Tutorial 5: Mathematical Induction

MATH 1200A02: Problems, Conjectures, and Proofs

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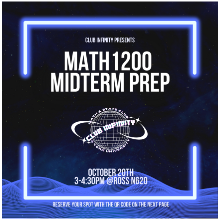
Announcement

Club Infinity is hosting a **MATH 1200 Midterm Review Session**! This is a good opportunity students to prepare for the MATH 1200 midterm coming up.

Details of Event

- **Date:** Friday, October 20, 2023
- **Time:** 3:00 PM-4:30 PM
- **Location:** Ross North 620



Mathematical Induction

Theorem (Principal of Mathematical Induction)

Let $P(n)$ be the statement for $n \in \mathbb{Z}$. Assume that

- *There exists a $k_0 \in \mathbb{Z}$ such that $P(k_0)$ is true.*
- *For any $k \geq k_0$, if $P(k)$ is true, then $P(k+1)$ is true.*

Then for all $n \geq k_0$, $P(n)$ is true.

Theorem (Strong Induction)

Let $P(n)$ be a statement for $n \in \mathbb{Z}$. Assume that

- *There exists some integer $k_0 \in \mathbb{Z}$ such that $P(k_0)$ is true.*
- *If $k \geq k_0$ is an integer such that $P(k_0), P(k_0+1), \ldots, P(k)$ are all true, then $P(k+1)$ must be true as well.*

Then for $n \geq k_0$, $P(n)$ is true.

Example

Show that for all $n \in \mathbb{N}$,

$$1 + 2 + \cdots + n = \frac{n(n+1)}{2}$$

Example

Show that for all $n > 1$,

$$n! < n^n$$

Activity

Question 1

Prove that for every positive integer n, $\sum_{r=1}^{n} \frac{1}{\sqrt{r}} \leq 2\sqrt{n}$.

Question 2

Show that the Fibonacci numbers satisfy $f_n < 2^n$ for all $n \in \mathbb{N}$.

Question 3

Prove that the Lucas numbers satisfy $L_n = \left(\frac{1+\sqrt{5}}{2}\right)^n + \left(\frac{1-\sqrt{5}}{2}\right)^n$

Group 1: Lizhi, Tong, Giuseppina, Ananya, Taya, David
Group 2: Stefania, Sarah, Emily, Nicholas, Oliver, Caitlyn
Group 3: Alysa, Zakariya, Huiru, Christian, Tyandy

Challenge!

Challenge

Use strong induction to show that for all $n \in \mathbb{N}$

$$\sqrt[n]{x_1 x_2 \cdots x_n} \leq \frac{1}{n}\sum_{i=1}^{n} x_i$$

Hint: Use the fact that $(\sqrt{x_1} - \sqrt{x_2})^2 \geq 0$ to show that the base case $n = 2$ is true.