show

$$\neg Q \to \neg P$$

Summary of Proof Methods

Proof by Contradiction

We aim to get a contradiction of the original assumption, by assuming the hypothesis and the contrary of the conclusion. So we assume $P \wedge \neg Q$ to be true, i.e. P and the contrary of Q to be true. We proceed to find that $\neg P$ to be true, that is,

$$(P \wedge \neg Q) \to \neg P$$

P	Q	$\neg P$	$\neg Q$	$P \wedge \neg Q$	$\neg Q \to \neg P$	$(P \wedge \neg Q) \to \neg P$	$P \to Q$
T	T	F	F	F	T	T	T
T	F	F	T	T	F	F	F
F	T	T	F	F	T	T	T
F	F	T	T	F	T	T	T

The last three columns are identical in truth values, hence their logical equivalences:

$$P \to Q \equiv \neg Q \to \neg P \equiv (P \wedge \neg Q) \to \neg P$$

Tutorial Groups For Today

Group A

Lizhi, Ananya, David, Nicholas, Christian, Zakariya

Group B

Alysa, Sarah, Tong, Giuseppina, Caitlyn, Oliver

Group C

Taya, Stefania, Tyandy, Emily, Huiru

Tutorial Problems

Exercise 1 (Group A)

Prove the following

1. Show that for each integer a, if $a \not\equiv 0 \pmod 3$, then $a^2 \equiv 1 \pmod 3$.
2. Using (1) show that for each natural number n, $\sqrt{3n+2}$ is not a natural number.

Exercise 2 (Group B)

Prove that the product of three consecutive integers is divisible by 3.

Exercise 3 (Group C)

Prove that the difference of a rational number and an irrational number is irrational.